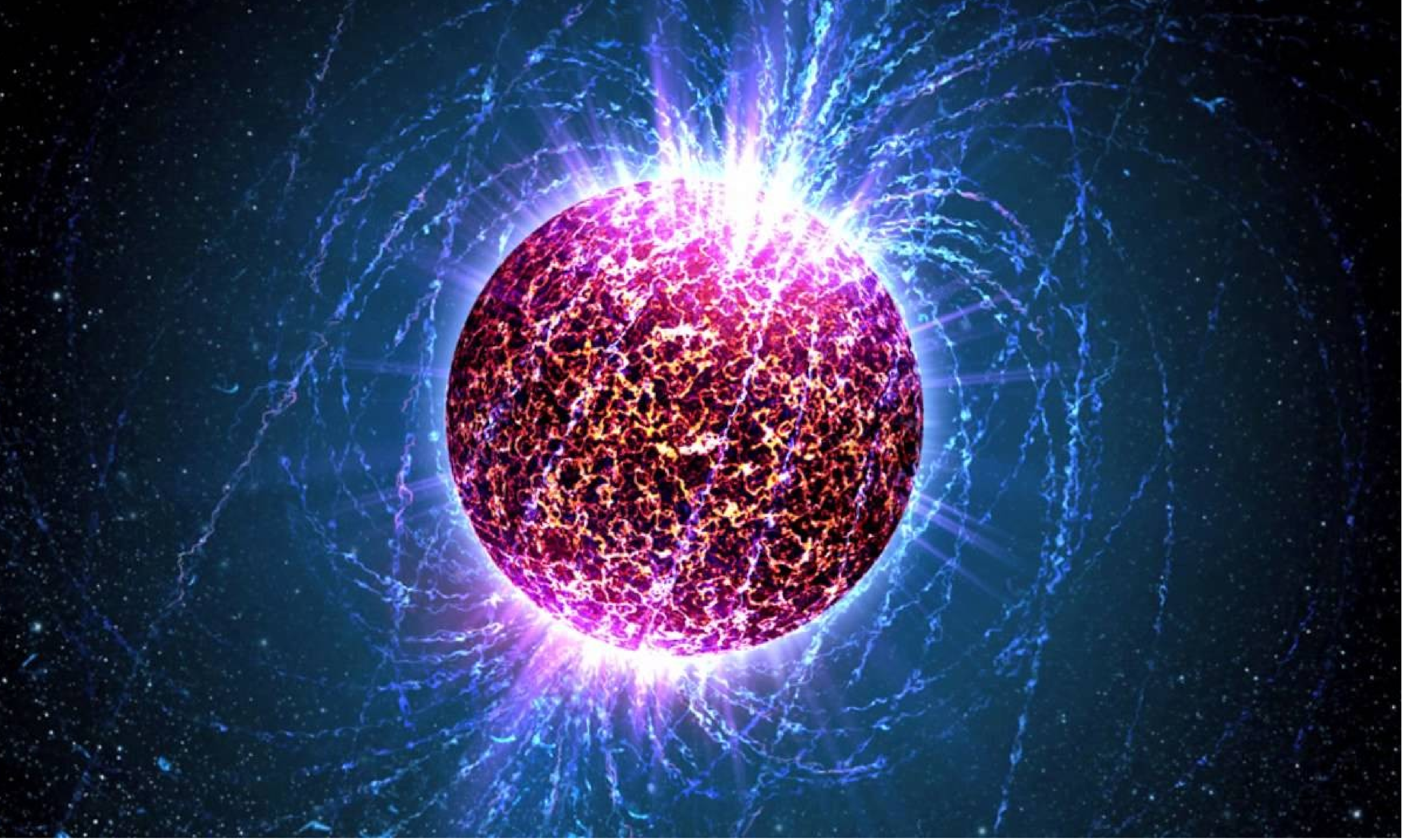
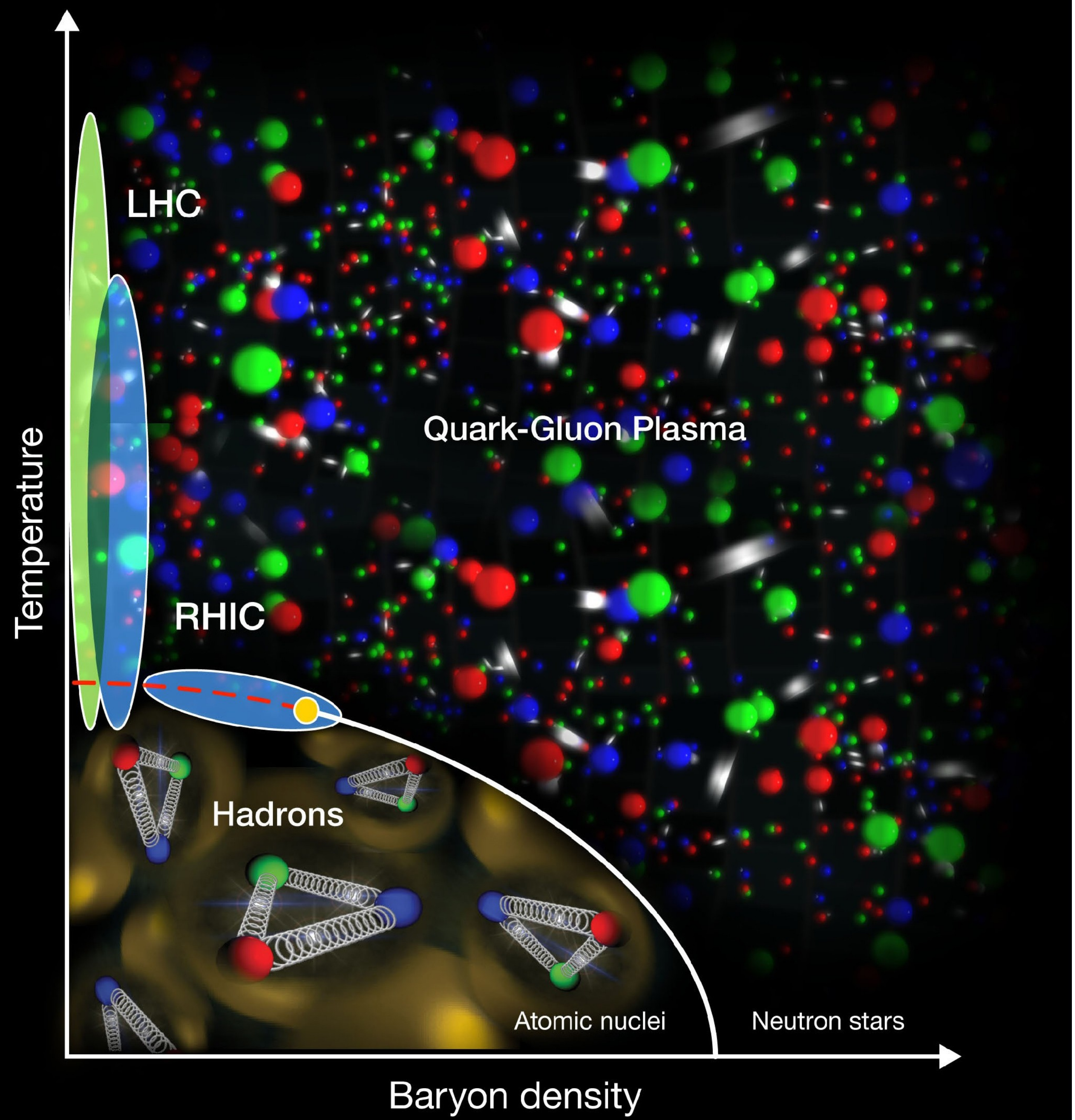


Neutron Stars



The strong force is still poorly understood at high densities and low temperatures.

QCD phase diagram shows what we know so far and what might happen to matter in different regimes of density and temperature.

Neutron stars are at the high density/low temperature end of the QCD phase diagram, in a region poorly understood.

Understanding the composition of neutron stars can thus give important information on the behavior of the strong interaction

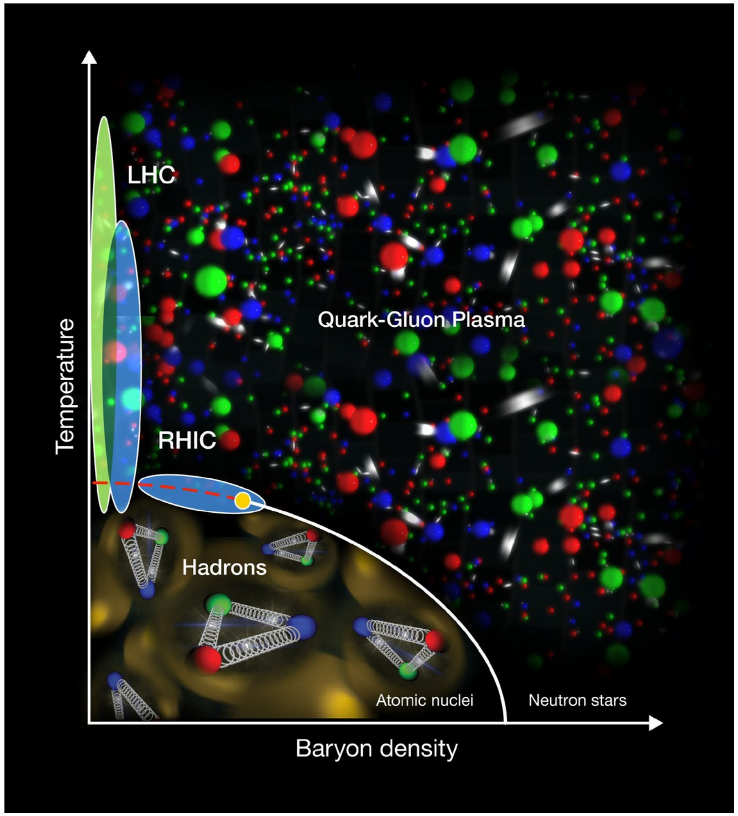
Quantum Chromo-Dynamics: Current Uncertainties

Important: QCD calculations are difficult only in a certain regimes

$\rho \lesssim \rho_0$ **NS crust; nuclear theory methods valid, but some uncertainties**

$\rho_0 \lesssim \rho \lesssim 10\rho_0$ **NS cores: uncertainties growing with density.** Deconfinement of quarks? Maybe, but in the non-perturbative QCD regime. Fortunately NS exist and are available for measurements!

$10\rho_0 < \rho \lesssim 100\rho_0$ no stars in this density range. Strong coupling QCD plasma, reliable calculations not possible



Above 100x nuclear density perturbative QCD can be applied and calculations become relatively easy to perform.

Hydrostatic Equilibrium

with Hydrostatic Equilibrium + EoS you can build a model for your star (and calculate Mass and Radius)

$$m(r) = \int_0^{r'} \rho\, 4\pi r^2 dr \rightarrow \frac{dm(r)}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = \frac{-G\, m(r)\, \rho}{r^2}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G \rho$$

+

Equation of State

Sructure of a star (e.g., M-R relation)

Hydrostatic Equilibrium in General Relativity

Neutron stars are very compact (M/R extremely large) so that strong gravity needs to be considered.

Tolman-Oppenheimer-Volkoff Equation:

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2r}\right]^{-1}$$

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Special Relativity corrections

General Relativity corrections

Note: Both SR and GR “strengthen” the gravitational field with respect to the Newtonian limit.

General Relativity Strengthens Gravity

GR destabilizes stars: collapse is easier

$$E = E_{Newt.} + \Delta E_{GR} < E_{Newt.}$$

$$\downarrow$$

Maximum mass of neutron star smaller than in Newtonian case

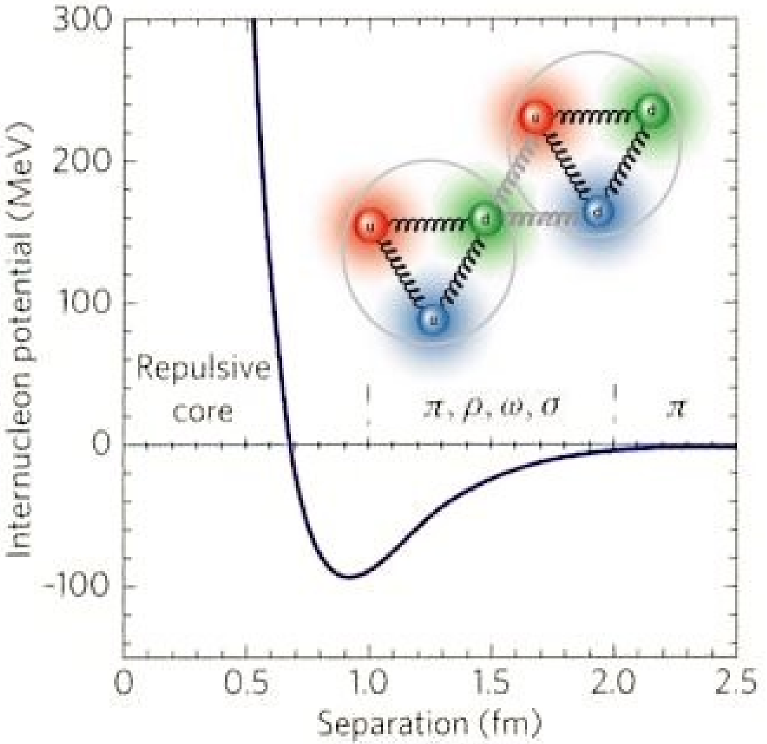
Oppenheimer and Volkoff assumed a polytropic equation of pure neutron gas

They got: $M_{max} = 0.7\, M_{sun} \quad R = 9.6\, km \quad \rho_c = 5 \times 10^{15}\, g\, cm^{-3}$

Maximum mass too small ! Observed values are around 1.2-2.0 Solar masses !

Pure ideal neutron gas too simple to be realistic !

Nucleon-Nucleon Interaction



Radial part of nucleon-nucleon interaction is relatively well known.

$$V_{\mathrm{Yukawa}}(r) = -g^2 \frac{e^{-kmr}}{r},$$

$$\alpha_s = \frac{g^2}{(\hbar c)}$$

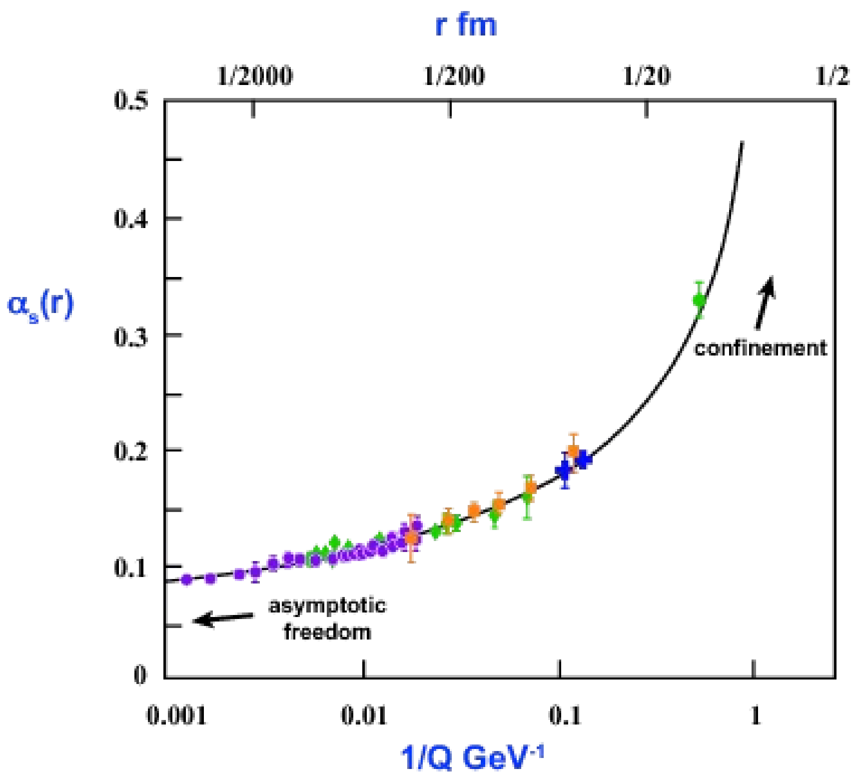
However other components (spin, isospin) and the many-body problem remain difficult to solve.

1. Calculations are prohibitive in this regime and one needs to rely on simplifications of the nuclear potential.
2. Poor constraints from observations. E.g. symmetry energy is not well constrained.

The symmetry energy of nuclear matter can be defined as the difference between the energies of pure neutron and symmetric nuclear matter as a function of density.

Neutron Stars enter in this context to help us understand in which direction to move.

Quarks and Asymptotic Freedom



Quarks are **confined** inside nucleons.
This means that no free quarks can ever be observed.

However, when the length-scale of the interaction becomes very small, the coupling constant of the strong interaction vanishes.

This means that quarks are “free”, i.e., the strong force goes to zero (**asymptotic freedom**).

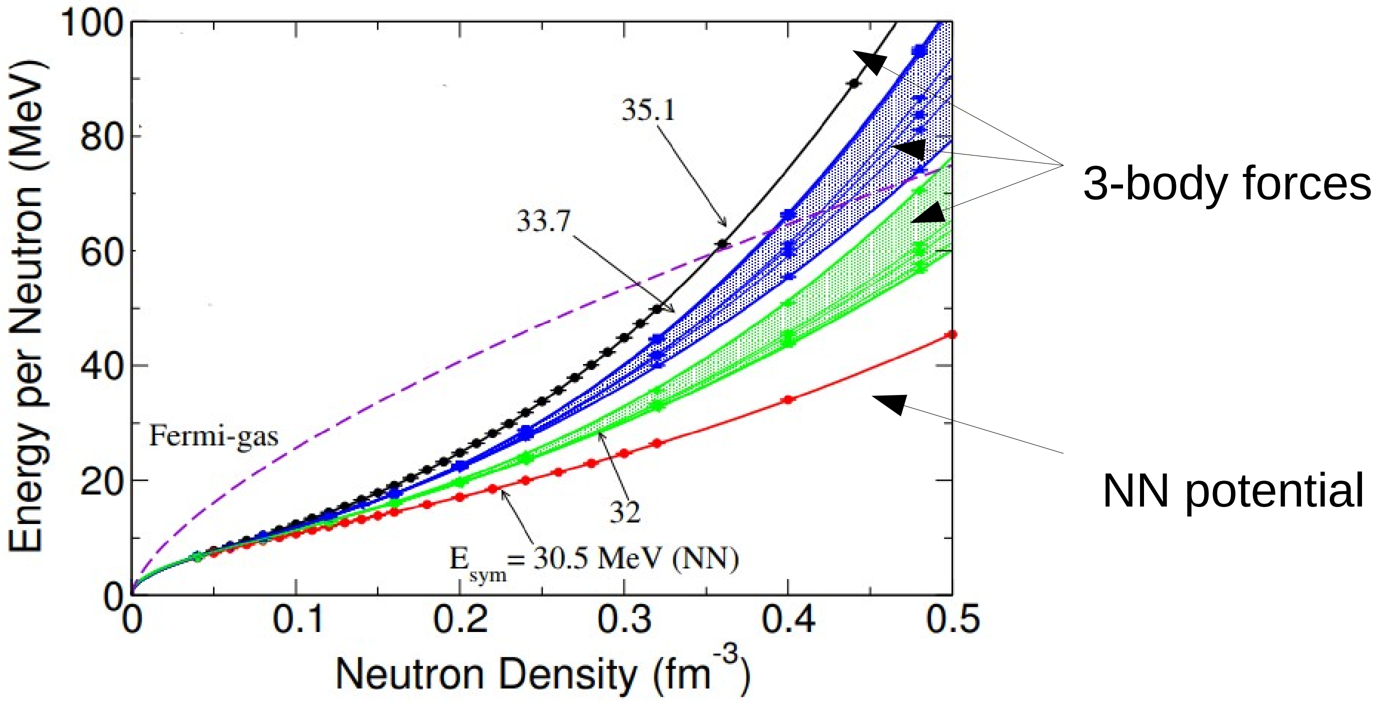
It is currently unclear whether neutron stars might contain “free quarks” in their cores.
If this is the case then these might be the only objects in the universe where one can “observe” free quarks. This of course adds more uncertainty to the EoS since we do not know (yet?) at which density the transition to free quarks might occur.

Summary

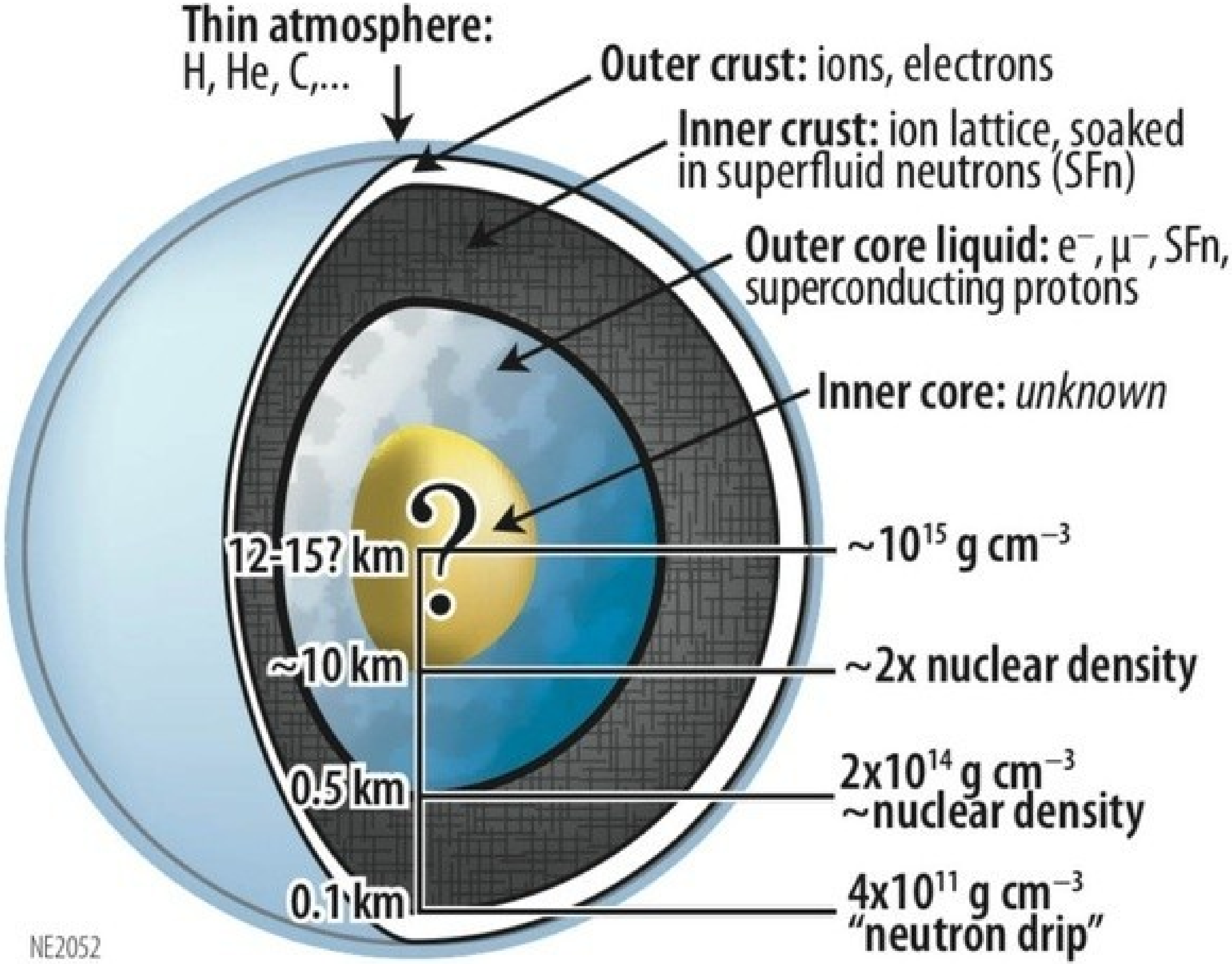
1. We don't know how the strong force behaves at high densities and low temperatures because QCD calculations are prohibitive

2. We don't know exactly the form of the nucleon-nucleon potential and therefore we need to rely on approximations (e.g., effective field theories)

3. We don't know which particles compose the core of neutron stars with the further complication that at sufficiently high densities we might have free quarks.

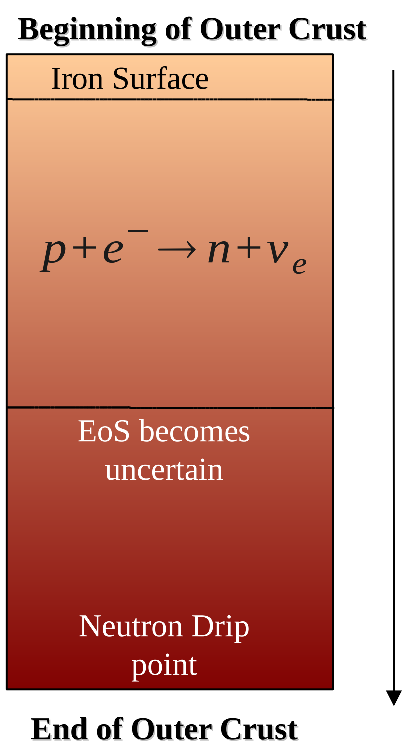


Neutron Star Structure



The Outer Crust

In neutron stars, neutron heavy nuclei are found as relativistic electrons penetrate the nuclei and produce inverse beta decay, wherein the electron combines with a proton in the nucleus to make a neutron and an electron-neutrino.



ρ_{max} [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^6	^{56}Fe	26	30	1404.05
2.71×10^8	^{62}Ni	28	34	449.48
1.33×10^9	^{64}Ni	28	36	266.97
1.50×10^9	^{66}Ni	28	38	259.26
3.09×10^9	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92

Experimental values

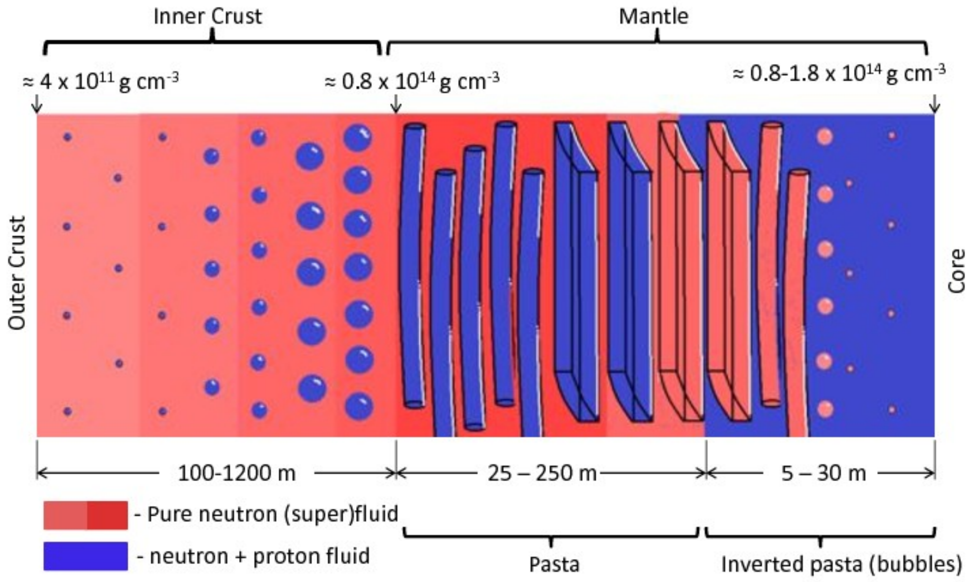
Theoretical values

The Inner Crust: the Neutron Drip Point

As more and more neutrons are created in nuclei the energy levels for neutrons get filled up to an energy level equal to the rest mass of a neutron. At this point any electron penetrating a nucleus will create a neutron, which will "drip" out of the nucleus.

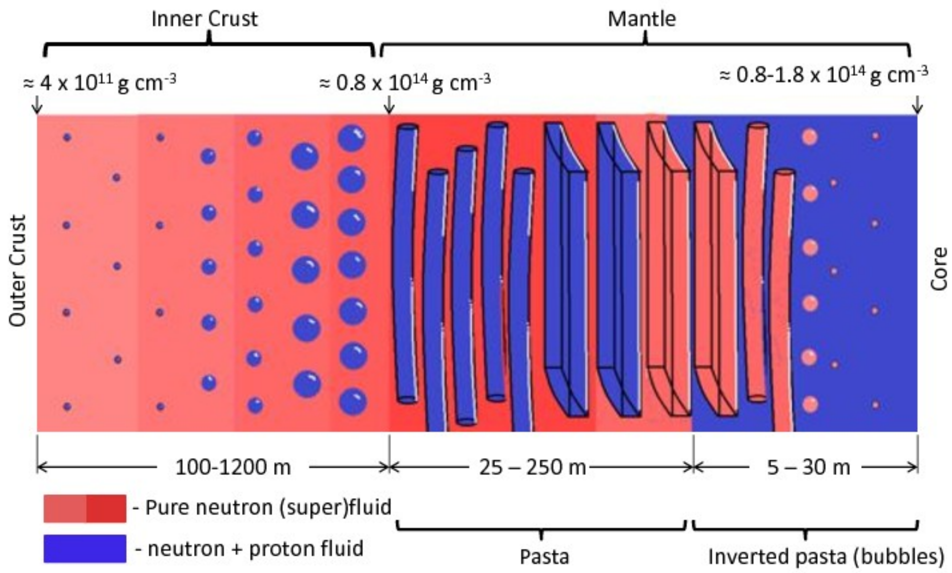
$$E_n = m_n c^2$$

At this point it is energetically favorable for the neutron to “drip out” of nuclei.



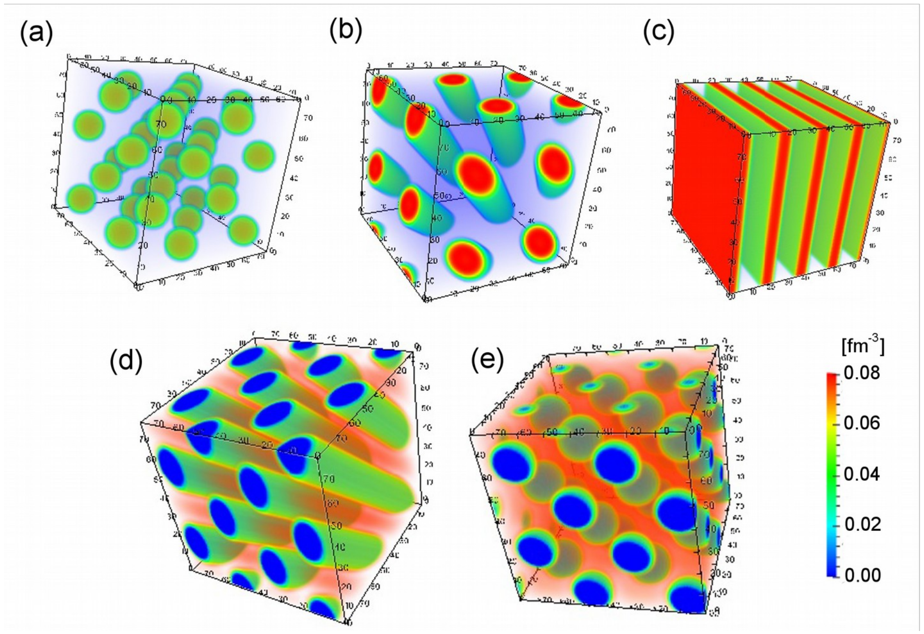
Mantle: Nuclear Pasta

After the inner crust the nucleons arrange in strange structures called "nuclear pasta" and "inverted nuclear pasta".

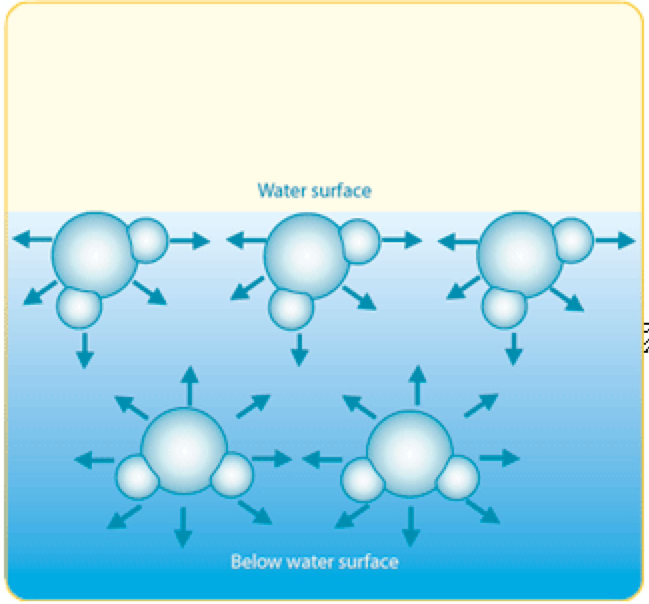


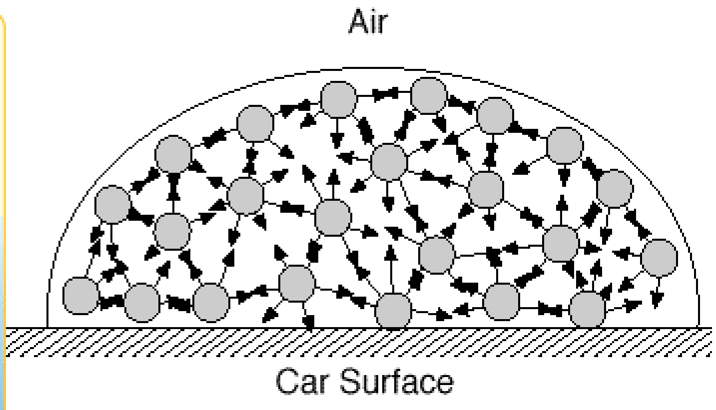
This nuclear phase is not observed anywhere else in the Universe beside neutron stars.

Nuclear pasta is an arrangement of nucleons in structures that resemble clusters (gnocchi), rods (spaghetti) and slabs (lasagna) of nuclear material

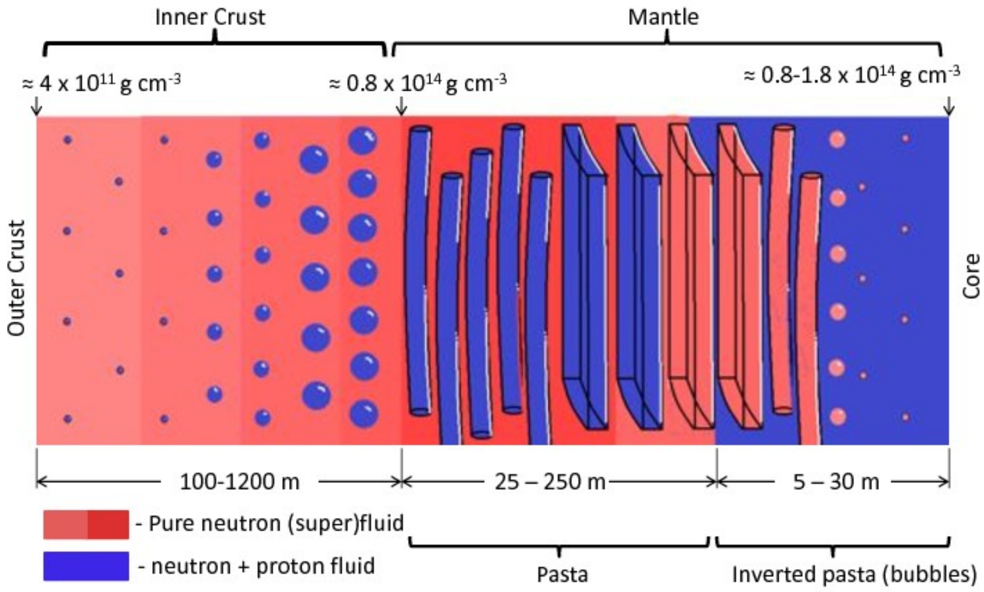


The Outer Core



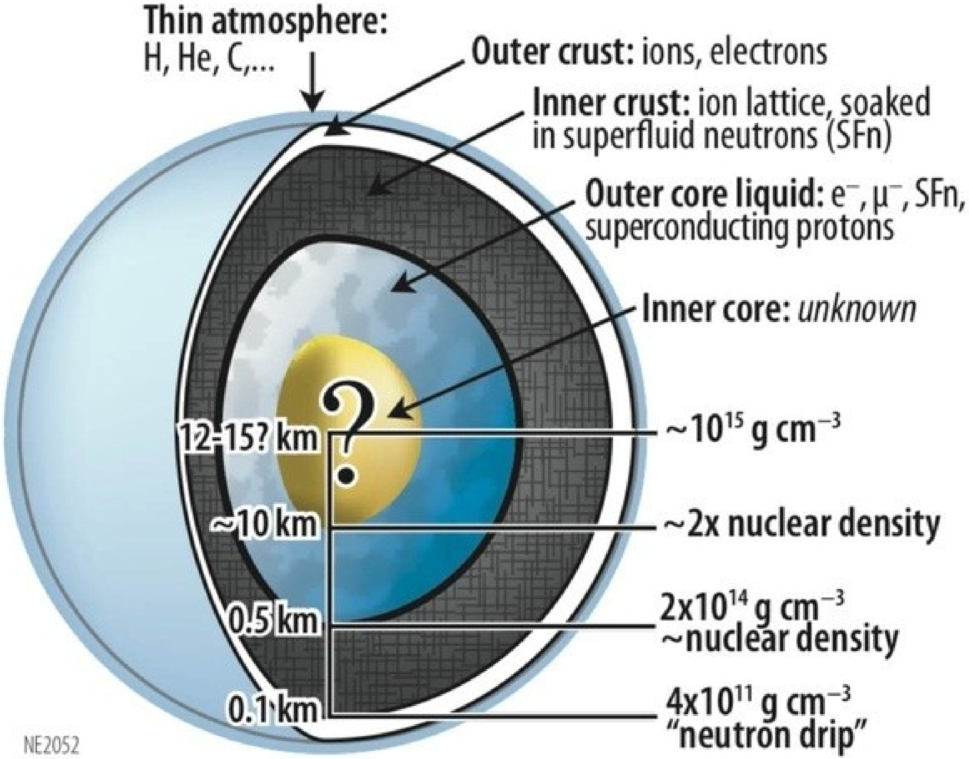


Molecules inside a water drop are attracted in all directions. Drops on the surface are attracted to the sides and inward.



Nuclei have a surface tension.
When you inject free neutrons in the environments then the surface tension decreases until it becomes equal to zero. At this point nuclei disappear and a soup of free protons, neutrons and electrons (and muons) appears.

The Outer Core



The outer core constitutes ~50-90% of the neutron star mass.

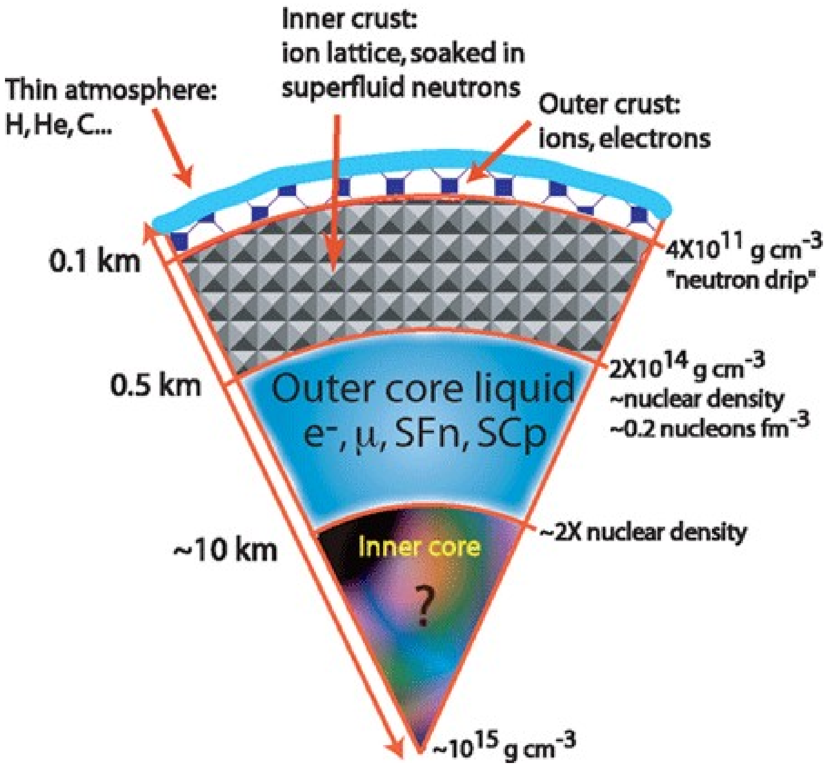
It is a soup of n,p,e and muons.

$$\mu^- \leftrightarrow e^-$$

Neutrons and protons are in a superfluid state. This means that protons are superconductors arranged in flux tubes whereas the neutrons are arranged in vortices that carry (quantized) angular momentum.

The edge of the outer core is uncertain. At about 2x nuclear density a transition to the inner core might occur.

The Inner Core



Inner core appears if the density exceeds ~4×10^14 g/cm3.

The extension of this region might reach up to 5 km.

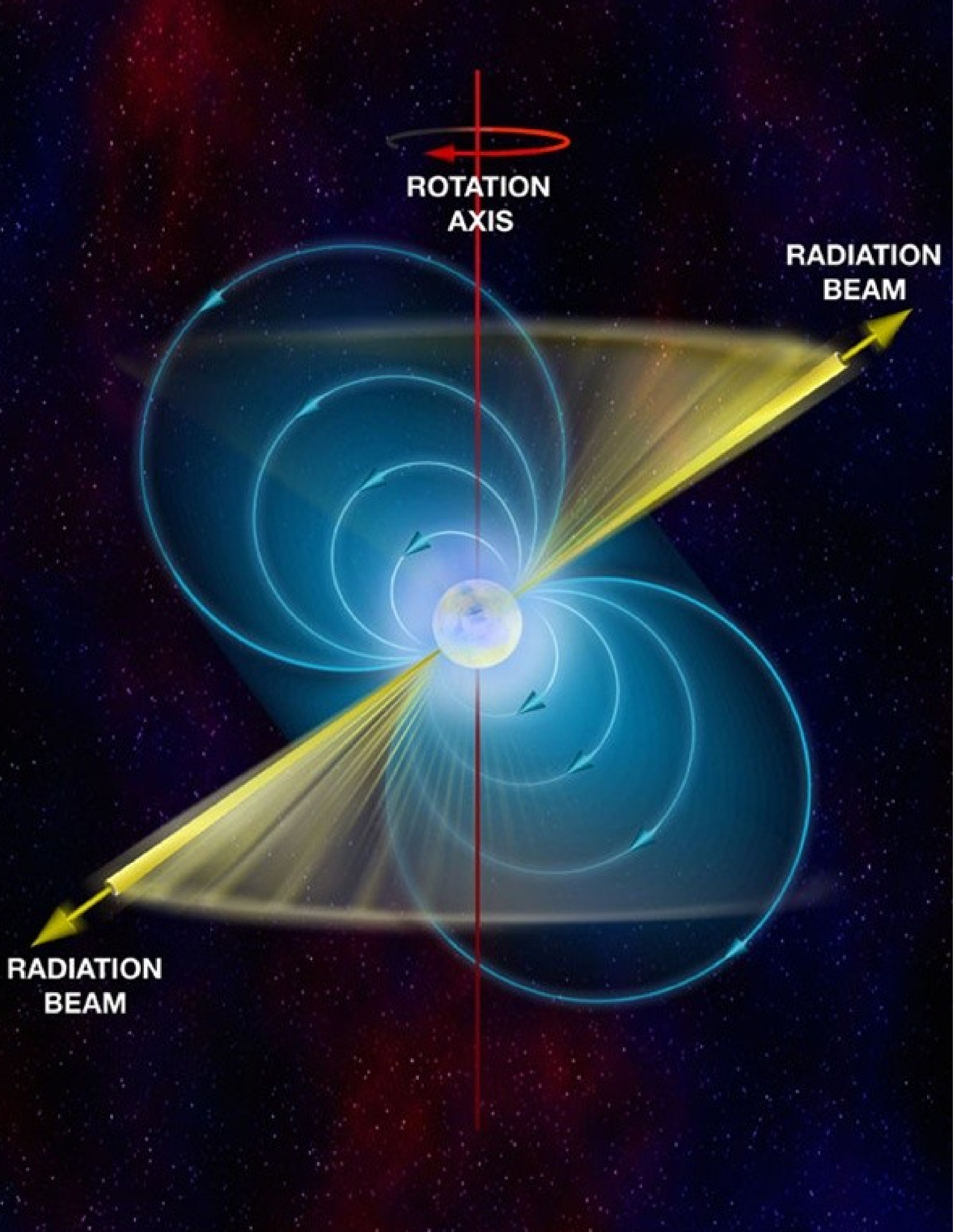
Exotic matter might appear in the inner core:

- meson condensates (π, κ)

- hyperons (strange baryons)

- quark matter (hybrid star)

Problem: solve many body problem for the particles in non-perturbative QCD.
This is not possible at the moment and so effective nucleon-nucleon interaction is used.
However, the choice is very large and so the solution to the problem are many. Which one is correct (or at least closest to reality)?



Radio Pulsars

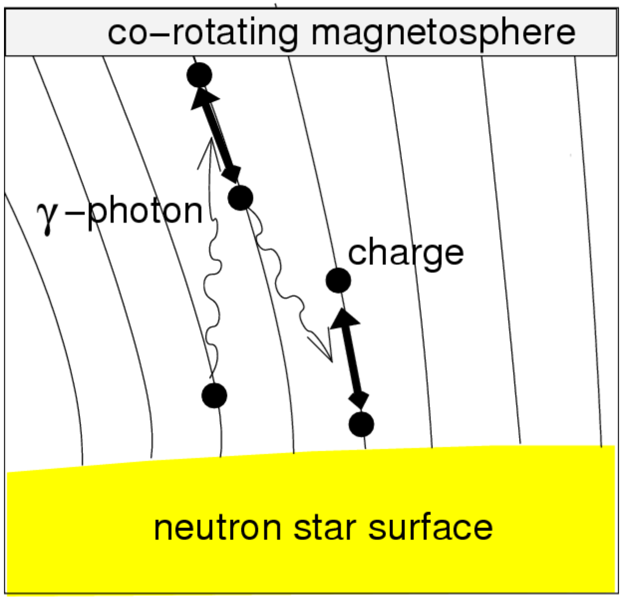
Radio pulsars are the most common type of neutron stars (because they are relatively easy to observe, not because they are the most abundant type of neutron stars).

A strongly magnetized neutron star (B~10^8 – 10^15 G) can emit a beacon of radio-waves.

The rotation of the neutron star modulates the radio emission and you observe pulses of radio waves.

Pulsar Emission Mechanism

Faraday's law: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

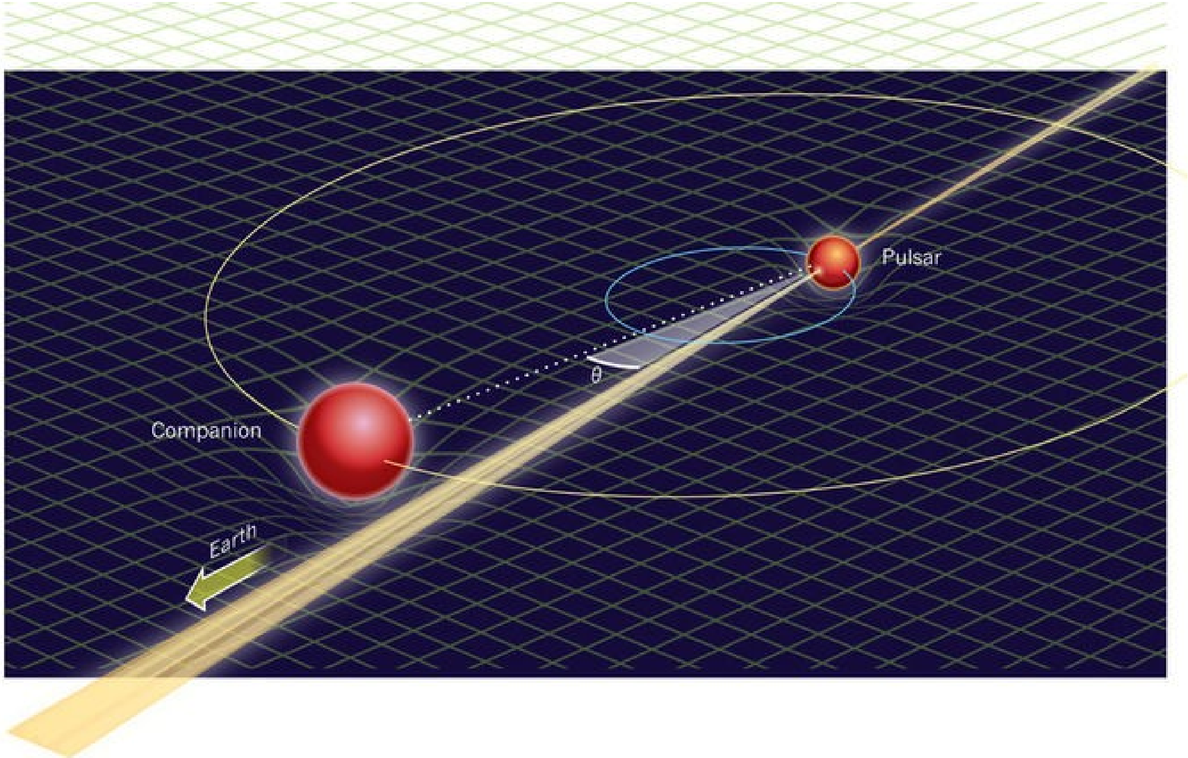


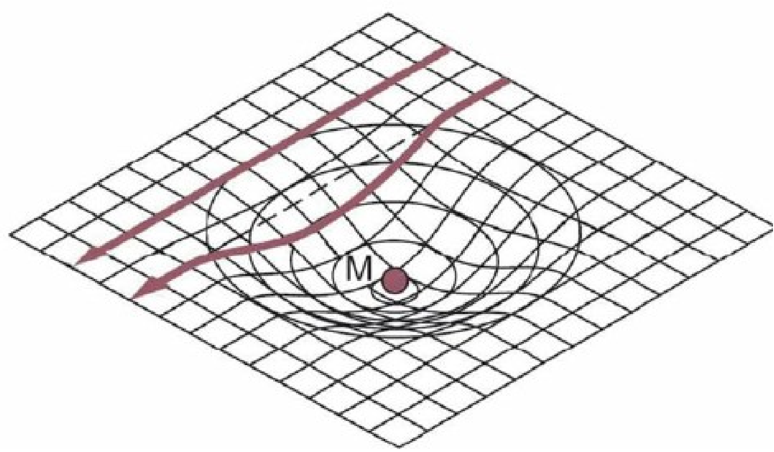
The enormous **B** field of the pulsar rotates in vacuum → generation of **E** field (up to 10^17 V). This tremendous voltage extract charges from the NS surface (electrons and protons).

Electrons are readily accelerated.
Electrons in the polar cap are magnetically accelerated to very high energies along the open but curved field lines, where the acceleration resulting from the curvature causes them to emit **curvature radiation**.

High-energy photons produced by curvature radiation interact with the magnetic field and lower-energy photons to produce electron-positron pairs that radiate more high-energy photons. The final results of this cascade process are bunches of charged particles that emit at radio wavelengths.

Observables: Neutron Star Mass



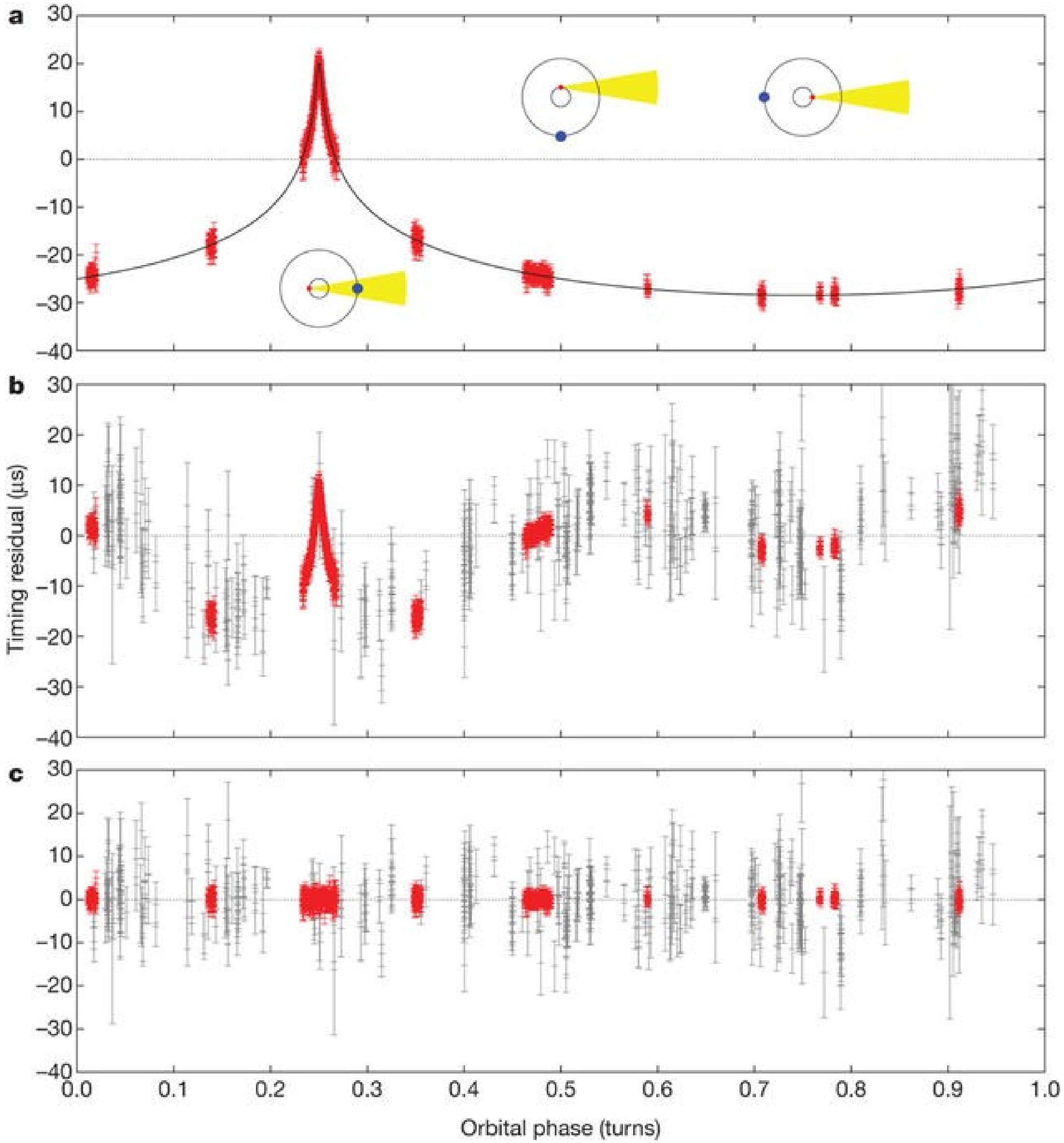


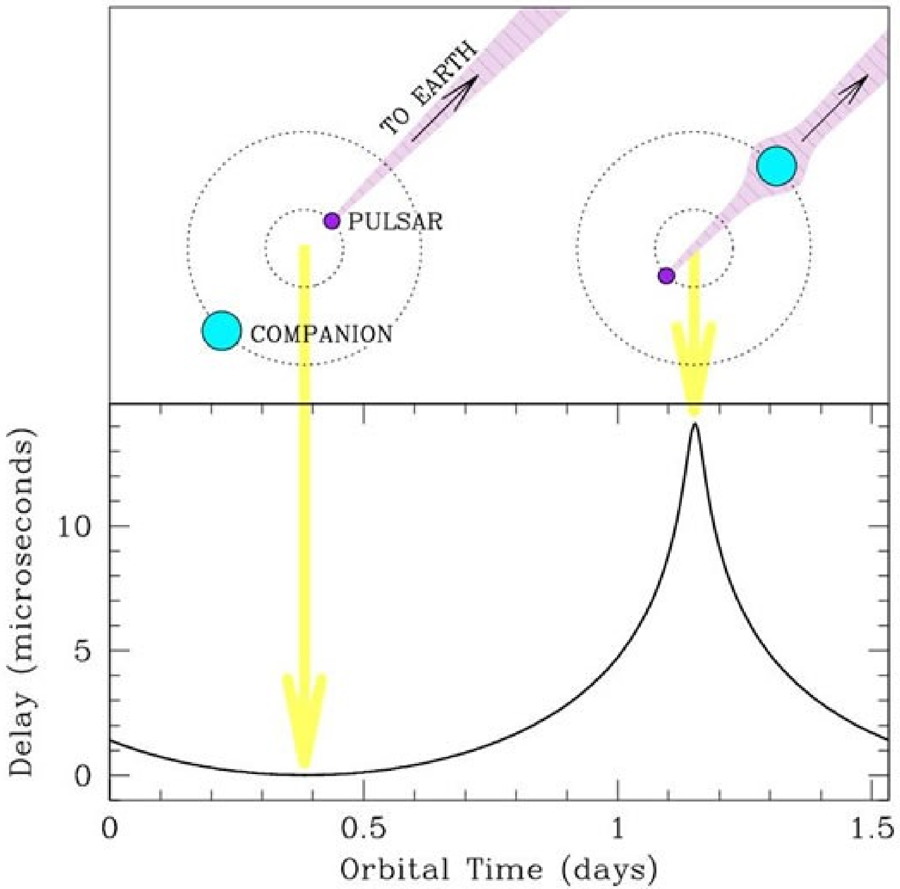
General Relativity predicts that a photon will take a longer path when crossing a strong gravitational field than in flat space-time. This effect is called Shapiro delay and can be directly observed using pulsar timing.

$$\Delta t = -\frac{2GM}{c^3}\log(1 - \mathbf{R}\cdot\mathbf{x})$$

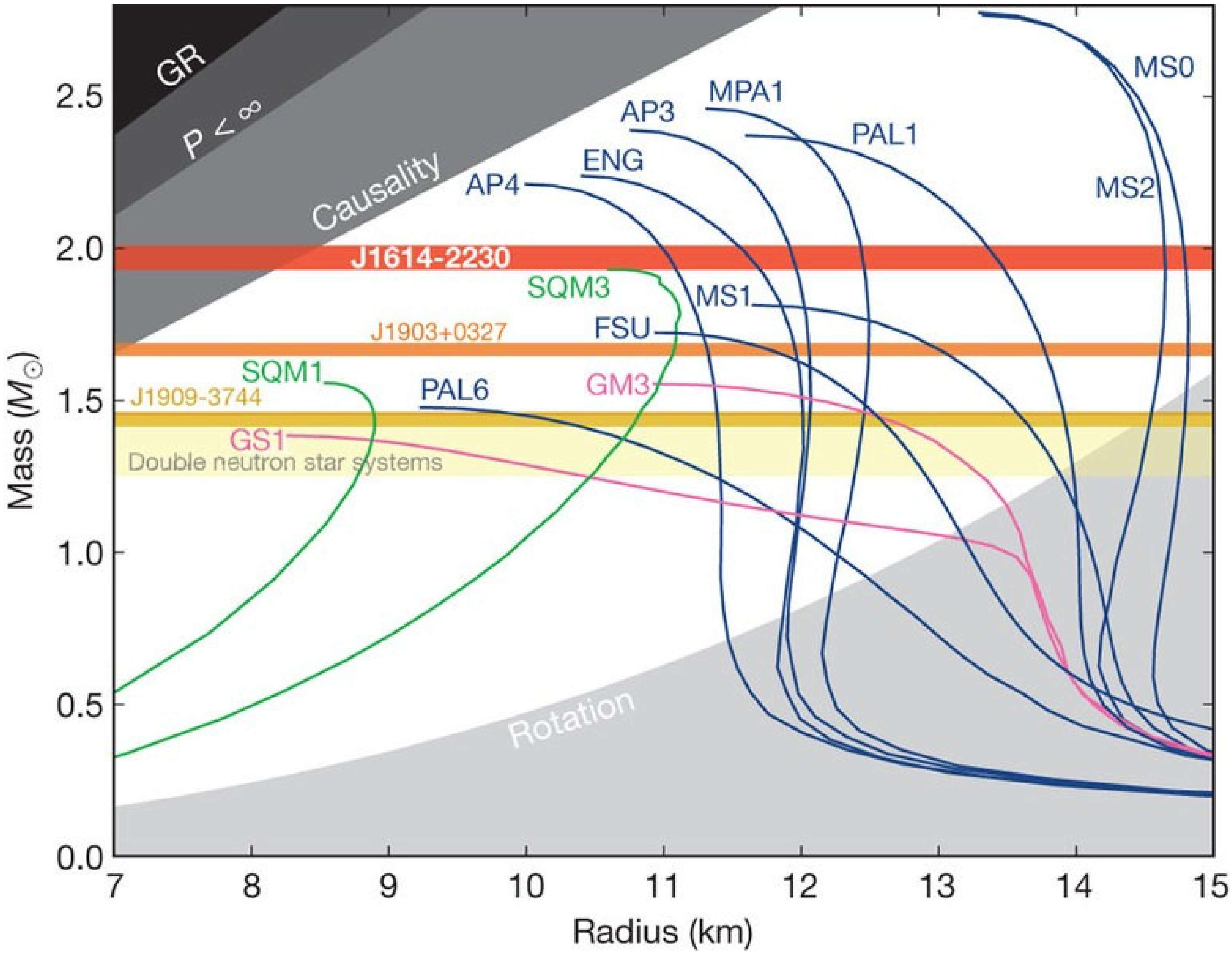
R: unit vector from observer to source
x: unit vector from observer to gravitational mass M

A Two Solar Mass Neutron Star

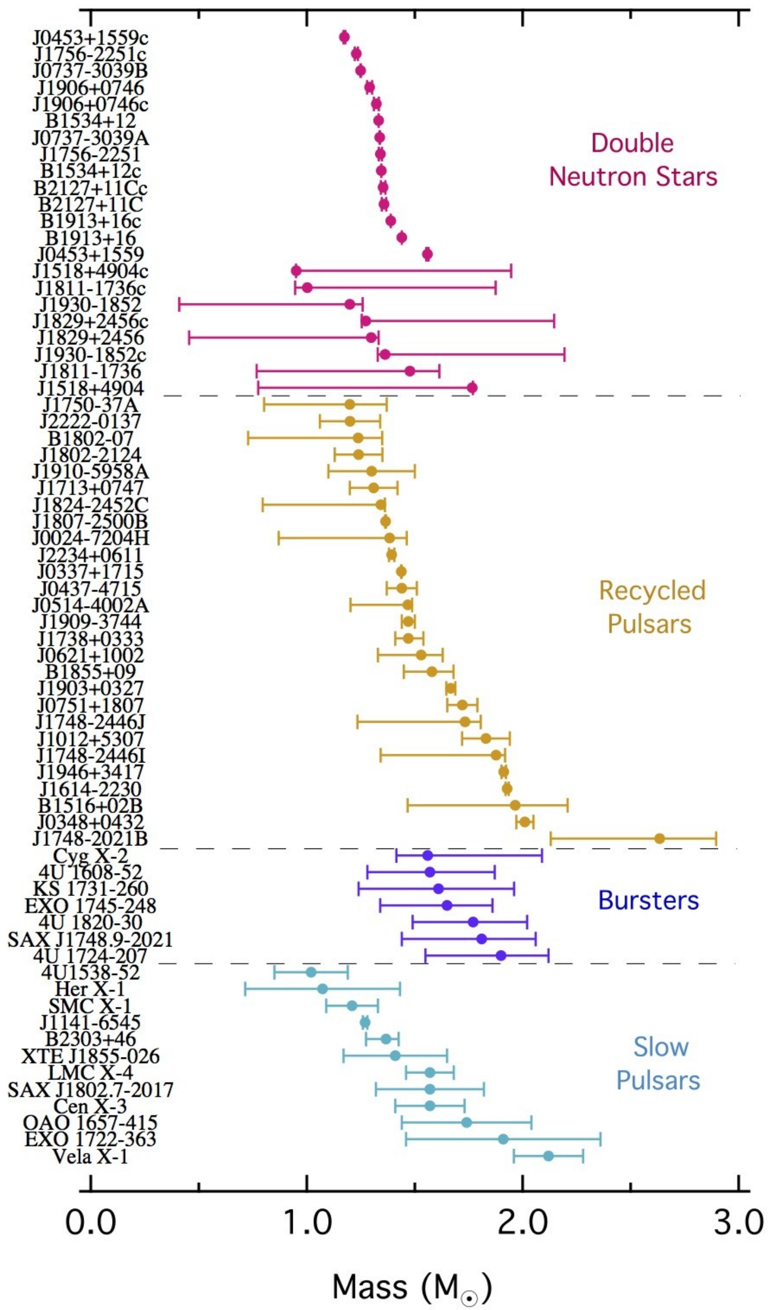




Constraints on the EoS of ultra-dense matter



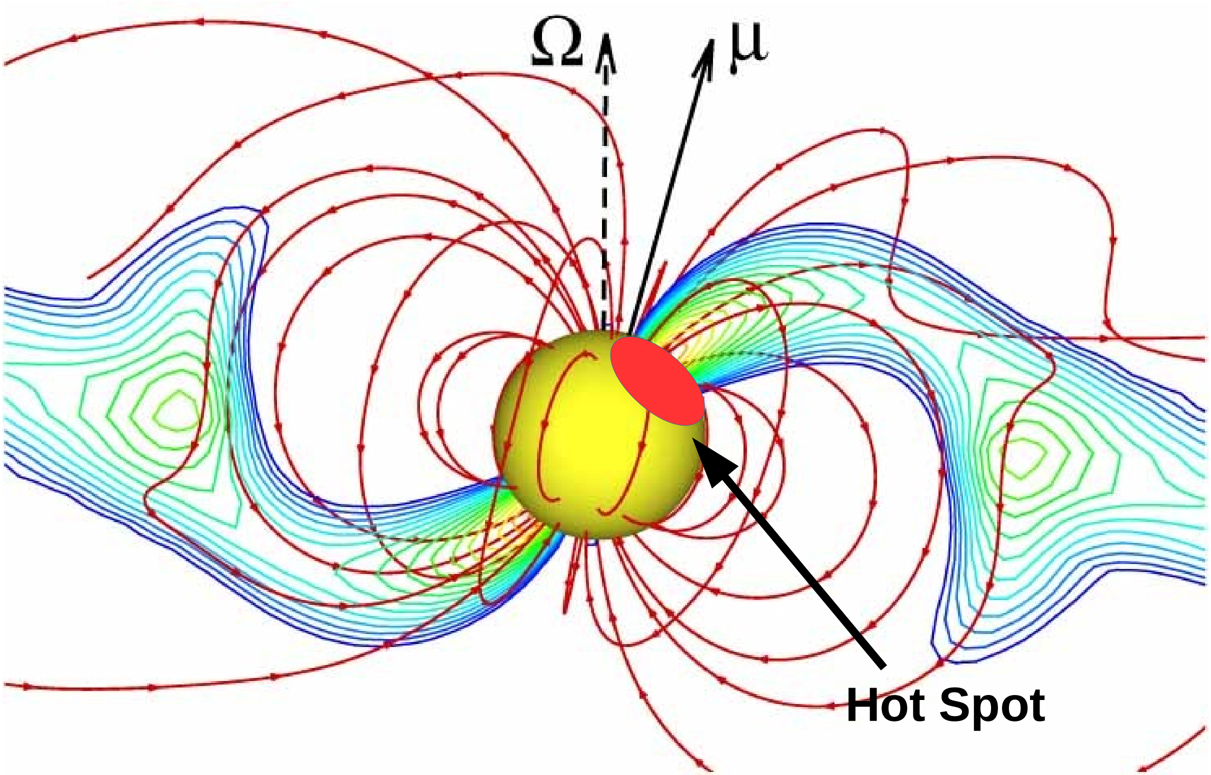
Neutron Star Masses



Recycled pulsars are fast rotating (ms) radio pulsars in a binary system.

The “Bursters” and “Slow Pulsars” are accreting objects and beside the larger error bars they might also suffer of several systematic errors (not reported in this figure).

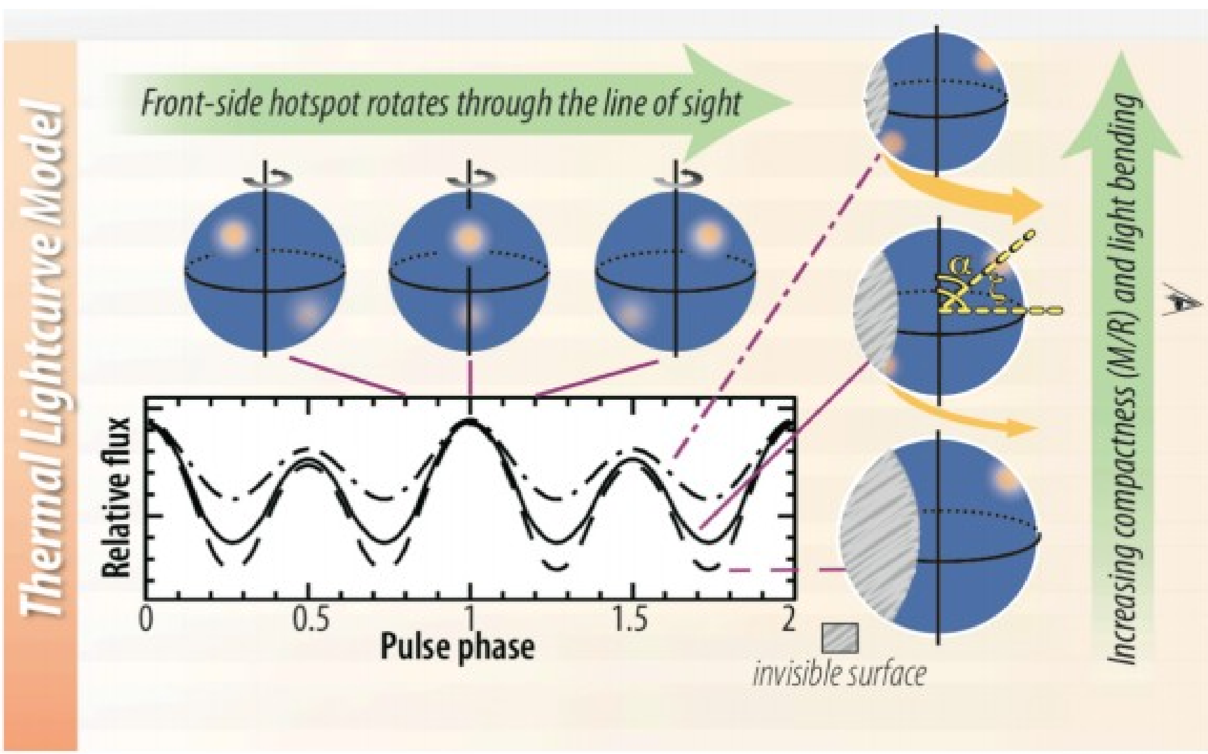
Electromagnetic Signals: Hot Spots and Accretion

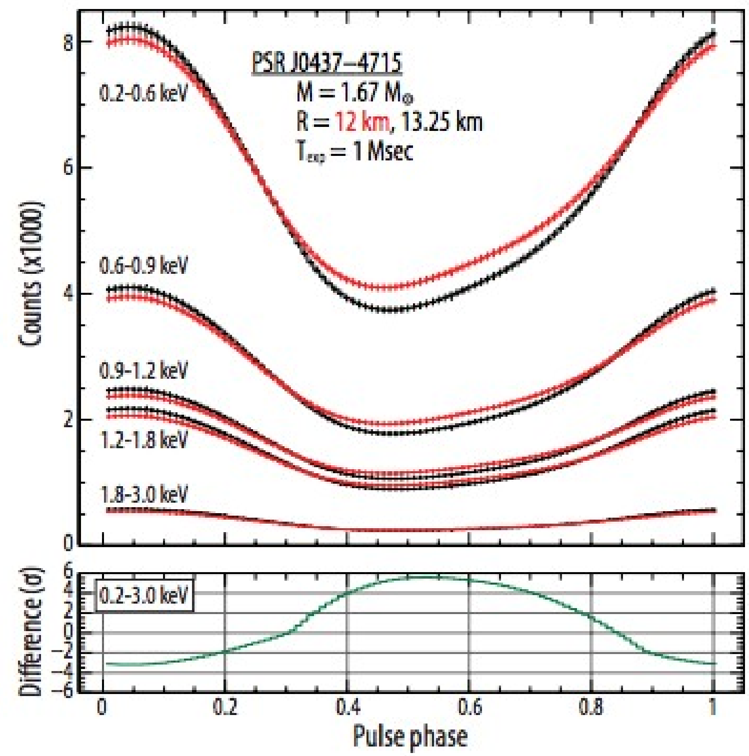


The hot spot emits X-rays that are coming from strongly curved regions of space-time.
Indeed the surface of a neutron star is just at 2-3 Schwarzschild radii.
This situation is more favorable than in non-rotating black holes!

Electromagnetic Signals: Hot Spots and Accretion

A “hot-spot” can be created also without accretion. In a normal radio pulsar the intense electric field can create discharges that heat up the surface of the neutron star around the magnetic poles.



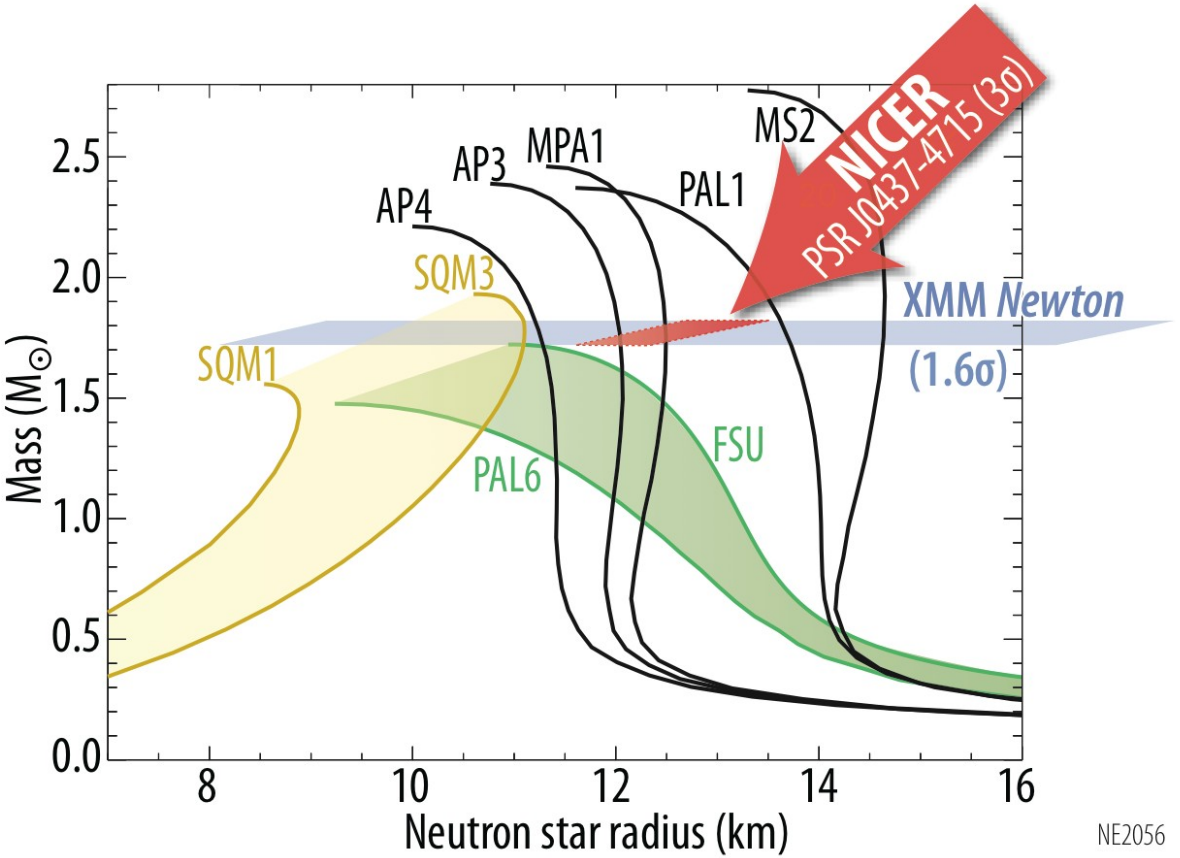


Observables: Radius

Measuring the tiny radius of a neutron star is a formidable challenge.

Indeed several methods exist but they all suffer from strong systematic errors.

Current (NICER) and future missions (STROBE-X, eXTP) are trying to overcome this problem by measuring the mass and radius from pulse profile modeling.



First results should be available late this year (2018)!

Electromagnetic Signals: Sub-ms Pulsars

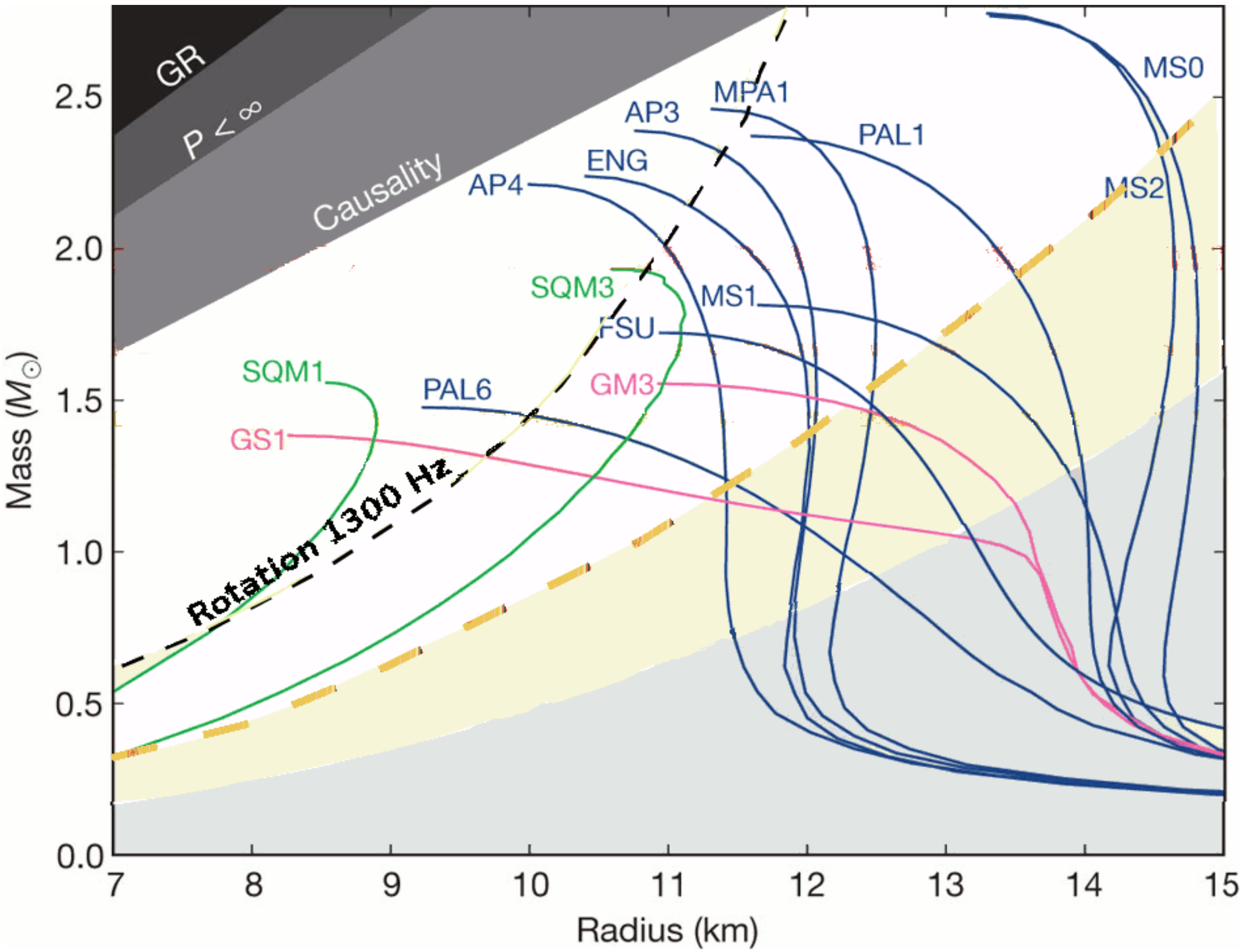
A final method to constrain the EoS of ultra-dense matter is with the neutron star rotation.

Neutron stars are held together by gravity, but the fast rotation generates a centrifugal force that wants to break them apart.

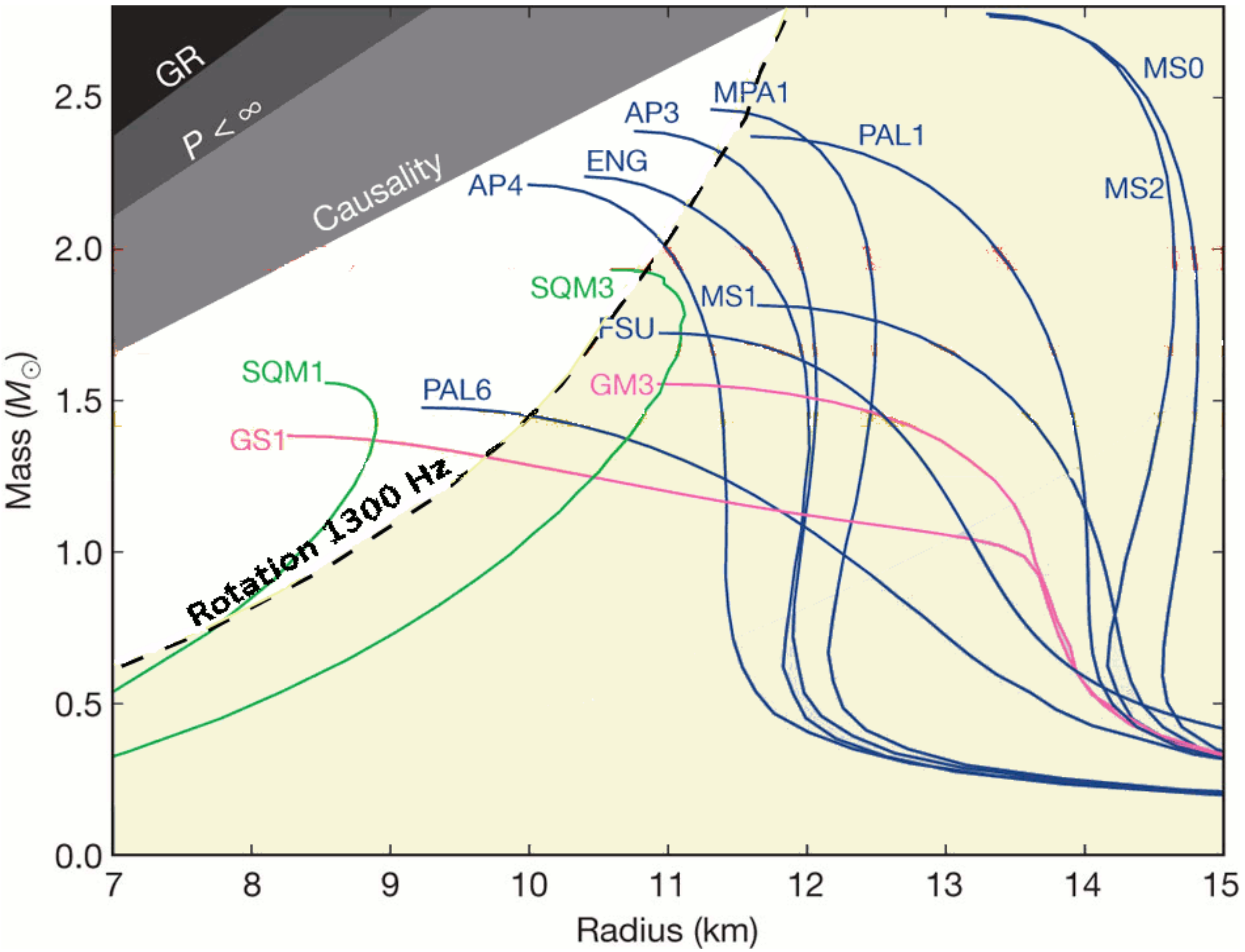
There is a maximum spin frequency that a neutron star can reach, that depends on the mass-radius relation of the neutron star (and thus on EoS of ultra-dense matter):

$$\nu_{max} = 1230 \left(\frac{M_{NS}}{1.4\, M_{Sun}} \right)^{1/2} \left(\frac{R_{NS}}{10\, km} \right)^{-3/2} Hz$$

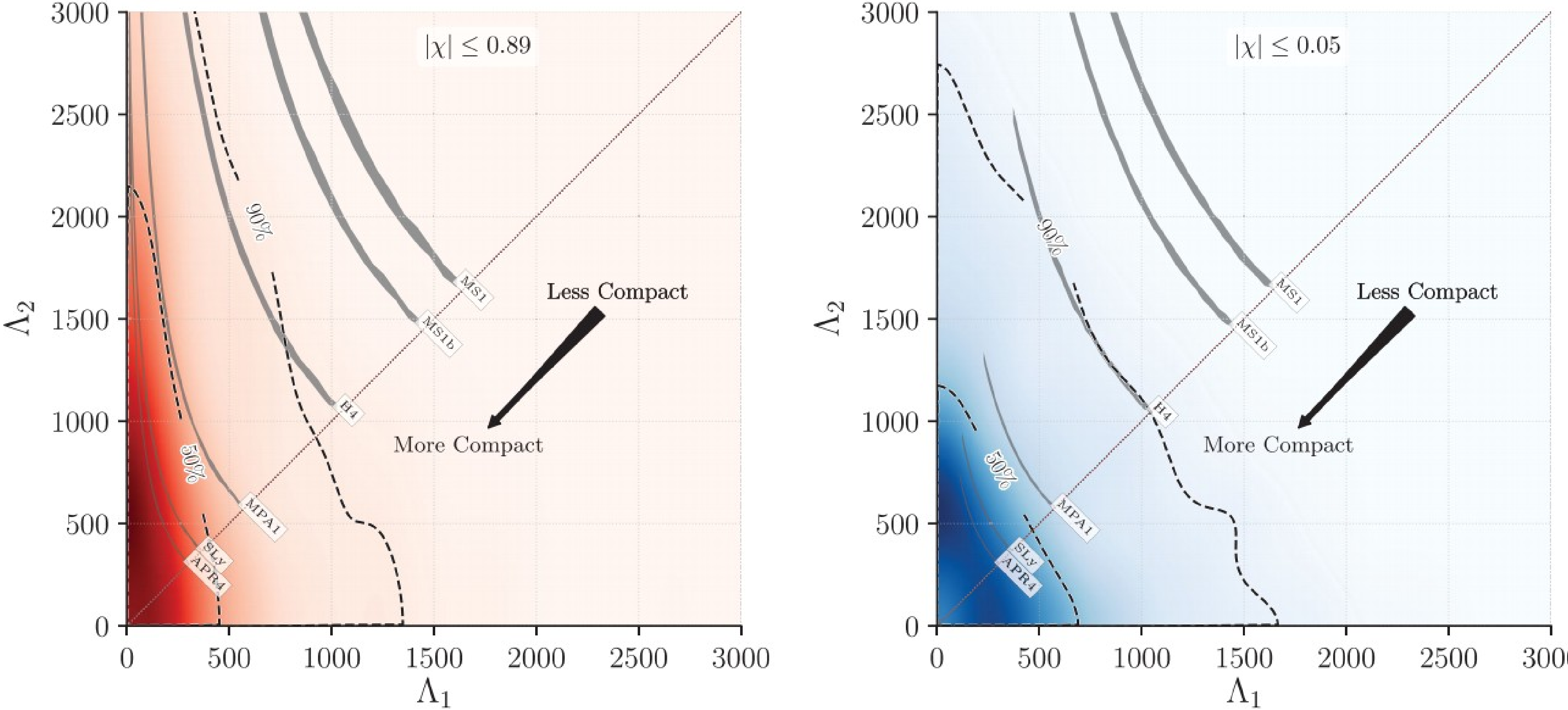
Electromagnetic Signals: Sub-ms Pulsars



Electromagnetic Signals: Sub-ms Pulsars



Gravitational Waves: Tidal Distortions and mergers



Deformability: $\Lambda = (2/3)k_2[(c^2/G)(R/m)]^5$

k2: Second Love number (k2=0 for rigid bodies and/or black holes)

Rotation of neutron stars not well constrained.

References:

Shapiro & Teukolsky “Black Holes, White Dwards and Neutron Stars

Gandolfi et al. (2013) [https://arxiv.org/pdf/1308.6002.pdf]

Lattimer & Prakash (2010) “What a Two Solar Mass Neutron Star Really Means” [https://arxiv.org/abs/1012.3208]