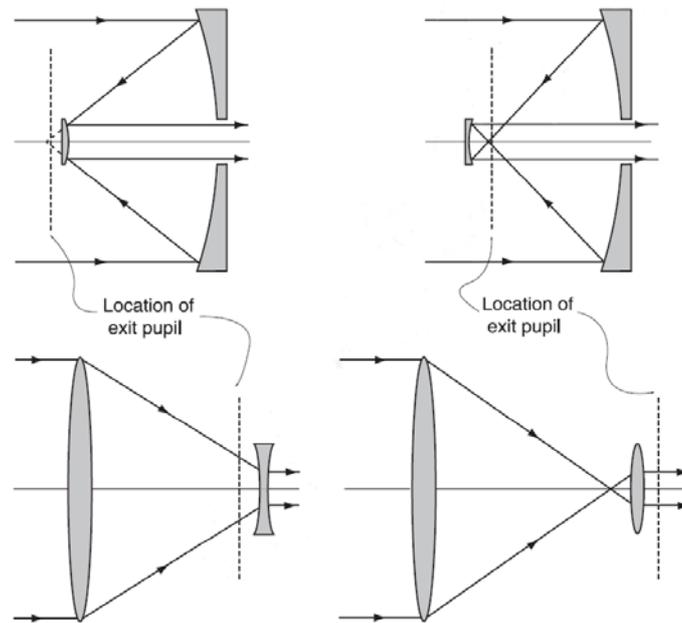


Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

4th Lecture: 08 October 2008



Based on "Observational Astrophysics" (Springer) by P. Lena, F. Lebrun & F. Mignard, 2nd edition - Chapters 2 and 4

1. Atmospheric Turbulence

A Model of Turbulence

Turbulence develops in a fluid when the **Reynolds number** exceeds a critical value ($Re \sim 2200$; transition from **laminar** to **turbulent** flow).

The Reynolds number is defined as: $Re = \frac{VL}{\nu}$

where V is the flow velocity, ν the kinematic viscosity of the fluid ($\nu_{\text{air}} = 1.5 \cdot 10^{-5} \text{ m}^2 \text{ s}^{-1}$), and L the characteristic length (e.g., a pipe diameter).



Example: wind speed $\sim 1 \text{ m/s}$, $L = 15 \text{ m} \rightarrow Re = 10^6 \rightarrow$ turbulent!

The Power Spectrum of Turbulence

In turbulence the kinetic energy of large scale ($\sim L$) movements is gradually transferred to smaller and smaller scales, down to a minimum scale length l (at which the energy is dissipated by viscous friction).

At any time the local velocity field can be decomposed into spatial harmonics of the wave vector κ . The reciprocal value $1/\kappa$ represents the scale under consideration.

The **mean kinetic energy** is then $dE(\kappa) \propto \kappa^{-2/3} d\kappa$

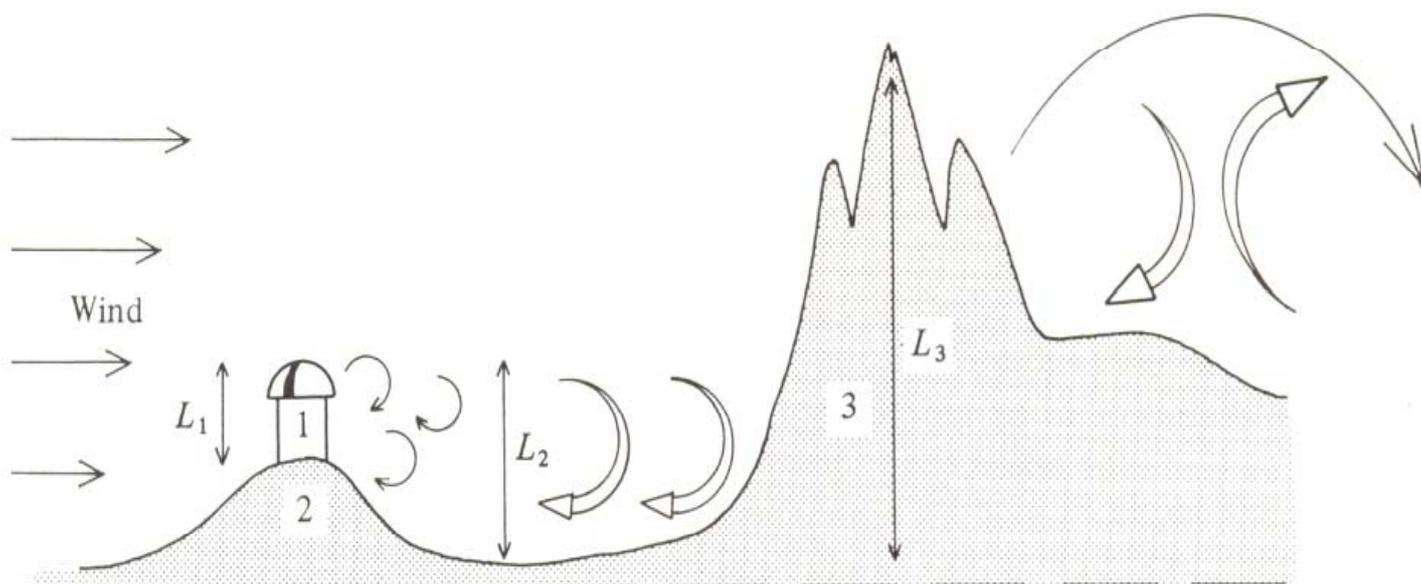
and by integration we obtain the spectrum of the kinetic energy, or **1D Kolmogorov spectrum**:

$$E(\kappa) \propto \kappa^{-5/3}$$

with $L_0^{-1} < \kappa < l_0^{-1}$

where l_0 is the **internal scale** and L_0 the **outer scale** of the turbulence.

Turbulence on many Scales

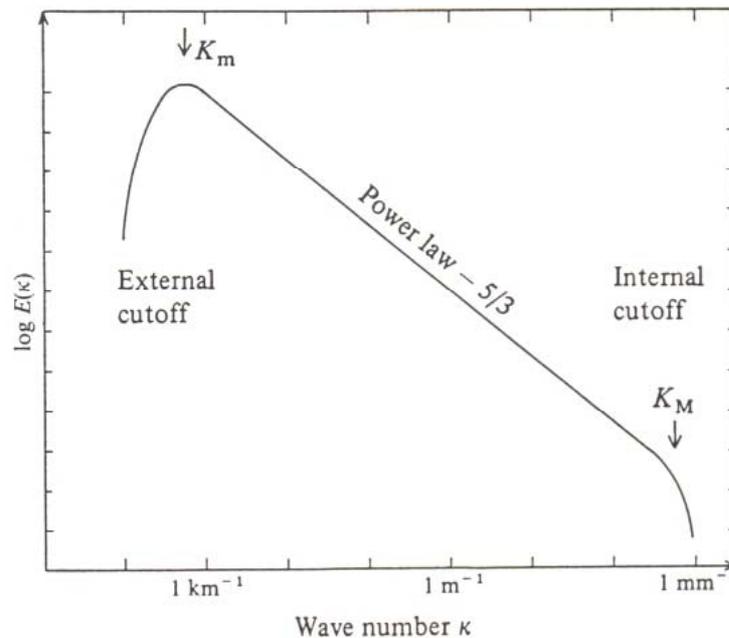


The scales L_1 , L_2 , L_3 are characteristic of the outer (external) scales of turbulence caused by the wind around the obstacles 1, 2, 3.

1D Kolmogorov Turbulence

Within $L_0^{-1} < \kappa < l_0^{-1}$ a Kolmogorov spectrum is said to be **homogeneous**.

Near ground the outer scale varies between several meters and several hundred meters.



Temperature Fluctuations

Turbulence mixes air of different temperature \rightarrow fluctuations of temperature at the same altitude.

The **temperature fluctuations** about the mean $\langle T(r) \rangle$ are given by:

$$\Theta(r) = T(r) - \langle T(r) \rangle$$

The **covariance** of the temperature fluctuations is given by:

$$B_T(\rho) = \langle \Theta(r) \Theta(r + \rho) \rangle$$

The **structure function** of the temperature fluctuations is:

$$D_T(\rho) = \langle |\Theta(r + \rho) - \Theta(r)|^2 \rangle = \dots = C_T^2 \rho^{2/3}$$

where C_T^2 is called the **structure constant** of the temperature fluctuations. It characterizes the intensity of the turbulence.

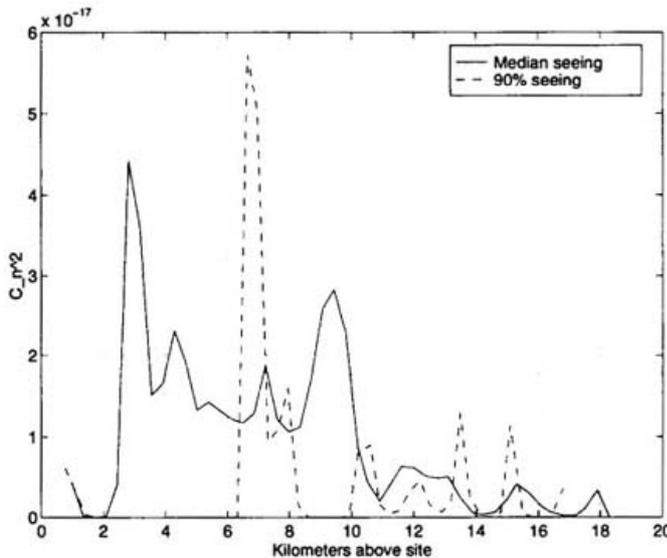
Temperature Fluctuations and Refractive Index

Temperature and refractive index are related: $C_n = \frac{80 \cdot 10^{-6} P[\text{mb}]}{T^2[\text{K}]} C_T$

Usually, one is only interested in the integral of fluctuations along the line of sight: $C_n^2 \cdot \Delta h$.

Typical value: $C_n^2 \cdot \Delta h \sim 4 \cdot 10^{-13} \text{ cm}^{1/3}$ for a 3 km altitude layer

But: always several layers of turbulence

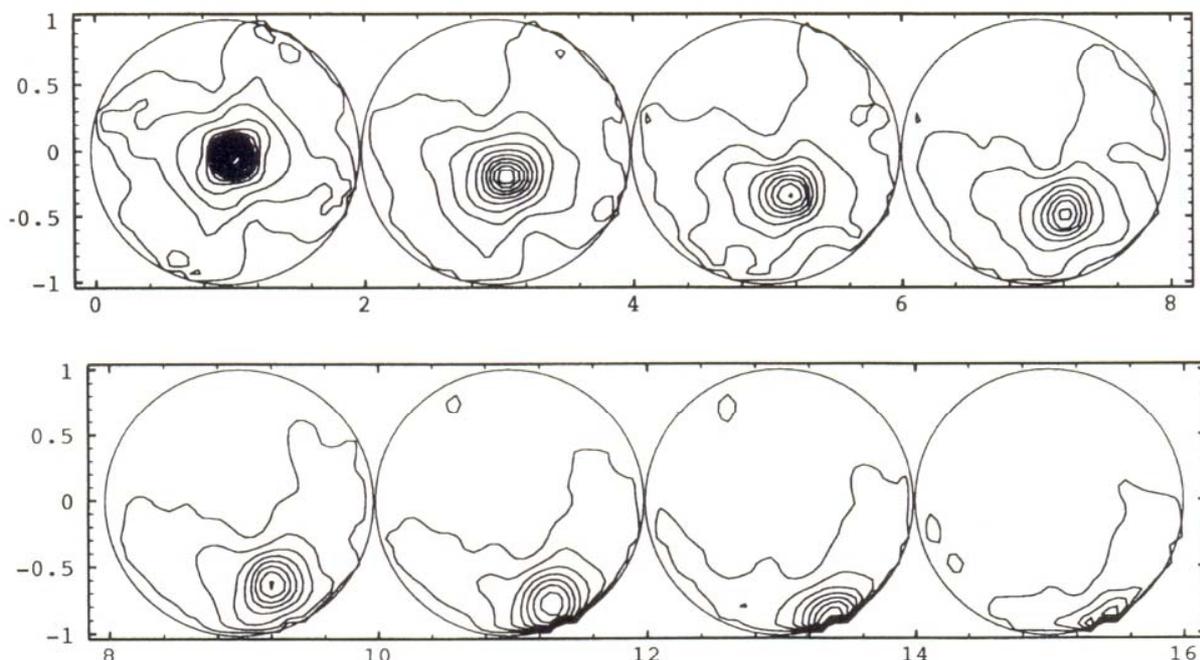


Median seeing conditions on Mauna Kea are taken to be $r_0 \sim 0.23$ meters at 0.55 microns. The 10% best seeing conditions are taken to be $r_0 \sim 0.40$ meters. Figure taken from a paper by Ellerbroek and Tyler (1997).

Frozen Turbulence

Time scales to generate turbulence are much longer than the time for the turbulent medium to pass the telescope aperture (\leftarrow wind speed).

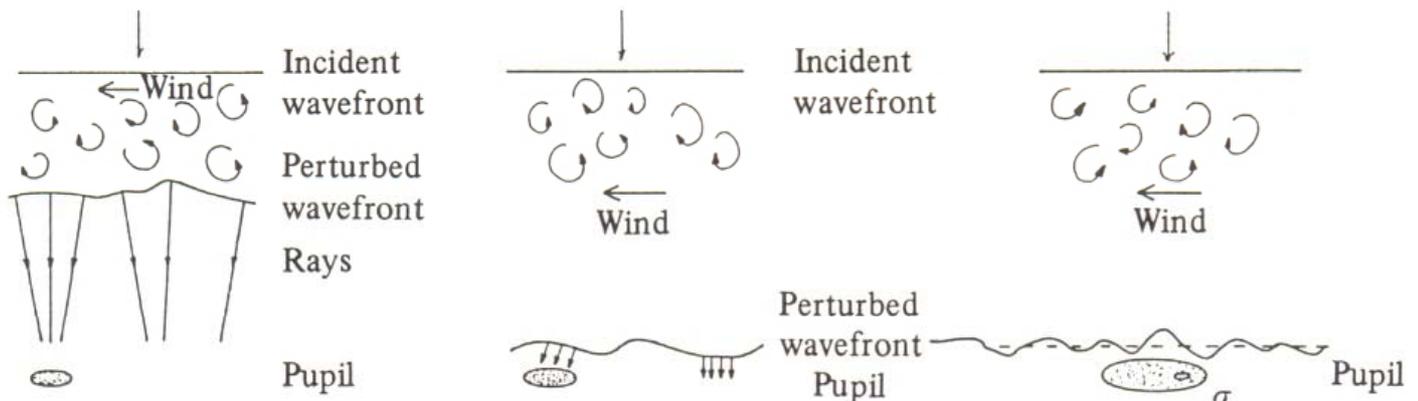
\rightarrow correlation time τ_c .



Motion of a frozen patch of atmosphere across the 3.6m telescope aperture. Pictures by E. Gendron (1994)

Image Degradation by the Atmosphere

Three main effects:



SCINTILLATION

the energy received by the pupil varies in time

AGITATION

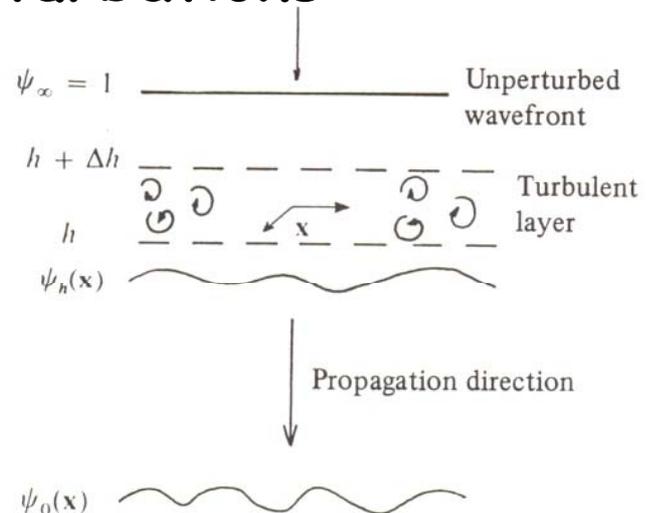
the average slope of the wavefront at the pupil varies

SMEARING

the spatial coherence of the wavefront is reduced

Wavefront Perturbations

Consider now a monochromatic, plane wave $\psi_\infty = 1$ which passes through a turbulent layer of thickness Δh .



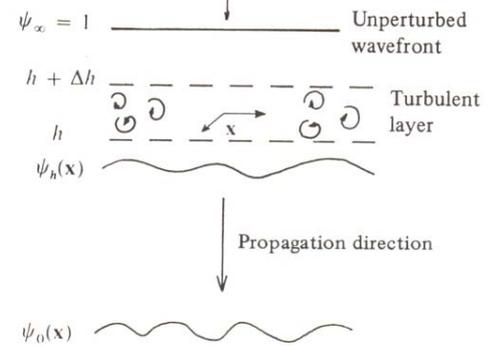
We want to know the spatial correlation function of the wave (e.g., across our telescope aperture):

$$\langle \psi_h(x + \xi) \psi_h^*(\xi) \rangle = \exp(-1.45k^2 C_n^2 \Delta h x^{5/3})$$

where $k = 2\pi/\lambda$ (see Lena p. 169 for detailed derivation).

Wavefront Perturbations (2)

Note: the wave field at $h=0$ results from **Fresnel diffraction** ("near field") of the wave on leaving the layer. Fresnel diffraction has to take phases into account.



Consequences:

- near the layer only the phase is disturbed → smearing & image motion
- further away both phase and amplitude are disturbed → scintillation
- near the layer the wavefront displays a correlation function with complex amplitude → isotropic profile in the 'near Gaussian' plane → define the **correlation length** x_c via:

$$\frac{\langle \psi_h(x + x_c) \psi_h^*(\xi) \rangle}{\langle |\psi_h(0)|^2 \rangle} \approx \frac{1}{e}$$

so that
$$x_c \approx (1.45k^2 C_n^2 \Delta h)^{-3/5} = \left(1.45 \left(\frac{2\pi}{\lambda} \right)^2 C_n^2 \Delta h \right)^{-3/5} \propto \lambda^{6/5}$$

Image Formation: Long Exposures

When $t_{\text{int}} \gg \tau_c$ the image is the mean of the instantaneous intensity:

$$I(\theta) = \langle I_0(\theta) * T(\theta, t) \rangle$$

The **modulation transfer function** (MTF) becomes (for $D \gg x_c$):

$$\langle \tilde{T}(\omega) \rangle \approx \exp[-1.45k^2 C_n^2 \Delta h x^{5/3}]$$

The image is smeared or **spatially filtered** (loss of high spatial frequencies).

The angular dimension now has order of λ/x_c rather than λ/D .

In other words: a bigger telescope D will not provide sharper images.

The Fried Parameter r_0

Goal: compare diffraction-limited imaging and imaging through turbulence.

Calculate the diameter r_0 of a diffraction-limited pupil which gives the same resolution as the long exposure image $\langle I \rangle$.

It can be shown that:
$$r_0(\lambda) = 0.185 \lambda^{6/5} \left[\int_0^\infty C_n^2(z) dz \right]^{-3/5}$$

r_0 is called the **Fried parameter**.

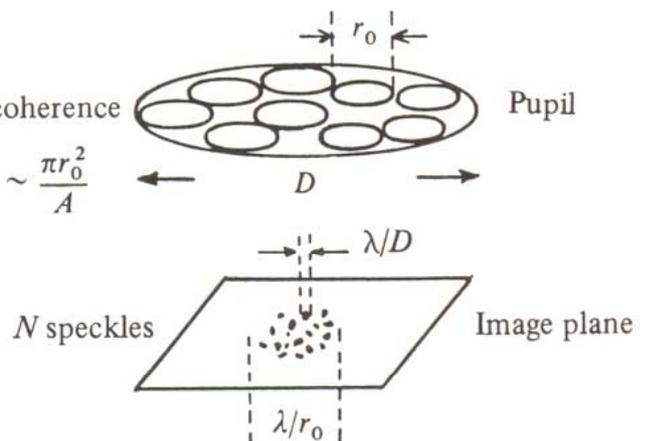
The angle $\Delta\theta = \frac{\lambda}{r_0}$ is often called the **seeing**.

Image Formation: Short Exposures

Random intensity distribution in the focal plane:

Presence of **speckles**. (high frequency components) $N \sim \frac{\pi r_0^2}{A}$

Speckles move randomly and their superposition produces the long exposure image.



Let I_0 be an arbitrary source. The observed quantity is the

convolution: $I(\theta) = I_0(\theta) * T(\theta)$ and hence: $\langle |I(\omega)|^2 \rangle = |I_0(\omega)|^2 \langle |T(\omega)|^2 \rangle$

All we need is to observe a point source through the same r_0 and we

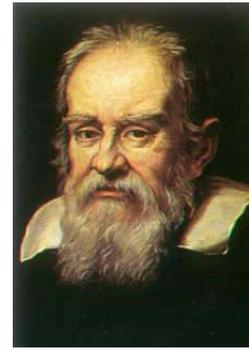
can calculate:
$$|I_0(\omega)| = \left(\frac{\langle |I_0(\omega)|^2 \rangle_{obs}}{\langle |T(\omega)|^2 \rangle_{obs}} \right)^{1/2}$$

This is called **speckle interferometry**.

2. Telescopes



- Hans Lippershey 1608 - first patent for "spy glasses"



- Galileo Galilei 1609 - first use in astronomy

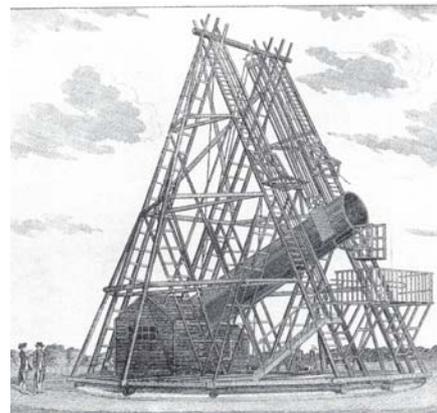
- Newton 1668 - first refractor



- Kepler - improves reflector

- Herschel 1789 - 4 ft refractor

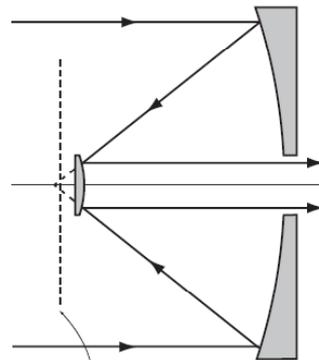
- and many, many more ...



Basic Optical Telescope Types

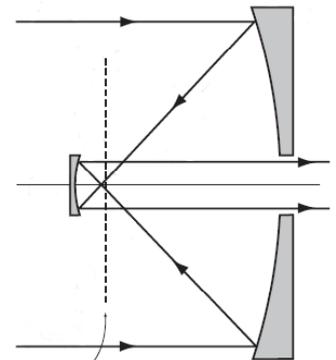
Fundamental choices:
 Refractor ↔ Reflector
 Location of exit pupil

a) Mersenne reflecting afocal Cassegrain form

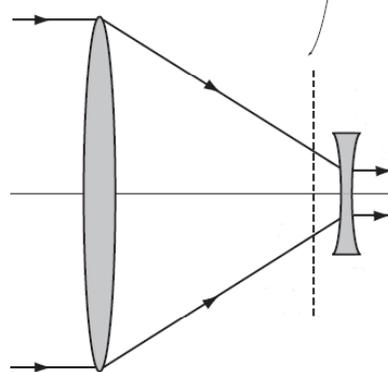


Location of exit pupil

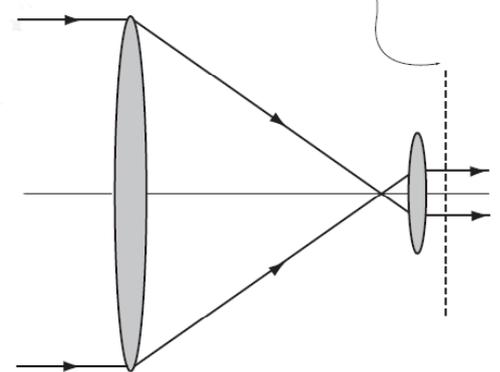
b) Mersenne reflecting afocal Gregory form



Location of exit pupil

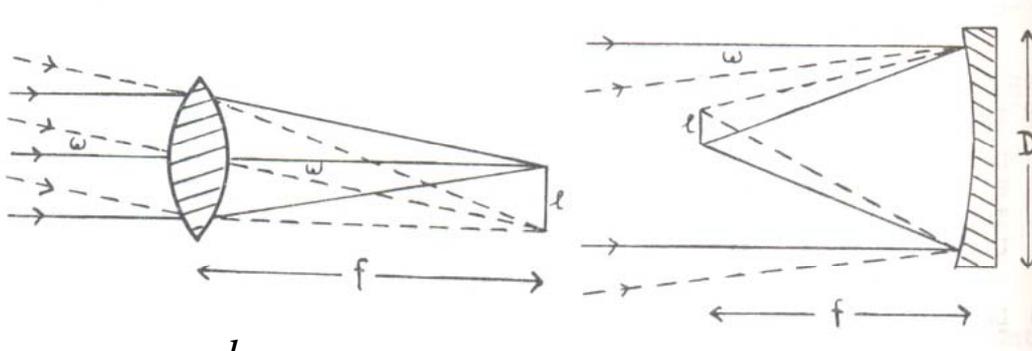


c) Galileo-type refractor

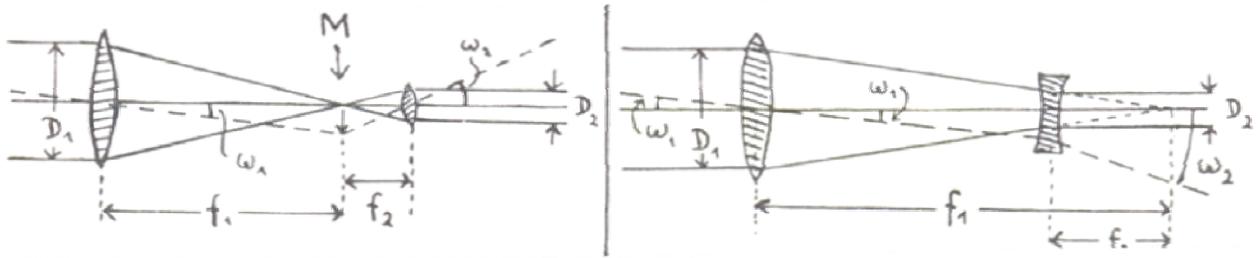


d) Kepler-type refractor

Basic Considerations



1. **Scale:** $\tan \omega = \frac{l}{f}$ and for small ω : $l \approx 0.0175 \omega f$

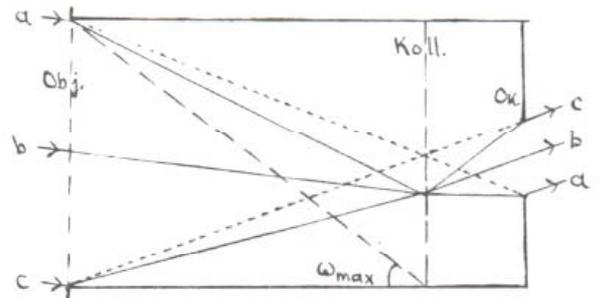


2. **Magnification:** $V = \frac{f_1}{f_2} = \frac{D_1}{D_2} = \frac{\omega_2}{\omega_1}$

Basic Considerations (2)

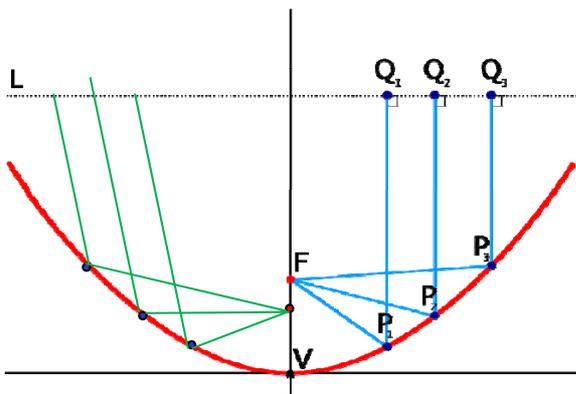
3. **Field of view**

geometrically: $\tan \omega_{\max} = \left(\frac{D}{f} \right)_{\text{Camera}}$

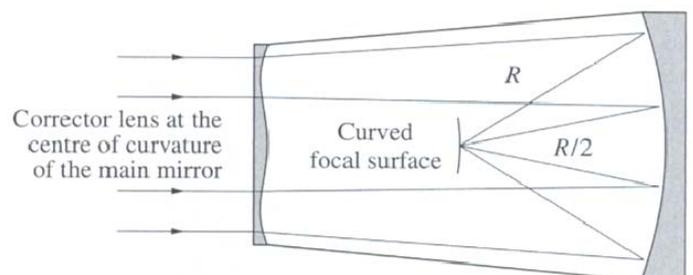


but practically given by aberrations (=bigger problem for bigger mirrors where [parabola - sphere] becomes more significant.

Parabolic primary:



Schmidt telescope: uses spherical primary + corrector plate

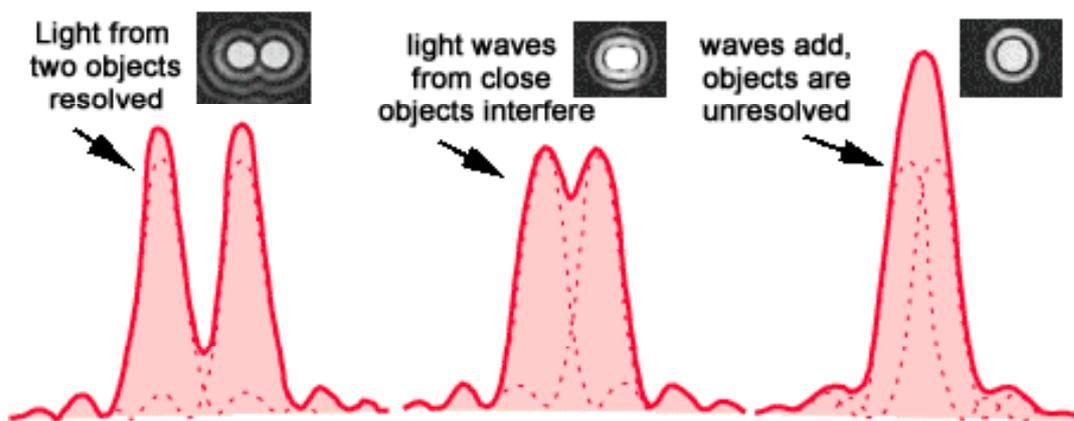


Basic Considerations (3)

4. Light gathering power

$$S/N \propto \left(\frac{D}{f}\right)^2 \quad \text{for extended objects, and for point sources: } S/N \propto D^2$$

5. Angular resolution $\sin \Theta = 1.22 \frac{\lambda}{D}$ or $\Delta l = 1.22 \frac{f\lambda}{D}$ (given by the Rayleigh criterion)



Nomenclature for the Ritchey-Chrétien Config.

Optical parameters

Primary mirror diameter

$$D_1$$

Primary mirror f -ratio

$$N_1$$

Primary mirror focal length

$$f_1 = N_1 D_1$$

Backfocal distance

$$b = \beta f_1$$

Normalized back focal distance

$$\beta = b/f_1$$

Magnification of secondary mirror

$$m = f/f_1$$

Primary-secondary separation

$$s = (f - b)/(m + 1)$$

Secondary mirror focal length

$$f_2 = m(f_1 + b)/(m^2 - 1)$$

Primary mirror conic constant

$$\kappa_1 = -1 - \frac{2(1+\beta)}{m^2(m-\beta)}$$

Secondary mirror conic constant

$$\kappa_2 = -\left(\frac{m+1}{m-1}\right)^2 - \frac{2m(m+1)}{(m-\beta)/(m-1)^3}$$

Secondary mirror dia. (zero field)

$$D_2 = D_1(f_1 + b)/(f + f_1)$$

Obscuration ratio (no baffling)

$$D_2/D_1$$

Final f -ratio

$$N$$

Final focal length

$$f = ND_1 = \frac{f_1 f_2}{f_1 + f_2 - s}$$

Field radius of curvature

$$\frac{f_1 f_2^2 (f_1 - s)}{f f_1^2 + s(f^2 - f_1^2)}$$

Aberrations

Angular astigmatism⁻²

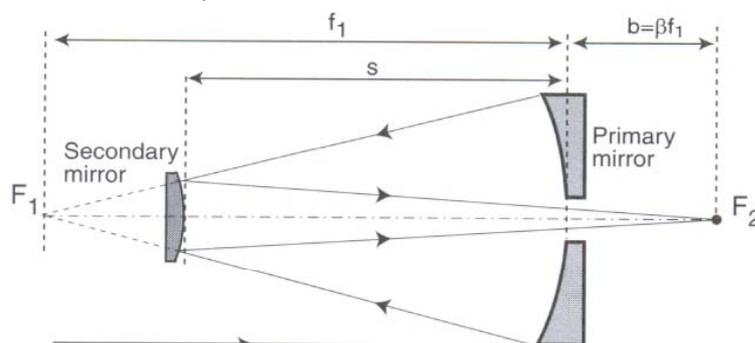
$$\frac{\theta^2}{2F} \frac{m(2m+1)+\beta}{2m(1+\beta)}$$

Angular distortion

$$\theta^3 \frac{(m-\beta)}{4m^2(1+\beta)^2} (m(m^2 - 2) + \beta(3m^2 - 2))$$

Median field curvature

$$\frac{2}{R_1} \frac{(m+1)}{m^2(1+\beta)} (m^2 - \beta(m-1))$$



Note: RC telescopes

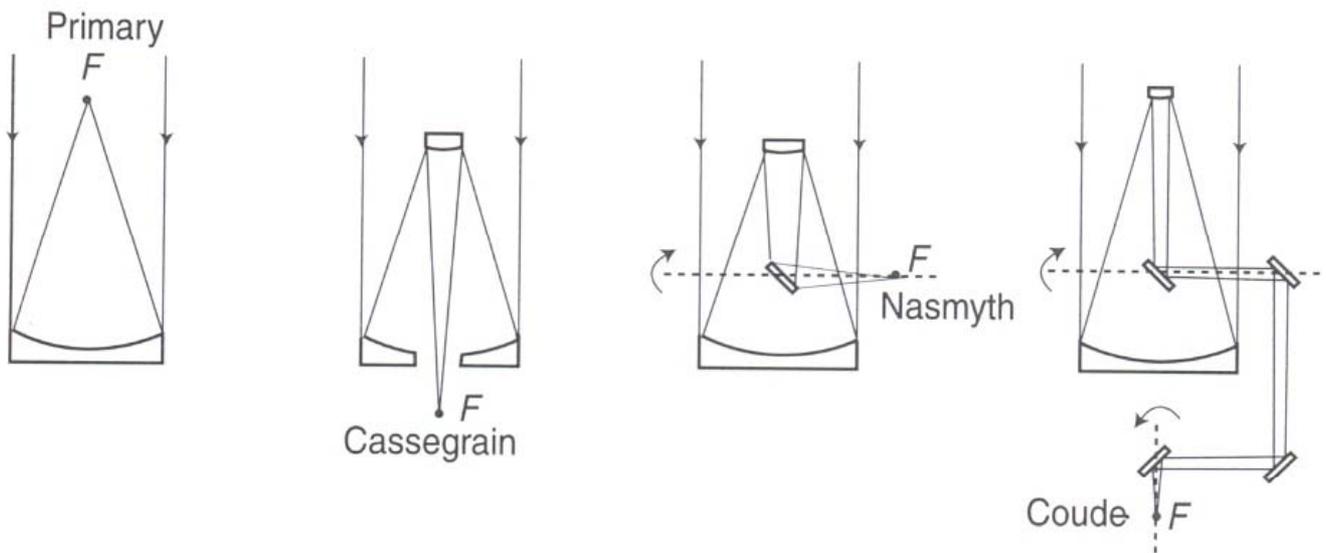
use two hyperbolic

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{mirrors}$$

instead of a parabolic

$$y - ax^2 = 0 \quad \text{one.}$$

Telescope Foci



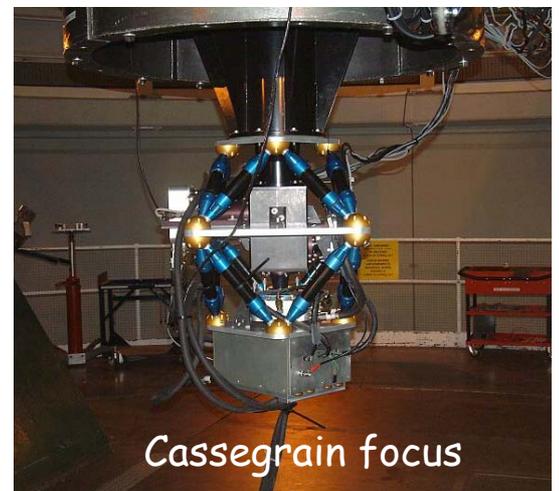
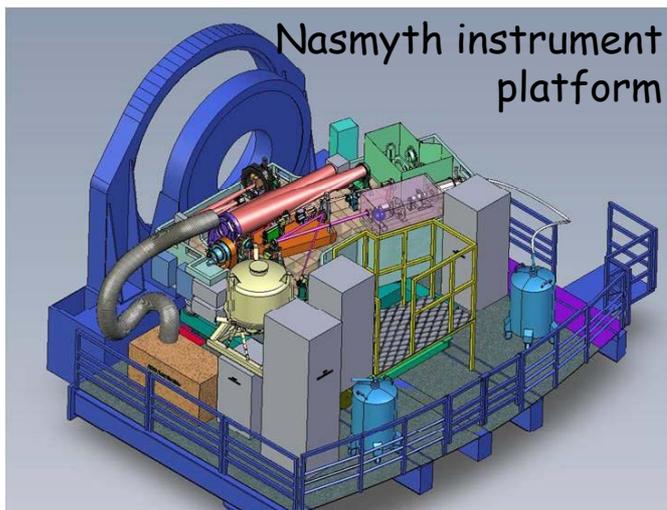
Prime focus - wide field, fast beam

Cassegrain focus: moves with the telescope, no image rotation

Coude - slow beam, usually for large spectrographs in the "basement"

Nasmyth - ideal for heavy instruments

Telescope Foci (2)



Coude focus

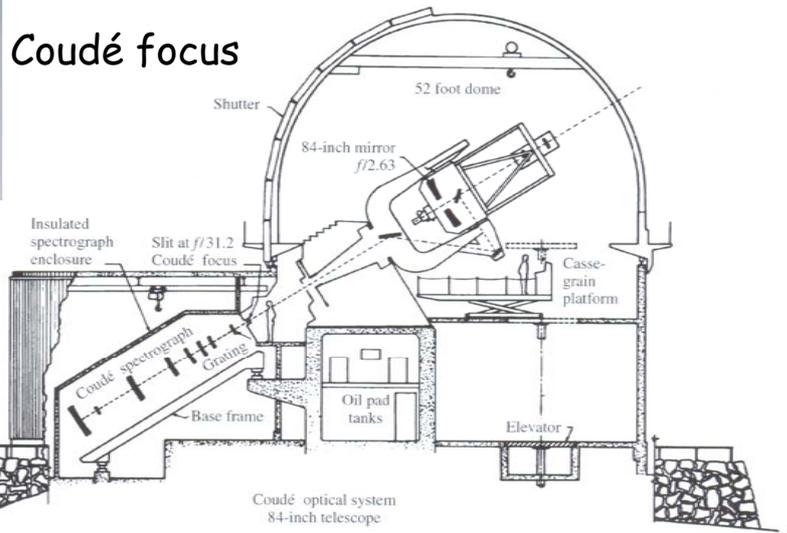
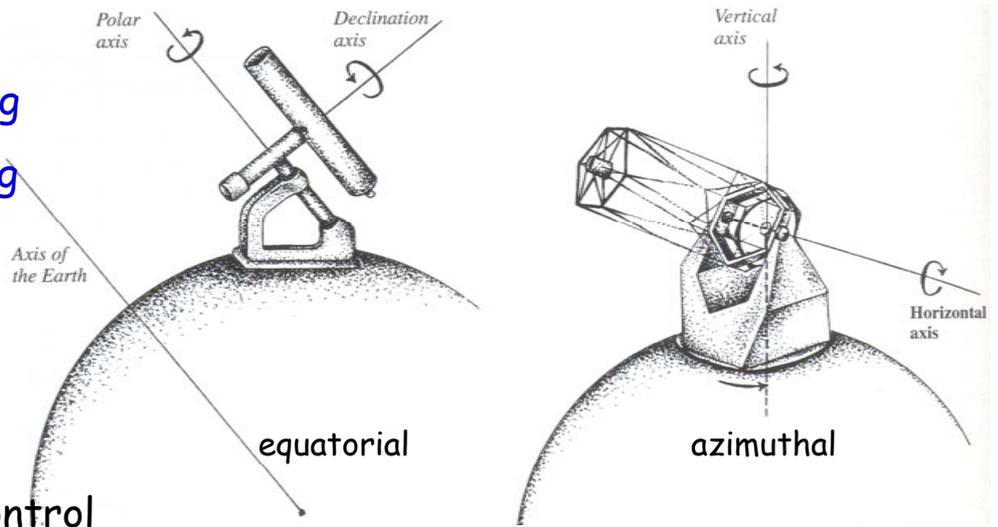


Fig. 3.13. The coude system of the Kitt Peak 2.1 m reflector. (Drawing National Optical Astronomy Observatories, Kitt Peak National Observatory)

Telescope Mounts

Two main types:

1. Equatorial mounting
2. Azimuthal mounting

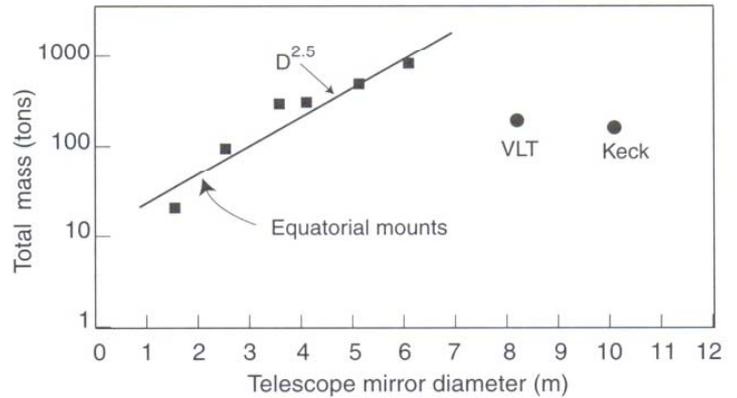


Azimuthal:

- light and symmetric
- requires computer control

Equatorial:

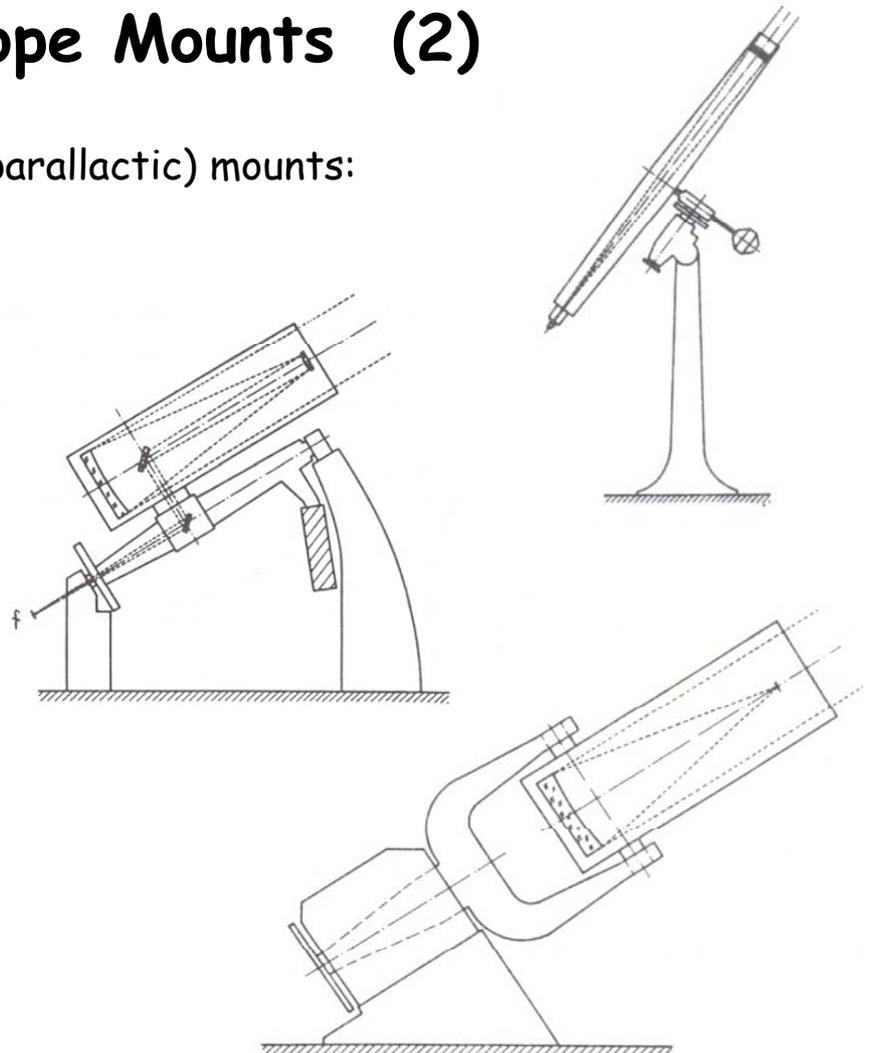
- follows the Earth rotation
- typically much larger and massive



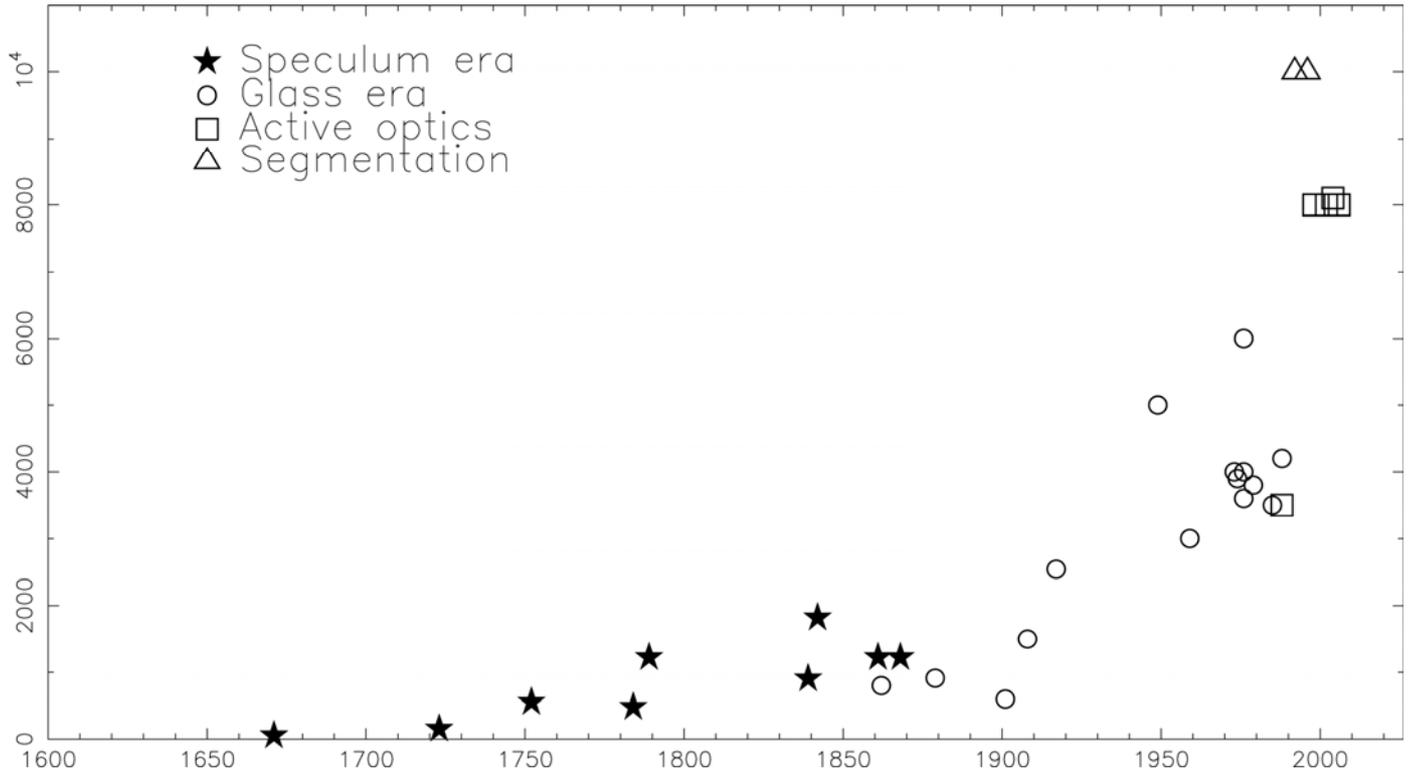
Telescope Mounts (2)

Examples of equatorial (or parallactic) mounts:

- German mount
- English mount
- Fork mount



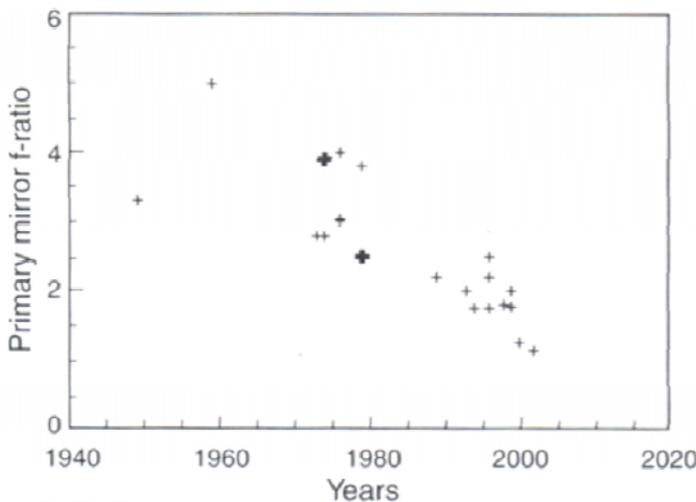
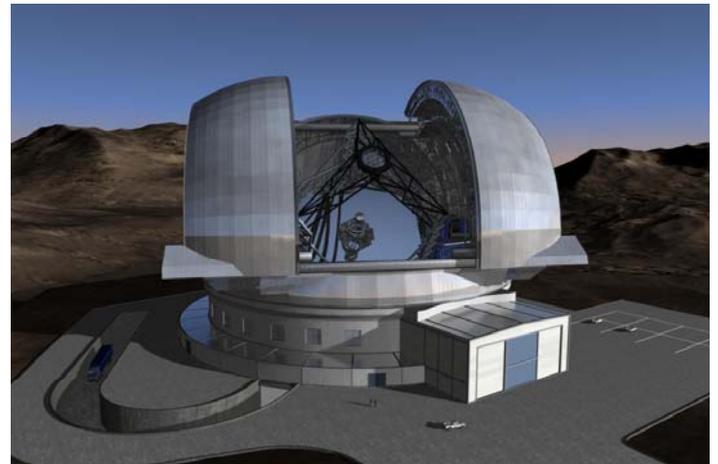
Growth of Telescope Collecting Area



...and Shrinkage of Telescope Size

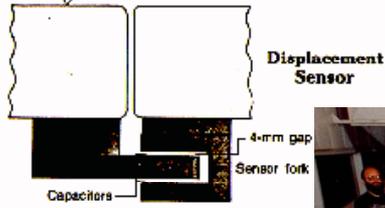
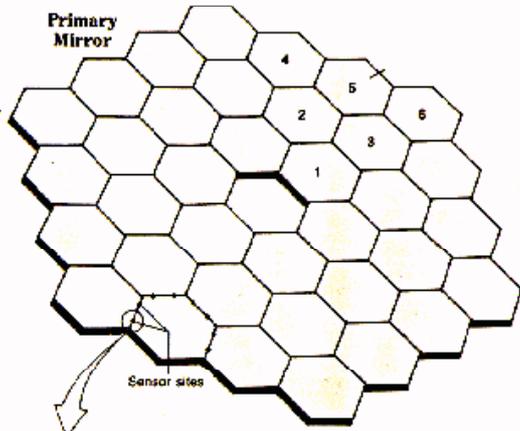
Most important:

- faster mirrors
- lighter mirrors
- new polishing techniques

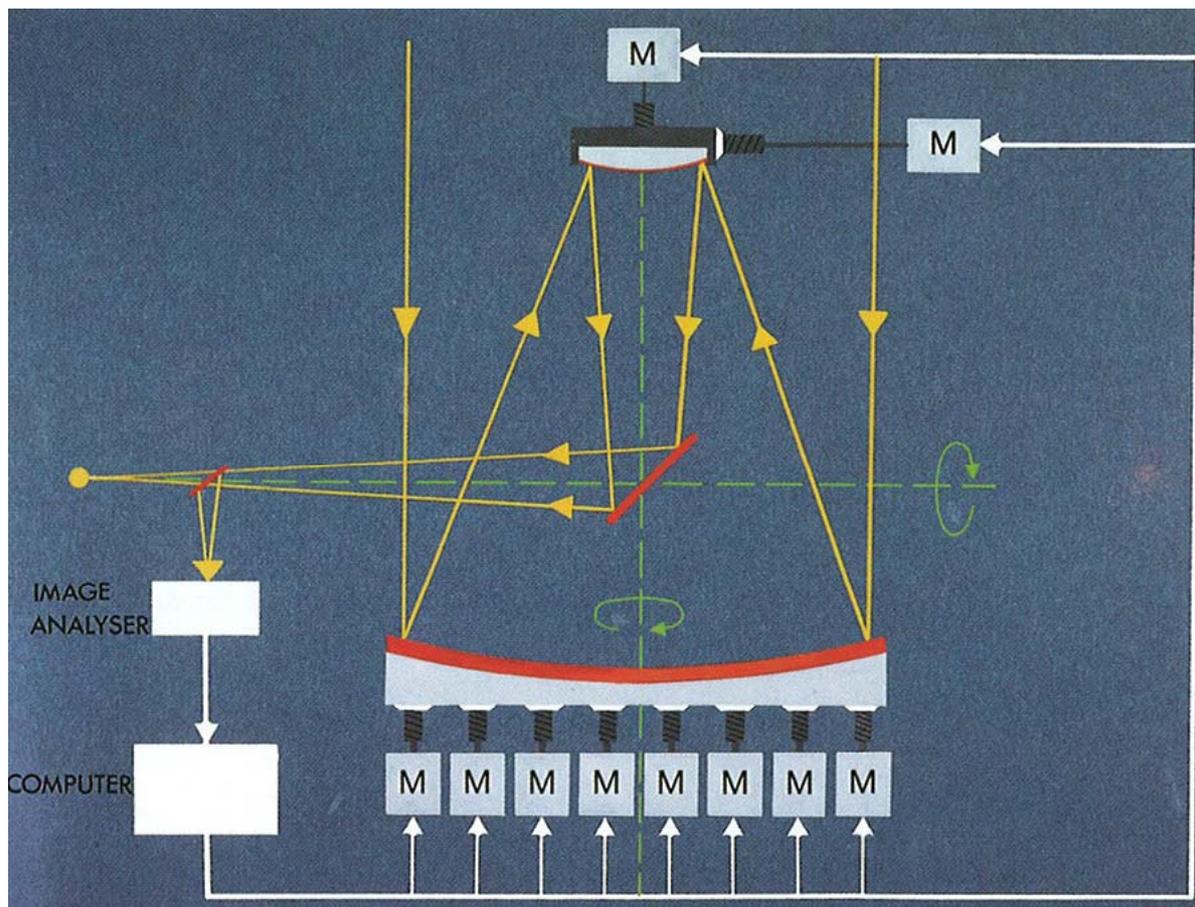


	Diameter	Thickness	Mass
	60 inch	200 mm	0.6 tons
Hooker	100 inch	400 mm	7 tons
Hale	200 inch	650 mm	15 tons
Zelenchuk	6 m	700 mm	45 tons

Segmented, Thin and Honeycomb Mirrors

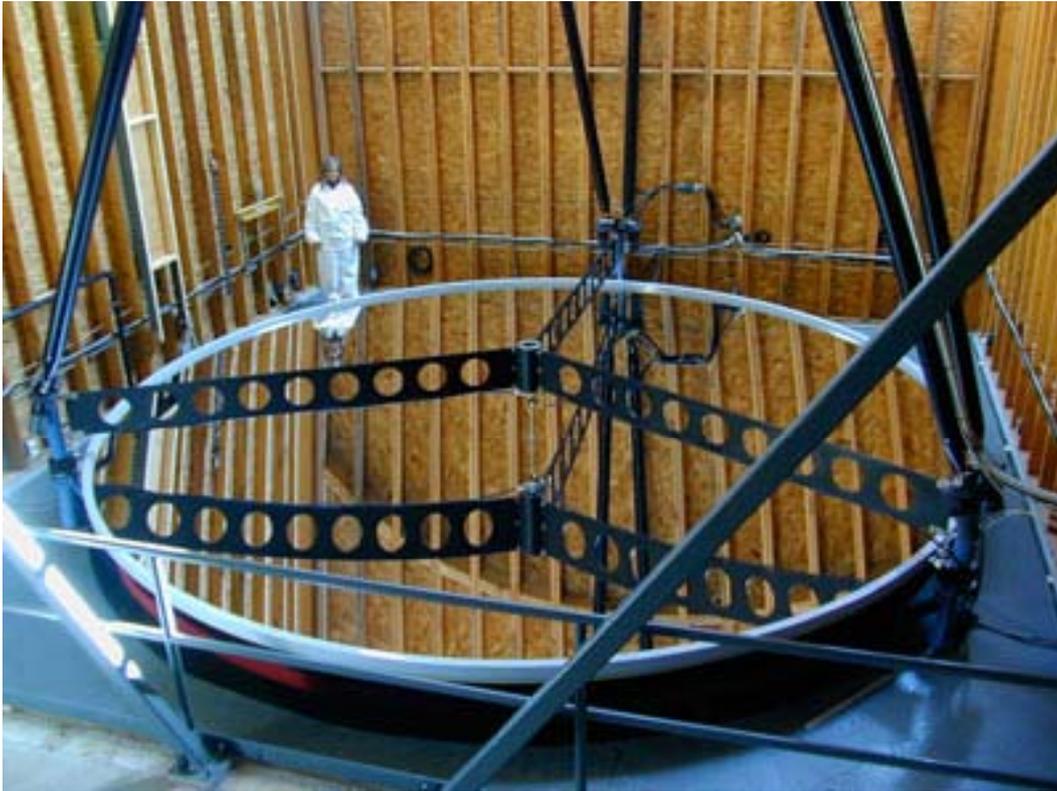


A key element: Active Optics

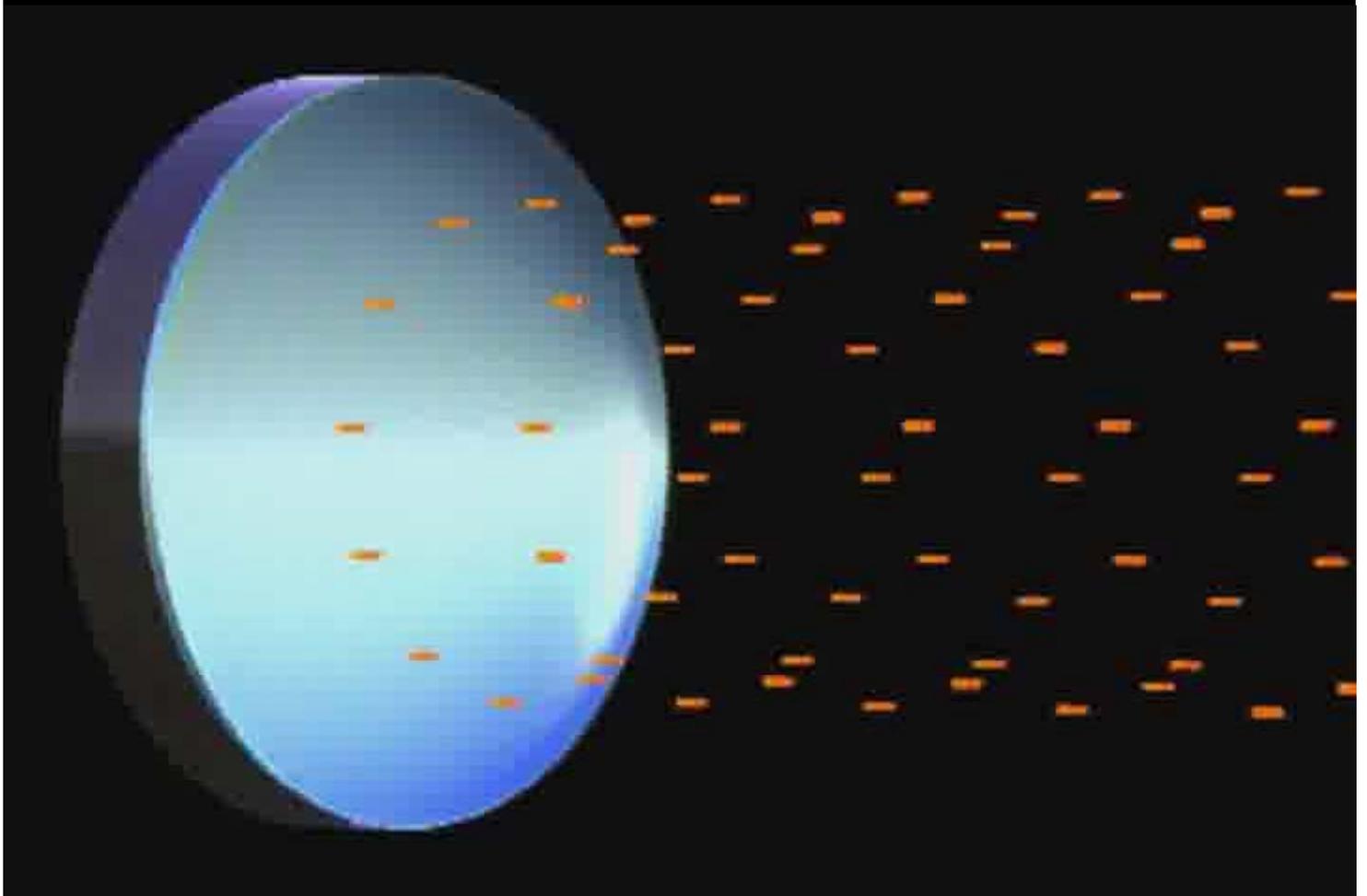


Liquid mirror telescopes

- First suggestion by Ernesto Capocci in 1850
- First mercury telescope built in 1872 with a diameter of 350 mm
- Largest mirror: diameter 3.7 m



X-rays need a different kind of optics...



Other Types of Telescopes

- X-ray telescopes

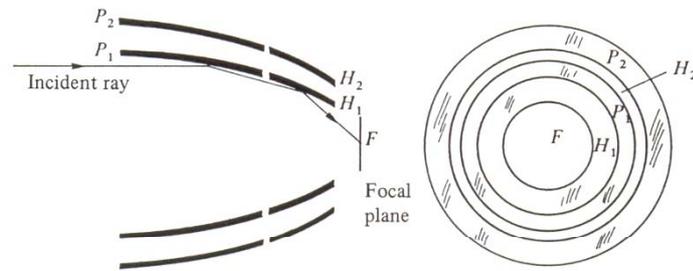
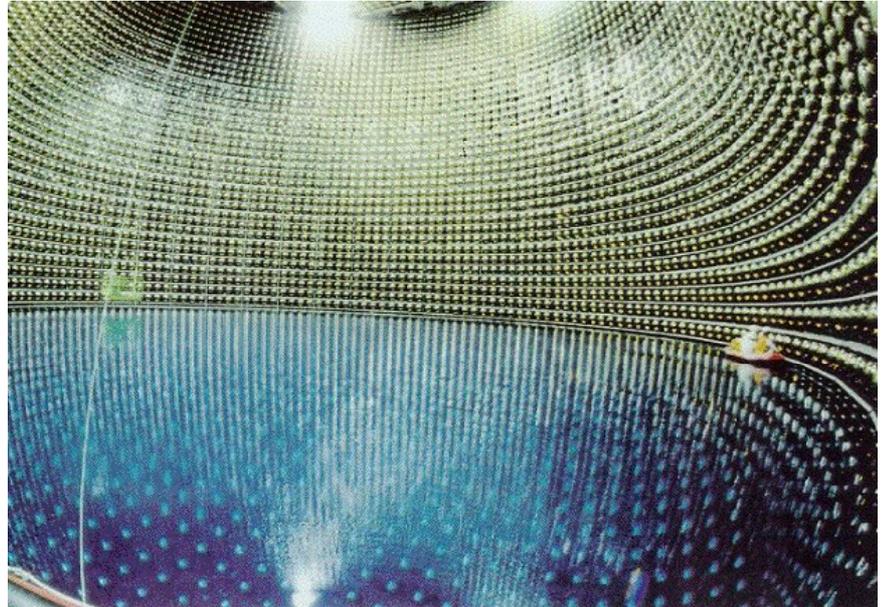


Fig. 4.33. Side and front views of a Wolter X-ray telescope. P and H denote parabolic and hyperbolic surfaces of revolution, whose common axis points to the source

- Neutrino detectors

- Gamma-ray telescopes



And of course: Radio Telescopes

