Stellar density distribution in a Keplerian Potential

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1 Introduction

We are interested in the stellar density distribution in galactic center, within the sphere of influence of a supermassive black hole. The SMBH dominates the potential within a radius which encompasses a stellar mass comparable to its own. For our Galactic center, where the SMBH has $M \approx 4 \times 10^6 M_\odot$, the sphere of influence is ≈ 3 pc. The stellar distribution depends on the ratio of the relaxation timescale $t_{\rm rx}$ to the age of the system.

1.1 Relaxation timescale

We derive the relaxation time as the inverse of the "large angle" collision rate. For simplicity, we will assume in the following that all the stars have the same mass m_*. We thus neglect long distance interactions, which will give just a logarithmic correction to our result. The collision rate is

$$t_{\rm rx}^{-1} \simeq n(r)v(r)\Sigma(r), \tag{1}$$

where $n(r) \propto r^{-s}$ is the stellar number density, $v(r)^2 = GM/r$ is the Keplerian velocity and

$$\Sigma(r) \simeq \pi \left(\frac{Gm_*}{v^2}\right)^2, \tag{2}$$

is the cross section, given by the radius Gm_*/v^2 at which the stellar potential overcomes the velocity dispersion in the star field. This gives,

$$t_{\rm rx}^{-1} \propto r^{3/2-s}. \tag{3}$$

1.2 The Wolf-Bahcall cusp

In this section we consider a system for which the relaxation time is smaller than the its age. Thus the system can maintain a steady state.

Within the tidal radius $r_{\rm t} = R_* \left(M/m_*\right)^{1/3} \approx 1$ AU, stars are continuously tidally disrupted. This removal of negative energy must be transported outward in order to maintain a steady state.

The luminosity (energy passing through a shell per unit time)

$$L(r) \simeq N(<r)E(r)t_{\rm rx}^{-1} = \text{constant}, \tag{4}$$

where $N(r) \propto n(r)r^3 = r^{3-s}$ is the total number of stars within r and $E \propto 1/r$ is the binding energy per star. Equating the exponent of the left and right hand side of eq. 4, we derive the density profile that guarantee the steady state solution

$$n(r) \propto r^{-7/4}. \tag{5}$$

This is called the Wolf-Bahcall profile (Bahcall, J. N. & Wolf, R. A. 1976, ApJ, 209, 214).

1.3 Distribution when the relaxation timescale is longer than Hubble

to be completed...