At present the CMB and matter are decoupled; the universe is transparent to 2.73K radiation
Note that the Universe has been reionized, therefore the relevant photon scattering process
is Thomson scattering:

Charged particles oscillate in electromagnetic wave and radiate
The cross-section for scattering is $\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$ $\Rightarrow$ largest for electrons

$$\Rightarrow \sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = 6.65\times10^{-25}\,\mathrm{cm}^2$$

The collision time now: # collisions of a photon / unit time
= $\#e^-$ in volume of length c and area σ_T
= $c\sigma_T n_e$ $\Rightarrow$ very small.

The temperature evolution of matter and radiation is different: We already saw that $\varepsilon_{rad} = \sigma_r T^4 \propto (1+z)^4$
and $T \propto (1+z)$
This can also be seen by considering adiabatic expansion $TV^{\gamma-1}$ = constant
For relativistic gas $\gamma = 4/3$ (and $V \propto a^3$) $\Rightarrow T \propto \frac{1}{a} \propto (1+z)$

For matter $\gamma = 5/3$ $\Rightarrow T \propto \frac{1}{a^2} \propto (1+z)^2$

$\Rightarrow$ back in time the matter temperature rises faster than the radiation temperature, until t_d
when matter and radiation were coupled. Before t_d the two temperatures are equal
by thermal interactions $\Leftarrow$ again Thomson scattering

After t_d the matter density evolves as $\propto (1+z)^3$ and the radiation density $\propto (1+z)^4$
Before t_d they evolve $(1+z)^{4+\varepsilon}$ $\quad\varepsilon(z)$ due to exchange of energy with matter

~~Effect~~ Adiabatic gas + radiation in a comoving volume evolves with $dE + p\,dV = 0$
(no heat flow)

$$d\left[\left(\rho_m c^2 + \frac{3\rho_m kT}{2m_p} + \sigma_r T^4\right)a^3\right] = -\left(\frac{\rho_m kT}{m_p} + \frac{\sigma_r T^4}{3}\right) d(a^3)$$

we assume that the matter component has the equation of state
of a perfect gas $p = \frac{\rho_m kT}{m_p}$ and $\rho_m a^3$ = constant
because of mass conservation

$\Rightarrow$ if we define $\sigma_{rad} = \frac{4 m_p \sigma_r T^3}{3k\rho_m}$ one can show that:

$$\frac{dT}{T} = -\frac{1+\sigma_{rad}}{\frac{1}{2}+\sigma_{rad}}\frac{da}{a}$$

Because $\sigma_{rad}(T)$ depends on the unknown $T(a)$
we cannot integrate this analytically

If we interpret T as T_{rad} then $\sigma_{rad}(T)$ does not depend on a

$$\Rightarrow \sigma_{rad} = \sigma_{rad}(t=t_0) \quad \text{which is } \sigma_{rad}(t_0) = \frac{4 m_p \sigma_r T_{0r}^3}{3 k_B \rho_{m_0}} \simeq \frac{3.6}{\eta_0} \simeq 1.35 \times 10^8 (\Omega_{b,0} h^2)^{-1}$$

which is a very large number as $\Omega_{b_0} \sim 0.04$

This value also applies at t_d $\Rightarrow \frac{dT}{T} \approx -\frac{da}{a}$ $\Rightarrow \varepsilon \approx 0$ because of the large $\frac{1}{\eta}$

At higher temperatures the matter also becomes relativistic and total equation of state $w \sim 1/3$ and $T \propto (1+z)$ $\Rightarrow$ the temperature keeps rising

In hotter plasmas more particle reactions are possible $\Rightarrow$ composition changes

if $T \gg T_p$ such that $kT_p \simeq 2 m_p c^2$ where p is a particle of mass m
then pair creation / annihilation is possible

For instance $\gamma + \gamma' \leftrightarrows e^+ + e^-$ which has an equilibrium which lies to the right
$\Rightarrow$ many $e^+ e^-$ pairs are created; this occurs at ~ 1 MeV or $\sim 5 \times 10^9$ K

The lepton era is the period when $5 \times 10^9 < T < T_\pi$

e^+, e^-; $\mu^+ \mu^-$; $\tau^+ \tau^-$ dominate the energy density

130 MeV where pions are created

Hadron era: combinations of 2 quarks (mesons) and 3 quarks (hadrons)

When $kT > 200-300$ MeV the hadrons are separated into their component quarks

quark soup era

During these periods $P = \frac{1}{3} \rho c^2$ and $\rho c^2 = A \sigma T^4$

many different particles contribute

$[1 \text{eV} = 1.6 \times 10^{-19} \text{J} = k(1.16 \times 10^4 \text{K})]$

Entropy per baryon

The high value of σ_{rad} ensures that the temperature and density of the radiation evolve as a pure radiation universe

σ_{rad} is actually related to the entropy of the radiation per unit volume

$$S_r = \frac{\rho_{rad} c^2 + P_{rad}}{T} = \frac{4}{3} \frac{\rho_{rad} c^2}{T} = \frac{4}{3} \sigma_r T^3$$

$$\Rightarrow \sigma_{rad} = \frac{S_r}{k_B n_b} \qquad \text{where } n_b = \frac{\rho_m}{m_p} \text{ number density of baryons}$$

as $\rho_r c^2 = \sigma_r T^4$ and $\eta^{-1} = \frac{n_\gamma}{n_b}$ $\Rightarrow \sigma_{rad} \simeq 3.6\, \eta^{-1}$

σ_{rad} is also proportional to the ratio of heat capacities c of radiation and plasma

radiation $c_r = \frac{dE}{dT} = 4\sigma_r T^3$ (per unit volume)

plasma $c_m = \frac{dE}{dT} = \frac{3}{2} \rho k/m_p$ (per unit volume)

$\Rightarrow \frac{\rho_{rad} c_r}{\rho_m c_m} = 2\sigma_{rad}$ $\Rightarrow$ radiation dominates the heat budget during matter-radiation coupling $\rightarrow T \propto (1+z)$

· Baryon asymmetry: why is there mostly matter now, not anti-matter?

During the hadron era there must have been lots of proton-antiproton pairs; these annihilate as the universe cools, but a small residual of matter remained

$(n_b - n_{\bar{b}})\, a^3$ remains constant because baryon number is conserved (below $T \sim 10^{15}$ GeV) ← GUT scale
As we do not see a large γ-ray background the baryon number per comoving volume is $n_{b,0}$ $\Rightarrow n_{b,0}\, a_0^3$

Above GUT temperature $n_b \sim n_{\bar{b}} \sim n_\gamma \propto T^3$

$$\Rightarrow \frac{n_b - n_{\bar{b}}}{n_b + n_{\bar{b}}} \simeq \frac{n_b - n_{\bar{b}}}{2 n_\gamma} \simeq \frac{n_{b,0}}{2 n_{\gamma,0}} \propto \frac{1}{\sigma_{rad}}$$

The asymmetry is very small: for every 10^9 anti baryons there are 10^9+1 baryons
Or σ_{rad} is large because the asymmetry is so small

Matter or radiation dominated universes decelerate $\rightarrow$ we expect a finite age of the Universe

$\Rightarrow$ at $t=0$ the density diverges and the proper distance between two points $\rightarrow 0$
This singularity is called the "Big Bang"

It is the consequence of
- the cosmological principle
- the Einstein equations in the absence of a cosmological constant
- the expansion of the Universe $(\dot{a}/a)_0 = H_0 > 0$
- the assumed form of the equation of state $(0 \leq w \leq 1)$

The singularity could be avoided if the equation of state of matter in the very early universe would be different from a perfect fluid with $P/\rho > -1/3$

Fluids with $w < -1/3$ violate the strong energy condition $\rho + 3P \geq 0$

Current observations show that $|\Lambda| < \left(\frac{H_0}{c}\right)^2 \sim 10^{-55}\ \mathrm{cm}^{-2}$ too small to be relevant in the early universe

If the dynamics of the early universe are dominated by a homogeneous and isotropic scalar ~~field~~ quantum field then it may have been important early on.
Such a field has a Lagrangian $L_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\varphi)$

kinetic term effective potential

One can define effective density and pressure (it is not a fluid)

$$\rho_\Phi c^2 = \frac{1}{2}\dot{\Phi}^2 + V(\varphi) \quad \text{and} \quad P_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

If the kinetic term is negligible compared to potential term $P_\Phi = -\rho_\Phi c^2$

$\Rightarrow$ It behaves like a fluid with $w=-1$ (thus violating the strong energy condition) or as an effective cosmological constant $\Lambda = \frac{8\pi G}{c^2}\rho_\Phi$

This could happen in a false vacuum at $T > 10^{12}$ K when quantum effects become important

Whether or not the singularity can be avoided is an open question: we do not understand the origin of the Universe.

There is a fundamental limit in our understanding of physics when quantum mechanical effects and strong gravity occur on the same scales: we do not have a theory of quantum gravity.

When does this occur?

We can define the Compton time for a body of mass m (or energy mc^2) to be

$$t_c = \frac{\hbar}{mc^2}$$ this represents the time permissible to violate energy conservation by an amount $\Delta E \sim mc^2$

The corresponding Compton length is $l_c = c t_c = \frac{\hbar}{mc}$
Note that t_c & l_c increase as $m \downarrow$: these scales indicate when quantum mechanics is important

The Schwarzschild radius is $l_S = \frac{2Gm}{c^2}$ and time $t_S = \frac{l_S}{c} = \frac{2Gm}{c^3}$

We need quantum gravity when $l_S = l_c$ or $m = \sqrt{\frac{\hbar c}{2G}} \approx \sqrt{\frac{\hbar c}{G}} \equiv m_p$

where m_p is the Planck mass.

Our definition of the unit of time is arbitrary, but it is possible to derive a time that is "natural" on which everybody in the Universe agrees: there is a unique combination of fundamental constants that yields a time

$$t_P = \left(\frac{\hbar G}{c^5}\right) \sim 10^{-43} s$$ the Planck time

Similarly we can define
$$l_P \equiv c t_P = \left(\frac{G\hbar}{c^3}\right)^{1/2} \sim 1.7 \times 10^{-35}\, m$$ Planck length

$$m_P = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.5 \times 10^{-8}\, kg$$ Planck mass

$$E_P = m_P c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2} = 1.2 \times 10^{19}\ GeV$$ Planck energy

$$T_P = \frac{E_P}{k_B} = \left(\frac{\hbar c^5}{k_B^2 G}\right)^{1/2} \simeq 1.4 \times 10^{32}$$ Planck temperature

The first $t_P \sim 10^{-43}$ seconds cannot be described by GR or quantum mechanics
The horizon $c t_P \sim$ Planck length and particle pairs are created that have Planck mass seperated by less than Planck length $\rightarrow$ particles/black holes at once, with quantum effects on the scale of the horizon $\rightarrow$ we cannot describe this with known physics.

Dark energy problem

A non-zero lamba introduces a natural length and time scale

$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ if $\Lambda > 0$ then eventually $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \Rightarrow a = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$

The horizon is $d_{Hor} = c\int_0^{t_0} \frac{dt}{a} \approx c\int_0^{t_\Lambda} \frac{dt}{a} + c\int_{t_\Lambda}^{t_0} dt\, e^{-\sqrt{\Lambda/3}\,t}$

t_Λ time when Λ dominates

$\approx d_{Hor}(t_\Lambda) + c a_0 \sqrt{\frac{3}{\Lambda}}$ if Λ is large then $d_{Hor}(t_\Lambda)$ is small as is the second term $\Rightarrow$ the horizon is small

The scale $r_\Lambda = \sqrt{\frac{3}{\Lambda}}$ is observed to be large, much larger than l_P; the Universe is also old in terms of t_P $\Rightarrow$ Λ must be small

If $\Lambda < 0$ then it acts as an attractive force; if we take a flat Universe then $\Omega_\Lambda = 1 - \Omega_m$

$\Rightarrow \frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + (1-\Omega_m)$ it will expand until $a_{max} = \left(\frac{\Omega_m}{\Omega_m - 1}\right)^{1/3}$

and will collapse to $a=0$ at a cosmic time $t_{crunch} = \frac{2\pi}{3H_0}\frac{1}{\sqrt{\Omega_m - 1}}$

For comparison a positively curved $\Omega_m = 1.1$ Universe undergoes big crunch at $t \approx \frac{110}{H_0}$ but $\Omega_m = 1.1$ & $\Omega_\Lambda = -0.1$ has lifetime $\approx \frac{7}{H_0}$

$\Rightarrow$ This has not happened $\Rightarrow |\Lambda|$ has to be small

$\Rightarrow -3t^{-2} \lesssim \Lambda \lesssim 3r^{-2}$ in Plank units $r > 10^{60}$ and $t > 10^{60}$

$\Rightarrow |\Lambda| < 3\times 10^{-120}$ in Planck units

However vacuum energy has the form of a cosmological constant so we can define

$$\rho_\Lambda = \rho_{vacuum} + \frac{\Lambda_{Einstein}}{8\pi} < 10^{-120}$$

Like a harmonic oscillator in the ground state, every mode of free energy of every free field contributes a zero-point to the energy density of the vacuum.

Energy is $\sim \frac{m^4}{(\hbar/c)^3}$ where the cut-off mass depends on the physics and is $\sim 10^{60}\,(GeV)^4$ for GUT; alternatively one could choose $m = m_{Planck}$

We would therefore expect Λ to be much larger then is observed: This is the cosmological constant problem.