Notes on Galaxy Formation course February 5

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1 Introduction

This lecture provided a whirlwind tour of the content of the course and these notes are likewise very brief. We will come back to most of this at later stages of the course. I have included a bit more than I covered in the lecture but this is stuff that we will return to later or that you already should know from previous courses.

Throughout this and later notes I will make a number of references to literature for further reading. I will use these abbreviations for these book, some of these I brought to the lecture but not all.

MvdBW "Galaxy formation and Evolution", Hojun Mo, Frank van den Bosch & Simon White (ISBN 978–0–521–85793–2).

Pdm3 "Theoretical astrophysics, Vol 3: Galaxies and cosmology", T. Padmanabhan (ISBN: 0521566304).

Longair "Galaxy Formation", Malcolm S. Longair (ISBN 978–3–540–73477–2).

Peacock "Cosmological physics", John Peacock (ISBN 0–521–42270–1).

K&T "The early Universe", Edward W. Kolb & Michael S. Turner (ISBN: 0201626748).

R&L "Radiative processes in Astrophysics", George B. Rybicki & Alan P. Lightman (ISBN: 978–0–471–82759–7).

A&T "Dark Energy — Theory and Observations", Luca Amendola and Shinji Tsujikawa (ISBN 978–0–521–51600–6).

C&L "Cosmology — The Origin and Evolution of Cosmic Structure", Peter Coles & Francesco Lucchin (ISBN 978–0–471–48909–2).

2 A homogeneous Universe

Literature for this and the following five sections: Longair Part II; A&T Chapter 2; C&L Part I; Pdm3 Chapter 1; MvdBW Chapter 2& 3

If we make the reasonably-looking assumption that the Universe on large enough scales is isotropic and homogeneous, the velocities of particles can scale with the distance away from us, but cannot have a preferred direction, and they can only scale uniformly with time, which gives us a relation like:

$$\vec{v} = f(t)\vec{r}. \tag{1}$$

Furthermore, if the Universe is expanding we also need that the positions scale uniformly with expansion (with possibly a different scaling function, $a(t)$) and we can then write

$$\vec{r} = a(t)\vec{x}, \tag{2}$$

where $\vec{x}$ are constant (co-moving) coordinates for the particle (we are assuming test-particles here with no proper motion). If we combine these two equations with $\vec{v} = \dot{\vec{r}}$, we get

$$\vec{v} = \left(\frac{\dot{a}}{a}\right)\vec{r}, \tag{3}$$

where I have suppressed the time dependence of a. It is also useful to introduce the redshift, z, through:

$$a = 1/(1+z), \tag{4}$$

where I have also made use of the convention that sets the value of a today, $a(t_0)$, to 1. This corresponds to $z = 0$.

Observationally it is found that at $t = t_0$, $z = 0$, we have

$$\left(\frac{\dot{a}}{a}\right)(t_0) = H_0 \quad = \quad 0.3 \times 10^{-17}\, h\,\mathrm{s}^{-1} \tag{5}$$

$$\quad = \quad 100h\,\mathrm{km/s/Mpc}, \tag{6}$$

where $h \approx 0.71$ (See for example the WMAP collaboration's recommended set of cosmological parameters[1]). This then defines a natural time-scale:

$$t_H = H_0^{-1} \sim 10^{10} h^{-1}\mathrm{yr}, \tag{7}$$

called the Hubble time, and a distance given by the distance light can travel in this time:

$$d_H = cH_0^{-1} \sim 3h^{-1}\mathrm{Gpc}, \tag{8}$$

called the Hubble size/Hubble radius. Note that although in many scenarios this is similar to the horizon size

$$h = \chi = \int_0^t \frac{c}{a(t)}\,\mathrm{d}t, \tag{9}$$

it is *not* the same. The Hubble radius is a local quantity, whereas the horizon is an integral over the history of the Universe and the two can differ vastly depending on the expansion history of the Universe.

3 Energy densities

The key driver of the dynamics of the Universe is the various energy densities of matter and radiation. It is therefore useful to review these. To get the densities, we need the volume evolution which comes naturally from the expansion

$$V(t) = V_0(t)a^3, \tag{10}$$

[1] http://lambda.gsfc.nasa.gov/product/map/current/best_params.cfm

and if we assume that the kinetic energy of matter is negligible in comparison to its internal energy we have that the energy density of N particles of matter is:

$$\rho_{\rm matter} \approx \frac{Nmc^2}{V_0} a^{-3} \propto a^{-3}, \tag{11}$$

while for N_γ photons we need to recall that not only the volume, but also the wavelength of the photons will be stretched, thus the energy density is:

$$\rho_\gamma = \frac{N_\gamma h\nu}{V_0} a^{-3} = \frac{N_\gamma h\nu_0}{V_0} a^{-1} a^{-3} \propto a^{-4}. \tag{12}$$

In general if your matter has an equation of state

$$w = \frac{P}{\rho}, \tag{13}$$

we have (for constant w):

$$\rho \propto a^{-3(1+w)}, \tag{14}$$

which agrees with the above since for pressure-less matter we have $w = 0$, for photons $w = 1/3$. For the cosmological constant we have $w = -1$ and this leads to a constant energy density with time.

For photons we also know that $\rho_\gamma \propto T^4$ from Stefan-Boltzmann's law, this then tells us that

$$T \propto a^{-1}. \tag{15}$$

4 Ionization balance

Literature: See e.g. MvdBW section 3.5

Since T increased back in time, it is clear that at some point sufficiently early the Universe will have been ionized. This event, looking the other way, is known (rather unfortunately) as *recombination*. Let us estimate quickly when that took place.

If we assume a simple system with binding energy ΔE, then it is a simple application of Saha's equation to find that 50% ionization occurs when

$$k_B T_{50\%} \sim \frac{\Delta E}{50}. \tag{16}$$

For Hydrogen $\Delta E \approx 13.6$eV which gives

$$T_{50\%} \sim \frac{13.6\,\mathrm{eV}}{50} \frac{1}{8.6 \times 10^{-5}\,\mathrm{eV/K}} \approx \frac{1}{4} 10^4 = 2.5 \times 10^3. \tag{17}$$

From measurements we know the temperature of the microwave background today to be $T_0 \approx 2.7$K so we get

$$T_0(1 + z_{\rm req}) = T_{50\%} \tag{18}$$

$$z_{\rm req} \approx 10^3 \tag{19}$$

Note that the exact location of this "event" depends on what ionization fraction you adopt. As a consequence this has a somewhat arbitrary nature and compilations of cosmological parameters often do not include this.

5 Matter-radiation equality

Literature: See e.g. MvdBW section 3.5

I will from now on tend to refer to matter as 'non-relativistic matter' and use a subscript NR, and for relativistic species I will use R as subscript. From $\rho_{\rm NR} \propto a^{-3}$ and $\rho_{\rm R} \propto a^{-4}$ we see that as long as $\rho_{\rm NR} > \rho_{\rm R}$ today, there would be a time in the past when they had equal contribution to the energy density of the Universe. This is generally called matter-radiation equality and we can easily estimate the redshift.

Today

$$\rho_{\rm NR} \sim 10^{-30}{\rm g/cm}^3 \tag{20}$$

from observations and we also know from observations that the background radiation has $T \approx 2.7{\rm K}$ with a black body shape. We can then get the energy density from the Stefan-Boltzmann law:

$$\begin{aligned} \epsilon_{\rm R} &= \frac{4\sigma}{c}T^4 \approx \frac{(k_B T)^4}{(\hbar c)^3} &(21)\\ &\approx 3\times 10^{-13}\,{\rm erg/cm}^3 &(22)\\ &\Downarrow \\ \rho_{\rm R} &= \frac{\epsilon_{\rm R}}{c^2} \approx 3\times 10^{-34}\,{\rm g/cm}^3 &(23)\end{aligned}$$

Setting this equal to the NR energy density gives:

$$\begin{aligned} \rho_\gamma(t) &= \rho_{\rm NR}(t) &(24)\\ 1+z_{\rm eq} &\approx \frac{10^{-30}}{3\times 10^{-34}} \approx 3000 &(25)\end{aligned}$$

which means that the redshift of equality was roughly at $z_{\rm eq} \sim 3000$.

6 The dominance of the cosmological constant

We can also ask when the energy density due to the cosmological constant will dominate over the energy in matter. From observations we know that $\Omega_{\Lambda,0} \approx 0.7$ and $\Omega_{\rm NR} \approx 0.3$, so we have

$$(1+z)^3 = \frac{\Omega_{\Lambda,0}}{\Omega_{\rm NR}} \Rightarrow z \approx 0.33, \tag{26}$$

thus we are now in a period where the dynamical evolution of the Universe is very significantly affected by a cosmological constant.

It is of course not well-known what this cosmological constant is, or indeed whether it is a dynamically changing quantity, more commonly referred to as dark energy (see A&T if you want to know much more about this topic).

7 Evolution of the scale-factor

To make further progress we need to have an estimate of the scale factor of the Universe. We can do this by solving the Friedmann equations. A simplified derivation of this can be done using Newtonian arguments and was given in the lecture. The result is

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho, \tag{27}$$

where a is the scale factor, ρ is the total density, k is a constant which from the general relativistic derivation is seen to express the curvature of space. In the Newtonian derivation k is related to the total energy of the system. There is also a second equation which does not follow from a Newtonian derivation. However if one is willing to accept a relativistic formulation of internal energy, it is easy enough to derive from the first law of thermodynamics

$$dU + PdV = 0, \tag{28}$$

with U the internal energy, $U = \rho c^2 V$, $V(t) \propto a^{-3}$ and P the pressure of the matter under consideration. Using that $dV/dt = 3V\dot{a}/a$, we can write equation 28 in the form

$$\frac{d}{dt}\left(\rho c^2 V\right) + 3p\frac{\dot{a}}{a} = 0, \tag{29}$$

which by expansion and division of V gives:

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right). \tag{30}$$

If we evaluate equation 27 today, we have:

$$kc^2 = H_0^2\left[\frac{8\pi G}{3H_0^2}\rho_0 - 1\right], \tag{31}$$

where subscript 0 refers to today, H_0 is the value of $\dot{a}/a$ at the present time. This makes it clear that there is a characteristic density, $\rho_{\rm crit}$,

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G}, \tag{32}$$

for which $k = 0$. Since the Universe has this characteristic density, we often normalise densities relative to this. Quantitatively it is

$$\rho_{\rm crit} \approx 3 \times 10^{11} \frac{h^{-1}\,M_\odot}{(h^{-1}\,{\rm Mpc})^3} \tag{33}$$

$$\approx 1.879 \times 10^{-29} h^2\,{\rm g/cm^3}. \tag{34}$$

It is also useful to rewrite the Friedmann equation

$$H(z) = H_0 E(z), \tag{35}$$

where we have used $a = (1+z)^{-1}$ and $E(z)$ is given by

$$E(z)^2 = \left[\Omega_\Lambda + (1 - \Omega_{\rm tot})(1+z)^2 + \Omega_{\rm NR}(1+z)^3 + \Omega_{\rm R}(1+z)^4\right], \tag{36}$$

where $\Omega = \rho/\rho_{\rm crit}$ for each constituent of the Universe. Λ corresponds to a cosmological constant and $\Omega_{\rm tot}$ is the sum of the other Ω's.

We can solve the Friedmann equations analytically in several situations. By setting $\Lambda = 0$, $k = 0$, $\Omega_\mathrm{R} = 0$, we get a particularly simple equation.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_\mathrm{crit}}{3}a^{-3} \tag{37}$$

$$\Downarrow$$

$$\dot{a} = H_0 a^{-1/2} \tag{38}$$

$$a(t) = \left(\frac{3}{2}H_0 t\right)^{2/3}. \tag{39}$$

If instead we focus on the radiation dominated regime, we have find $a(t) \propto t^{1/2}$, and in the regime dominated by the cosmological constant you will find $a(t) \propto e^{H_0 t}$. It is a good idea to do this yourself!

8 Growth of structure — simple introduction

Literature: C&L Part 3; A&T Chapter 4; Longair Part III; MvdBW Chapter 4

The Friedmann equations describe a uniform Universe — this is not the real Universe but it is found to be a good description of the average properties. What we want now is the evolution of perturbations around this smooth background.

We will adopt the same Universe as above, ie. a flat Universe with matter dominating the dynamics. This means that

$$\rho \propto a^{-3} \Rightarrow d\rho = -3a^{-4}da, \tag{40}$$

and the density contrast δ is then given by

$$\delta = \frac{\rho - \rho_0}{\rho_0} = -3\frac{\Delta a}{a} \tag{41}$$

If we perturb the Universe slightly we can write

$$a(t) \to a(t) + \Delta a(t) \qquad \text{and} \qquad \rho(t) \to \rho(t) + \Delta\rho(t). \tag{42}$$

We then insert this into the time-derivative of the Friedmann equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\dot{a}^2 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{8\pi G}{3}\rho_0 a^{-1} \tag{43}$$

$$\ddot{a} = -\frac{4\pi G\rho_0}{3}\frac{1}{a^2}. \tag{44}$$

Inserting equation 42 into this equation and expanding the right-hand side to first order in $\Delta a/a$ we get

$$\frac{d^2 a}{dt^2} + \frac{d^2\Delta a}{dt^2} = -\frac{4\pi G\rho_0}{3}\frac{1}{a^2} + \frac{8\pi G\rho_0}{3}\frac{\Delta a}{a^3}. \tag{45}$$

The first term on each side cancel each other because they satisfy equation 44. We are then left with

$$\frac{d^2\Delta a}{dt^2} = \frac{1}{2}H_0^2\frac{\Delta a}{a^3} = \frac{4}{9}\frac{\Delta a}{t^2}, \tag{46}$$

where we inserted $a(t)$ from equation 39.

This can be solved by a power-law in t, t^n and inserting this gives a quadratic equation in n and we find that the growing solution is $n = 4/3$ which corresponds to

$$\Delta a \propto t^{4/3} \qquad \Rightarrow \qquad \delta \propto \frac{\Delta a}{a} \propto t^{2/3} \propto a(t). \tag{47}$$

9 Cooling

MvdBW Chapter 8 and Appendix B; R&L Chapter 5 (for Bremsstrahlung); the derivation of typical mass follows Burrows & Ostriker (2014, astro-ph:1401.1814).

Note that this will be covered in much more detail later in the course and this is a set of rough and approximate results:

For further collapse to take place we need cooling. At high temperature ($T > 10^6$K) we expect Bremsstrahlung to dominate. This causes an energy loss

$$\frac{dE}{dt\, dV} = \frac{\alpha^3 \hbar^2}{m_e} \left(\frac{k_B T}{m_e}\right)^{1/2} n^2, \tag{48}$$

where α is the fine-structure constant, m_e the electron mass and n the particle density. Writing n as

$$n \sim \left(\frac{M}{m_p}\right) \frac{3}{4\pi R^3}, \tag{49}$$

where m_p is the proton mass and we are considering a gravitating source of mass M and radius R. And taking T from that for a virialized halo

$$k_B T \sim \frac{GMm_p}{R}, \tag{50}$$

we get a cooling time ($E/\dot{E}$) of

$$t_{\rm cool} = 1.24 \times 10^{-16} \left(\frac{R}{1\rm m}\right)^{5/2} \left(\frac{M}{1\rm kg}\right)^{-1/2} \text{ seconds} \tag{51}$$

$$= 467 \left(\frac{R}{1\rm pc}\right)^{5/2} \left(\frac{M}{1M_\odot}\right)^{-1/2} \text{ years} \tag{52}$$

$$= 1.48 \left(\frac{R}{100\rm kpc}\right)^{5/2} \left(\frac{M}{10^{12}M_\odot}\right)^{-1/2} \text{ Gyrs} \tag{53}$$

At lower temperature the main cooling mechanism is typically due to collisional excitation and recombination, while at high redshift he main cooling is due to inverse Compton scattering. The former two do not have closed forms as does Bremsstrahlung but it turns out (see e.g. Burrows & Ostriker 2014) that you can write the cooling due to recombination in a hydrogen plasma as

$$\Lambda_{\rm bf} \sim A_{\rm bf} \frac{\rho^2}{T^{1/2}}, \tag{54}$$

and if we write the cooling due to Bremsstrahlung in a similar form

$$\Lambda_{\rm ff} \sim A_{\rm ff} \rho^2 T^{1/2}, \tag{55}$$

the two are related through

$$A_{\rm bf} \sim \alpha^2 \frac{m_e c^2}{k_B} A_{\rm ff}, \tag{56}$$

with α being the fine-structure constant, $\alpha = e^2/\hbar c$. Putting in numbers, it turns out that the Bremsstrahlung cooling dominates at temperatures above $\sim 3 \times 10^5$ K.

Now it turns out that typical dark matter halos of interest to us have virial temperatures below this, so to get *typical* masses of galaxies the appropriate cooling would be something like the recombination cooling above, while to get a maximum mass one could use the Bremsstrahlung equation. Either way one can derive an equation for a mass in terms of fundamental constants.

10 Gravitational collapse time

This is given by

$$t_{\rm grav} \sim \left(\frac{GM}{R^3}\right)^{-1/2}, \tag{57}$$

and leads to time-scales of

$$t_{\rm grav} = 1.22 \times 10^5 \left(\frac{R}{1{\rm m}}\right)^{3/2} \left(\frac{M}{1{\rm kg}}\right)^{-1/2} \text{seconds} \tag{58}$$

$$= 1.5 \times 10^7 \left(\frac{R}{1{\rm pc}}\right)^{3/2} \left(\frac{M}{1M_\odot}\right)^{-1/2} \text{years} \tag{59}$$

$$= 0.47 \left(\frac{R}{100{\rm kpc}}\right)^{3/2} \left(\frac{M}{10^{12}M_\odot}\right)^{-1/2} \text{Gyrs} \tag{60}$$

11 The criterion for forming galaxies

Armed with the preceding discussion we can then outline three main regimes:

1. $\mathbf{t}_{\rm cool} > \mathbf{t}_{\mathbf{H}}$. In this case the Universe expands faster than the gas can cool and no significant collapse can take place.

2. $\mathbf{t}_{\rm dyn} < \mathbf{t}_{\rm cool} < \mathbf{t}_{\mathbf{H}}$. In this case the system evolves adiabatically at essentially constant mass. Since the dynamical time is shorter than the cooling time, the system can react to reduced pressure by contracting. The system contracts on a time-scale given by $t_{\rm cool}$.

3. $\mathbf{t}_{\rm cool} < \mathbf{t}_{\rm dyn}$. In this case energy can be removed sufficiently quickly to allow rapid gravitational collapse to take place. This is to be expected to be the main regime for galaxy formation.

Moving from the preceding discussion it is possible to write the cooling time more generically as

$$t_{\rm cool} = \frac{E}{n^2 \Lambda(T)}, \tag{61}$$

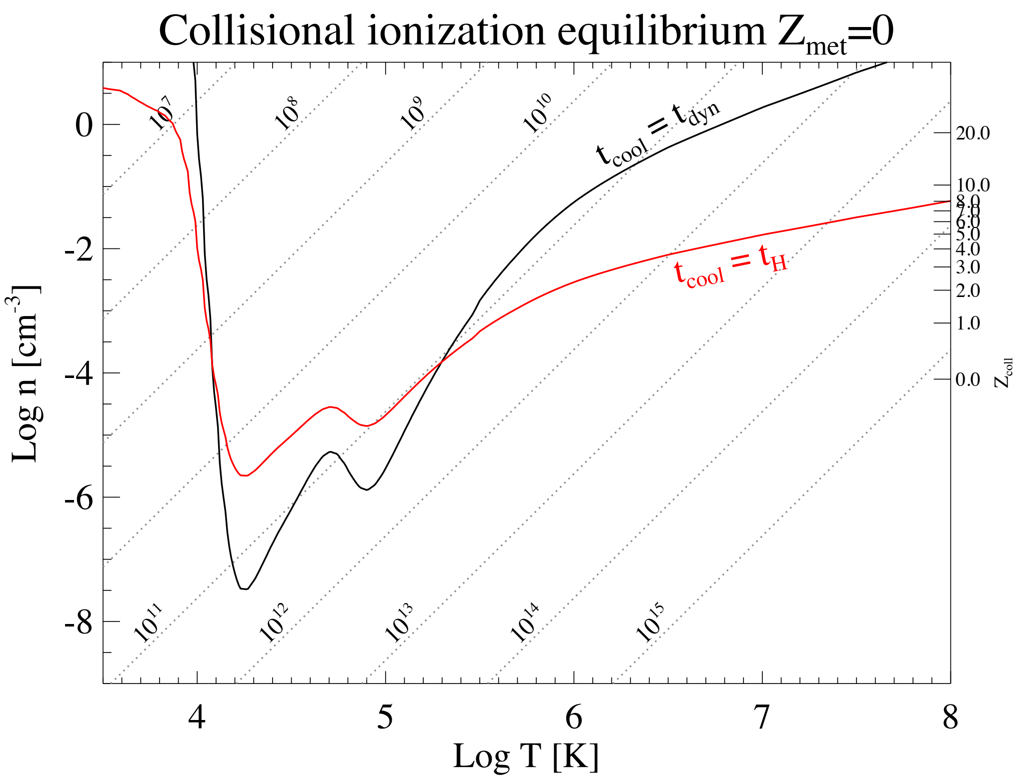

Figure 1: The density-temperature diagram for primordial gas. The black line shows the locus for $t_{\rm cool} = t_{\rm dyn}$ and the red line $t_{\rm cool} = t_H$, drawn for $z = 3$. Where relevant I have set $\Omega_m = 0.3$ and $f_{\rm gas} = 0.15$. Objects that lie above both the red and black lines are able to cool efficiently. The right y-axis shows the redhshift at which the density of a collapsed object is the value of n given on the main y-axis. The dashed lines show lines of constant $M_{\rm gas}$.

where we have isolated the cooling in the *cooling function* $\Lambda(T)$. This has typical values of $10^{-23}\mathrm{erg\,cm^3\,s^{-1}}$. Thus gives a characteristic cooling time of

$$t_{\rm cool} \approx 3.3 \times 10^9 \left(\frac{T}{10^6\,\mathrm{K}}\right) \left(\frac{n}{10^{-3}\,\mathrm{cm^{-3}}}\right)^{-1} \left(\frac{\Lambda(T)}{10^{-23}\,\mathrm{erg\,cm^3\,s^{-1}}}\right)^{-1} \text{ years,} \tag{62}$$

and if we equate this to the gravitational collapse time $t_{\rm dyn}$, we get

$$\left(\frac{n}{10^{-3}\,\mathrm{cm^{-3}}}\right) \approx 245 \left(\frac{T}{10^6\,\mathrm{K}}\right)^2 \left(\frac{\Lambda(T)}{10^{-23}\,\mathrm{erg\,cm^3\,s^{-1}}}\right)^{-1}. \tag{63}$$

This then provides a locus in the $n--T$ plane — illustrated in Figure 1. The details of this figure and how you calculated all the ingredients in it is the subject of later lectures so although I did mention it briefly in the lecture I will not include it further here.

What you can see in the figure is that the constant mass lines (the inclined dashed lines) do asymptote to the $t_{\rm cool} = t_{\rm dyn}$ curve around a mass of $10^{12}M_\odot$ which therefore is an approximate upper limit for monolithic collapse of gas clouds and hence in such a scenario provides an upper limit to the mass of a galaxy.