# 1 Dynamical Friction

A massive object travelling through a gas cloud will feel besides the regular aerodynamic friction, an additional friction called "Dynamical Friction". This friction arises from the fact that the massive object changes slightly the distribution of matter around it due to its gravitational field. In proto-planetary discs, dynamical friction is thought to cause the inward migration of the proto-planets. During the merger of two galaxies, it helps supermassive black holes to sink towards the centre of the newly formed galaxy.

An easy way to imagine where this friction arises from, it is to imagine an object of mass $M$ moving with a velocity $V_{\rm M}$ relative to the gas (see Fig. 1). The object will gravitationally attract the gas towards it, and as a result a small over-density of gas will form where the object was. The object will then feel a gravitational pull behind him, which will cause it to decelerate. We are now going to calculate this deceleration and its dependencies on the physical properties of the body and the external medium.

Let's assume for simplicity, that the background medium is at rest and of constant density $\rho$. Let's consider a small displacement $\Delta r = V_{\rm M}\Delta t$ of the body in a time interval $\Delta t$. The mass $\Delta M$ accumulated behind the moving body in a time $\Delta t$ is the mass within the region where its free-fall time $\sqrt{r^3/GM}$ is shorter than $\Delta t$ ($r$ is measured from the *initial*, position of the body). The radius $r_{\rm ff}$ of this spherical region is

$$r_{\rm ff}^3 = \Delta t^2 GM, \tag{1}$$

and therefore

$$\Delta M \simeq \frac{4\pi}{3} r_{\rm ff}^3 \rho = \frac{4\pi}{3} GM\rho(\Delta t)^2. \tag{2}$$

Incidentally, we notice that the mass inflow can be described through an "accretion rate" $\dot{m} \simeq \frac{4\pi}{3} GM\Delta t$ independent of radius. It can be, for example, evaluated as a flux of matter through a surface of radius $\Delta r$, where the instantaneous radial velocity acquired by the falling mass around $\Delta r$ is $v = (GM/\Delta r^2)\Delta t$ (see Fig.2). This would be an equivalent way of calculating $\Delta M$,

$$\Delta M \simeq 4\pi \Delta r^2 \rho v \Delta t = 4\pi GM\rho(\Delta t)^2. \tag{3}$$

Finally, the gravitational force that the object feels because of the over-density is:

$$F_{\rm DF} = -\frac{GM\Delta M}{\Delta r^2} \propto -\frac{G^2M^2\rho}{V_{\rm M}^2}. \tag{4}$$

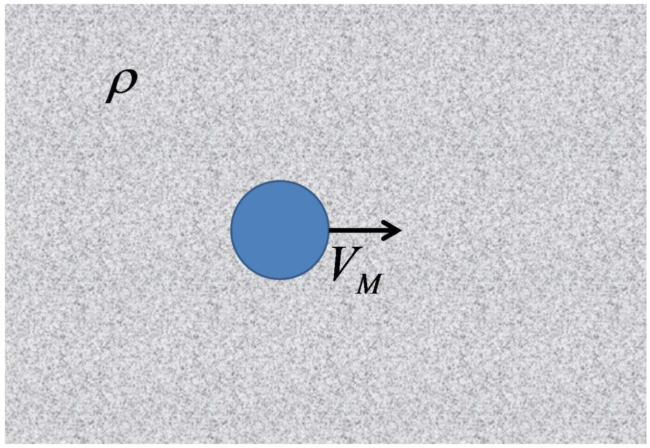


Figure 1: The object moving through the homogenous cloud

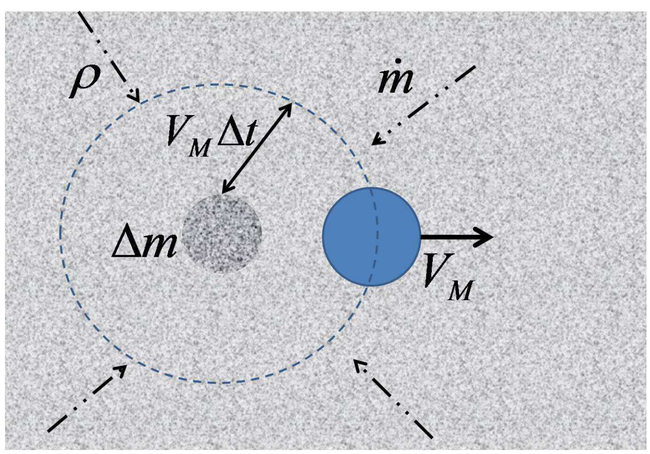


Figure 2: The over-density that develops behind the object