Atmospheric Effects

What will we learn?

- What physical effects of the atmosphere affect imaging
- How do we describe them statistically
- What are their effects on the images
- Maybe? – How do we try to correct for them

Light from a distant point source arrives at the Earth as a series of *flat* wavefronts. If there were no atmosphere these could be focused to sharp images, no bigger than λ / D, where D is the telescope size. In the atmosphere the speed of light is no longer *c* but *c/n* where *n* is the *index of refraction.* If the atmosphere were plane parallel in structure this would only cause a tilt of the wavefronts (Snell's law) but the atmosphere is in fact turbulent and inhomogeneous. The result is that bits of the wavefront arrive at the surface earlier than other bits: the wavefront is *corrugated* and this causes blurring of the image.

Refractive index

When a light wave passes an atom it causes the electrons in the atom to oscillate at the frequence of the wave. This acceleration causes the electrons to radiate waves that interfere with the original wave. If the material is uniform on scales smaller than the original wavelength, the new radiation all has a definite phase relation with respect to the original wave and is expressed as a slowing down (or speeding up) and absorption of the original wave. That is, the wave is proportional to $\exp(i\omega t + i\omega x[n_r + i\, n_i]/c)$, where n_r, n_r are the *real* and *imaginary* parts of the refractive index. We can see that the *effective wave number* is now $k = \omega n_r / c$, or the wavelength is shortened to λ / n_r, and the *absorption coefficient* is $\omega n_i / c$. The absorption and refraction coefficients are closely related. Not only are they the real and imaginary part of one number, but the are related to each other by what is called the Kramers-Kronig relation that is true for all *causal* systems, i.e. those where the output occurs *after* the input:

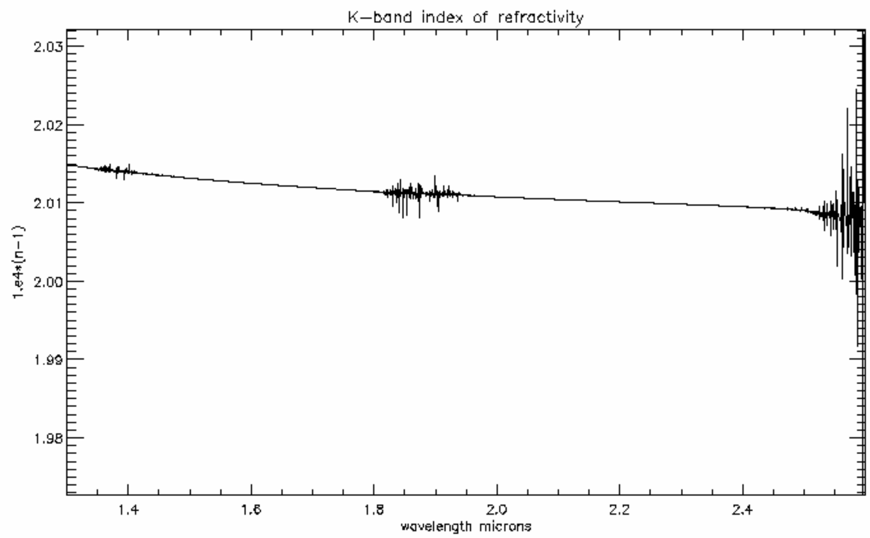
$$n_r(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{n_i(\omega')d\omega'}{\omega' - \omega}, n_i(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{n_r(\omega')d\omega'}{\omega' - \omega}$$

This means that if I know the *absorption* lines of a substance; these are usually a series of more or less narrow lines, then I know the index of refraction. These relations are actually used in practise (e.g. the program HITRAN) to calculate indices of refraction.

In classical physics it is usually assumed that an atom's electrons are attached by little springs and they act like little damped oscillators. This results in a formula for the refractive index (due to Lorenz):

$$n^2 = 1 + \frac{Ne^2}{m}\sum_k \frac{f_k}{\omega_k^2 - \omega^2 + i\gamma_k\omega}$$ **where *N* is the space density of atoms, *e* is the electron charge and *m* its mass, ω_k is the oscillator frequency of the *k*th transition, and f_k is a dimensionless *oscillator strength* that gives the relative contribution of each transition. γ_k gives the width of the absorption feature and its depth at its center. If *N* is small, we can get *n* from the sqrt of the right side, i.e. multiplying the last part by ½. You see that this includes both a real and imaginary part, which should satisfy the above relation. It is useful to look at a single transition and to note that at frequencies much *higher* than the center frequency, the index drops as $1/\omega^2$, but at frequencies much *lower* than the center frequency, the index approaches a constant value: $1 + \frac{Ne^2}{2m}\frac{f}{\omega_k^2}$. Thus the index of refraction in the *visible* region is largely constant and is caused by the very strong electronic absorption bands of Oxygen and Nitrogen in the ultraviolet. The index in the *infrared* is altered by the molecular vibration**

lines of water and carbon dioxide and is not so constant with



Why are the wavefronts irregular?

The turbulent atmosphere is not entirely uniform in pressure. The pressure variations are essentially *adiabatic* and cause small density fluctuations: $d\rho = dP/c^2$, where *c* is the sound speed. These turbulent fluctuations are of a random nature and can only be described statistically. There is in fact no very good complete theory of turbulence. The most important results are due to a Russian mathematician, Kolmogorov, who made what is essentially a dimensional analysis.

He assumed that turbulent energy is inserted into the atmosphere at some large scale *L* which turns around into smaller and smaller eddies until at very small lengths it is dissipated by viscosity. At an intermediate scale *l* between these extremes he assumed that there was a *constant rate of flow of energy down the scalelengths*, *W(l)*. This *W* can only depend on the density, the typical velocity at this scale, *v(l)* and *l* itself. The only combination that has the units of *power* (per unit volume) is $W = \rho v^3 / l$ and if this is constant, we find $v(l) \propto l^{1/3}$. The *pressure* fluctuations are given by *Bernoulli's law* that along a flow line: $dP + \rho d v^2 = 0$, so $dN \propto dP \propto l^{2/3}$. Following my rules from Huygen's law, if the fluctuations are small, we can calculate the electric

field in the aperture plane by integrating all of the phase changes along the propagation lines from the source to the aperture. If we take two points separated by a distance *h* in this plane, the integration adds one factor of distance to the above relation. In otherwords if I define the *Optical Path OP* at *h* as $\int n(z,h)dz$ through the vertical direction *z* I will find that $dOP \propto h^{5/3}$. This formula is very commonly assumed in atmospheric physics as "Kolmogorov turbulence". Slightly more formally we describe the index variation in terms of a *Structure Function (closely related to the Autocorrelation Function)* of some variable q:

$SF(h) \equiv < (q(x) - q(x+h))^2 >$, averaged over all positions *x* (and possibly over time). So we write that the structure function of the *OPD* (Optical Path Difference) is proportional to h^5/3, and (in the optical) essentially independent of wavelength. The SF of the *phase fluctuations* in the aperture plane will then be proportional to the *OPD* fluctuations time k^2, and still proportional to h^5/3. This is commonly written as:

$SF(\varphi) = 6.88(h/r_0)^{5/3}$ where r0 is called *Fried's scale length*. In this formulation r0 depends on the wavelength:

$r_0 \propto \lambda^{6/5}$. You may recall that I said that the achieved resolution under the air is approximately λ / r_0 so this is proporitional to $\lambda^{-1/5}$. Surprisingly, the seeing resolution get better (slowly) as the wavelength increases.

R0 depends on the wind turbulence structure. There is often a lot of turbulence generated by the wind as it travels over the buildings and hills around the telescope. This generates *ground seeing*, which should not be strictly Kolmogorov. Other sources of turbulence are heating of the ground, which generates convention eddies , and the jet stream at altitudes of 10 km or so.The first two sources can be minimized by good site design, and not observing while the sun is shining (!). The second is more or less unavoidable.

At a *reasonable* site, r0 is about 10 cm in the blue, so the seeing resolution is about 1". At good sites (hawaii, paranal) the median seeing is about 0.8" (occasionally 0.2") and at the South Pole often <0.2". The seeing depends on the wind structure, but also on the altitude (less air above) and a little on the temperature and humidity (freezing out of water vapor).

If the wind speed at the turbulent level is larger than the typical turbulent velocity, we can assume that the turbulence is carried along a fair distance by the wind before it changes pattern. We can also define a *temporal* structure function of phase at one point as a function of time difference, and for this *frozen in turbulence* approximation, this SF has exactly the same form: $<(\phi(t)-\phi(t+\Delta t))^2>=6.88(\Delta t/t_0)^{5/3}$ where t0 is now called Fried's characteristic time.

If we take this formula, or something like it, we can try to find the statistical properties of the image. Remember that the electric field in the image plane is the Fourier transform of that in the aperture plane: $E(p) \sim \int \exp(ikpx/f + i\phi(x))dx$. Since ϕ is a random variable we can't say much about the exact value of E, but we can look at the average value of the power |E^2| by multiplying the integral by its complex conjugate:

$$<E^2(p)>=\iint \exp(ikp(x-x')/f + i(\phi(x)-\phi(x')))dxdx'$$

If we write x-x'=h and change the variable of integration:

$$=\int dx \int \exp(ikph) < \exp(i(\phi(x)-\phi(x+h))) > dh$$

The exponential on the right is average of a random variable. It can Be shown that if *z* is a gaussian random variable, with standard deviation σ that $<\exp(iz)>=\exp(-\sigma^2/2)$. We will also assume that the turbulence statistics do not depend on the position *x* so the integral over *x* can be done by just multiplying the the range in x, i.e. *D*.:

$$<I>=D\int \exp(ikph/f)*\exp(-SF(h))dh =$$

$$D\int \exp(ikph/f)*\exp(-h/r)^{5/3}dh =$$

$$Dr\int \exp(ikpry/f - y^{5/3})dy$$

I don't have an analytic expression for the last integral but it is clearly a function of the variable *kpr/f*. If you change the exponent to 6/3 I can do the integral, and I get $Dr*\exp(-(kpr/f)^2)$. That gives the characteristic angular size on the sky, $p/f = 1/kr \sim \lambda/r$.

Can we do anything about the seeing?

The first thing we could do is not just look at the long term average value of the intensity, but the details of its structure in a single image. As I explained earlier, the speckles in the image have a typical size of λ/D for a point source, and larger if the source is extended. So by looking at the autocorrelation function of the individual images you can reconstruct the autocorrelation function of the original source. This is called speckle interferometry. It only works for fairly bright targets.

Adaptive Optics:

A better solution is to *measure* the form of the wavefront in the aperture at every instant and correct for the bumps and wiggles with a *rubber mirror* or something like it. This is called Adaptive Optics (AO). Normally you extract part of the light coming into the telescope with a (dichroic?) half-silvered mirror and direct it into a *wavefront sensor.* This creates an image of the *aperture* (or pupil), not of the sky, and uses various techniques to estimate the form of the wavefronts from a point source. This information is digested by a fast computer, and used to move *actuators* on a flexible, or segmented, mirror that flattens the wavefront.

To get this to work you have to have a point source to work on. Because the process has to work quickly (on order Fried's t0) and the mirror area that you have to correct is relative small (order r0^2), there are not many photons to test the wavefront unless the target (*guide star*) is quite bright. This limits the usefulness of the system to science sources that are near bright stars. A solution is to provide *artificial guide stars* by shining a laser up from the telescope and catching the photons that come back after scattering from the ionosphere (above the atmospheric regions that cause seeing). These are called *laser guide stars*.