At $T \sim 200-300$ MeV the quark-hadron transition binds quarks into hadrons
- protons / neutrons made of 3 u, d quarks
- mesons, in particular π^+, π^0, π^- which are combination of 2 u, d quarks

In this hadron era pion-pion interactions are very important and the equation of state of the hadron fluid is very complicated

Once the pions disappear ($\sim 10\,\mu s$ after the Big Bang when $T \sim 130$ MeV $\sim 10^{12}$ K) the lepton era starts. It lasts until the $e^+ e^-$ pairs annihilate at $T \sim 500$ keV $\sim 5 \times 10^9$ K (1 s after Big Bang)

At the start of the lepton era the Universe comprises
- photons
- small excess of non-relativistic baryons as $mc^2 \gg kT$
- leptons / antilepton

$e^+ \; e^- \qquad \nu_e \; \bar{\nu}_e$

$\mu^+ \; \mu^- \qquad \nu_\mu \; \bar{\nu}_\mu$

perhaps $\tau^+ \; \tau^- \qquad \nu_\tau \; \bar{\nu}_\tau$ but most likely already annihilated
$\rightarrow$ the corresponding neutrinos remain

Before we study the neutrino background we need to discuss the statistical distribution of particles in equilibrium

A relativistic particle species in thermal equilibrium has a Fermi-Dirac (+) or Bose-Einstein (−) distribution, with particle density:

$$n(T) = \int_0^\infty \frac{g}{2\pi^2 \hbar^3 c^3} \frac{E^2 dE}{e^{E/kT} \pm 1} \simeq 0.1216 \overset{\swarrow \text{Fermion}}{\underset{\uparrow \atop \text{Boson}}{\begin{pmatrix} 3/4 \\ 1 \end{pmatrix}}} \; \underset{\nwarrow}{g} \left(\frac{kT}{\hbar c}\right)^3$$

statistical weight = # independent states (spin states, polarization, etc)

Similarly the energy density is obtained by calculating ($n \times E$):

$$\rho(T) c^2 = \int_0^\infty \frac{g}{2\pi^2 \hbar^3 c^3} \frac{E^3 dE}{e^{E/kT} \pm 1} = \begin{pmatrix} 7/8 \\ 1 \end{pmatrix} \frac{g}{2} \sigma_r T^4$$

a photon is a boson with $g = 2$ $\Rightarrow$ we obtain the familiar Stefan-Boltzmann law

So to get the total energy density sum up all species ($7/8\, g$ for fermion, g for boson) $\times \sigma_r T^4/2$

The total energy density $\rho(T) c^2 = \underset{\uparrow \atop \text{effective weight}}{g^*} \sigma_r T^4/2$

At the start of the lepton era $g^*(T<T_\pi) = \underset{\nearrow \atop e^+e^-\mu^+\mu^-:\, g=2}{4\times 2\times\frac{7}{8}} + \underset{\uparrow \atop \text{neutrinos } g=1}{N_\nu \times 2\times\frac{7}{8}} + \underset{\uparrow \atop \text{photons } g=2}{2} \simeq 14.25$ if $\underset{\nearrow \atop \text{number of families}}{N_\nu} = 3$

The particles are still in thermal equilibrium because the relevant collision time τ_{coll} is much smaller than the Hubble time : at $T\sim 10^{11}K$ ($t\sim 10^{-4}s$) $\tau_{coll} = (\sigma_T n_e c)^{-1} \sim 10^{-21}s$ for photons

As the ~~temperature~~ decreases neutrinos will decouple.

Important events during the lepton era are
- annihilation of muons at $T \lesssim 10^{12}K$ (early)
- " " electrons at the end
- nucleosynthesis at $T\sim 10^9K$

These represent ~~phase~~ transitions where a particle species disappears. However entropy is conserved for components still in thermal equilibrium (reactions are reversible)

$$S = \frac{(\rho c^2+p)V}{T} \underset{\nearrow \atop \text{relativistic particles}}{=} \frac{4}{3}\frac{\rho c^2 V}{T} = \frac{4}{3} g^*(T)\frac{1}{2}\sigma T^3 V$$

$\Rightarrow g^* T^3$ is conserved: as a particle species annihilates g^* falls and T rises a bit

The first instance in the lepton era is when muons annihilate at $T\sim 10^{12}K$

$$g^* \text{ falls from } \underset{\nearrow \atop \nu}{6\times\frac{7}{8}} + \underset{\nearrow \atop \mu^\pm,\, e^\pm}{4\times\frac{7}{8}\times 2} + \underset{\nearrow \atop \text{photon}}{2} = 14.25 \text{ to } 6\times\frac{7}{8} + 2\times\frac{7}{8}\times 2 + 2 = 10.75$$

$\Rightarrow T$ rises by 9.8%

At the start of the lepton era the neutrinos are still in thermal equilibrium through weak force reactions such as

$$\nu_e + \mu^- \leftrightarrows \bar{\nu}_\mu + e^- \qquad \bar{\nu}_\mu + \mu^+ \leftrightarrows \nu_e + e^+$$

The cross section for these weak interactions is $\propto T^2$: decreases quickly as T decreases. When the rate for the interactions falls below the expansion rate they cannot maintain equilibrium and the neutrinos become decoupled

$$\frac{\tau_H}{\tau_{coll}} \simeq \left(\frac{T}{3\times 10^{10}K}\right)^3 < 1 \Rightarrow \text{neutrino decoupling occurs at } T\sim 3\times 10^{10}K$$

They do remain relativistic as $mc^2 \ll k_BT$

Note this occurs after $\mu^\pm$ annihilation, but before $e^\pm$ annihilation

At the time of decoupling the temperature of the neutrinos coincides with the temperature of the other constituents; after decoupling the neutrinos continue to evolve as a relativistic gas $T \propto \frac{1}{a}$ and $\rho \propto 1/a^4$

The e^+, e^-, γ component follows the same evolution, but at e^+, e^- annihilation the temperature is increased again; this ratio of temperatures persists until today $T_\gamma / T_\nu = \text{const} > 1$

Once e^+/e^- annihilation is over the Universe is dominated by radiation with a thermal Planck spectrum.

Towards the end of the lepton era nuclear physics starts to take place, ultimately resulting in H, He and traces of D, Li. This phase is not important for the thermal history of the Universe, but clearly important for our existence!

The element abundance by mass is the ratio of the mass of a particular element to the total mass in baryons

The abundance of ^4He : $Y = 0.25$ → 6% of the nuclei
^3He : $\sim 10^{-3} Y$
^2H or D : $\sim 2 \times 10^{-3} Y$

Could the helium be produced in stars : no! If our Galaxy maintained a constant luminosity for $\sim 10^{10}$ years the total energy radiated would correspond to the fusion of 1% of the nucleons.

Atmospheres of main-sequence stars consist of ~25% He by mass with only weak trends with age and metallicity → This He was not made in stars
Furthermore D is photodissociated in stars; note that essentially all elements heavier than ^7Li are made in stars.

The ratio of neutron and proton number densities is $\frac{n_n}{n_p} \simeq e^{-Q/kT} = e^{-1.5 \times 10^{10} K / T}$ ← mass difference $m_n - m_p = 1.29$ MeV
as long as proton and neutrons are in thermal equilibrium

The equilibrium is maintained by weak nuclear reactions :

$$n + \nu_e \leftrightarrows p + e^- \quad \text{and} \quad n + e^+ \leftrightarrows p + \bar{\nu}_e$$

Because of the mass difference there are more protons than neutrons

The equilibrium is maintained for $T_{EW} > 10^{10}$ K when the neutrinos decouple

After this happens $X_n = \frac{n}{n+p} \simeq \frac{n}{n_{tot}} \simeq (1+e^{1.5})^{-1} \simeq 0.17 = X_n(0)$

More detailed calculations show that equilibrium is maintained until $T \sim 1.3 \times 10^9$ K (20s after the Big Bang), after which neutrons decay via beta-decay: $n \to p + e^- + \bar{\nu}_e$ which has a mean lifetime of 900s

$$\Rightarrow X_n(t) = X_n(0)\, e^{-(t-20s)/900s)} \simeq X_n(0)$$

Note that $m_n - m_p \sim k T_{EW}$ if T_{EW} would be much lower, only protons would be formed, and no nuclear reactions would occur.

Still the scarcity of neutrons relative to protons explains why BBN is so incomplete, leaving 75% of baryons unfused.

$$p + p \to D + e^+ + \nu_e$$ which is a weak force interaction $(\sigma_W \sim 10^{-47} m^2 (\frac{T}{MeV})^2$

whereas $p + n \leftrightarrows D + \gamma$ is a strong force interaction

As $p + n \to D + \gamma$ is so much more efficient ($n + n \to D + e^- + \bar{\nu}_e$ is also a weak reaction) BBN proceeds until of neutrons are bonded into nuclei

$\Rightarrow$ This allows us to compute the maximum value for Y. If we take $n_n / n_p = 0.2$ and consider a group of 2 neutrons and 10 protons $\to$ 1 ^{4}He + 8 p

$$\Rightarrow Y_{max} = \frac{4}{12} = \frac{1}{3}$$ generally if $f = n_n / n_p \Rightarrow Y_{max} = \frac{2f}{1+f}$

The observed value is indeed smaller; this is because nucleosynthesis takes a while and some neutrons will decay; some will end up in ^{3}He or D or heavier nuclei such as Li

To compute Y accurately, as well as other abundances ~~also~~ all reactions need to be considered

At $t \simeq 2s$ the proton-neutron freeze-out occurs, the neutrinos are decoupled but the photons are still strongly coupled. To build the heavier nuclei we need a series of 2 particle interactions

$$p + n \leftrightarrows D + \gamma$$

the binding energy $B_D = (m_n + m_p - m_D)c^2 = 2.22\,\text{MeV}$
γ with $E > B_D$ can destroy D.

Around the time of D formation, the relative number densities are given by a Saha-like equation

$$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} e^{B_d/kT}$$

$g_D = 3$
$g_p = g_n = 2$

as $m_p \simeq m_n \simeq m_D/2$

$$\Rightarrow \frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} e^{B_d/kT}$$

$\Rightarrow$ deuterium is favoured in the limit $T \to 0$, whereas p^+, n are favoured when $T \to \infty$

Define T_{nuc} is the temperature where $n_D/n_n = 1$ (half the neutrons have been fused)

$$\Rightarrow \frac{n_D}{n_n} = 6 n_p \left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} e^{B_d/kT}$$

Currently 75% of baryons are in the form of H, and before deuterium synthesis 83% were

$$\Rightarrow n_p \approx 0.8\, n_{baryon} = 0.8\,\eta\, n_\gamma = 0.8\,\eta\left[0.243\left(\frac{kT}{\hbar c}\right)^3\right]$$

$$\Rightarrow \frac{n_D}{n_n} \approx 6.5\,\eta\left(\frac{kT}{m_n c^2}\right)^{3/2} e^{B_d/kT}$$

This ratio is unity for T_{nuc} 8×10^8 K $\Rightarrow$ 200s after the Big Bang.

The time delay until the start of nucleosynthesis is not negligible compared to the decay time of neutrons (900s)

At the start of nucleosynthesis $\frac{n_n}{n_p} \approx \frac{e^{-200/900}}{5.9\,(1-e^{-200/900})} \approx \frac{0.8}{5.2} \approx 0.15$

$\Rightarrow$ This lowers Y_{max} to ~ 0.27

The ratio n_D / n_n does not remain at the equilibrium value; once there is enough D we get

$$D + p \rightleftarrows {}^3He + \gamma \quad \text{or} \quad D + n \rightleftarrows {}^3H + \gamma$$

${}^3H$ decays into ${}^3He + e^- + \bar{\nu}_e$ but a decay time of 18 years ⇒ stable during BBN

${}^3H$ and ${}^3He$ are quickly converted into ${}^4He$ trough

$$\left.\begin{aligned} {}^3H + p &\rightleftarrows {}^4He + \gamma \\ {}^3He + n &\rightleftarrows {}^4He + \gamma \\ {}^3H + n &\rightleftarrows {}^4He + \gamma \\ {}^3He + D &\rightleftarrows {}^4He + p \end{aligned}\right\} \text{there are all strong force reactions (no neutrinos involved)}$$

The binding energy per nucleon is very high for ${}^4He$, whereas there are no stable nuclei with $A=5$ (${}^5He$ and ${}^5Li$ are not stable) ⇒ very difficult to make heavier elements

$$\begin{aligned} {}^4He + D &\rightleftarrows {}^6Li + \gamma \\ {}^4He + {}^3H &\rightleftarrows {}^7Li + \gamma \\ {}^4He + {}^3He &\rightleftarrows {}^7Be + \gamma \end{aligned}$$

Synthesis of nuclei with $A > 7$ is hindered by the absence of stable nuclei with $A = 8$

Initially at $T >> 10^9$K all the baryons are in the form of protons and neutron
As the deuterium density increases ${}^3H$, ${}^3He$ and ${}^4He$ are formed
At $t \sim 10$ min ($T \sim 4 \times 10^8$ K) BBN is essentially over.

The yields of D, ${}^3He$, ${}^4He$, ${}^6Li$ & ${}^7Li$ depend on η: a high value for η (baryon to photon ratio) increases the value for T_{nuc}

As BBN is a race against the clock, an earlier start means more ${}^4He$ is formed, but less D and ${}^3He$ as leftovers

The deuterium abundance can be used to estimate η

$$\Rightarrow n_{baryon,0} = \eta \, n_{\gamma,0} = 0.23 \pm 0.02 \text{ m}^{-3} \Rightarrow \Omega_{baryon,0} = 0.04 \pm 0.01$$

Also CMB gives constraints on Ω_{baryon} which are in good agreement.