Interpretation with incomplete data

Interpretation of interferometric data is considered by many to be more of an art than a science—it has the repuration of requiring a great deal of skill and expertise. The reasons for this are:

1. The measurement in the UV-plane are incomplete so that the synthesized image is the “real” image convolved with a complicated pointspread function.
2. The observations are acquired with a large number of systems (e.g. telescopes) over a long period of time, so that the calibration process is complicated—there are many “degrees of freedom” in the instrumental response that have to be measured.

In the first case the difference may be more apparent than real; “ordinary” observations with single dish telescopes also only measure a limited piece of the UV-plane but the effects of this are more intuitive—all the high spatial frequency information is missing. We are accustomed to this and it only becomes a sophsticated problem if we try to push the resolution to the limits by “superresolution” techniques which emphasize the high frequency information, perhaps at the expense of signal/noise.

With interferometers the holes in our information set are more clearly visible, namely all the UV-points that we didn’t measure, be they short spacings, long spacing or intermediate spacings.

The first case is where we have only a few UV-points. This is now commonly the case for optical interferometry, and used to be the case for VLBI radio astronomy. in the extreme case we have only one or two visibilities. Then it is clear that “imaging” is futile and our only alternative is model fitting. Indeed there is then an art to interpretation; we should choose models that are physically motivated, physically plausible and less importantly, easy to fourier-transform so that we can compare them directly with the observations. Favorate among models which represent the morphology of the sky with or without much physical motivation, are point sources, uniform disks, and gaussians, singularly or in combinations. Gaussians and disks may be circular or ellipsoidal. In the first case they require two defining parameters: size and flux (and possibly two position coordinates), in the second they require 4 parameters: two axes, an orientation and a flux. More complicated models may have several disks/gaussians…

The modelling procedure is fairly simple at this level, you just specify the parameters, calculate the visibilities, compute a chi-squared error, and fiddle the parameters until this is minimized. The complications are only that the measurement errors are sometimes hard to estimate and the fitting process is decidedly non-linear.

An important distinction in modelling is whether phase information is present or reliable. This is often the case in radio astronomy, but often not the case in optical interferometry because the atmospheric phase effects are not calibratable. However if three or more telescopes observe simultaneously it is possible to determine *closure phases* which are specific differences of the phases on various baselines, chosen so that the atmospheric phases cancel out. In the simplest case of three telescopes, there are three baselines, and three phases:

$$\varphi_{12} = \psi_{12} + \theta_1 - \theta_2 ; \varphi_{23} = \psi_{23} + \theta_2 - \theta_3 ; \varphi_{31} = \psi_{31} + \theta_3 - \theta_1$$

Where φ is the measured phase, ψ is the true phase from the target, and the θs are the unknown atmospheric phases above the three telescopes. Then $\varphi_{12} + \varphi_{23} + \varphi_{31} = \psi_{12} + \psi_{12} + \psi_{31}$; the atmospheric terms cancel out.

Often the phases provide better information about source structures than the visibility amplitudes, so determining the closure phases are valueable constraints on the model fitting. The number of closure phases increases as the cube of the number of simultaneously used telescopes.

Despite these limitations, the technique is invaluable when there is no other way to get the information. It is the basis of many important discoveries like the superluminal motion of quasars, measurements of the surface temperature of Titan, the oblateness of rapidly rotating stars, the tilt of AGN disks…

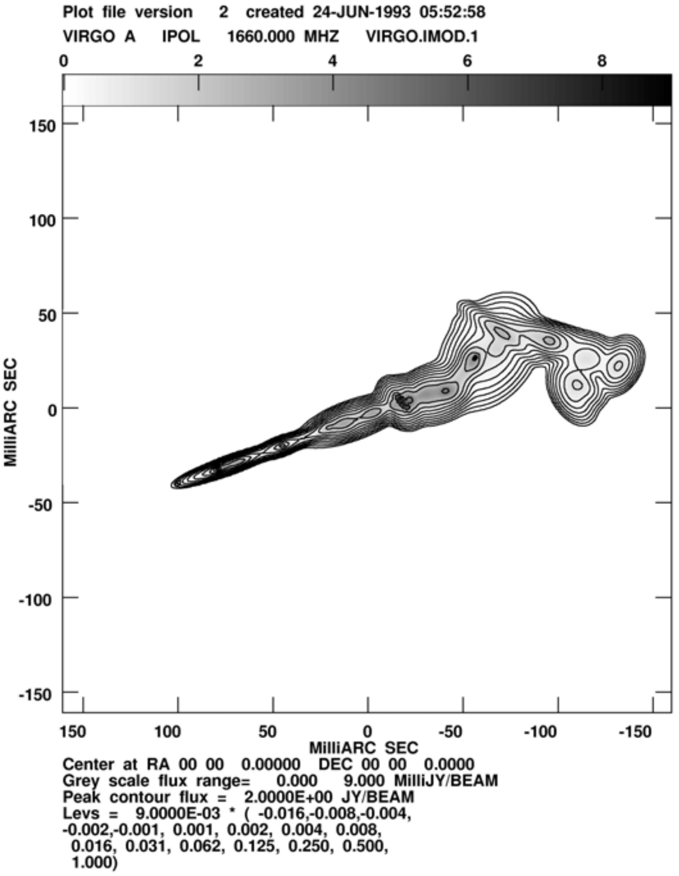
The more confusing situation is when there are many UV-points (hundreds or tens of thousands) where we can create the illusion of an image with a simple fourier transform. The point spread function, and the “dirty” map then have artefacts due to the missing information. This is a classic example of the “inverse problem”—we have transformed information from reality to a representation, losing information along the way. How can we “invert” the representation to recover reality?

We can’t without additional assumptions (positivity, smoothness, blackness…). Each of the image improvement (“deconvolution”) techniques incorporates these assumptions explicitly or implicitly.

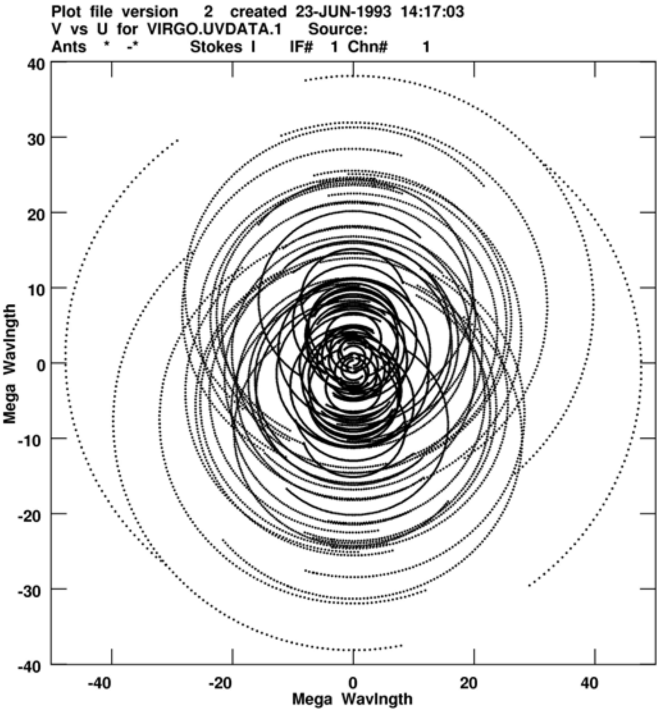
EXAMPLES: CLEAN and Maximum Entropy (MEM)

Clean makes the assumption that the sky is mostly black and deconvolves the image into a number of delta functions. This often fails badly for smooth sources. MEM assumes that the sky has limited variations and that we can best represent it by minimizing some measure of the information content (thus maximizing the “entropy”). Mathematically this is done by maximizing the sum over the pixels of

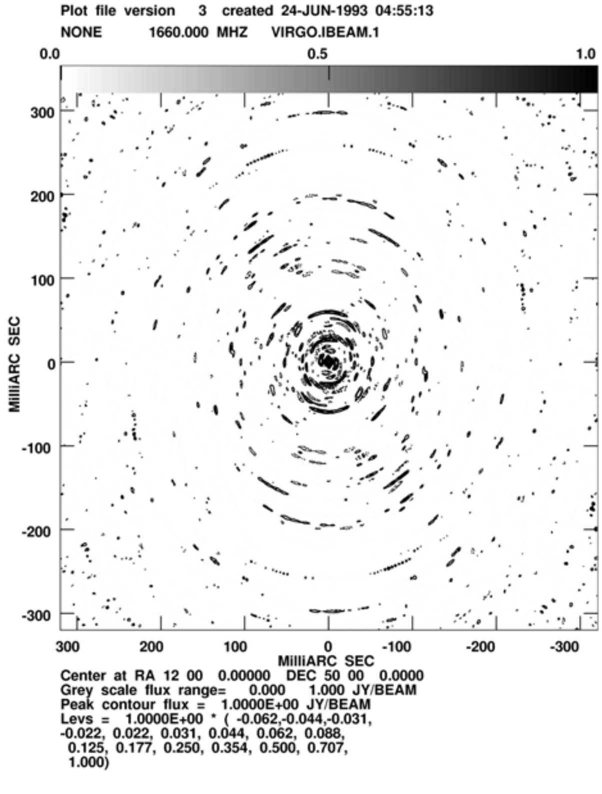
$\sum \boldsymbol{B}_i \ln(\boldsymbol{B}_i)$ under the constraint that the fourier transform of B still looks like the original data. Both methods are highly non-linear but MEM is more obnoxiously nonlinear.



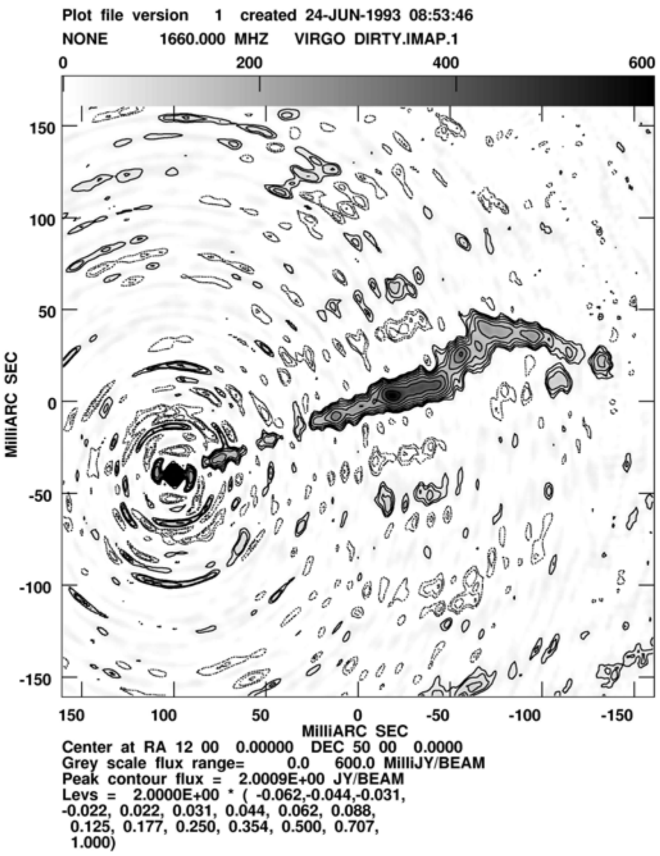
Model image of jet of M87



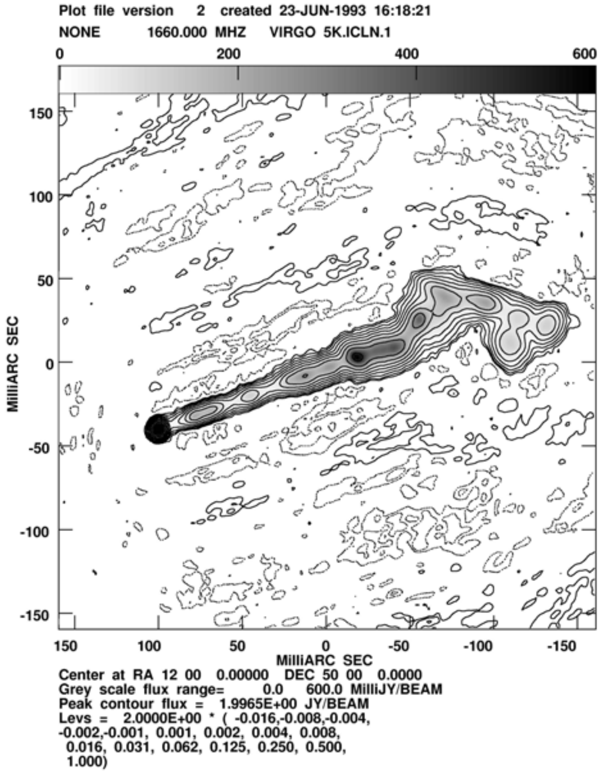
UV-coverage for VLA example



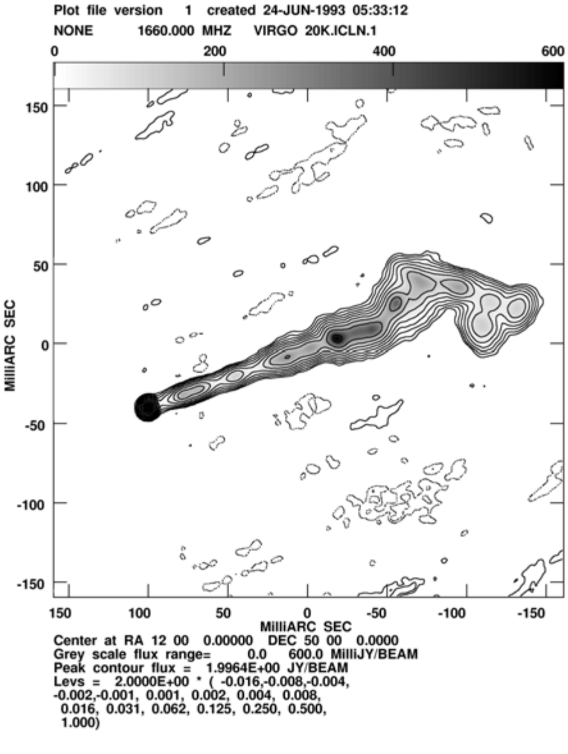
Point spread function



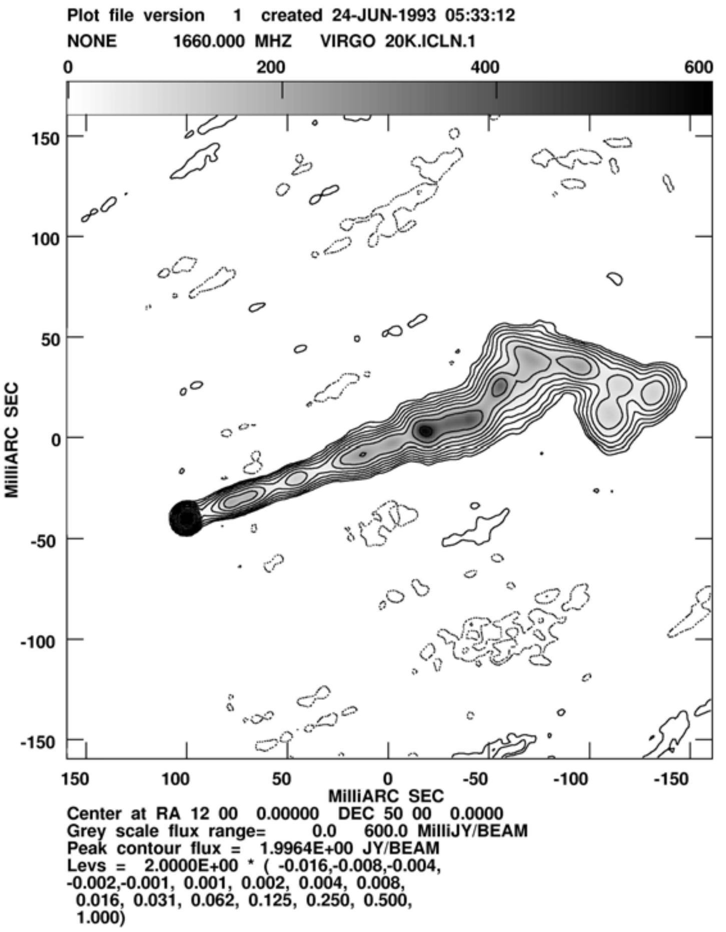
Dirty image



Clean 5000 iterations

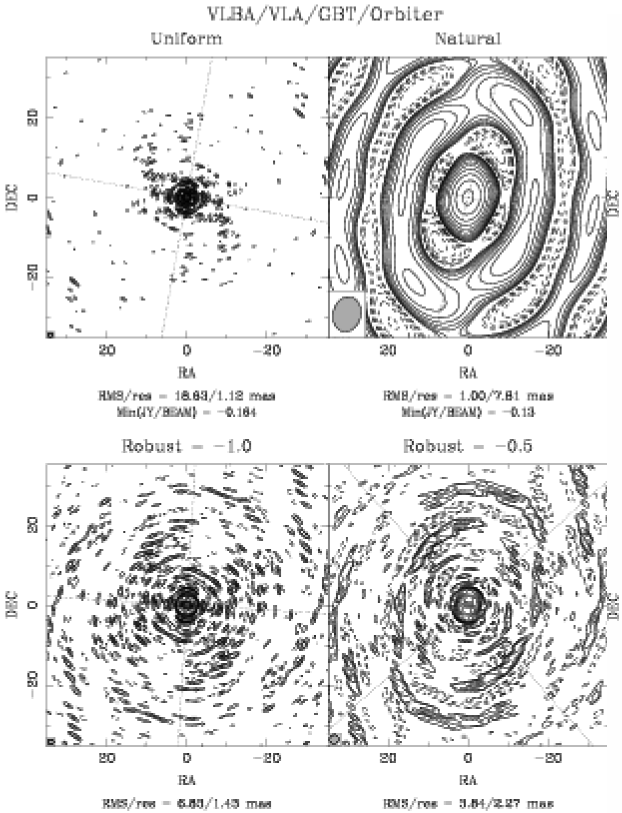


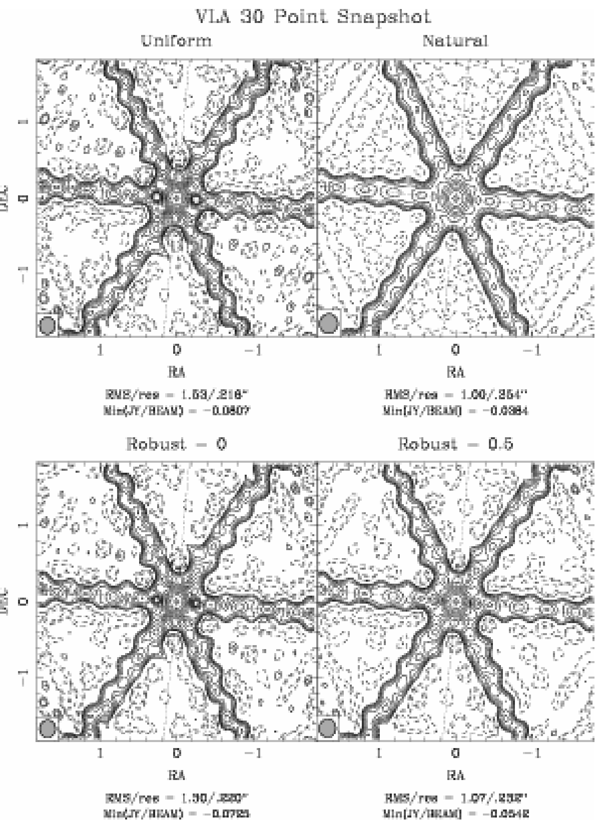
Clean 20000 iterations

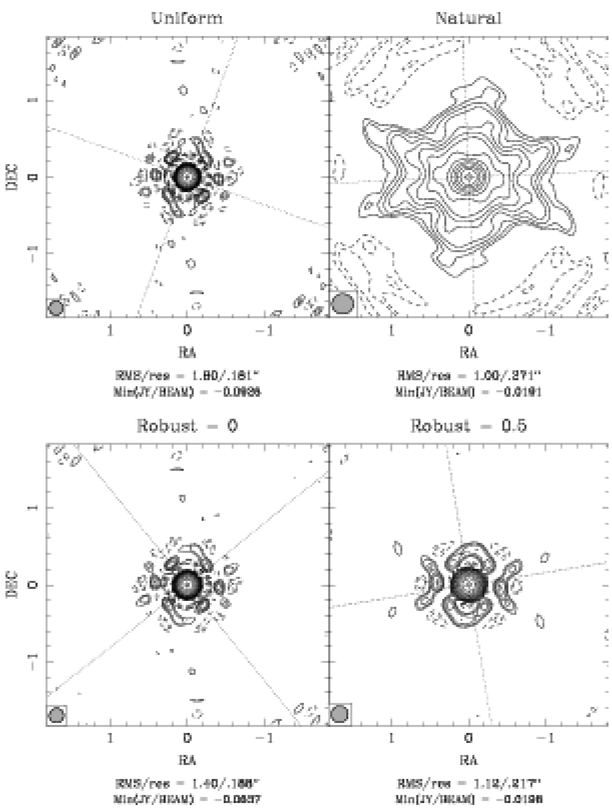


MEM

The nastiness of the dirty image can be influenced to some extent by reweighting the UV-points to give the best approximation of a uniformly weighted, “flat” uv-plane. This improves the ugliness of the psf, but makes the signal/noise worse.

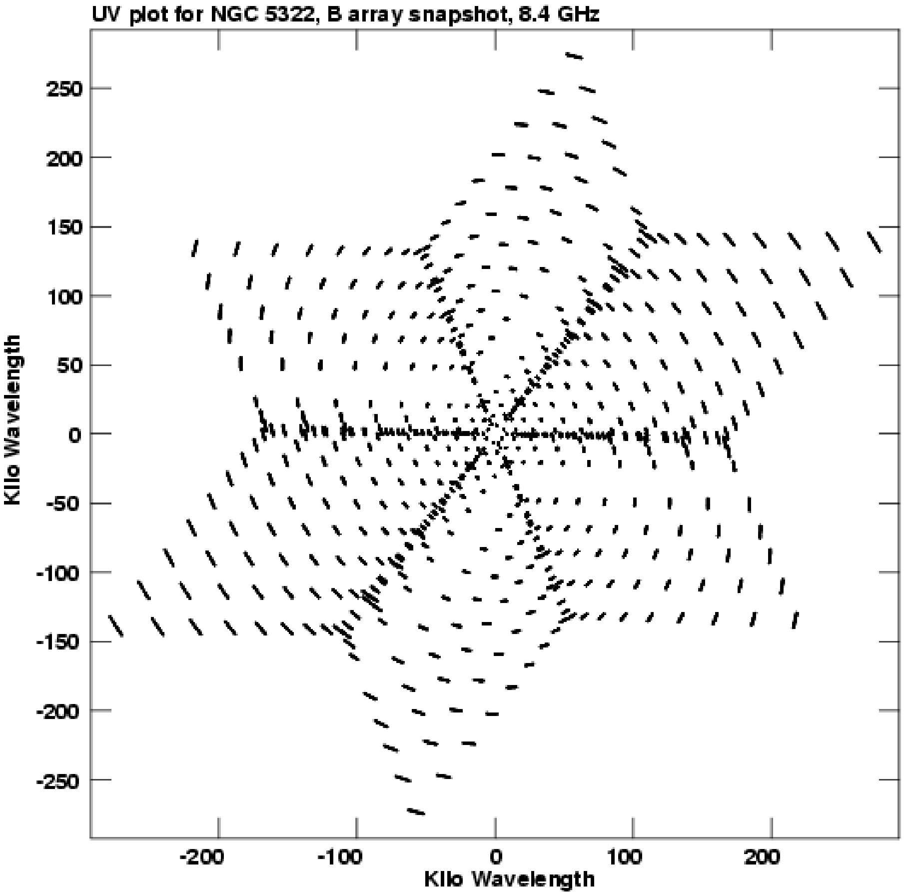




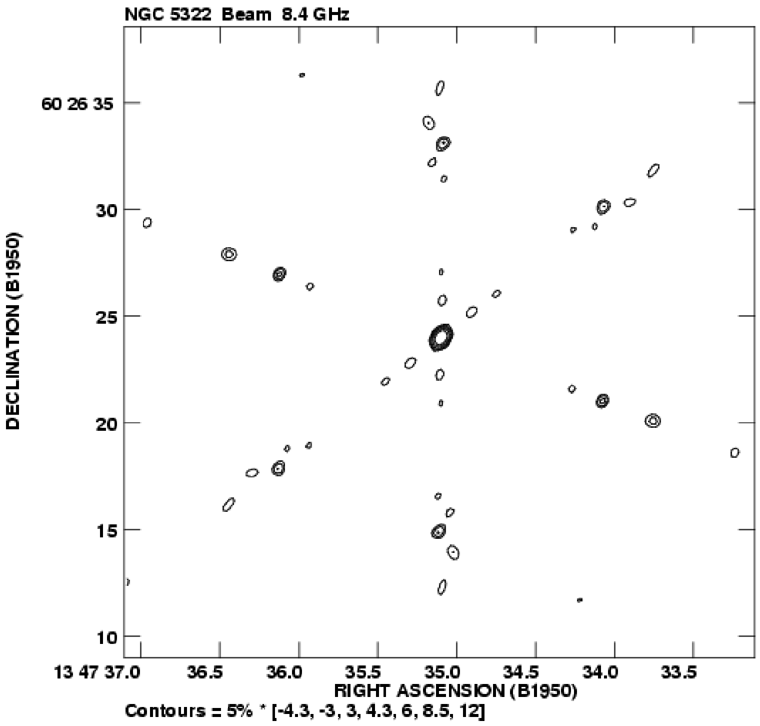


Self calibration:

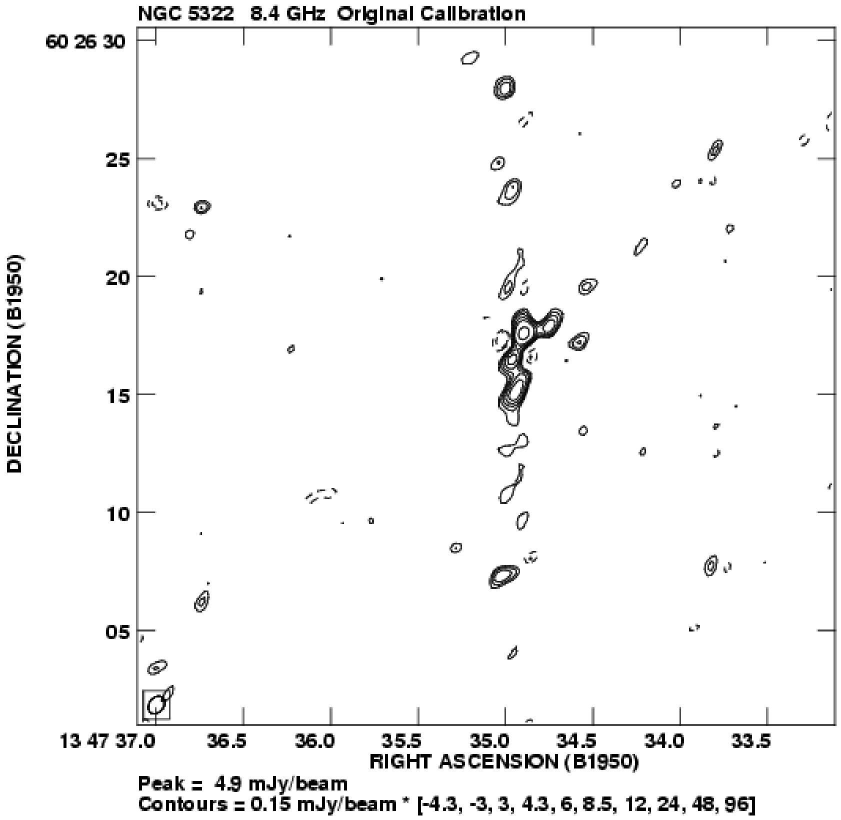
1. Iterative Self-Calibration
2. Create an initial source model, typically from an initial image (or else a point source)
3. Use model to convert observed visibilities into a “pseudo-point source”



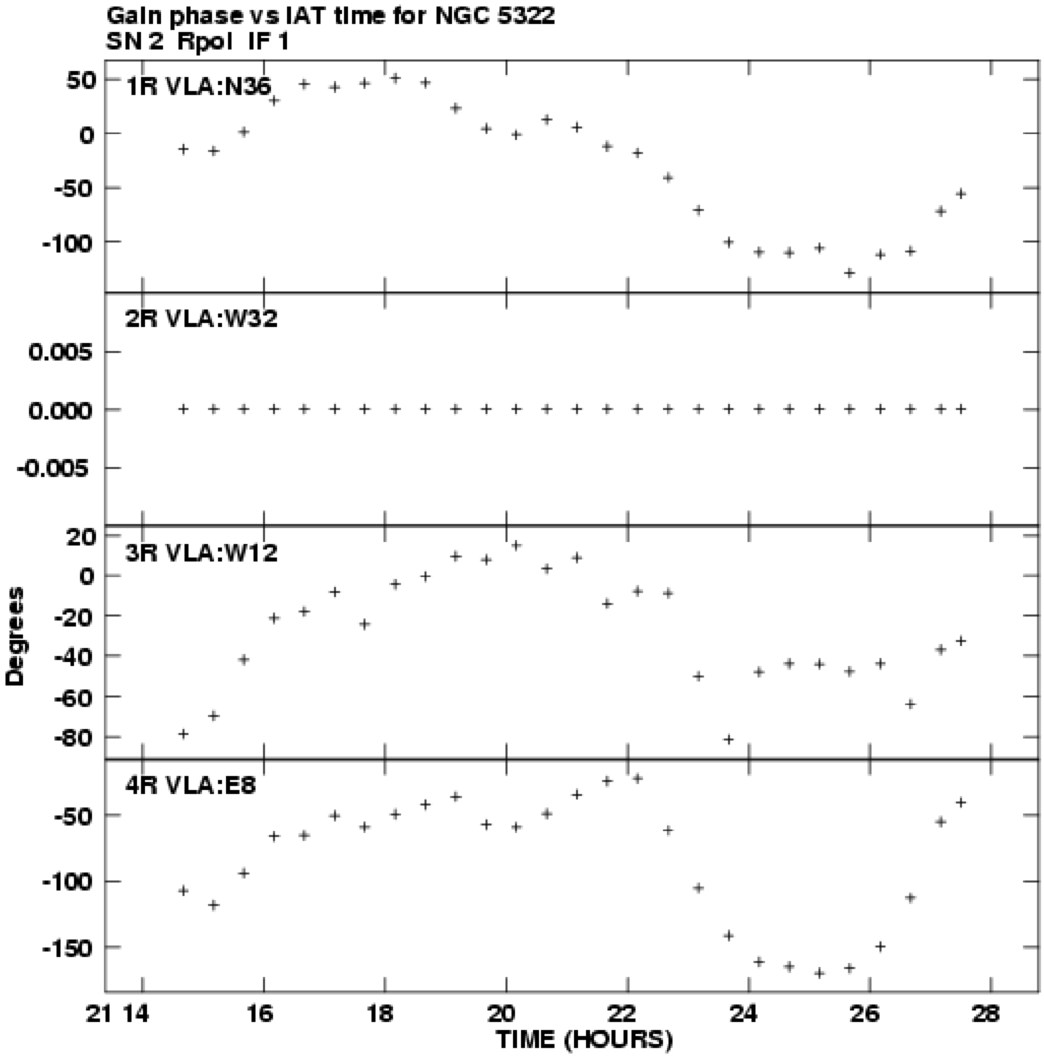
UV-coverage, short VLA measurement



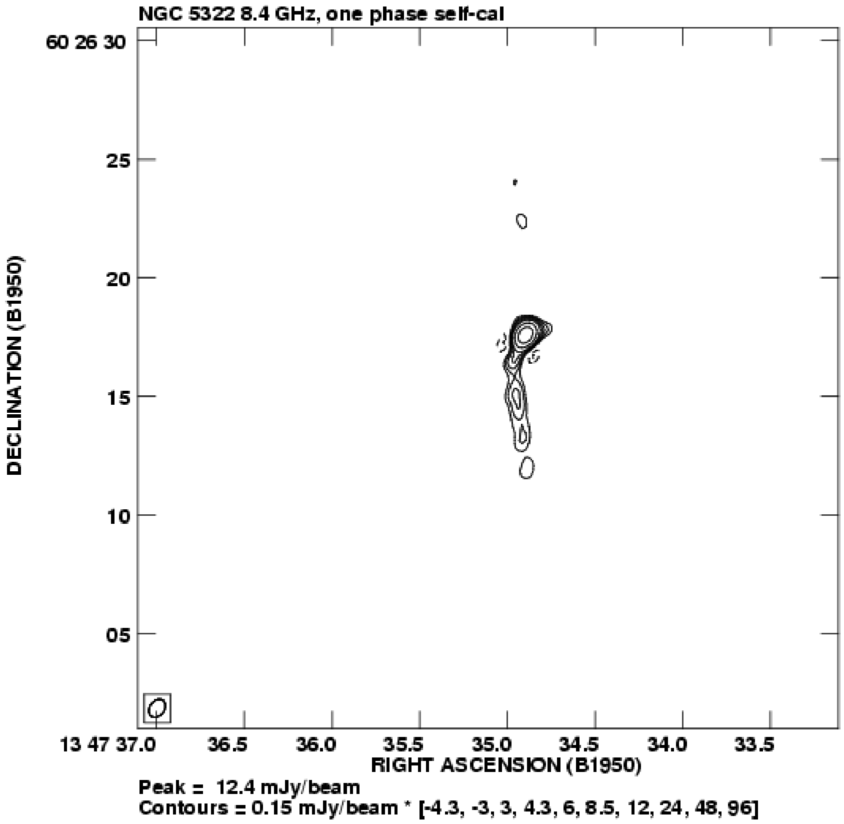
Point spread function (“dirty beam”)



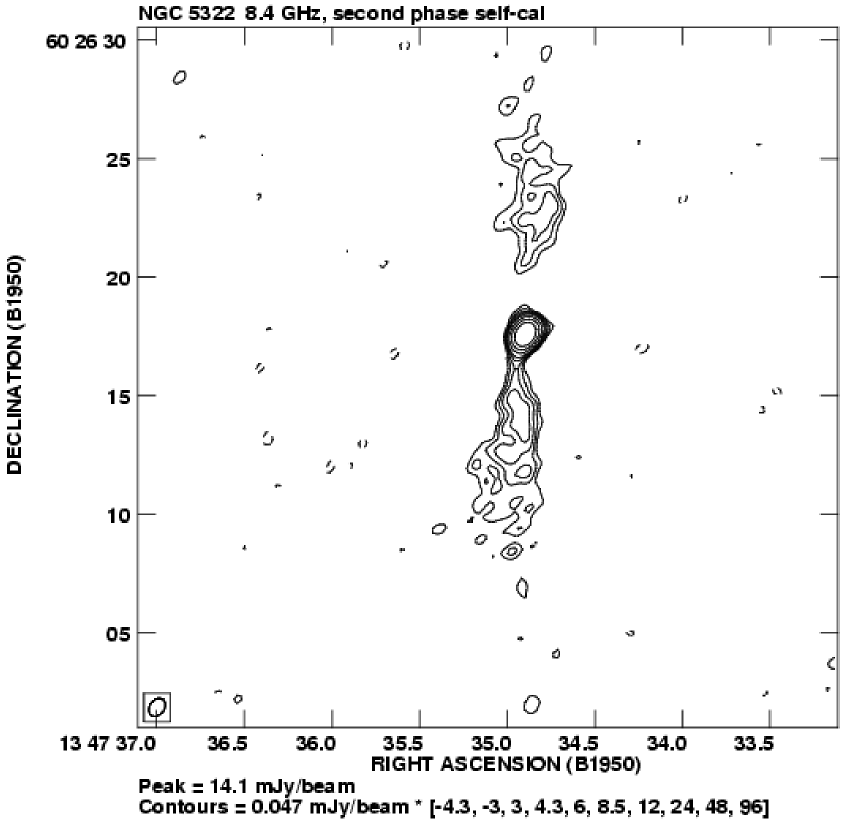
First calibrated image



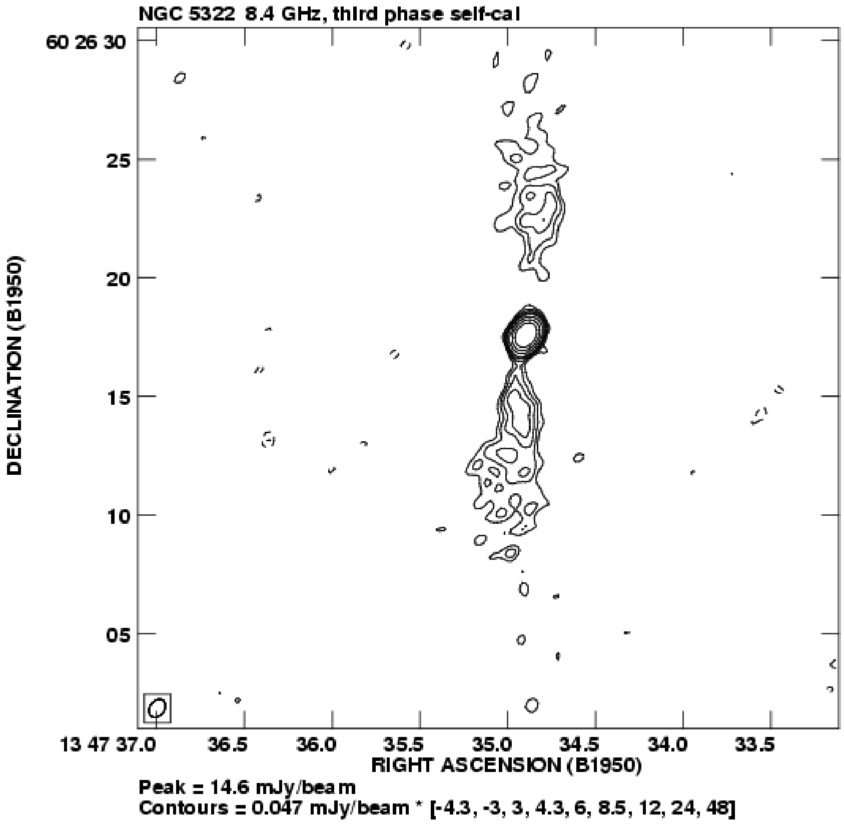
First model included 4 point sources. First self-cal solution assumes that all visibility amplitudes are correct but phases may vary.



Map after 1st recalibration.

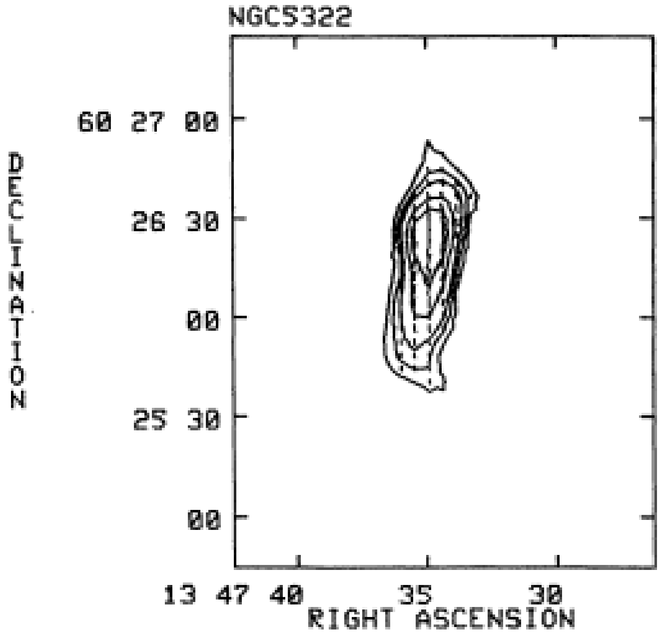


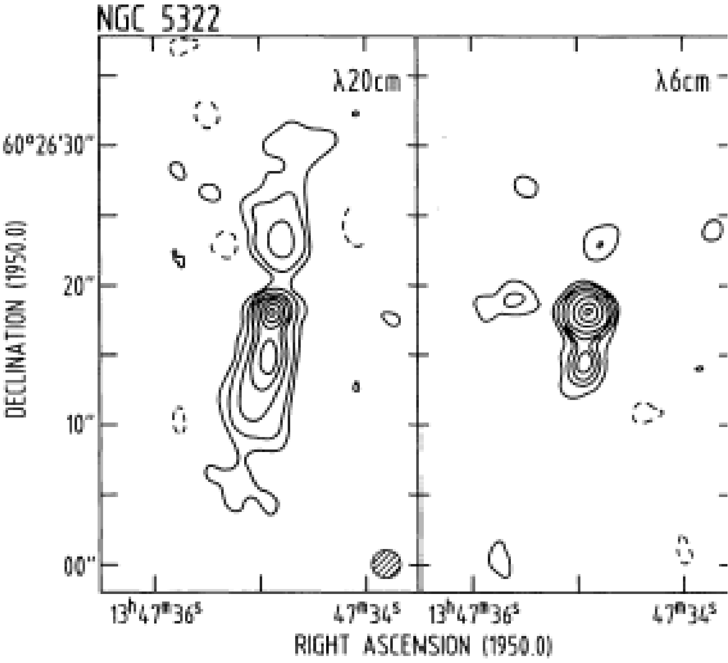
Map after 2nd self-cal (3 components)



3rd self-cal (11 components, not much improvement).

Is the structure real? Compare to map at lower frequency with better phase stability:





At the end of the day, clean+selfcal (or MEM…) are not much different than model fitting. Our “model” in this case is an arbitrary collection of intensities on the sky, and we vary them, along with the calibration unknowns until a “best fit” is achieved, including both the actual measurements and any regularization criteria, such as smoothness, maximum entropy, positiveness, spectral information, low rate of changes of phase…