1 Superluminal Motion

Can an object move faster then the speed of light? According to Einstein's Special theory of Relativity this is impossible. However, occasionally, if one looks at the night sky he can see objects which are moving faster then the speed of light? Was Einstein wrong? Of course no! In fact, the possibility to observe a "superluminar motion" was predicted by M. Rees in 1966 (Nature 211, 468) and it has a very simple geometric explanation.

The geometrical set up we consider is shown in Figure 1, where a point source is moving with velocity v at an angle θ with respect to the line of sight. If a $t = 0$ a first photon is emitted at A, it takes a time

$$t_{\mathrm{A}} = \frac{AO}{c} \tag{1}$$

to reach the observer. After a time delay of $\frac{AC}{v}$ the source has reached C and it emits a second photon that reaches the observer after a time

$$t_C = \frac{AC}{v} + \frac{BO}{c}. \tag{2}$$

The *observed* time lag is thus

$$\Delta t_{\mathrm{ob}} = t_C - t_A = \frac{AC}{v} + \frac{BO}{c} - \frac{AO}{c} = \frac{AC}{v} - \frac{AB}{c} = \frac{AC}{v}\left(1 - \frac{v}{c}\cos\theta\right) \tag{3}$$

The distance seen by the observer is:

$$\Delta x = AC \sin\theta \tag{4}$$

Using eq.3, the apparent velocity is:

$$v_{\mathrm{ob}} = \frac{\Delta x}{\Delta t_{\mathrm{ob}}} = \frac{AC\sin\theta}{\frac{AC}{v} - \frac{AC\cos\theta}{c}} = \frac{v\sin\theta}{1 - \frac{v}{c}\cos\theta} \tag{5}$$

The angle corresponding to the maximum apparent velocity can be found by differentiating eq.5 and equating to zero:

$$\cos\theta_{\mathrm{max}} = \frac{2v}{2c + v} \tag{6}$$

For a relativistic source $v \sim c$,

$$\theta_{\mathrm{max}} \simeq 48, 2^\circ$$

and the maximum observed velocity is

$$\left.\frac{v_{\mathrm{ob}}}{c}\right|_{\mathrm{max}} \simeq 2.24.$$

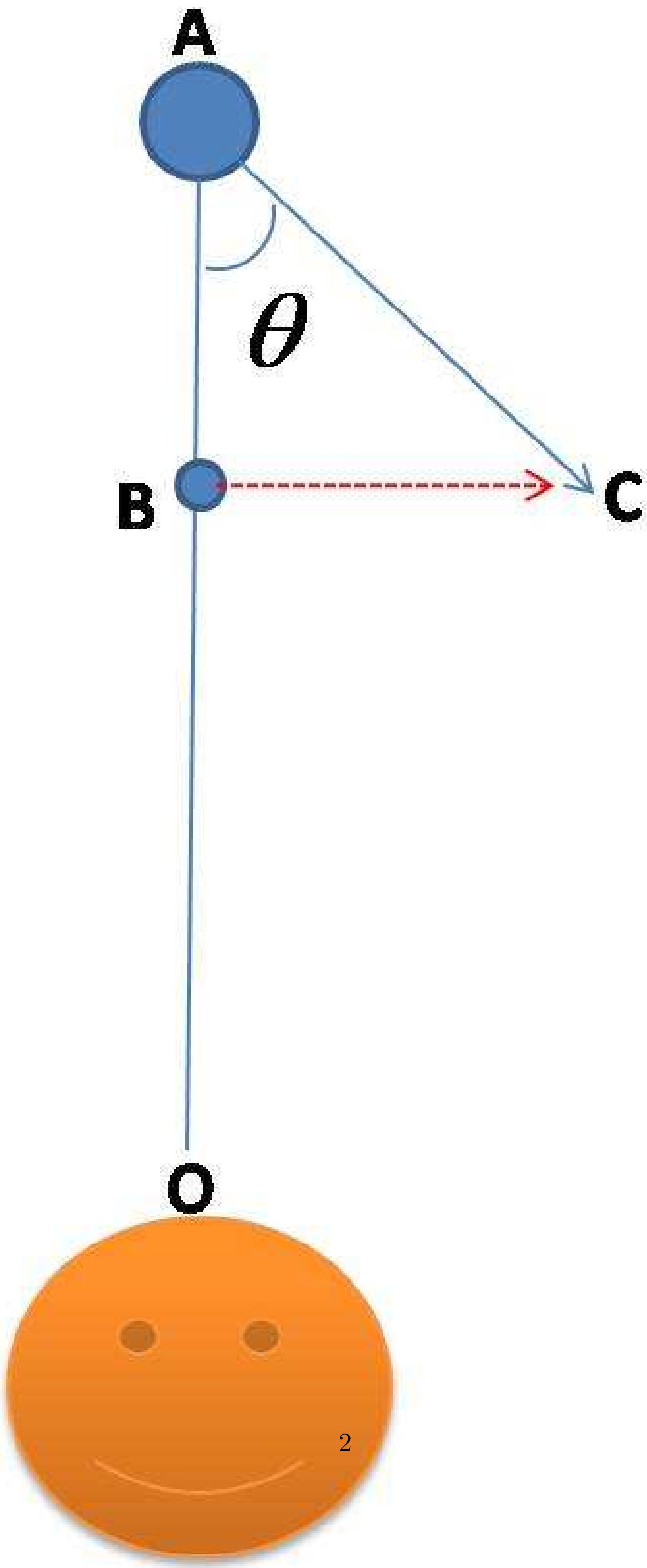

Figure 1: The motion as seen on the plane of the sky by an observer.