THE INFLUENCE OF SYSTEMATIC ERRORS ON THE CONSTRAINTS OF COSMOLOGICAL PARAMETERS

NADIEH BREMER & DAVID HUIJSER
Leiden Observatory, Leiden University, Netherlands

SUPERVISOR: PROF. DR. K.H. KUIJ肯KEN
Leiden Observatory, Leiden University, Netherlands
June 30 2009

Abstract

We present the influence of three systematic errors with a redshift dependency on the constraints of cosmological parameters. We used the Union dataset of SN Ia (Kowalski 2008). This dataset consists of 307 SN Ia (0 < z < 1.55). The Hubble constant $H_0$ is taken from the CMB 5-year WMAP (Komatsu 2009) and is set as 70.5 km \( \text{s}^{-1} \text{Mpc}^{-1} \). Using $\chi^2$-minimalization we calculated the best fitting values of $\Omega_\Lambda$, $\Omega_M$ and $w$ for a flat and non-flat $\Lambda$ Universe. For a $\Lambda$ Universe where $w = -1$ the best fit corresponds to $(\Omega_\Lambda, \Omega_M) = (0.93, 0.42)$, for a flat $\Lambda$ Universe where $w = -1$, we calculated $(\Omega_\Lambda, \Omega_M) = (0.73, 0.27)$ as the best fit. For a flat $\Lambda$ Universe with an adjustable $w$ the best fit was $(\Omega_\Lambda, w) = (0.62, -1.35)$. To see how the cosmological fit would respond to possible systematic errors we implemented three power law errors; a square-root, linear and quadratic dependency on redshift. Each was calculated with a positive or negative difference with the distance modulus in two general models; a $\Lambda$ Universe with $w = -1$ and a flat $\Lambda$ Universe with $w$ as a parameter. All the error models showed an almost linear trend of movement over the $\Omega_\Lambda$ and $\Omega_M$ plane. The systematic errors in a flat $\Lambda$ Universe with a variable $w$ did not give reliable results when the best fit came close to a Universe without vacuum energy. In the $\Lambda$ Universe where $w = -1$ it was only the positive square-root error which showed a trend towards a Universe without vacuum energy while the corresponding fit remained reliable.

Subject headings: Cosmology, cosmological parameters, systematic errors, vacuum energy, supernovae:

Union set

1. INTRODUCTION

Most people are interested in what they are made of, what makes them what they are and how does this relate to the world in which they live. Knowing what makes up a certain object can give you insight to its workings, origins and the properties of the building blocks. This applies from the smallest structures, such as protons and quarks, to the largest structures around and what could be bigger than the Universe itself? Knowing what the Universe is made of can tell you something of its past, the way in which it is headed and why it behaves the way it does. All very interesting questions to find the answer to. One might think that basic questions like the composition of the Universe have already been answered for years, and it is true that people have been trying to find the answer for some time. But the Universe is not so easily figured out, it seems to have a few hidden ‘tricks’ up its sleeve, some of which have only recently been discovered, such as the fact that a large part of all the matter in the Universe is Dark Matter, something we cannot even see (Freese, Fields & Graff 2000).

The question about the composition of the Universe is thus still very alive today and one that we were very eager to try and help solve.

In §2 we will introduce the basics of cosmology that are needed to calculate the relative abundance of the different components that make up the Universe. In §3 we will explain our research questions in more detail. Why the research is important and for what reasons. Afterwards we will inform the reader about the statistics (§4) that come into play to answer our research questions and the methods (§5) used. In §6 our results will be displayed after which we will discuss and summarize our conclusions (§7 & §8).

2. COSMOLOGY

The general idea of cosmology is based on the cosmological principle which states that the Universe is isotropic and homogeneous on large scales. From this assumption follows that an invariant interval in space-time between two events at coordinates $(t, r, \theta, \phi)$ and $(t + dt, r + dr, \theta + d\theta, \phi + d\phi)$ is defined by

$$ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right]$$

(1)

which is known as the Robertson-Walker Metric (RWM) (Coles & Lucchin 2002), where $c$ is the speed of light. The $K$ in eq. (1) is a dimensionless quantity which stands for the curvature of the Universe and has a negative value for an open space, zero for a flat space and a positive value for a closed space.

We have known for some time that the Universe is not static. Recent cosmological observations indicate an accelerated expansion (Riess 1998; Perlmutter 1999). The relative expansion of the Universe is represented by the scale factor $a(t)$, which has the dimension of length. It is a measure of the size of the Universe at a certain time.

To clarify, if one were to place a grid over the Universe,
all the galaxies could be represented by grid coordinates. The implication of the expanding Universe is that the grid itself is what expands\(^1\). It is the scale factor which represents the length scale of this expanding grid.

### 2.1. Comoving and proper distance

The \( r \) in the RWM is the comoving distance. It is a distance whose length scale grows with scale factor. Thus if we return to the idea that there is a grid over the Universe, the comoving distance of a certain galaxy would be its (comoving) grid coordinates. The Universe expands, nevertheless a galaxy keeps the same grid coordinates. Thus the comoving distance is independent of the Hubble expansion and the value can only change because a galaxy can move over the grid due to its peculiar velocity.

However the physical distance between two objects on the grid does change with expansion. It is known as the proper distance \( d \) and its value expands with scale factor. The relation between these two distances at the present time is given by the following formula

\[
d = a_0 r
\] (2)

where \( a_0 \) is the present day value of the scale factor and is usually defined as \( a_0 \equiv 1 \).

### 2.2. Friedmann equation

The description of the dynamics of the Universe is derived from General Relativity and is known as the Friedmann equation

\[
H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{K c^2}{a^2} + \frac{\Lambda c^2}{3}
\] (3)

Here \( H \) is the Hubble-parameter which gives the expansion rate of the Universe in \([\text{km s}^{-1} \text{ Mpc}^{-1}]\). \( G \) is the gravitational constant, \( \rho \) is the density of matter and radiation, and \( \Lambda \) is the cosmological constant which has the dimension of \([\text{m}^{-2}]\).

\( \Lambda \) was originally proposed early in the development of general relativity by Einstein (1917) in order to allow a static Universe solution. But it was abandoned when the Universe was found to be expanding. At the present the cosmological constant is invoked to explain the observed acceleration of the expansion of the Universe (Riess 1998; Perlmutter 1999).

If we are considering the cosmological constant as a form of vacuum energy then \( \Lambda \) can also be written as a density \( c^2 \Lambda = 8\pi G \rho_{\Lambda} \), where \( \rho_{\Lambda} \) is the density of vacuum energy.

### 2.3. Density parameter \( \Omega \) equation of state

It is usually more convenient to work with a density parameter instead of a density. The density parameter \( \Omega \) is defined as the ratio of the density to the critical density

\[
\Omega = \frac{\rho(t)}{\rho_{\text{crit}}(t)} = \frac{\rho(t)}{3H^2(t)}
\] (4)

The critical density \( \rho_{\text{crit}}(t) \) is the average density in the Universe at time \( t \) that would be needed to exactly halt, at some point in the future, the cosmic expansion.

The Universe is composed of several components: (dark)matter, radiation and vacuum energy, each of which can be represented by a density parameter.

The density of each component depends on the scale factor \( a(t) \) in a different way. This dependency comes from the equation of state which describes the relation between pressure and density for an ideal perfect fluid. If the mean free path between particle collisions is much less than the scales of physical interest, the fluid may be treated as perfect. This is a realistic approximation for the components in which we are interested. For a perfect ideal fluid the equation of state is

\[
p = w p c^2
\] (5)

where \( p \) is the pressure and \( w \) is the equation of state parameter, which is different for each component. It takes on the value 0 for pressureless matter, \( \frac{1}{3} \) for radiation and \( w \) has to be \(-1\) for a cosmological constant (corresponding to vacuum energy), i.e. a density independent of scale factor, which you can infer from eq. (6) below.

The first law of Thermodynamics tells us that \( dQ = dU + p dV \) where \( Q \) is the energy added to the system, \( U \) is the internal energy, \( p \) is the pressure and \( V \) is the volume of the system. Since the expansion of the Universe is presumably adiabatic (no energy is added) we can set \( dQ = 0 \). The first law of Thermodynamics can then be written as:

\[
\frac{dU}{U} = -p dV
\]

The total internal energy in a volume \( a^3 \) is given by \( U = p c^2 a^3 \). Using eq. (5) and the first law of Thermodynamics we can derive the following relation

\[
d(\rho c^2 a^3) = -p d(a^3)
\Rightarrow c^2 a^3 d\rho + p(c^2 a^3) da = -w p c^2 a^3 da
\Rightarrow \frac{d\rho}{\rho} = -3(1+w)\frac{d a}{a}
\Rightarrow \rho \propto a^{-3(1+w)}
\] (6)

For arbitrary \( K \) and \( \Lambda \) and a mixture of matter and radiation, the density \( \rho \) in eq. (3) can be rewritten. By using eq. (4) to express \( \rho_0 \) in terms of \( \Omega_0 \) and \( \rho_{\text{c,0}} \) and eq. (6) to write \( \rho \) in terms of \( \rho_0, a \) and \( \rho_{\text{c,0}} \), while using the corresponding values for \( w \) it becomes

\[
\rho = \rho_{\text{c,0}} \left[ \Omega_{M,0} \left( \frac{a_0}{a} \right)^3 + \Omega_{R,0} \left( \frac{a_0}{a} \right)^4 \right]
\] (7)

where \( \Omega_{M,0} \) and \( \Omega_{R,0} \) are the cosmological (density) parameters corresponding to matter and radiation, respectively, evaluated at the present time. Henceforth we will drop the suffix ‘0’ from the density parameters and the critical density as they will be evaluated at the present time further on.

### 2.4. Redshift

The redshift of an object is the shifting of its spectral features to longer wavelengths. It is mainly due to the combination of the Doppler effect and the expansion of the Universe. To be more precise, the term radial velocity is used primarily for Doppler motions, which are often the result of gravitational interactions between objects, while redshift is reserved for the expansion of the Universe. However, it is not generally possible to separate out cosmological expansion and Doppler velocities.
except for nearby objects and those known to be members of galaxy clusters.

Redshift, \( z \), is defined as

\[
  z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e}
\]

(8)

where \( \lambda_o \) and \( \lambda_e \) are the observed and emitted wavelengths, respectively. The redshift at the present time is zero.

Redshift is also a measure of the scale factor \( a(t) \) at the epoch when the source emitted the radiation

\[
  \frac{\lambda_e}{\lambda_o} = \frac{1}{1 + z} = \frac{a(t)}{a_0}
\]

(9)

For example, when a source such as a galaxy is observed with redshift \( z = 1 \), the scale factor of the Universe, when the light was emitted, was \( a(t) = 0.5 \), since \( a_0 \equiv 1 \). Hence the separation between galaxies at \( z = 1 \) was half its present day value. However, the redshift does not give us any information about when the light was emitted. Because it is possible to obtain the spectra of the SN Ia from measurements, a redshift can be labeled to each one, therefore in observational astronomy it is customary to work with redshift instead of scale factor.

2.5. Supernovae

Supernovae (SNe) are exploding stars which on their peak magnitude can outshine an entire galaxy. Because of their tremendous luminosity SNe can be seen up to great distances. They are distinguished in two main types; I and II, which were classified by Minkowski (1941). The classification is based on absorption lines of different elements in their spectra, where the first division is the presence or absence of hydrogen. If the spectrum contains hydrogen it is classified as a type II, otherwise it is type I. The subdivisions of the two types are both based on the presence of other elements and the shape of their lightcurves. Type Ia, for example, has, besides the absence of hydrogen, a strong singly ionized silicon(Si II) line at 615.0 nm. However it is only the type Ia SN that explodes through a different physical process. The SN Ia ignites through a thermonuclear explosion of a white dwarf while the other types of SNe are caused by core collapse (Woosley & Weaver 1986). An important aspect of the SN Ia is that this process depends on a physical limit on their mass, known as the Chandrasekhar limit, which is approximately 1.4\( M_\odot \) (Mazzali 2007). It is believed that the peak luminosity of the light curve is consistent across SN Ia, having a maximum absolute magnitude (Branch & Tammann 1992). SN Ia are thus classified as standard candles and can therefore be used to measure distances in the Universe.

The data that we use in our research comes from the Union set\(^2\). It contains 307 SNe ranging up to a redshift of 1.55. It contains the name, redshift, distance modulus and observational uncertainty in the distance modulus of each SN.


2.6. Luminosity Distance

To calculate the distance to a supernova, the luminosity distance \( d_L \) is used. It is defined as the distance at which an object would lie if the Universe were static. In a static Universe, the propagation of light follows the inverse square law

\[
  F = \frac{L_{\text{obs}}}{4\pi d^2}
\]

(10)

where \( F \) is the measured flux, \( L_{\text{obs}} \) is the observed luminosity and \( d \) is the proper distance to the source. But the Universe is not static and if we take the expansion into account there will be two additional factors of redshift \( 1/(1+z) \). The first comes from the fact that the Universe has gotten bigger while the emitted photons were travelling towards Earth. The expansion causes the wavelength of the photons to increase with a factor \( 1/(1+z) \) which leads to a lower observed luminosity.

The second factor comes from a time-dilatation. The rate at which the photons are emitted by the source and the rate at which they are observed by a telescope are not the same. Since the distance between the photons has increased due to the expansion of the Universe, it again leads to an observed luminosity which is lower than the emitted luminosity by a factor \( 1/(1+z) \). In the end we can say that \( L_{\text{obs}} = L_e/(1+z)^2 \), where \( L_e \) is the luminosity emitted by the source.

The distance in eq. (10) is given as the proper distance \( d \). Preferably, we use eq. (2) to express the luminosity distance in terms of the comoving distance \( r \). Substituting the above changes into eq. (10) leads to a following equation for flux which we define \( d_L \) to be the luminosity distance

\[
  F = \frac{L_e}{4\pi a_0 r^2 (1+z)^2} = \frac{L_e}{4\pi d_L^2}
\]

\[
  d_L \equiv a_0 r (1+z)
\]

(11)

Since we are assuming that all SN Ia have the same maximum luminosity \( L_e \), we can calculate \( d_L \) by measuring the received flux \( F \) here on Earth, once the value for \( L_e \) has been established.

2.6.1. Distance Modulus

In order to calculate the luminosity distance as a function that depends on the density parameters, let us use a Robertson-Walker metric in which we are at the center of coordinates, and consider a light ray coming towards us along the radial direction \(( d\theta = 0, d\phi = 0 \)). A light ray always obeys the relation \( d\sigma^2 = 0 \). For such a ray eq. (1) becomes

\[
  \int^r_0 \frac{dr}{\sqrt{1-Kr^2}} = \int^{t_o}_0 \frac{dt}{a}
\]

(12)

where \( t_0 \) and \( t_e \) are the observed and emitted time, respectively.

We would like to express \( dt \) as a function of the scale factor \( a \). This requires the introduction of two more cosmological parameters

\[
  \Omega_K = -\frac{Kc^2}{a_0^2 H_0^2} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3 H_0^2}
\]

(13)
where \( \Omega_K \) and \( \Omega_\Lambda \) are the density parameters for the curvature of the Universe and vacuum energy, respectively. Although curvature has no real density it is still common to use this substitution with a density parameter \( \Omega \) for clarity in the equations following. Using equations (7) and (13) to replace \( \rho, K \) and \( \Lambda \) and \( \rho_c \) from eq. (4) and that it is being evaluated at the present time. Combining all density parameters

\[
H^2 = \frac{8\pi G \rho_c}{3} \left[ \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_K \left( \frac{a_0}{a} \right)^4 \Omega_R \left( \frac{a_0}{a} \right)^2 + \Omega_\Lambda \right]
\]

Where the fraction before the square brackets is now apparent to be \( H_0^2 \), remembering the definition of \( \rho_c \) from eq. (4) and that it is being evaluated at the present time.

Going back to the equation for \( d_L \), eq. (11), and replacing \( r \) by eq. (20) we can finally express the luminosity distance as a function of the density parameters and redshift, where \( dz \) is calculated in Mpc.

\[
d_L(z) = \frac{c(1+z)}{H_0 |\Omega_K|^{1/2}} \times \sin \left[ |\Omega_K|^{1/2} \int_0^z \frac{dz}{H(z)} \right] \]

3. Research Questions

The rate of the expansion of the Universe is governed by its composition. For example, matter feels gravity which attracts. If the Universe were to consist primarily out of matter, one would assume the Universe to slow down and eventually collapse again. But the Universe is made up of more components, thus the evolution of the Universe depends on the values of all these parameters. An important aspect in determining the values of the cosmological parameters is thus finding the way in which the Universe has expanded throughout time.

3.1. General models

As could be perceived from eq. (6) all cosmological parameters evolve differently with scale factor. The speed of the expansion is not constant throughout time. The observations of the SNe should support this theory, and with the use of eq. (25), we want to find the values of the density parameters which are best described by the SNe.

3.1.1. Cosmological Constant

In the equations of luminosity distance (22, 23, 24) we have assumed a true cosmological constant, meaning that \( \Lambda \) corresponds to an equation of state parameter \( w \) of -1. This gives a constant value for the density of vacuum energy: \( \rho_\Lambda \propto a^{-3(1+w)} = a^0 = \text{constant} \). However vacuum energy is still a part of cosmology that is not entirely understood. Thus to take into account the possibility that the vacuum energy does not have a constant density, we will also try to vary the equation of state parameter for vacuum energy \( w \) and see what the effect will be on the best fitting values of the density parameters.

This would mean that \( \Omega_\Lambda \) has to be replaced by \( \Omega_\Lambda(1+z)^{3(1+w)} \) in eq. (18).

3.2. Systematic errors

One of the basic assumptions in cosmology is that SNe are standard candles. For SNe this means that they all have approximately the same maximum absolute magnitude. But the workings of a SN are still not very well understood. Without more extensive knowledge on SNe
one cannot really be sure that the assumption of a constant maximum absolute magnitude is correct. That is why, besides determining the density parameters in the general model, we want to find out what the effect would be if we introduce three systematic errors \( f(z) \) which depend on redshift.

There are several reasons why there might be a deviation from the assumption of standard candles. The first could thus be a lack of understanding of SNe, for example, the composition of the progenitors of the SNe might digress at higher redshift. A different metallicity might lead to a change in the Chandrasekhar-limit or it could influence the way in which the white dwarf collapses. Extinction could have a redshift dependency which is different from the commonly used ‘color correction.’ There is also the possibility that there is some unknown effect which would cause a systematic error.

There is no physical ground as to why we should choose a certain systematic error, therefore we decided to start with three basic power-law models, which include a square-root, linear and quadratic dependency on redshift. The errors are added to the basic formula for distance modulus, eq. (25).

\[
\mu(z) = 5 \log \left( \frac{d_L(z)}{Mpc} \right) + 25 + f(z) \tag{26}
\]

where

\[
f(z) = \begin{cases} 
C \cdot \sqrt{z} & \text{square root redshift dependency} \\
C \cdot z & \text{linear redshift dependency} \\
C \cdot z^2 & \text{quadratic redshift dependency}
\end{cases}
\tag{27}
\]

where \( C \) is a constant which has to be calculated.

Because we do not know whether the reasons for the systematic error might influence the observed luminosity to be brighter of fainter, we try both possibilities. A positive \( C \) would mean that SNe at higher redshift \( z \) appear more luminous and a negative \( C \) would mean that SNe appear to become less luminous at increasing redshift.

There are several subjects we would like to investigate with the introduction of the systematic errors. We are interested in the way the best fit will migrate for increasing values of \( C \). In what direction will density parameters change.

To determine whether the errors would significantly change the cosmological models, we compare the original and the systematic error model. What would be the minimal value of \( C \) for which the original and systematic error best fit will have a \( 6\sigma \) error distance?

We have chosen this distance because the contourplots shown in §6 are presented with a \( 3\sigma \) error contour. Thus if these two contour sets are disjoint we define the change in the model at value \( C \) to be significant and there will be a \( 6\sigma \) distance in total between the best fitting values.

At this distance the systematic error will already be greater than the statistical one, but it will give a good visualization for the previous research question about the migration of the best fitting values. It will also give an upper value of the allowed \( C \), for which the statistical error might still be comparable to (or preferably be greater than) the systematic error.

Since vacuum energy seems to be needed to explain the past expansion of the Universe, we are interested how the constraints on vacuum energy will change with the introduction of the systematic errors. Is there a certain systematic error where the best fit migrates towards a Universe without vacuum energy?

### 4. STATISTICS

To answer our research questions we must make use of statistics and probabilities. Starting with a set of measurements of distance modulus \( \mu \) and corresponding redshift \( z \) and an assumed relation between \( \mu \) and \( z \) given by eq. (25), we would like to determine if the assumed relation accurately predicts the measurements. If this is the case, what values of the density parameters in eq. (25) would best describe the observed data.

#### 4.1. Maximum likelihood

If we begin by assuming that the statement above is true, we can find the best fitting values of the (cosmological) parameters by using \( \chi^2 \)-minimalization. We use the observed value \( y \) and an assumed relation between the dependent variable \( x \), and the independent variable \( y \), given by the function \( f(x) \).

The resulting difference between the predicted value \( f(x_i) \) and the observed value \( y_i \) is the deviation \( \Delta y_i \). It is typical to assume that these deviations will follow some distribution. If the deviations from the mean follow a Gaussian distribution, which is correct for (most) physical processes\(^3\), the probability of observing \( y_i \) is given by the probability function

\[
P(x_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2 \right] \tag{28}
\]

Here \( \sigma_i \) is the total error, which can consist of point dependent errors, such as the measurement precision of a particular data point and sample dependent errors, which are the same for all the measurements, such as an intrinsic error.

If we can assume that the measurements are independent of each other, we can calculate the total probability of observing the entire set of \( N \) measurements. This is equal to the product of the probabilities of each data point

\[
P_{\text{tot}} = \prod_{N} P = \left[ \prod_{N} \frac{1}{\sigma_i \sqrt{2\pi}} \right] \times \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2 \right] \tag{29}
\]

The probability function \( P_{\text{tot}} \) thus expresses how likely the observed distribution is, given our assumed relation. We must maximize \( P_{\text{tot}} \) to find the optimum description of our set of measurements.

As can be seen from eq. (29) maximizing the probability is equivalent to minimizing the sum in the exponential term. Finally, we can define the \( \chi^2 \) statistic by

\[
\chi^2 \equiv \sum_{i=1}^{N} \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2 \tag{30}
\]

The method of \( \chi^2 \) is thus built on the hypothesis that the best fit between the measurements and the values

\(^3\) According to the central limit theorem
calculated according to the assumed relation is one which minimizes the weighted sum of squares of the deviations $\Delta y_i$.

4.1.1. Reduced $\chi^2$

If the observed values $y_i$ were to agree exactly with the predicted values $f(x_i)$, one would find $\chi^2 = 0$. However, since we are working with a probability, this is not a very likely outcome. For any physical experiment where the observed values will be different for each time the experiment is done, we expect $\chi^2_{\min} \approx N$, where $N$ is the number of data points. The true expectation value for $\chi^2$ is in fact

$$\langle \chi^2_{\min} \rangle = \nu = N - N_f$$

where $\nu$ is the degrees of freedom (DoF) and is equal to the number $N$ of datapoints minus the number $N_f$ of fitted parameters.

It comes from the fact that if the fitting function accurately predicts the means of the observed distribution, the estimated variance $s^2$, should agree well with the variance of the observed distribution $\sigma^2$. Thus their ratio should be close to one. The ratio of $s^2/\sigma^2$ can be estimated by $\chi^2/\nu$.

It is convenient to define the reduced chi-square as $\chi^2_r \equiv \chi^2/\nu$, with expectation value $\langle \chi^2_r \rangle = 1$. A $\chi^2$ near 1 usually means that within the error bars you appear to have a good fit to the model.

If the $\chi^2$ is much larger than 1, it could have several reasons. For one, you could have done a lousy job making measurements (i.e. something is wrong with the technique or machinery). It could also mean that you are working with a bad model, one that is an inappropriate fitting function. There could be a systematic error that remains unaccounted for in the model. Or it could be that you have been overly optimistic about the observational uncertainties (the error bars are too small.)

If on the other hand the $\chi^2$ is much smaller than 1, it probably means that you have somehow been too pessimistic about the observational errors and that you have overestimated the size of the errors.

4.2. $\chi^2$-minimalization

To clarify we will expand the way in which the $\chi^2$ is written, by showing the parameters that are used in the research explicitly and replacing the total error by the sample (intrinsic) and data-point dependent components.

$$\chi^2 = \sum_{SNe} \left( \frac{\mu_{obs,i} - \mu(z_i; \Omega_M, \Omega_K, w)}{\sigma_{obs,i}^2 + \sigma_{int}^2} \right)^2$$

where $\mu_{obs}$ is the observed distance modulus and $\sigma_{obs}$ represents the uncertainty in $\mu_{obs}$ due to measurement errors, lensing, the uncertainty in the Milky-Way dust extinction correction and a velocity dispersion error due to host galaxy peculiar velocities of 300 km/s, where both are given in the Union set (Kowalski 2008). $\mu$ is the distance modulus calculated with eq. (25) and $w$ is the equation of state parameter corresponding to vacuum energy. The intrinsic dispersion $\sigma_{int}$, which is sample dependent, contains a dispersion due to possible unaccounted-for intrinsic errors. For example, the different circumstances of the progenitors, such as rotation speed, which could cause the white dwarf to explode at a mass somewhat different than the Chandrasekhar limit, thus giving a slight different maximum absolute magnitude.

At first it is set to 0.15 because algorithms at the present day, which correct the peak magnitude, are capable of standardizing SNe Ia to a level of ~ 0.15 magnitudes (Kowalski 2008). To obtain self-consistency we would like to get a $\chi^2_{\min}$ of 1. With the fitting parameters corresponding to the $\chi^2_{\min}$ of the first fit (where $\sigma_{int}$ was set to 0.15) we iteratively calculate the value of $\sigma_{int}$ until it corresponds with a $\chi^2$ of 1. With this more precise value for $\sigma_{int}$ we redetermine the best fitting cosmology using eq. (32).

4.3. Constraints

The Friedmann eq. (2) tells us that the following has to be true: $\Omega_\Lambda + \Omega_M + \Omega_K = 1$. This can be seen by choosing $z = 0$, the redshift at the present time, in eq. (18) and remembering that $H(z=0) = H_0$.

$$H(z=0) = H_0 \left[ \Omega_\Lambda + \Omega_K + \Omega_M + \Omega_R \right]^{1/2}$$

According to Weinberg (2009) the value of $\Omega_R$ is extremely small ($\approx 4.15 \times 10^{-5} h^{-2}$), thus we have decided to neglect the density parameter corresponding to the radiation in our research.

Since all the density parameters together have to add up to one and we have neglected $\Omega_R$, the remaining two density parameters determine the last one. Thus the calculated $\mu$ depends only on redshift $z$, $\Omega_\Lambda$ and $\Omega_K$, where $\Omega_K$ follows from: $\Omega_K = 1 - \Omega_\Lambda - \Omega_M$.

Because we cannot try all possible values in $\mathbb{R}$ for a parameter, boundary values of an interval have to be decided. Sometimes a boundary value is provided by physics, such as the matter density parameter. It is not possible to have a negative or zero value for $\Omega_M$ since it is acknowledged that we are here.

Other times we only have a vague idea about where to expect the best fit to lie. In this second case, we start out by taking a large interval with a big stepsize. After performing $\chi^2$-minimalization once, we will get a crude estimate of the best fit, but it will tell us in what interval we should focus to get a better value.

In our research we expected that the best fit for $\Omega_\Lambda$ and $\Omega_M$ would lie between 0 and 1, because when one of them reaches the boundary value of 1, it would have the same density as $\rho_c$, which is the only density that is really defined by the Friedmann equation. Because the resulting confidence contours, which are shown in §6, stretch beyond $\Omega_\Lambda = 1$, we increased the interval of this parameter to 1.5 to see how the entire $3\sigma$ contour would look.

The same reasoning of wanting to see the entire contour led us to set $w$ as $-3 < w < -0.5$ in the models where $w$ is an adjustable parameter.

4.4. Marginalization

If we want to see if there is a correlation between two parameters or if we want to calculate the 1$\sigma$ error for

\[ \text{which one expects for the } \chi^2_{\min} \]
It is only the smallest possible χ2 that remains at the set of unique parameters. To expand this to a more general explanation; of all the possibilities that χ2 might take on the number of fitted parameters of the distribution. A confidence interval gives the chance that the value of χ2 corresponds to a unique set of parameters. The choice for confidence levels is given in the Table 1. For example one might be interested in the confidence level of each parameter separately, in this case we need to marginalize out the unwanted parameters. Marginalization means reducing the number of parameters on which a distribution depends, leaving us with a distribution of the variables of interest that takes account of the range of plausible values of the unwanted parameters.

Of course, marginalization will broaden the distribution of the parameters we do want, because it is absorbing the uncertainty in the parameters we do not want.

4.4.1 Marginalization in χ2

For χ2-minimization this process takes on a straightforward implementation. You start with a discrete distribution where every χ2 corresponds to a unique set of parameters. Say that you have a distribution with parameters W, X, Y and Z, where each has an interval which has been divided into a values. You want to marginalize it to a distribution that only depends on W and X. Taking a unique set of a Wj and Xj value still leaves a2 possible χ2 values from the entire intervals of Y and Z. It is only the smallest χ2 value of the a2 possibilities that remains at the set of Wj and Xj. Repeat this step for each unique set of the W and X interval, until in the end you have a distribution where there is only one corresponding value of χ2 for each Wj and Xj. To expand this to a more general explanation; of all the χ2 values in the intervals of the unwanted parameters, it is only the smallest that remains at the set of unique values of the parameters of interest.

4.5 Confidence levels

According to Numerical Recipes (Press 1986) there is a natural choice for the shape of the χ2 confidence intervals. A confidence interval gives the chance that the actual best fit lies within that region and its size depends on the number of fitted parameters of the distribution. For example one might be interested in the confidence level of each parameter separately, in this case N = 1. The 1σ error or confidence level would be given by the distance between the parameter value Xmin corresponding to χ2|min and the parameter value Xj corresponding to a χ2j for which χ2j|min - χ2j = 1 ⇒ Δχ2 = 1.

The choice for confidence levels is given in the Table 1 which gives the Δχ2 distance to χ2|min as a function of the number of parameters of the distribution (Press 1986).

4.6 Statistical and systematic errors

One has to take into account several kinds of errors or uncertainties. Because these are often confused it is important to clearly state what each term means. Although even within the field of statistics there seem to be some dispute about what we should call errors and what we should call uncertainties. Therefore we will clearly state what each term means and rule out any ambiguity to avoid any obscurity over nomenclature. At first we will briefly summarize some of the terminology, mostly adopted from Bevington & Robinson (1992).

There is a difference between the terms accuracy and precision. The accuracy is a measure of how close the result of the experiment is to the true value. If a sample consists of multiple measurements it will form a distribution. The accuracy is the deviation of this distribution from the true value. The precision is a measure of the spread of the distribution around the average. The accuracy of an experiment is generally dependent on how well we can control or compensate for systematic errors that will make our results different from the true value. The systematic errors are errors that arise from consistently overestimating or underestimating a true value. They may result from faulty calibration of equipment, from a bias on the part of the observer or from a lack of understanding of the physical process. These errors can only be reduced by understanding the equipment, circumstances and processes involved.

The precision of an experiment is dependent on how well we can overcome random errors. These are fluctuations that yield results that differ from experiment to experiment. Any factor that randomly affects the measurement of the variable across the sample contributes. This error randomly alters the scores up and down, thus there is no consistent effect across the entire sample. Therefore if we would add all the errors, the sum would be zero, because there are as much positives as there are negatives. Thus it adds a spread to the distribution, but it does not alter the average. The random error shows a 1/√N diminution, where N is the number of observations or number of datapoints.

The statistical error has a similar effect, it forms a spread on the distribution and does not change the average. Although the random errors are caused by deviations in measurements, statistical errors arise from statistical calculations.

There is also an intrinsic dispersion which comes from different circumstances of the observed SNe. Because not all SNe occur under the same circumstances, there is a spread in the observations. The term error signifies a deviation from some true value. Often we do not know exactly what the true value is, therefore we can only consider estimates of the error. There is a finite precision in the measurements, thus there is a finite precision in the determination of the true value as well. Resulting in the fact that is impossible to determine the corresponding error. Nonetheless, with the proper mathematical tools we can estimate this error. This estimate is called an uncertainty.

Applied to our research we have an uncertainty which comes from observations. We have an intrinsic dispersion which comes from different circumstances of the observed SNe. And there is the statistical error which comes from the statistical calculations. All these should only give a spread in the distribution. Finally, we have a systematic error which comes from a lack of understanding of SNe. It alters the average of the distribution.

5 Technically this is an uncertainty.
5. PROGRAMMING IDL AND C

We start with a set of 307 SNe where for each of the SNe we have the name, redshift, distance modulus $\mu$ and observational uncertainty in $\mu$. To calculate the $\chi^2$ from eq. (32) we need two other values, namely the calculated distance modulus and the intrinsic dispersion. The method of calculating $\sigma_{\text{int}}$ is explained in §4.2. To find the theoretical value of the distance modulus for each set of parameters, the integral from equations (22, 23, 24) has to be calculated and the numerical ‘Adaptive’ method is used. The integral appears for every one of the 307 SNe from the Union set for every combination of the density parameters in the fit. We use a grid-method to calculate $\chi^2$ for each possible combination. To gain good accuracy a large grid size is preferred. All these factors combined can take up a lot of time using IDL only. Since the programming language of C is known to be faster than IDL when it comes to calculations, we split the programming in two parts. However, we also have to make several contour and Hubble plots. As a result we do not use C only, since IDL is much more versatile than C in visual programming.

It is of course not possible to use a continuous distribution for the adjustable parameters. Therefore we divide each of them into a certain number of discrete values inside the allowed parameter intervals, where each parameter is divided in the same amount of steps. Thus if the interval of one parameter is bigger than the other, the stepsize in the former is also bigger than the latter.

All our programs start in IDL where immediately a call_external command is executed to initiate a C program. The C program reads the SNe data and runs through several loops needed to calculate the $\chi^2$ for all possible sets of the adjustable parameters. While running through the loop, each $\chi^2$ is compared to the smallest $\chi^2$ that emerged from the parameter sets that have already been run through, to test if the new one is smaller yet. If this is the case, its value is saved along with its corresponding parameters. Thus after the entire grid has been covered, a smallest $\chi^2$ along with a best fit is known.

With these best fitting values for $\Omega_\Lambda$, $\Omega_\text{M}$ and $\Omega_k$, the distance modulus $\mu$ from eq. (25) is calculated for many points between redshift 0 and 5 to get a visualization of a continuous line in a plot. Finally, all calculated $\chi^2$ and distance moduli are saved in a textfile. IDL reads the new textfiles, makes the required contour and Hubble plots and calculates the $1\sigma$ errors.

In the beginning we tried passing on all calculated values of $\chi^2$ and $\mu$ directly from C to IDL, thus without first writing it away to a textfile. However, it turned out that C and IDL are very difficult to merge if it involves pointers to the places where the values are stored in the memory. Writing it to a textfile offers many benefits, besides the more foolproof method, because the calculated data can always be looked up. Also, if we want to use the data in IDL, the entire C program, which can take up a lot of time, does not have to be called again.

5.1. Incorporating the systematic errors

Our goal with the systematic errors is to find the minimal value of the constant $C$ from eq.(27) for which the $3\sigma$ contour from the original and systematic error model will have no more overlap. This process is executed by replacing the original equation for distance modulus $\mu$ from eq. (25) by eq. (26). The value of $C$ starts at 0.01 and a new best fit is calculated. The following step is the comparison of the original contour with the new contour to see if there is overlap. If the contours have a non-empty intersection the program raises the value of $C$ and a new best fit is calculated. The following step is repeated until the two sets become disjoint. After a $C$ is found for which both sets have no more overlap, the program runs out of its loop and the values corresponding to this new best fit are saved.

6. COSMOLOGICAL FIT RESULTS

Using the method of $\chi^2$-minimalization, we have calculated the best fitting values of the density parameters for several cosmological models, such as a flat and non-flat $\Lambda$ Universe and a flat $\Lambda$ Universe with a different value for the equation of state parameter corresponding to vacuum energy. The calculated values for the three general models which were evaluated, are combined in Table 2.

![Fig. 1. — The contourplot of a $\Lambda$ Universe where $w = -1$, with $\Omega_\Lambda$ against $\Omega_\text{M}$. The best fitting values are $(\Omega_\Lambda, \Omega_\text{M}) = (0.93, 0.42)$. The contours represent the 68.3% (white), 95.4% (black) and 99.7% (grey) confidence levels. The pink line represents a flat Universe.](image-url)
Influence of systematic errors on cosmological parameters

6.1. General Models

We have found the following best fitting values of a Λ Universe with a true cosmological constant, \((\Omega_\Lambda, \Omega_M) = (0.93, 0.42)\). The confidence level contours which belong to this fit are given in Fig. 1.

It is most remarkable that the contours seem to indicate that the majority of the preferred combinations of values lie in a closed Universe with \(K > 0\).

The corresponding Hubble plots of distance modulus against redshift is shown in Fig. 2.

In Fig. 2(a) we have plotted all 307 of the SNe. Although this does not give a good representation of the accuracy of our best fit, it does give a good idea about how the SNe are spread over a range of \(\sim 1.5z\). It shows that by far, most SNe have a redshift less than one.

To get a better view about how well our best fit describes the SNe, we have decided to bin the SNe into bins of 0.1 redshift, which is shown in Fig. 2(b). Each bin thus contains all the SNe between redshift 0.1a < z < 0.1a + 0.1, where a is the number of the bin. Because the distance modulus increases fast in the first few bins, we have centered the circle representing the mean \(\mu\) on the average redshift of all the SNe in the bin. Now it becomes much more apparent that our best fit accurately describes the binned SNe.

Fig. 2(c) shows, besides our best fit, three extreme models for which there would be only one component in the Universe (matter, vacuum energy or curvature). Up to a redshift of 1.5 the differences appear to be relatively small and some lines can hardly be distinguished. Therefore we made a residual Hubble plot, Fig. 2(d), which

\[w = -1\]

\[\Omega_K < 0, \text{ see eq. (13)}\]
Parameter are (ΩΛ, w) = (0.02, −1.35). The contours represent the 68.3% (white), 95.4% (black) and 99.7% (grey) confidence levels.

shows only the difference in distance modulus with an empty Universe (ΩK = 1), represented by the horizontal line. The redshift has been changed to a log and goes up to z = 5 to make the trends in z become more apparent. The best fit corresponding to a flat Universe where w = −1 and the corresponding values are (ΩΛ, ΩM) = (0.93, 0.42). The green line represents a flat Universe where w is an adjustable parameter, with values (ΩΛ, w) = (0.62, −1.35).

is usually assumed. Where in the former contour plot that was a flat Universe, here the inner most contour lies just below the predicted value of −1.

In Fig. 4 you can see the difference in distance modulus between the best fit for a non-flat Universe where w = −1, which is represented by the black horizontal line. The green line is the best fit for a flat Universe where w is an adjustable parameter. The deviations remain small and both lines give a good description of the SNe.

6.2. Introducing a systematic error when w = −1

Fig. 6 shows the six contourplots which belong to the three different systematic errors from eq. (27). The grey filled contour belongs to our general model of Fig. 1 and the empty colored contours correspond to the new systematic error models. For each error model we tried a positive and negative value for the constant C. Table 3 in the appendix shows the best fitting values and their 1σ errors for all the models that we have calculated in our research.

The systematic error contours correspond to the minimal value of C for which the entire 3σ contour from the original and error best fit have no more overlap. The value of C which belongs to this model can be found in the subtitles of each plot in Fig. 6. The upper three plots have a positive C and the bottom plots have a negative C.

It is helpful to see how these models relate to each other and the best fit from the general model, to find out if they give a good description of the SNe. Fig. 5 shows two residual Hubble plots of redshift against the difference in distance modulus to the general model (ΩΛ, ΩM) = (0.93, 0.42). The SNe have been binned again and we note that the error bars represent the observational uncertainties only, because the intrinsic uncertainties are calculated iteratively for each model and thus vary from model to model.

From the top plot of Fig. 5 one can see that all the lines still give a good representation of the SNe. The SNe with a redshift greater than one, which encompass only ~5% of the total set, might give you the idea that the best fit line should be further down for redshifts higher than one. However in Fig. 2(a) we could see that by far most SNe had redshifts lower than one, thus the best fit will be determined primarily by those SNe.

The six model lines in the top plot of Fig. 5 can barely be distinguished. Therefore a version which focuses on the area between the two dashed grey lines from the top plot is shown in the bottom plot. From this plot one can more easily predict the evolution of the model lines with increasing redshift.

6.2.1. Ratios of the error contours

Looking and comparing the error contours, one can see several similarities. The most obvious connection is the fact that it is the sign of C that defines if the error plot lies above or below the original contour. A positive C moves towards a Universe where there is a larger matter component, while a negative C moves towards a Universe without matter. Let us explain where this difference comes from for the case where C is positive. The SNe have a fixed combination of distance modulus and redshift. We have two equations to describe this
Influence of systematic errors on cosmological parameters

Thus in a Universe with a systematic error, the total distance modulus is actually a combination of a distance modulus like in the general model and a systematic error. Since the first equation of (34) must have the same answer as the third at the same redshift, we have \( \mu_{\text{Systematic}} \leq \mu_{\text{General}} \).

The distance modulus comes from subtracting the absolute magnitude from the apparent magnitude: \( \mu = m - M \). Since one has to assume a constant absolute magnitude of the SNe if they are to be used as standard candles, it will be the apparent magnitude that is different between the two models: \( m_{\text{Systematic}} < m_{\text{General}} \). A smaller apparent value of \( m \) means a brighter observed SNe. Thus in comparison with the general Universe, we would say that the SNe stands closer to us in the ‘systematic error’ Universe. The implication is that the Universe has expanded less in the systematic error Universe. For this to happen, matter domination should have ended more recently than in the general Universe, since it is vacuum energy that is the source of the accelerated expansion\(^8\). Thus the ratio of \( \Omega_M \) and \( \Omega_\Lambda \) would be larger today in a systematic error Universe. And that is exactly what can be seen from the upper three plots of Fig. 6.

For a negative \( \Lambda \) the Universe had to be vacuum dominated earlier in time, thus the ratio of \( \Omega_M \) and \( \Omega_\Lambda \) today would be smaller than in the general Universe, which is also visible in the bottom three plots of Fig. 6.

6.2.2. Migration of the Contours

By comparing the positive and negative models of a singular systematic error, one can see in what direction the contour would migrate for even higher values of \( \Lambda \). Thus extrapolating from the plots that we have, we can predict what sort of Universe will arise.

The contours of the square root of Fig. 6(a) & 6(d) seem to move over an almost vertically inclined line\(^9\). It is apparent that the square-root error has little effect on the best fitting value of \( \Omega_M \) and that it is \( \Omega_\Lambda \) which undergoes a great shift. Especially the positive square root plot in Fig. 6(a) shows a very interesting migration. We will come back to this systematic error in §6.2.3.

The contours in the linear error plots of Fig. 6(b) & 6(e) seem to move the same as the square-root contours, but with a 90° rotation, thus a horizontally inclined line. Meaning that this time it is the value of \( \Omega_\Lambda \) that is hardly affected by the error, while it does influence \( \Omega_M \) considerably. However, close inspection shows that the line has a slight negative slope. The positive linear error plot could therefore eventually migrate towards a Universe without vacuum energy. On the other hand, the slope of the migration is almost horizontal, meaning that before \( \Omega_\Lambda \) comes near 0, \( \Omega_M \) will become much greater than 1. This does not agree with the current knowledge of the Universe. Still, there is a more important reason why we think that such a Universe is not possible. Namely, the constant \( C \) would become very large. This implicates that the best line in a Hubble plot would, most likely, poorly describe the SNe. To compensate, the intrinsic uncertainty, which we calculate iteratively to get a \( \chi^2 \) of 1, would become too great. Combining the observational and intrinsic uncertainties would produce extreme error bars which would not give a physical result any more.

The quadratic error in Fig. 6(c) & 6(f) moves over a positively inclined slope. In the negative quadratic error plot, Fig 6(f), the contour seems to move towards a Universe where there would be no need for vacuum energy. However, before the contour reaches the line of zero \( \Omega_\Lambda \) it becomes a Universe with a negative matter density parameter. And as discussed in section 4.3, that is not a physically correct model.

\(^8\) Matter corresponds to decelerated expansion

\(^9\) It is slightly negatively inclined
The six contour plots of the three systematic errors from eq. (27), where each has two options: a positive and negative value of the constant $C$. The plots show $\Omega_{\Lambda}$ against $\Omega_M$. The grey filled confidence contours represent the general model best fit from Fig. 1. The colored empty confidence contours correspond to the systematic error fit given in the subtitle. For both contours the 68.3% (inner), 95.4% (middle) and 99.7% (outer) confidence levels are shown. The top half represents positive values of $C$ and on the bottom are the plots with a negative $C$. The values of $C$ which are given in the subtitles are the minimal values of the constant for which both the original and systematic error 3σ contour have no more overlap. The pink line represents a flat Universe.
6.2.2. Positive square-root error

In §6.2.2 we discussed the possibility that the positive square-root systematic error model seems to migrate towards a Universe without vacuum energy while the value of $\Omega_M$ does not become greater than one. Here we come back on this model to investigate if the Hubble plots resulting from this model could still give a good description of the SNe. First we calculated the best fitting values of $\Omega_\Lambda$ in models with increasing $C$ to find out which value would be closest to zero without actually being the boundary value. This led to a $C$ of 0.45 where the best fitting values are $(\Omega_\Lambda, \Omega_M) = (0.07, 0.65)$. Fig. 7 shows the corresponding contour plot, where the grey ellips is the general model and the dark blue empty contour represents the systematic error model where we have a Universe without vacuum energy.

In Fig. 8 one can see the residual Hubble plot where we have plotted the difference in distance modulus with our general model of $(\Omega_\Lambda, \Omega_M) = (0.93, 0.42)$ against redshift. Both the dark blue and red line correspond to a square-root systematic error with a positive $C$. The only difference lies in the values of $C$, thus the best fitting values of $\Lambda$ and $\Omega_M$ for both models differ as well. The dark blue line corresponds to the value of $C$ from Fig. 6(a), where $C = 0.17$, and red has $C = 0.45$. The red line does seem to give a slightly less correct fit to the SNe, but it does not deviate significantly. At a redshift of 1, SNe only have to be $\sim 0.45$ magnitude brighter according to the error model. This is almost comparable to the error of the algorithms that standardize peak magnitude\(^{10}\). The intrinsic error corresponding to the $C = 0.45$ model is indeed larger than the general model, but only by approximately a third\(^{11}\). Not a statistical significant difference.

Therefore according to the Hubble plot, the positive square-root model where $C = 0.45$ still gives a good description of the SNe without the need for vacuum energy. A very interesting result indeed.

6.3. Introducing a systematic error when $w$ is variable and $\Omega_K = 0$

Fig. 9 shows the six systematic error contours, where we have adopted the same method of display as in Fig. 6. The grey filled contour belongs to our general model of Fig. 3 and the empty colored contours correspond to the new systematic error models. Here we have assumed a flat Universe. It is common to display $w$ against $\Omega_M$ instead of $w$ against $\Omega_\Lambda$. However, in our research we are more interested in the migration of $\Omega_\Lambda$. The contour plots can be altered to show $w$ against $\Omega_M$ by using the following relation for a flat Universe: $\Omega_M = 1 - \Omega_\Lambda$. Thus the $w$ against $\Omega_M$ plots are identical to those shown in Fig. 9 only they are flipped around the vertical axis.

To see how these contours correspond to each other, we have made a residual Hubble plot, Fig. 10, which shows the difference in magnitude with the general best fit of $(\Omega_\Lambda, w) = (0.62, -1.35)$. The three lines above the black line are the positive $C$ systematic error models and the three lines below the black line come from the negative $C$ models. One can see that the inner four lines only have a slight deviation from the best fit of the general model. The lines with a downward slope even appear to fit the SNe better than the general model. In some way this can also be seen from the fact that the calculated value of the intrinsic dispersion is lower than the one from the general model. Meaning that there was less need to enlarge the error bars to give a good fit with $\chi^2 = 1$. On the other hand, the positive quadratic systematic error line gives a relatively poor representation of the outer SNe, which again becomes apparent during the calculations because the intrinsic dispersion of $\Omega_\Lambda$ is larger than 0.45.

\(^{10}\) Explained in §4.2

\(^{11}\) The values can be found in Table 3.
Fig. 9.—The six contour plots of the three systematic errors from eq. (27), where each has two options: a positive and negative value of the constant $C$. The plots show the equation of state parameter $w$ against $\Omega_\Lambda$. The grey filled confidence contours represent the general model best fit from Fig. 3. The colored empty confidence contours correspond to the systematic error fit given in the subtitle. For both contours the 68.3% (inner), 95.4% (middle) and 99.7% (outer) confidence levels are shown. The left half represents positive values of $C$ and on the right are the plots with a negative $C$. The values of $C$ which are given in the subtitles are the minimal values of the constant for which both the original and systematic error 3$\sigma$ contour have no more overlap.
The contours in Fig. 9 show many similarities with the contours for a Universe where \( w = -1 \). Again we see that it is the value of \( C \) that determines if the error contour moves up or below the original contour and again we see that a positive \( C \) means that the new best fit lies in a Universe with a larger ratio of \( \Omega_M \) over \( \Omega_\Lambda \) than in the general model. We have compared the positive and negative models of a singular systematic error again as well to find out in what direction the contour would migrate for even higher values of \( C \). The negative \( C \) models all move towards Universes where vacuum energy becomes very dominant.

Although the equation of state parameter moves towards the predicted value of \(-1\) for the \textit{linear} and \textit{quadratic} error model, it moves downward over the \( w \) axis for the \textit{square-root} model.

### 6.3.1. Migration of positive \( C \) models

The contours with a positive value of \( C \) seem to suggest that eventually they will reach the axis where \( \Omega_\Lambda \) is zero. However, \( w \) is connected to \( \Omega_\Lambda \) via eq. (6). \( \Omega_\Lambda \) changes into \( \Omega_\Lambda (1 + z)^{3(1+w)} \) if \( w \neq -1 \) in the equation for the luminosity distance (22, 23, 24). Thus if \( \Omega_\Lambda \) becomes very small, different values of \( w \) will not make a difference, since it will be the power of a very small number. The contours will begin to stretch across the entire \( w \) axis. This comes from the fact that \( \Omega_\Lambda \ll \Omega_M \) meaning that all values of \( w \) will give approximately the same value of the integral in \( d\phi \).

We have chosen to investigate the contour with a positive \( C \) which seems to go towards a Universe without vacuum energy the quickest for increasing \( C \). This appears to be the linear systematic error. Fig. 11 is a residual Hubble plot, showing the difference in distance modulus with the best fit from the general model \((\Omega_\Lambda, w) = (0.62, -1.35)\) against redshift. The \textit{red} line corresponds to the linear positive \( C \) systematic error model from Fig. 9(c), where \( C = 0.17 \), and the \textit{light blue} line is the distance modulus belonging to a \( C \) of 0.70 where the best fitting value of \( \Omega_\Lambda \) is very close to zero. One can see that the \textit{light blue} line gives a very poor description of the SNe, which also shows in the calculation. Because to compensate for the poor fit, the intrinsic uncertainty is almost four times as large as the \( \sigma_{int} \) from the general model. Thus although the best fitting values suggest a Universe without vacuum energy, the Hubble plot shows such a large deviation from the SNe, that this is no longer a reliable result.

#### 7. DISCUSSION

The best fitting values of the cosmological parameters \( \Omega_\Lambda, \Omega_M \) and \( w \) can be found in Table 2. It shows that according to SNe Ia the best fitting cosmological model is a closed Universe, where \( \Omega_\Lambda \) is the dominant component. Although the 2 and 3\( \sigma \) error contours do encompass a flat Universe. The constraints on the value of these parameters can be improved in a several ways. The statistical error can decrease by using a larger dataset with preferably more higher redshift SNe, because the statistical error decreases as \( 1/\sqrt{N} \), where \( N \) are the number of data points. A few extra high\( (\sigma) \) redshift SNe observations are even more important than increasing the number of low SNe, since the differences in the evolution of the Universe increase almost exponentially with higher redshift, as could be seen in Fig. 2(c). Another method is based not on getting a greater accuracy for the current confidence levels, but combining several confidence level contours from observations of other objects, such as the Cosmic Microwave Background (CMB) and Baryonic Acoustic Oscillations (BAO). The location where all three confidence contours overlap will give a much more accurate description of the best fitting values of the cosmological parameters, with a small error confidence level. The combination of the confidence contours from all three areas of research was beyond the scope of this research. However this has been executed by Kowalski (2008). The resulting contour lies along the
line of a flat Λ Universe with a slight inclination towards a closed Universe, thus implying that we most likely live in a flat Universe, with the following best fitting values $(\Omega_M, \Omega_K) = (0.285 \pm 0.02, -0.009 \pm 0.009)$. These are in agreement with our best fitting values when a flat Universe was assumed.

We did not expect both best fitting values to coincide exactly because in the research of Kowalski (2008) the total error, during the calculation of $\chi^2$ from eq. (32), has a third factor. This is the covariance matrix of fit parameters: peak magnitude, color and stretch. We did not have access to this matrix and could therefore not include these extra uncertainties in our research.

Looking at the best fit for a flat Universe model where $w$ is an adjustable parameter, Fig. 3, one sees that SNe do not give a small area of constraint for the value of $w$ since the error contours are stretched along quite a wide margin of this parameter. Observing more (high) redshift SNe will certainly decrease the size of the three confidence contours, but the best result can be found by combining the confidence contours from the SNe with the CMB and BAO contours which can be found in Kowalski (2008). The following best fitting values are given $(\Omega_M, \Omega_K, w) = (0.285 \pm 0.02, -0.010 \pm 0.010, -1.001 \pm 0.07)$, which indeed places $w$ at the predicted value of $-1$ for a true cosmological constant.

We cannot compare our exact results from a flat Λ Universe with $w \neq -1$ where we look only at the SNe constraints since Kowalski (2008) has not given these values, only the corresponding contour plot. However, the contour plots show a very strong resemblance and the best fit from Kowalski (2008) indeed seems to lie around the same values we found.

The systematic errors each show a linear trend of movement across the ΩΛ-ΩM plane, where models with a positive value of C all give a Universe where the ratio of $\Omega_M$ and $\Omega_\Lambda$ is larger than the general model. For all negative C models the ratio is smaller than the general model. We can say that the minimal value of C at which the original and systematic error contours have no more overlap is approximately $0.1 \sim 0.2$. For these values of C the systematic error is dominant over the statistical error, because there is already a 6σ error distance between the original and error best fit. However, it is preferred that the statistical error is larger than the systematic error, otherwise you are working with a model that is an inappropriate fitting function. Most of the systematic error models which move towards a Universe without vacuum energy give an unreliable or unphysical result before the best fit would give a $\Omega_\Lambda = 0$ value, such as a negative or very large $\Omega_M$, or a very big intrinsic error to compensate for the poor fit to the SNe. It is only the positive square-root error in a Λ Universe with $w = -1$ that gives a reliable fit to the SNe, Fig. 7, while having a best fit value of $\Omega_\Lambda \sim 0$. Thus if this ‘systematic error’ would in reality be an accurate description, our ideas about cosmological models would have to be altered.

8. CONCLUSIONS

To conclude, the best fitting values of $\Omega_\Lambda$, $\Omega_M$ (thus also $\Omega_K$) and $w$ have been calculated and can be found in Table 2. They agree well with the research of Kowalski (2008).

The systematic errors show trends in their movement across the contourplane. The sign of $C$ determines the above or below positioning of the new contour in comparison with the original one, which is in accordance with the theoretical prediction. We can say that the minimal value of $C$ at which the original and systematic error contours have no more overlap is approximately $0.1 \sim 0.2$.

There is only one systematic error that migrates towards a Universe without vacuum energy with a reliable fit, namely the positive square-root error. If this error were to exist, we would live in a very different model of a Universe than the one assumed today.

Thus in the end it will become increasingly more important to get a better understanding on the workings of SNe, extinction or discover an, as of yet, unknown cause of the systematic errors, if we want to ensure that our theoretical models describe reality.

We would like to thank to M. Kowalski for providing us with the Union supernova dataset and Dr. J. Brinchmann for providing us with IDL programs that enabled us to brighten up our plots with colors. We also would like to thank Dr. I. Snellen and Dr. H. Hoekstra for helping us when our supervisor was abroad. Of course we very much would like to thank our supervisor Prof. Dr. K. Kuijken for inviting us in the wonderful world of luminosity distances and density parameters. He always had, or else made, time in his busy schedule to answer all our questions. Thank you very much.

REFERENCES

Bevington, P. R. & Robinson, D. K., 1992, Data reduction and error analysis for the physical sciences, ISBN 0 07 911243 9


Einstein, A., Königlich Preussische Akademie der Wissenschaften, 142-152


Mazzali, P., A., K. et al., 2007, Science, 825-828 315

Minkowski, R., 1941, PASP, 53 224


Influence of systematic errors on cosmological parameters

<table>
<thead>
<tr>
<th>Universe model</th>
<th>Systematic error</th>
<th>Best fitting values</th>
<th>C</th>
<th>Intrinsic dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ωₐ</td>
<td>Ωₘ</td>
<td>w</td>
</tr>
<tr>
<td>General models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>—</td>
<td>0.93 ± 0.12</td>
<td>0.42 ± 0.08</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>Flat Universe Λ, w = −1</td>
<td>—</td>
<td>0.73 ± 0.02</td>
<td>0.27 ± 0.02</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>Flat Universe Λ, w ≠ −1</td>
<td>—</td>
<td>0.62 ± 0.07</td>
<td>0.38 ± 0.06</td>
<td>−1.35 ± 0.24</td>
</tr>
<tr>
<td>Systematic error models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>square root</td>
<td>0.69 ± 0.14</td>
<td>0.44 ± 0.11</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>square root</td>
<td>1.12 ± 0.12</td>
<td>0.37 ± 0.08</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>linear</td>
<td>0.90 ± 0.12</td>
<td>0.60 ± 0.11</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>linear</td>
<td>0.98 ± 0.10</td>
<td>0.27 ± 0.08</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>quadratic</td>
<td>1.24 ± 0.10</td>
<td>0.83 ± 0.10</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>quadratic</td>
<td>0.69 ± 0.12</td>
<td>0.06 ± 0.08</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>square root</td>
<td>0.57 ± 0.11</td>
<td>0.43 ± 0.09</td>
<td>−1.11 ± 0.28</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>square root</td>
<td>0.66 ± 0.06</td>
<td>0.34 ± 0.04</td>
<td>−1.58 ± 0.24</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>linear</td>
<td>0.46 ± 0.06</td>
<td>0.54 ± 0.05</td>
<td>−1.75 ± 0.43</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>linear</td>
<td>0.76 ± 0.06</td>
<td>0.24 ± 0.06</td>
<td>−1.20 ± 0.21</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>quadratic</td>
<td>0.39 ± 0.04</td>
<td>0.61 ± 0.02</td>
<td>−2.69 ± 0.54</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>quadratic</td>
<td>0.93 ± 0.06</td>
<td>0.07 ± 0.08</td>
<td>−0.83 ± 0.09</td>
</tr>
<tr>
<td>Interesting systematic error models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A Universe, w = −1</td>
<td>linear</td>
<td>0.07 ± 0.19</td>
<td>0.65 ± 0.14</td>
<td>−1 (fixed)</td>
</tr>
<tr>
<td>Flat A Universe, w ≠ −1</td>
<td>linear</td>
<td>0.06 ± 0.02</td>
<td>0.94 ± 0.03</td>
<td>−14.92 ± 8.25</td>
</tr>
</tbody>
</table>

*Discussed in §6.2.3
bDiscussed in §6.3.1