“Compact Objects and their merger”,

E.M. Rossi, Leiden Observatory

Neutron Stars

study material: Chapter 9, Shapiro Teukolsky
plus references in the slides

Neutron stars (NSs)

a few facts we already know

- Typical values for their physical characteristics

Radius	Mass	Central mass density
R	M	ρ_c
$\sim 10\,\mathrm{km}$	$\sim 1.4\,M_\odot$	$> \rho_{\mathrm{nuc.}} = 2.6 \times 10^{17}\,\mathrm{kg.m}^{-3}$

- Compactness of $\Xi \sim 0.2$

Neutron stars (NSs)

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NOTE: $n \rightarrow p + e^{-} + \bar{\nu_{\mathrm{e}}}$ is suppressed because electron degeneracy:

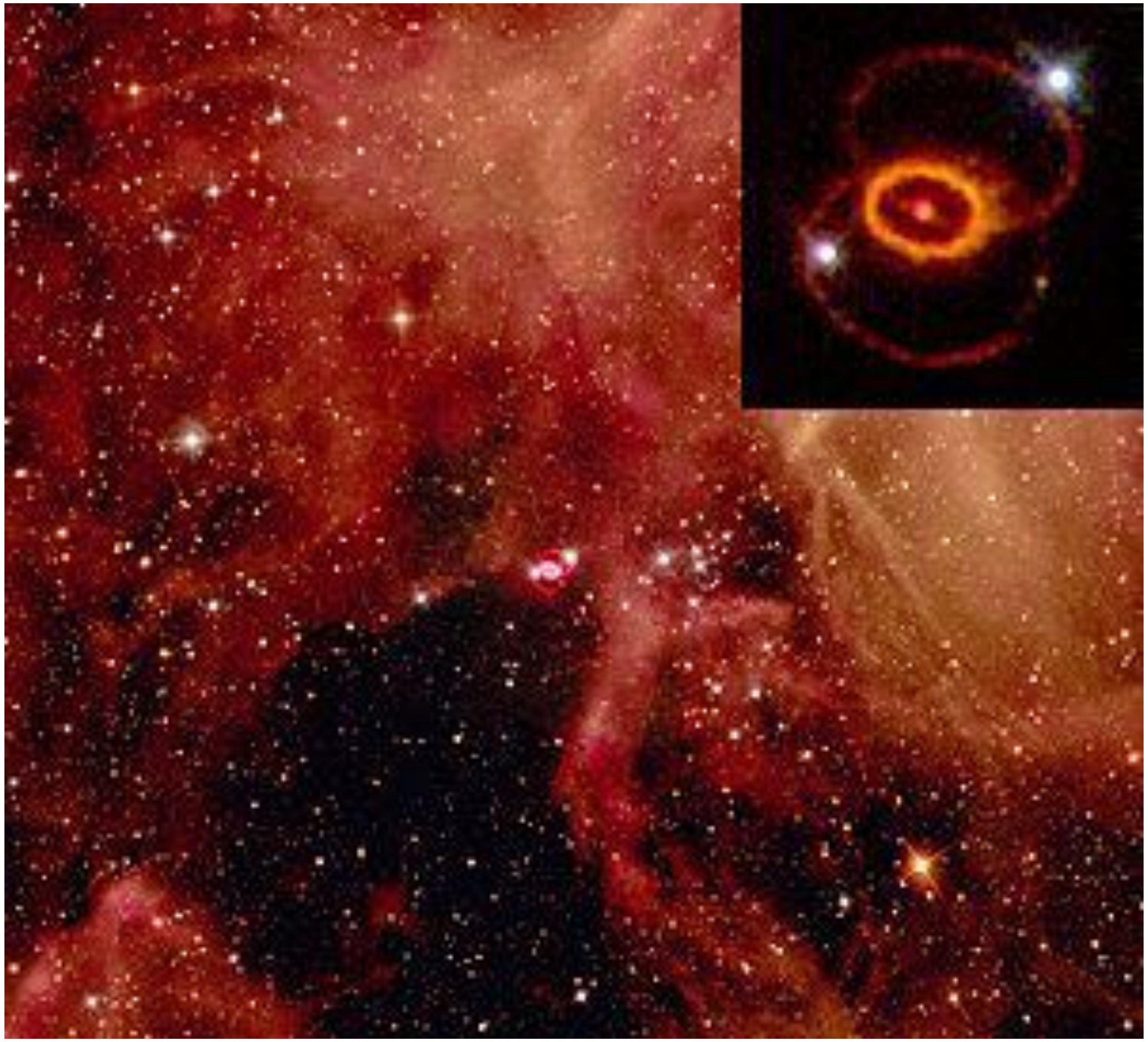
*Note, also electron capture $p + e^{-} \rightarrow n + \nu_{\mathrm{e}}$

Neutron stars (NSs)

a few facts we already know

- Matter remains transparent to neutrinos until a density of $\sim 10^{15}$ kg m^{-3} (R~300 km) and neutrinos escape and can be detected

- SN 1987A



HST image

Approximately ~2.5 hours *before* the visible light from SN 1987A reached Earth, a burst of neutrinos was observed at three separate neutrino observatories

This was the beginning of extra-solar neutrino astronomy !

Neutron stars (NSs)
a few facts we already know

- Matter remains transparent to neutrinos until a density of ~10^{15} kg m^{-3} (R~300 km) and neutrinos escape and be detected

- **Physics of neutrinos**: (see Volpe 16 for a review OPTIONAL)
- from supernova SN1987

1. constrain on superluminal neutrino: ($v-c/c < 10^{-9}$),
2. constraints on neutrino flavours,
3. upper bound on mass and charge

Neutron stars (NSs)

a few facts we already know

- Formation of a neutron stars is accompanied by a supernova explosion (Type II or Type I/b or macro/kilo Nova and/or short GRBs)

NSs: history

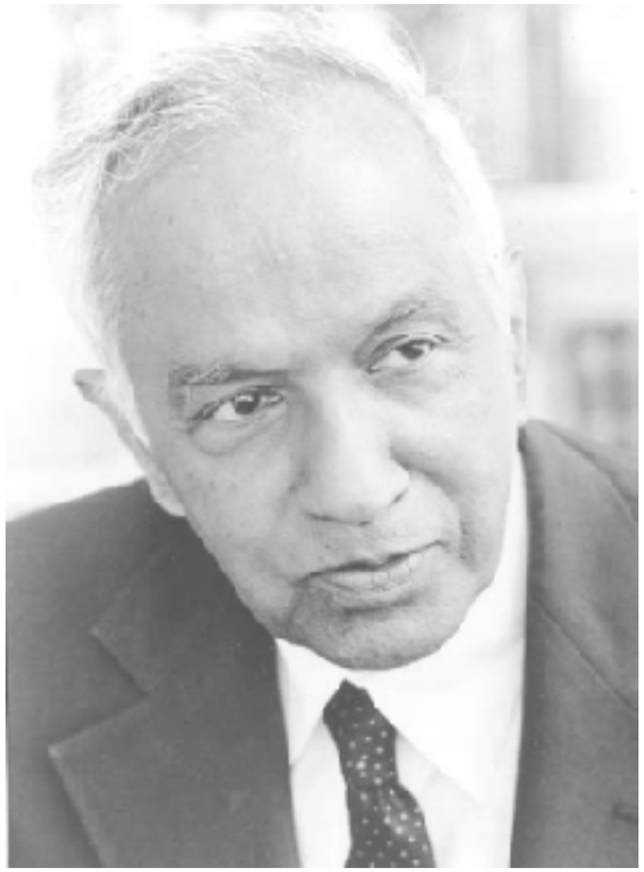
Discovery of Neutron

- *Neutrons were discovered by James Chadwick in 1932 in Cambridge (Nobel prize in 1935).*

James Chadwick

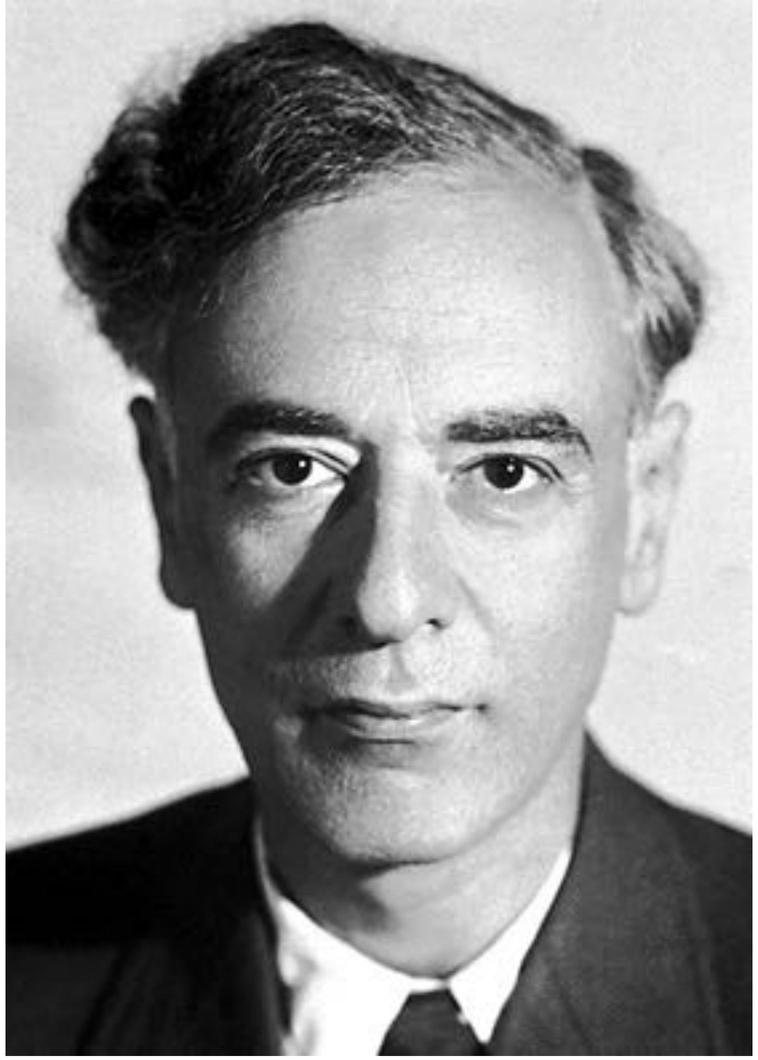
Maximum mass for WD

a.k.a. Chandrasekhar mass



- *First detected White Dwarf: 40 Eridani in 1914.*
- In 1930 S. Chandrasekhar introduces the relativistic corrections needed to describe the equation of state for degenerate electrons in high density environment He calculated the WD structure in this regime and derive that only one mass is possible. Thus <u>there is a limiting mass for WDs</u>

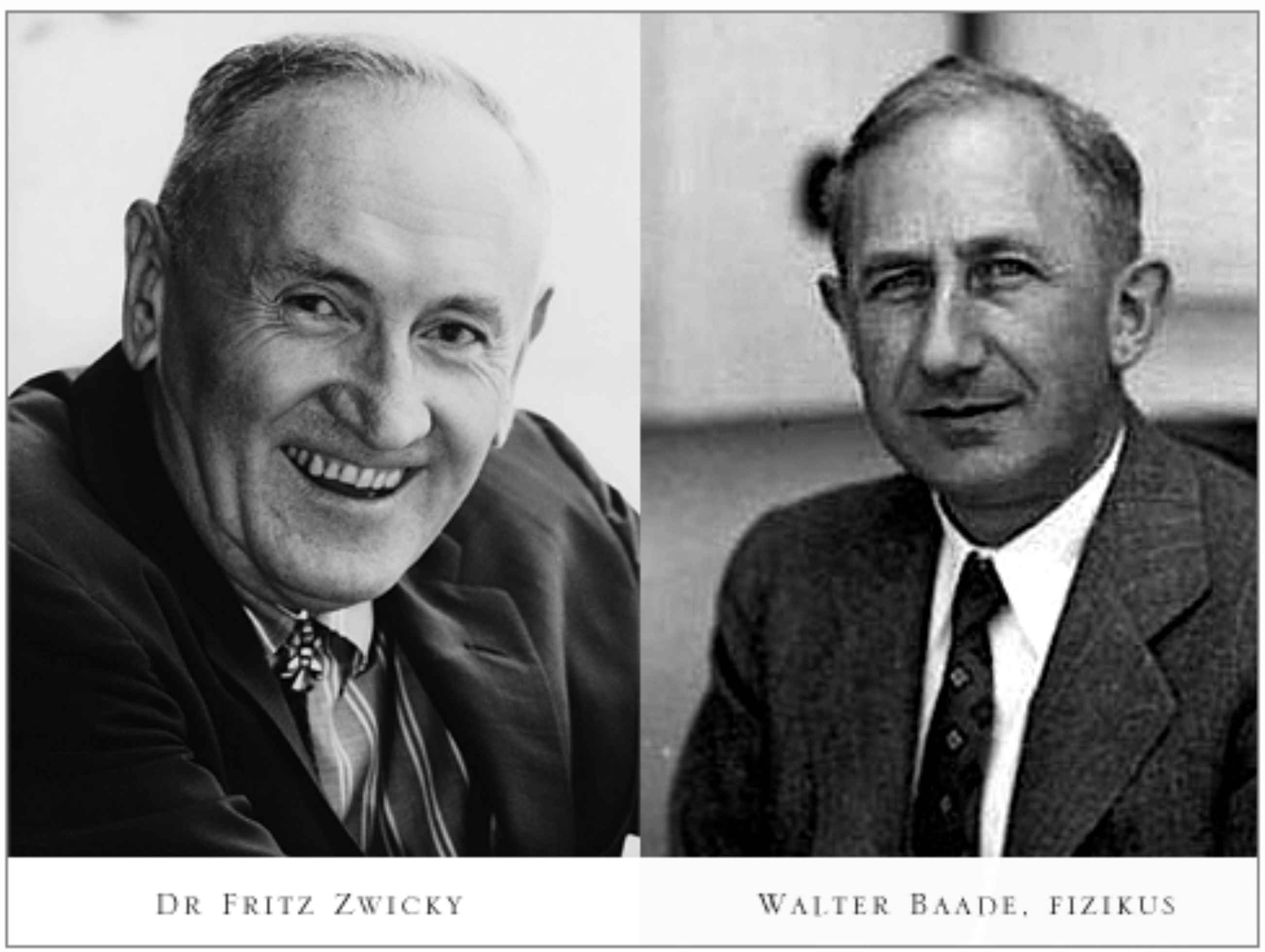
In 1931/1932 Landau proposed that the fate of star more massive than the limit would be an object "where the atomic nuclei were so close to each other to form a gigantic nucleus"



Lev Landau (Nobel Prize 62)

NS concept

1934 Baade & Zwicky proposed the existence of NSs. They had discovered that some “Novea” were extragalactic and therefore much more intrinsically luminous than Galactic Novea. They called them “Supernovae” and propose:



« ... with all reserve we advance the view that a supernova represents the transition of an ordinary star into a neutron star, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density » (Baade & Zwicky, 1934).

Equation of state

- 1939 first calculations of NS structure by Oppenheimer & Volkoff. They use general relativity and an equation of state for <u>a perfect gas of degenerate neutrons.</u>

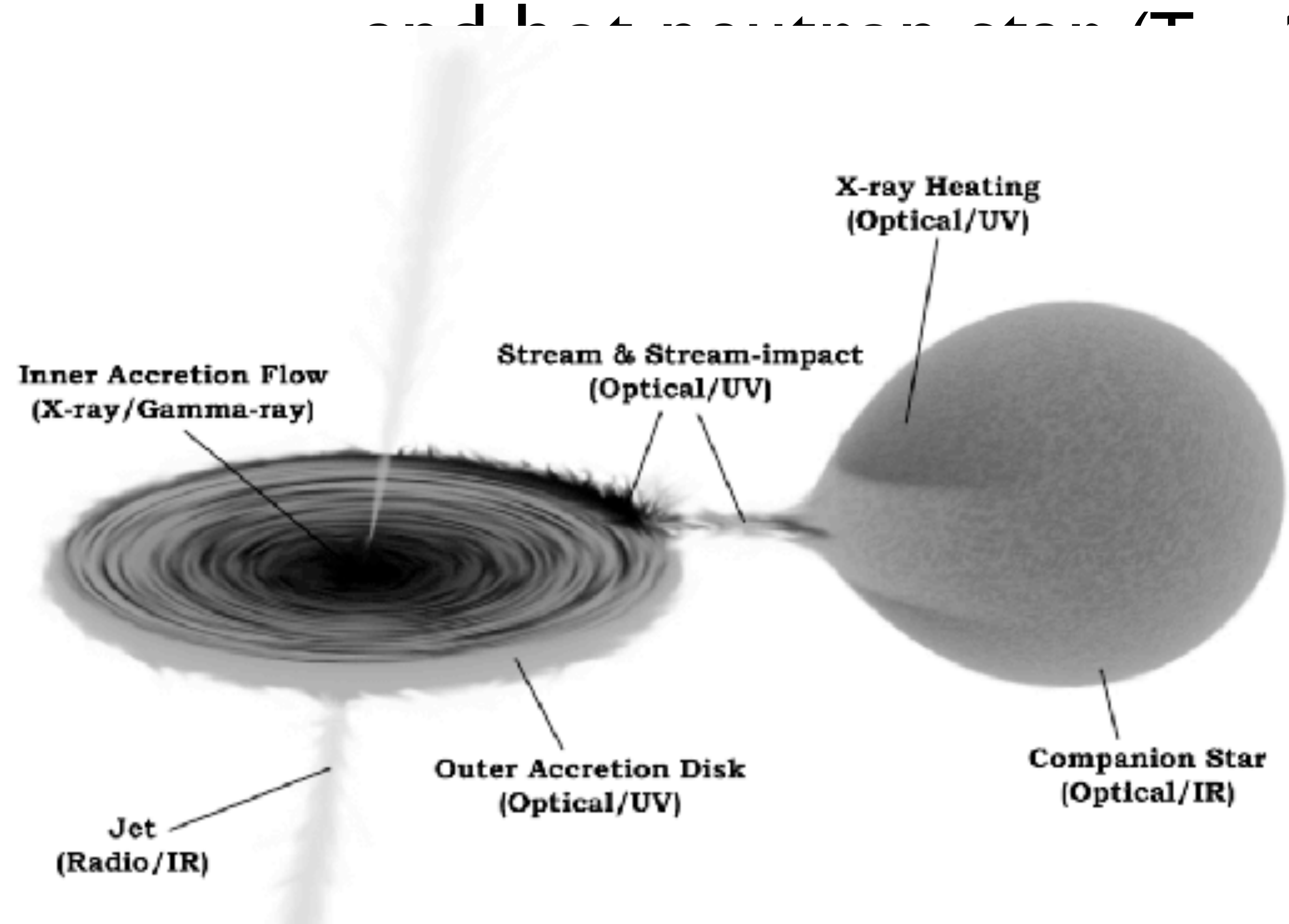
WWII stopped momentarily further advancements...

Equation of state cont.

- At the end of 1950s theoretical works on NSs re-started, in particular around the question of what is the equation of state for ultra-dense matter. Notable works by Harrison, Wikano & Wheeler (1958), Cameron (1959), Ambartsumyan & Saakyan (1960) & Hamada & Salpeter (1961).
- These more realistic equations of state showed that NS must have very small radii ~ 10 km. Despite their large surface temperature, the small size makes direct detection of surface emission challenging.
- NSs during these years remained a study subject for (a small group of) theoreticians.

The beginning of X-ray astronomy

In 1962, Giacconi and co-authors discovered the first X-ray source beyond the solar system (Sco X-1, L_X~ 6 10^4 L_{sun}), following the launch of the USA Arobee rocket (Nobel prize in 2002). It is interpreted as a young and hot neutron star (T ~ 2 10^{10} K). The question is then

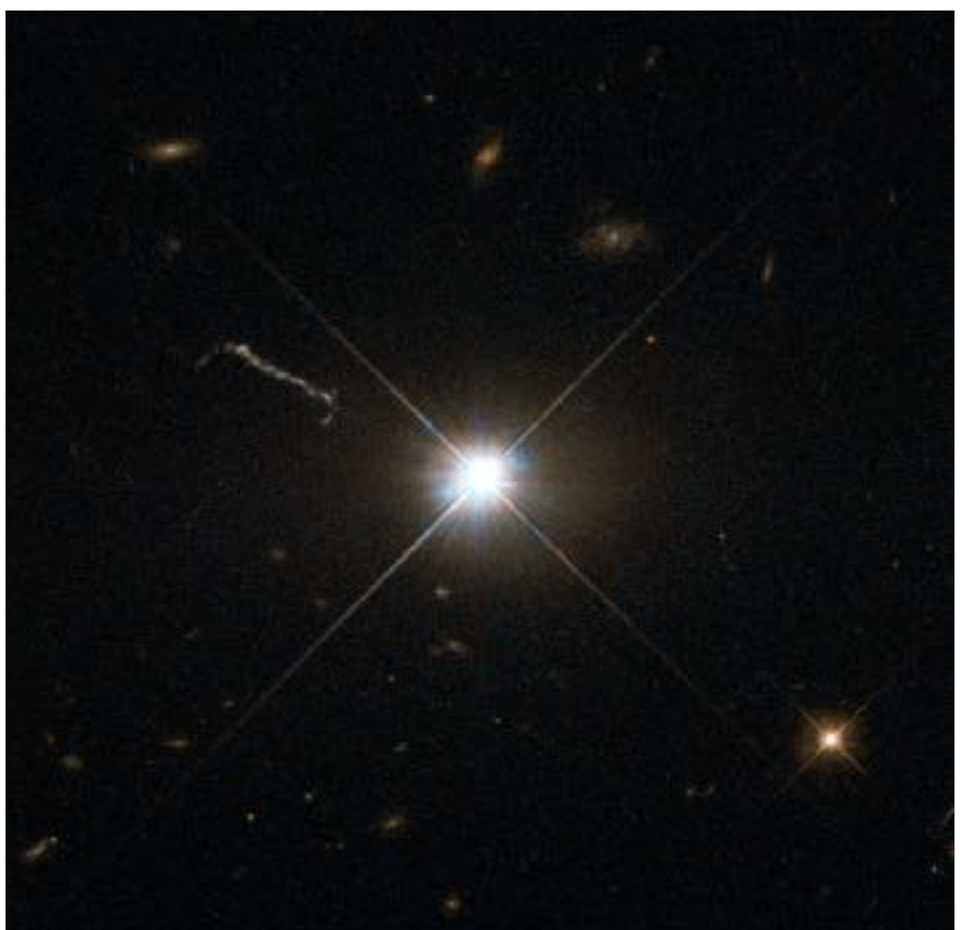


We now know that Sco X-1 is an accreting neutron star (not an isolated one)

At the same time, there is a debate on whether QSOs (discovered in 1963) are neutron stars.

how was this ruled out?

let’s open a parenthesis…(



3C 273 : the second optically identified Quasars by Maarten Schmidt, for which a redshift could be measured

Gravitational time dilation

Locally, an interval in proper time is related to the space-time metric by $c\,\mathrm{d}\tau = \sqrt{-g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu}$

An observer at rest on NS surface measures an interval in proper time

$$\mathrm{d}\tau_{\mathrm{source}} = \left(1 - \frac{2GM}{Rc^2}\right)^{1/2} \mathrm{d}t\,.$$

while an observer at infinity will measures $\mathrm{d}\tau_{\mathrm{obs}} = \mathrm{d}t\,.$

Therefore, an intrinsic period signal of the source on time scale T_0 will be observed with a period

$$\boxed{T_{\mathrm{obs}} = \frac{T_0}{\sqrt{1 - \frac{2GM}{Rc^2}}}\,.}$$

longer periods

Einstein's gravitational redshift

A signal of wavelength $\lambda_0 \sim T_0$ will then be redshifted

$$\frac{\lambda_{\rm obs}}{\lambda_0} = 1 + z = \frac{1}{\sqrt{1-2\Xi}}\,.$$

where the object compactness is $\Xi = GM/Rc^2$

For White Dwarfs $\quad z = (1-2\Xi)^{-1/2} - 1 \to \Xi \quad \Xi \to 0.$

For neutron stars $\quad z = (1-0.4)^{-1/2} - 1 \approx 0.3$

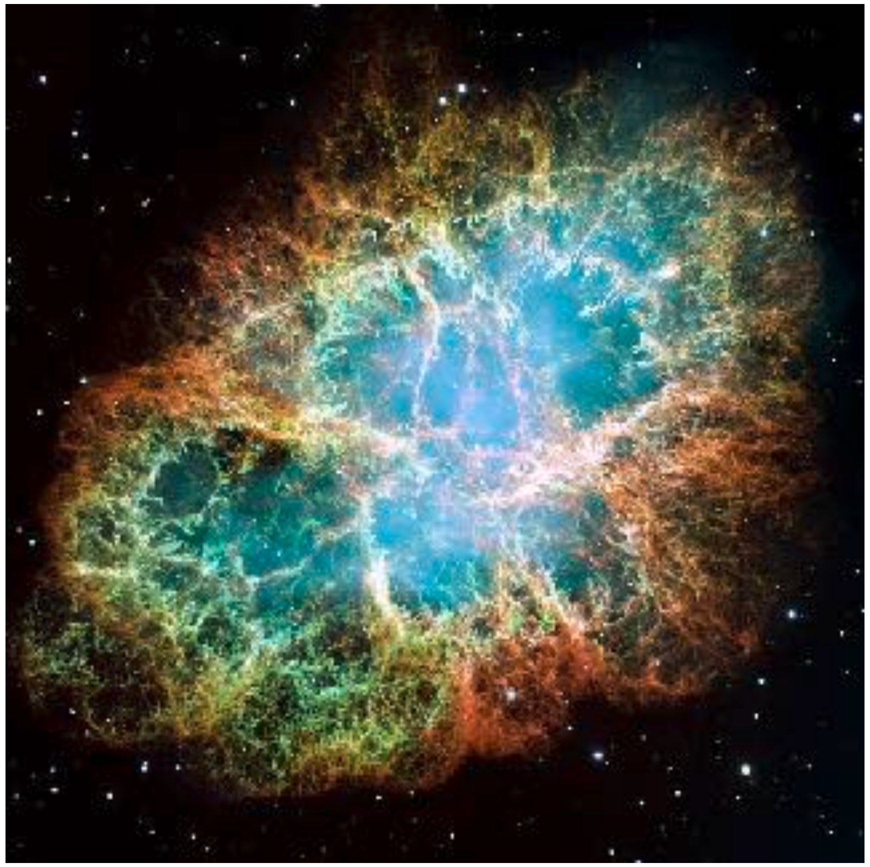
it was soon realised that the observed redshift in Quasars spectra could be larger than that: universe expansion!

…let’s close)

back to history!

The Crab

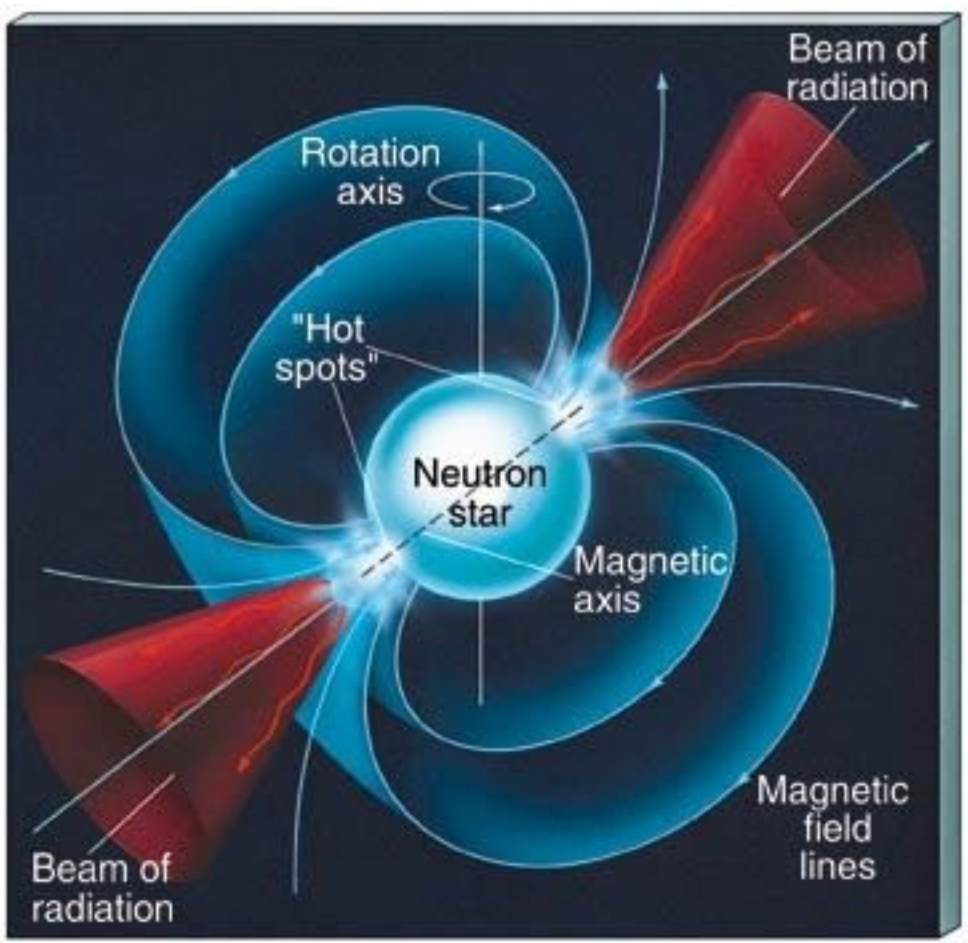
- In 1964, Hoyle, Narlikar & Wheeler proposed that a highly magnetised NS (~10^{10} Gauss) is at the centre of the Crab nebula.



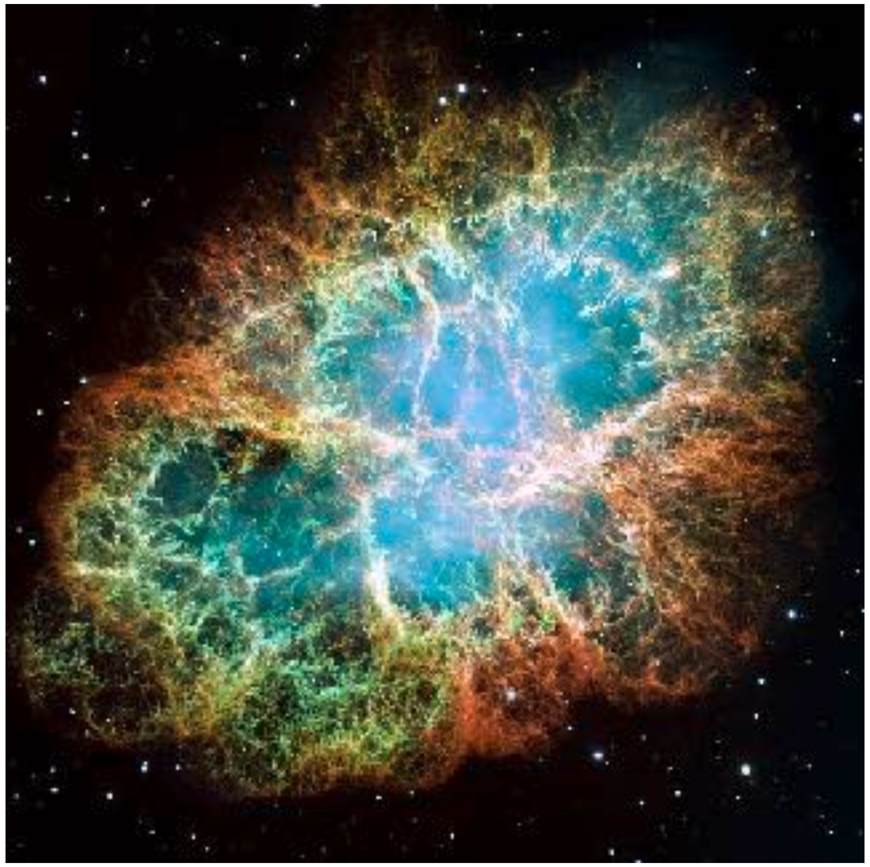
Optical

The Crab

- In 1964, Hoyle, Narlikar & Wheeler proposed that a highly magnetised NS (~10^{10} Gauss) is at the centre of the Crab nebula.
- In 1967, Pacini proposed that the source of energy for the Crab nebula is a highly magnetised and rotating NS.



Optical

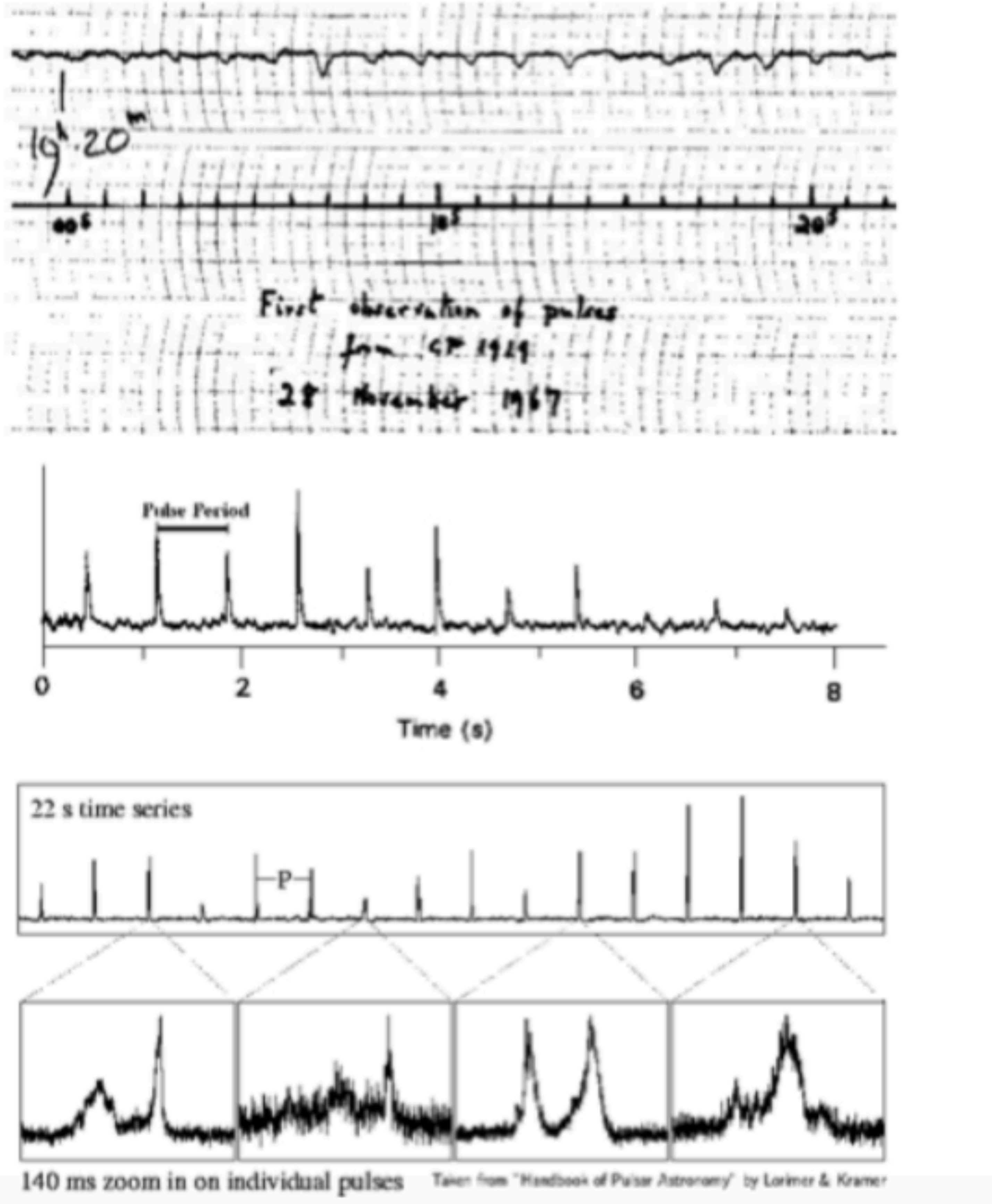


Pulsars discovery

- In 1967, Hewish and his student Bell discover the first radio pulsar: a period radio source, with an extremely short (modern value P = 1.337301 s) and stable period. (Nobel prize to Hewish in 1974).
- In 1968, they published their discovery after much double checking. Name: CP 1919 (Cambridge Pulsar, right ascension 19, declination 19. The Modern name is "PSR B 1919+21")
- the short duration period indicated an object with < 1000 km radio, white dwarf ??

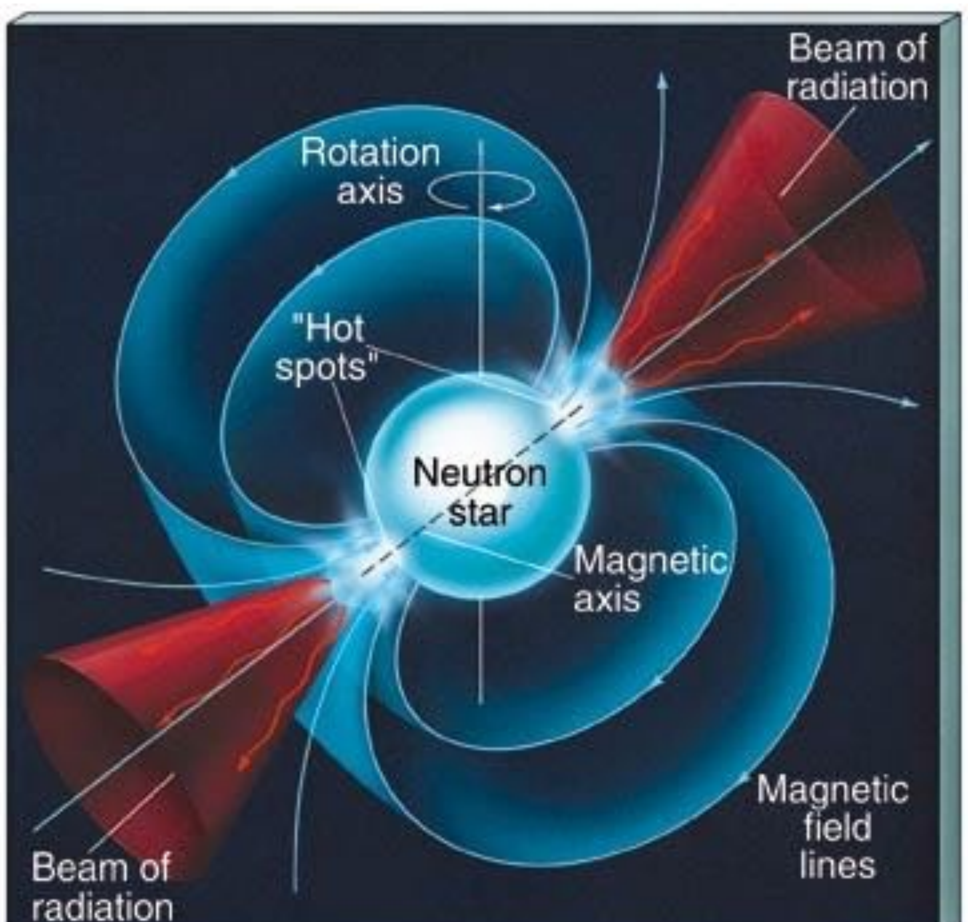
MS. JOCELYN BELL

historical recording of the pulses



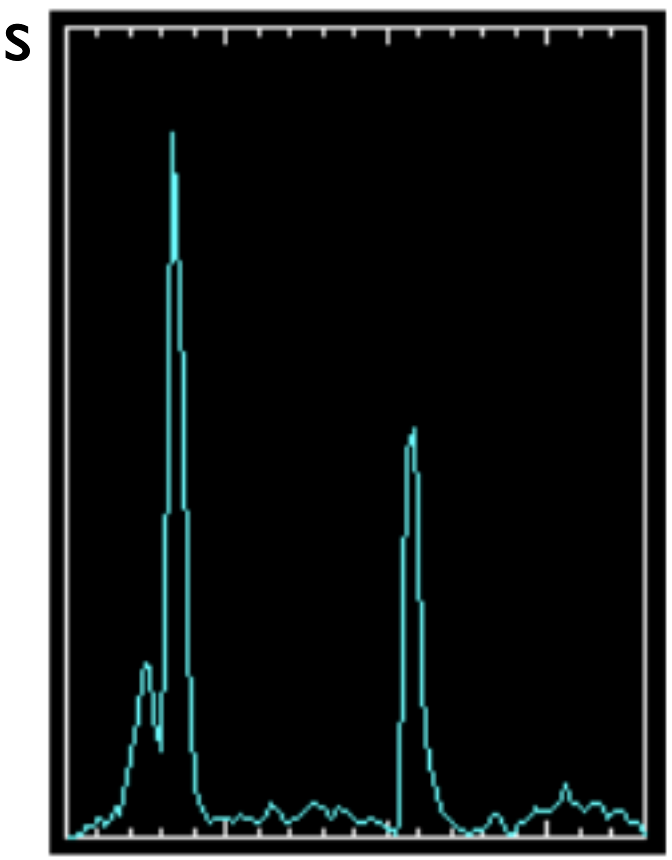
More pulsars...

- In 1968, discovery of the *radio* Pulsar Vela (PSR B 0833-45) and Crab (PSR B 0531+21). They are both within supernova remnants. These discovery validate Baade, Zwicky & Pacini's predictions.



1969 Crab's pulses observed in X-ray (here modern Chandra X-ray observations)

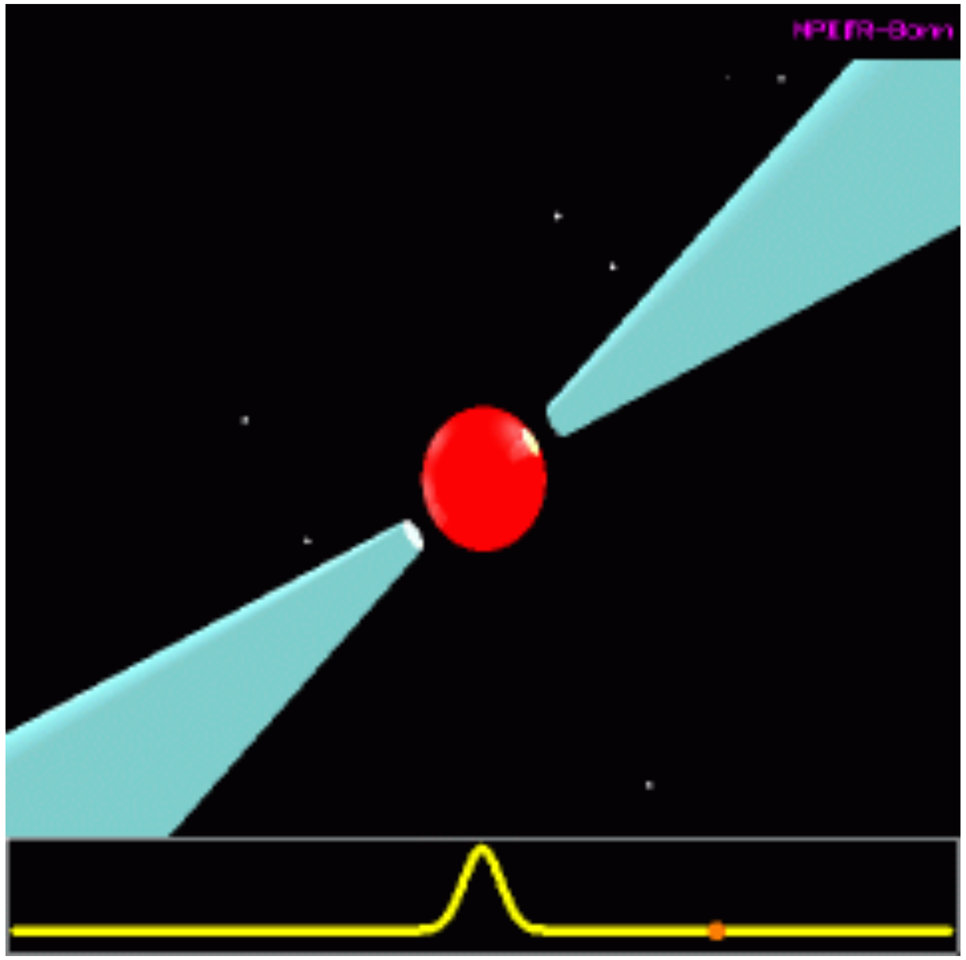
Crab's pulses radio



Pulsar	P	$\dot{P}$
Crabe	0.0334 s	$4.209599(2) \times 10^{-13}$
Vela	0.089328 s	1.2500×10^{-13}

More on pulsars...

- In 1968, Gold proposes a pulsar model as magnetised, rapidly rotating NS, with which he predicts with precision the Crab period and its derivative, that were only measured in 1969



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Exercise

The first radio pulsar has $\Omega \simeq 4.7 \ \mathrm{s}^{-1}$

The Crab pulsar has $\Omega \simeq 0.20 \ \mathrm{ms}^{-1}$

Show that this rapid rotation implies objects with density higher than a WDs

hint: think about equilibrium of forces in a star

tip: just consider equilibrium at the equator

solution: Pulsars, as NS

There are very little alternative to a NS to explain a Pulsar.

Given mass M, radius R and angular velocity Ω
the centrifugal force at the surface, at the equator is

$$f_{\mathrm{cent.}} = \Omega^2 R \quad \& \quad f_{\mathrm{grav}} = \frac{GM}{R^2}$$

The <u>maximum</u> rotation for balance (=negligible pressure force) is then

$$\Omega_{\max} = \Omega_{\mathrm{K}} = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{4\pi G\bar{\rho}}{3}} \quad \text{where} \quad \bar{\rho} = M/\left(\tfrac{4\pi}{3}R^3\right)$$

So $\Omega < \Omega_{\mathrm{K}}$ $\Rightarrow$ $\bar{\rho} > \bar{\rho}_{\min} = \dfrac{3\Omega^2}{4\pi G}$

solution Pulsars, as NSs

The first radio pulsar has $\Omega \simeq 4.7 \text{ s}^{-1}$ $\quad \bar{\rho} > \bar{\rho}_{\min} = \dfrac{3\Omega^2}{4\pi G}$

a *minimum* density of $\bar{\rho} > 8 \times 10^7 \text{ g cm}^{-3}$

The Crab pulsar has $\Omega \simeq 0.20 \text{ ms}^{-1}$

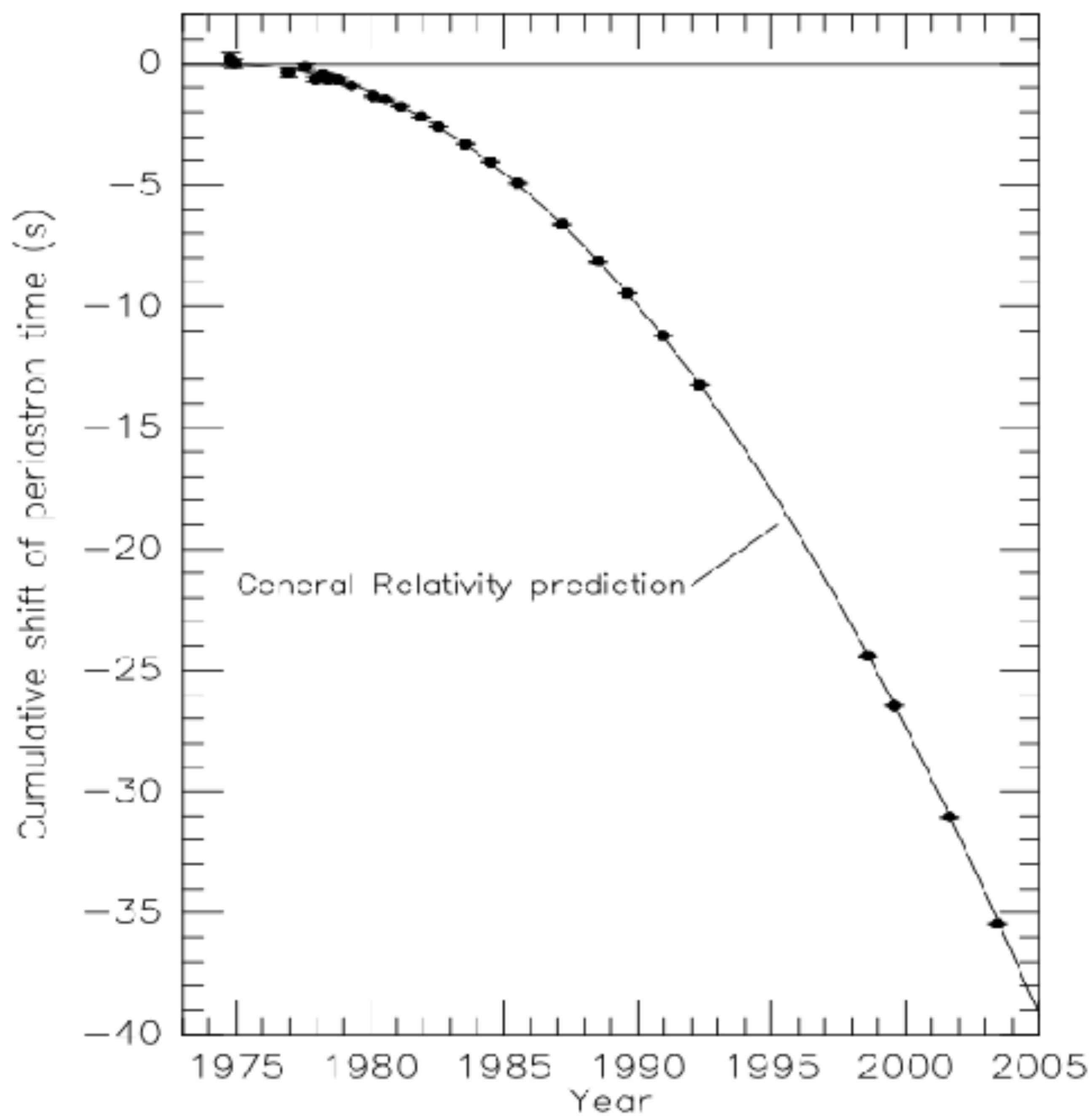
a *minimum* density of $\bar{\rho} > 1 \times 10^{11} \text{ g cm}^{-3}$

No system is known that is so dense, not even WDs

recall, for WD $\bar{\rho} \approx 10^6 \text{ g cm}^{-3}$

Other notable discovery

- 1975, Hulse& Taylor discover the first binary pulsar PSR B 1913+16 (pulsar period ~56 s; orbital period of 7.8 h). Determined that they are two NSs of ~1.4 M_{sun}. Only one is a pulsar. Monitoring the pulsar, they measured a decrease in the orbital period consistent with Gravitational Wave emission. First (indirect)detection of GW! Nobel prize in 1993



from Weisberg & Taylor 2004

they will coalesce in 300 million yr…

Currently, agreement with GR within 0.1 %

Other notable discoveries

- 1982, first pulsar discovered with P ~1.56 ms (millisecond pulsar) . Such a short period is difficult to justify within the standard formation scenario. We think now that they were "re-accelerated" via accretion from a companion. Recycled pulsars

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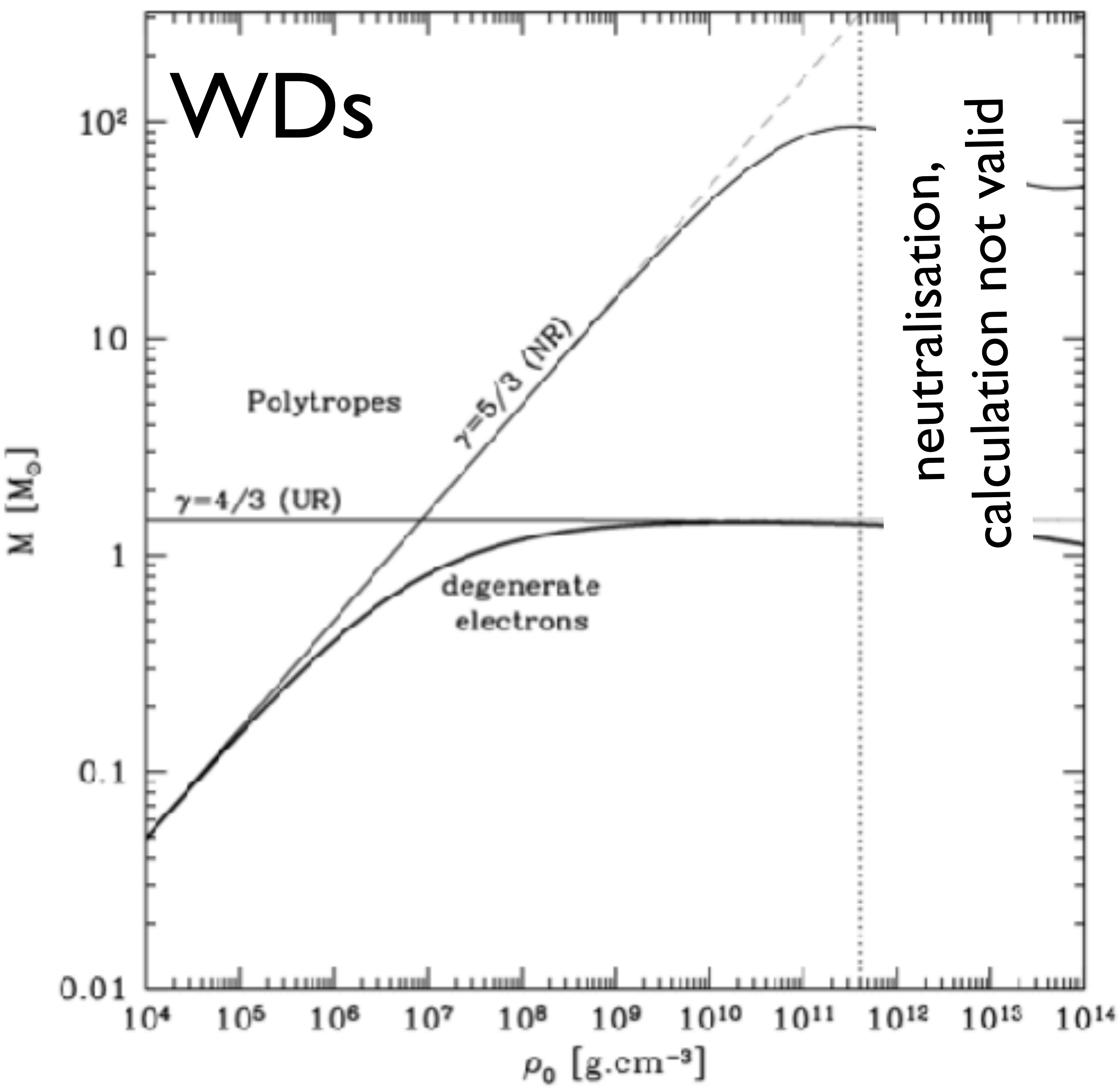
- In August 2017 first detection of GW from NS-NS+ E.M counterpart

cfr. Ch 9 Shapiro Teukolsky

NSs: Equation of state

Chandrasekhar mass

Equation of state given by electron degenerate pressure



two limits: Non Relativistic (NR)

$$P \approx 3 \times 10^{22}\ \mathrm{Ba}\ \ (Y_\mathrm{e}/0.5)^{5/3} \left(\frac{\rho}{10^6 \mathrm{g\ cm^{-3}}}\right)^{5/3}$$

Ultra Relativistic (UR)

$$P \approx 5 \times 10^{22}\ \mathrm{Ba}\ \ (Y_\mathrm{e}/0.5)^{4/3} \left(\frac{\rho}{10^6 \mathrm{g\ cm^{-3}}}\right)^{4/3}$$

1) for WD calculations in GR or Newtonian frame are indistinguishable

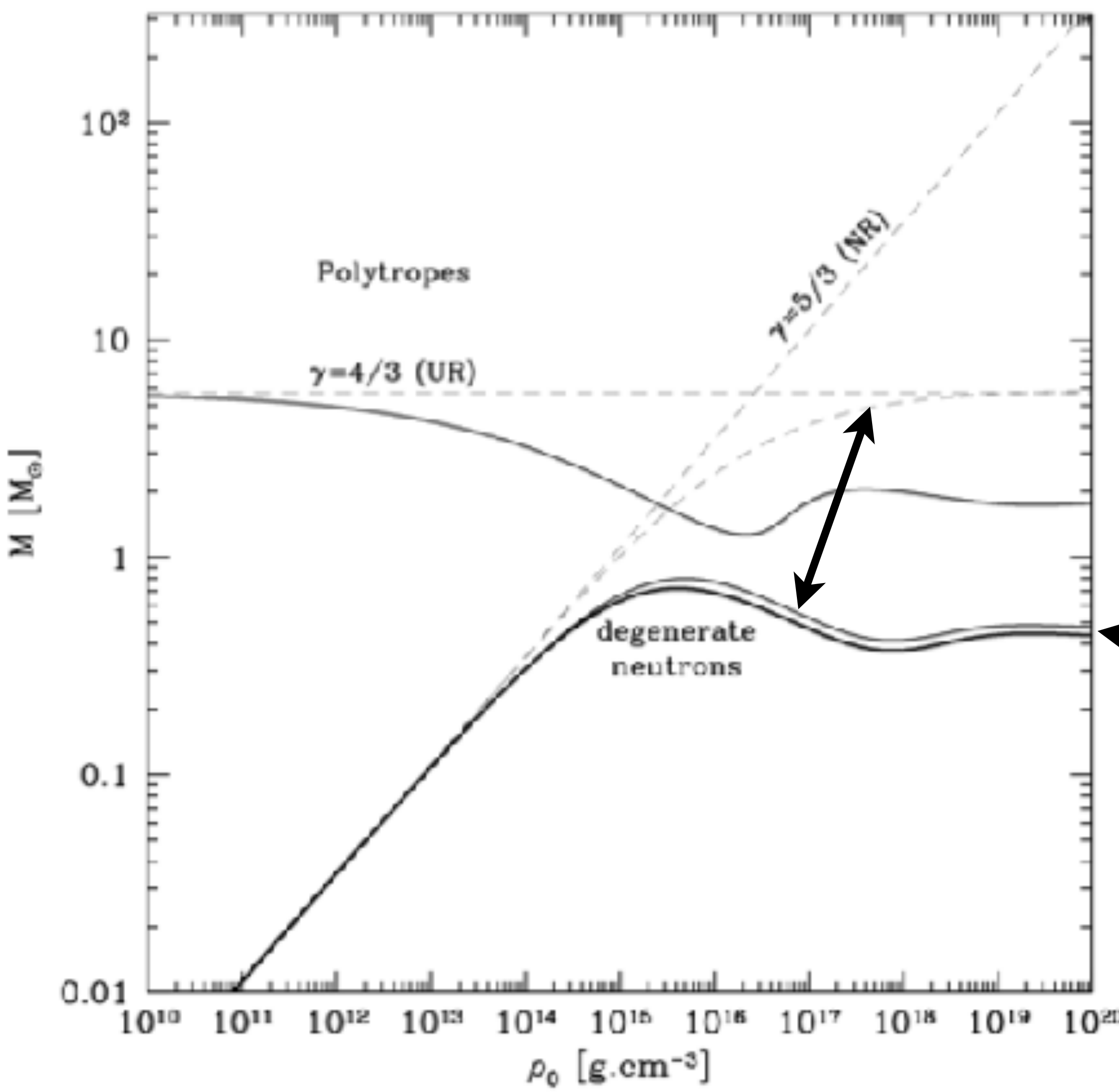
2) As the mass increases the electrons become relativistic and the equation of state allows only one Mass ~1.4 M_{sun}

full GR solution (solid) and Newtonian solution (dashed)

In analogy let's use ideal gas of T=0, fully degenerate neutrons.
In GR, this is the Oppenheimer & Volkoff (1939) calculation

$$P \propto \rho^{\gamma}$$

GR (solid) and Newtonian (dashed)



1) Solutions differ at high ρ

full GR treatment needed

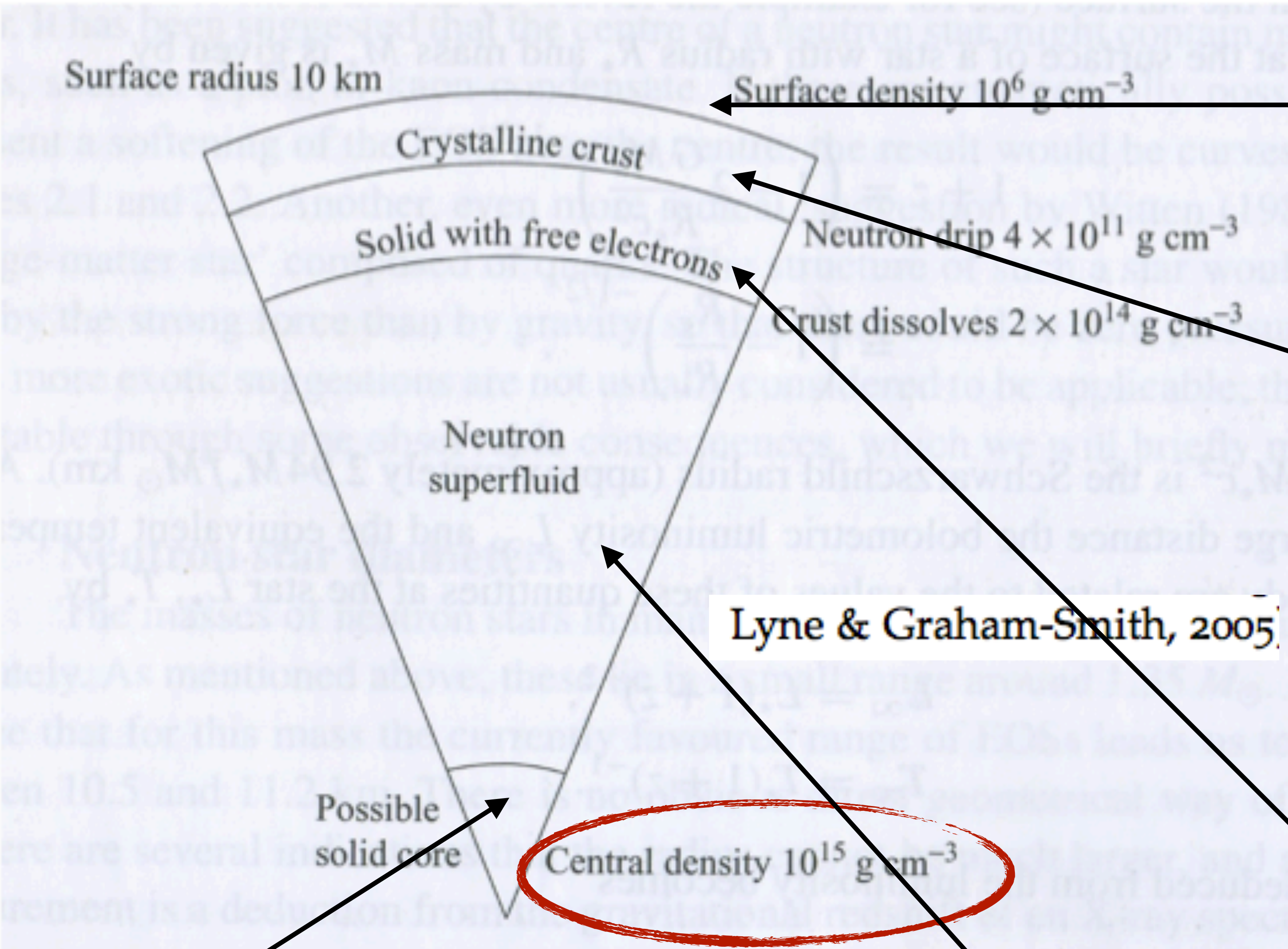
2) maximum mass is 0.7 M_{sun}, in contrast with observations

Realistic equation of state

Additional facts to consider:

- presence of protons an electrons: protons/neutrons ~10%
- presence of iron nuclei that can survive at law densities, but will progressively disintegrate beyond the neutron drip density ($4\ 10^{11}$ g/cm^3)
- strong force interaction beyond the nuclear density (~$2.8\ 10^{14}$ g/cm^3), when the mean particle distance is ~a Fermi (10^{-6} nm). Perfect gas approximation breaks down (perfect gas = non interacting gas)
- Strong Magnetic field

Equation of state within a NS



I) Surface, depth of ~100 m. EOS strongly influenced by magnetic field

II.)External crust of ~500 m. well understood: crystal structure of neutron rich nuclei (that interact via Coulomb force) + a fluid of free electrons and neutrons

III) Internal crust of ~1 Km. still below nuclear density. Nuclei progressively dissolve and neutron pressure become more important

IV) Interior: neutrons form a superfluid, nuclei completely dissolved, small fraction of e^- + p
<u>Strong force</u> between nucleons

V) Core. Most uncertain region: self-consistent QCD calculation difficult. 2 among several possibilities: formation of a solid core of neutron; phase transition towards a strange quark matter

NS structure: maximum mass

Note: despite the uncertainty on the equation of state, it can still be written as a barotropic relation $P = P(\rho)$

We are thus going to solve the NS structure as we did for WDs, but in a general relativistic framework

Einstein equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Source term is the energy-momentum tensor:

$$T_{\mu\nu} = \left(\rho c^2 + P\right) u_\mu u_\nu + P g_{\mu\nu}$$ with $$u^\mu = \frac{dx^\mu}{d\tau}$$ four-velocity

and Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$ where the space-time metric $$\mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu.$$

Gravity is a property of space-time, that depends on the local content of mass & energy

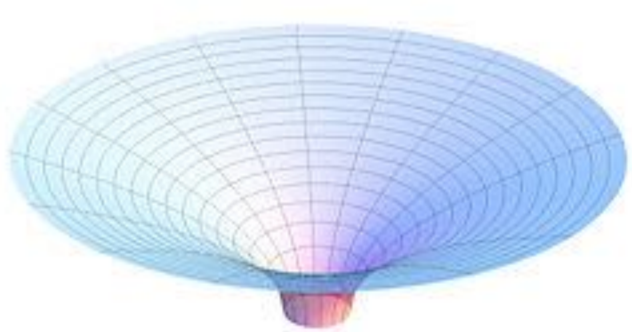
Schwarzschild solution

<u>At the outside of a neutron star</u> of M, we can assume vacuum $(T_{\mu\nu} = 0)$.

Static, spherical symmetric solution

$$
\begin{aligned}
N(r) &= \sqrt{1 - \frac{2GM}{rc^2}}, \\
A(r) &= \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}}, \\
\mathrm{d}s^2 &= -\left(1 - \frac{2GM}{rc^2}\right) c^2 \mathrm{d}t^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} \mathrm{d}r^2 \\
&\quad + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2\right)
\end{aligned}
$$

M is the gravitational mass, as measure by an observer at infinity



$r \to +\infty$ $\quad \mathrm{d}s^2 = -c^2 \mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2\right)$ Minkowski space

Tolman-Oppenheimer-Volkhoff (T.O.V)

Inside a NS. For a static spherical system, only the diagonal elements of $T_{\mu\nu}$ are non-zero. We thus have 3 independent equations:

with
$$
\begin{aligned}
N(r) &= e^{\frac{\Phi(r)}{c^2}}, \\
A(r) &= \frac{1}{\sqrt{1-\frac{2Gm(r)}{c^2}}}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}m}{\mathrm{d}r} &= 4\pi r^2 \rho(r), \\
\frac{\mathrm{d}P}{\mathrm{d}r} &= -\left(\rho(r) + \frac{P(r)}{c^2}\right)\frac{\mathrm{d}\Phi}{\mathrm{d}r}, \\
\frac{\mathrm{d}\Phi}{\mathrm{d}r} &= \frac{Gm(r)}{\mathrm{r}}\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}\left(1+4\pi\frac{P(r)r^3}{m(r)c^2}\right)
\end{aligned}
$$

Tolman-Oppenheimer-Volkhoff (T.O.V) cont.

In the limit: $Gm(r)/r \ll c^2$ and $P \ll \rho c^2$

They become the non-relativistic equations of 1) mass conservation, 2) hydrostatic equilibrium and 3) the equation for the gravitational potential

Boundary conditions

Relativistic regime

$$\rho(0) = \rho_c\,,$$

$$m(0) = 0\,,$$ Equivalent to dP/dr =0

$$\Phi(R) = \frac{c^2}{2}\ln\left(1 - \frac{2GM}{Rc^2}\right).$$ ensures continuity with solution inside and outside

Non relativistic regime

$$\rho(0) = \rho_c,\ \mathrm{d}P/\,\mathrm{d}r(0) = 0$$

For realistic equation of state the maximum mass is in the range

$$2\ M_{sun} < M_{max} < 3\ M_{sun}$$

mass measurements can be used to rule out e.o.s

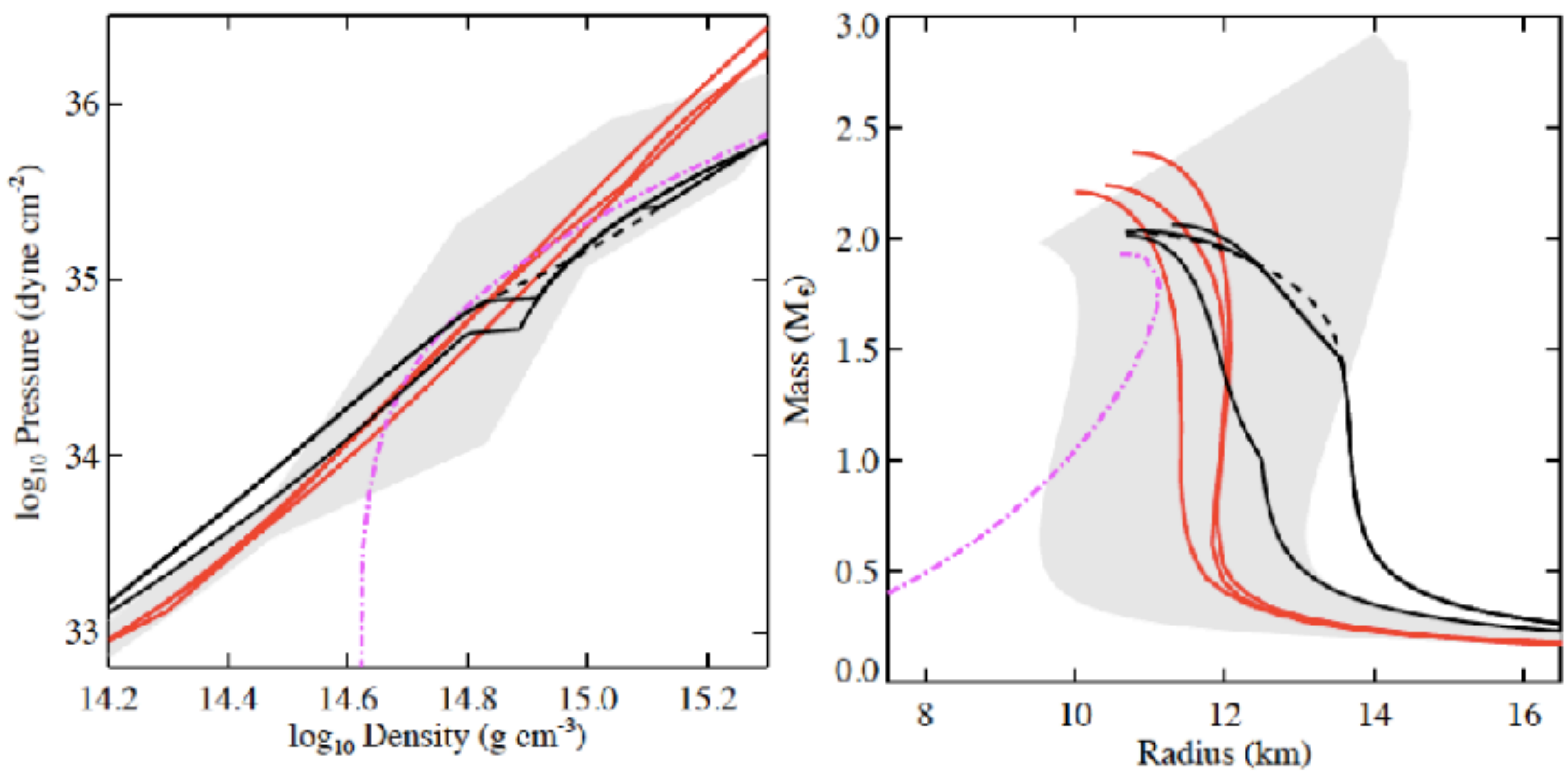


FIG. 3 The pressure density relation (EOS, left) and the corresponding M-R relation (right) based on models with different microphysics. Red: nucleonic EOS from Lattimer and Prakash (2001). Black solid: Hybrid models (strange quark core; Zdunik and Haensel 2013). Black dashed: Hyperon core models (Bednarek *et al.*, 2012). Magenta: A self-bound strange quark star model (Lattimer and Prakash, 2001). Grey band: range of a parameterized family of nucleonic EOS based on chiral effective field theory at low densities, which provides a systematic expansion for nuclear forces that allows one to estimate the theoretical uncertainties involved, combined with using a general extrapolations to high densities (see Figure 12 of Hebeler *et al.*, 2013, for examples of specific representative EOS lying within this band).

Anna L. Watts+16

See dedicated slides for NS EoS by A. Patruno