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Interferometers

General principle: Combine two (or more) beams coherently.

The angular resolution is determined by interference; the interference does not need a continuous aperture (see Young’s double slit experiment).

Some different types of interferometric concepts:

→ Hippolyte Fizeau (1868): basic concept of stellar interferometry

The Goal: increasing Resolution

Strictly speaking, the formula of $\theta = \frac{1.22\lambda}{D}$ only applies to a filled aperture; if only the outer regions contribute (similar to a very large central obscuration) the resolution is actually higher.
The Effect on the PSF

Interferometry is like masking a giant telescope:

Radio heterodyne receivers provide great flexibility for interferometry because their outputs retain the phase information about the incoming signal.

\[ \tau_g = B \cdot s/c \]
Main Components of an optical Interferometer
Main Components: 1) Telescopes

An optical interferometer typically consists of \( n \) telescopes of similar type and characteristics.

Keck interferometer (Hawaii) ↑
← VLTI (Paranal)

Main Components: 2) Delay Lines

Delay lines are needed to compensate the optical path difference between the various telescopes (depends on object location on the sky).

Challenge: travel over tens of meters, positioning to fractions of micrometers → dynamic range of \( > 10^9 \)!
Main Components: 3) Beam Combiner

Two main types:

- **multi-axial (image plane):** beams are placed adjacent to each other and form a fringe pattern in space.

- **co-axial (pupil plane):** beams are added on top of each other e.g. via a beam splitter.

- but also single-mode fibers and integrated optics.

Main Components: 4) Adaptive Optics

Adaptive optics (or for telescopes with $D < r_0$ tip-tilt correction) is essential to correct wavefront aberrations for good interference.

The amplitude of the fluctuations is: $\sigma = \sqrt{6.88 \left( \frac{B}{r_0} \right)^{5/6}}$ rads RMS

Hence, for a baseline $B = 100m$ and a seeing of $1''$ this amounts to $70\mu m$!

The MACAO (Multi Application Curvature Adaptive Optics) system on a 8m VLT. Can be used with natural guide stars with $1 < V < 17$, seeing $< 1.5''$, $r_0 < 1.5ms$ and airmass $< 2$. 
Fringe Visibility – Definition

The visibility is defined as

\[ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]

It is the Fourier transform of the object’s brightness distribution.

If the dark regions in the fringe pattern go to zero \( V = 1 \rightarrow \text{object is unresolved} \).

If \( V = 0 \) then there are no fringes \( \rightarrow \text{object is completely resolved} \).

Fig. 2. Left: examples of fringes with visibilities of 1, 0.5 and 0. Right: visibility as a function of baseline for a resolved star.
The observed pattern from a single star at the focal plane clearly changes as the distance between the two telescopes is gradually increased. The "fringes" disappear completely when the star is resolved.

**Fringe Visibility - Baseline**

Interferometric Fringes at Different Telescope Baselines (Simulation)

© European Southern Observatory

**Fringe Visibility and Power Spectrum**

Fringe Pattern  

| ~−λ/D' | 0 | ~λ/D' |

Power Spectrum  

| −B'/λ | 0 | B'/λ |

Ratio of energies = |V|^2/4

~ 2D'/λ
Fringe Visibility – another Example

Intensity profile and corresponding visibility for a 2-component model of
(i) An (almost) unresolved central star (hatched), and
(ii) a strongly resolved dust shell (solid black).

Visibility = sum of visibilities of the individual components

\[ \text{Dust shell} = \text{well resolved} \rightarrow \text{rapid decrease of visibility at low spatial frequency} \]

\[ \text{Compact central star creates broad visibility curve} \]
Fringe Tracking (Co-Phasing)

The fringes have to be actively tracked, which requires tracking fluctuations within a small fraction of wavelength in real-time.

Example: ESO’s FINITO scans the center of the fringe packet in H band with high speed and sends a co-phasing signal to the VLTI delay lines. FINITO operates on two channels, i.e. tracks three baselines.

White Light Fringes

Use of white light (= typical astronomical signal) will result in a pattern of colored fringes.

The term white light fringe refers to the central fringe.

The central fringe representing equal path length may be light or dark depending on the number of phase inversions experienced by the two beams as they traverse the optical system.
Closure Phase (1)

Fringe visibility tells one component of the objects Fourier transform = amplitude of the fringes.

The phase is determined by the position of the fringes.

Problem: due to atmospheric turbulence (which changes the optical path length), the fringes move constantly forward and backward.

Idea: use three telescopes \( \rightarrow \) three sets of fringes:
(1-2), (2-3), (1-3)
In all three sets the fringes move, but not independently!

\( \rightarrow \) this information is called closure phase (or self-calibration in aperture synthesis imaging - the standard technique in radio interferometry) and can be used to cancel out phase error terms.

Closure Phase (2)

\[
\begin{align*}
\Phi(1-2) &= \Phi_o(1-2) + [\Phi(2)-\Phi(1)] \\
\Phi(2-3) &= \Phi_o(2-3) + [\Phi(3)-\Phi(2)] \\
\Phi(3-1) &= \Phi_o(3-1) + [\Phi(1)-\Phi(3)] \\
\end{align*}
\]

\[
\text{Closure Phase} = \Phi_o(1-2)+\Phi_o(2-3) + \Phi_o(3-1)
\]

The error terms cancel out!

<table>
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<th>Number of telescopes</th>
<th>Number of Fourier phases</th>
<th>Number of closing triangles</th>
<th>Number of independent closure phases</th>
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(Radio) Aperture Synthesis

The limited information about the structure of a source provided by a two element interferometer can be expanded by moving the telescopes to change the baselines.

Even better: use $N$ telescopes and combine their outputs: $N$ telescopes provide $N(N-1)/2$ baselines. Each baseline adds a new Fourier component (or fringe spacing) →

This technique was pioneered in the radio by M. Ryle (Cambridge)

Figure 9.2. Improvement in field pattern quality (the images are the auto-correlation) with increasing number of interferometer baselines. Based on Condon and Ransom (2010).
Optical: VLTI Baselines

At good sites such as Mauna Kea in Hawaii, \( r_0 \) at 0.5 micron is 10 - 30 cm. The three ATs move on rails between the thirty observing stations above the holes that provide access to the underlying tunnel system. The light beams from the individual telescopes are guided towards the centrally located, partly underground Interferometry Laboratory.

Baseline Coverage (1)

A smooth reconstruction of the object's intensity distribution \( I \) requires a good coverage of the \((u,v)\) plane.

Remember: \( N \) telescopes provide \( N(N-1)/2 \) baselines.

Note: This is the uv-plane for an object at zenith. In general, the projected baselines have to be used.
Baseline Coverage (2)

The Earth's rotation helps to fill the $(u,v)$ plane. Example: source at 45° declination, observed for 3 hr both before and after transit.
**Field of View**

Typically, the field of view is limited to a few arcseconds only:

\[ \theta_{\text{max}} \leq \frac{\lambda}{D} \cdot \frac{\lambda}{\Delta \lambda} \]

- the size of the complex transfer optics. Larger field = larger optical elements
- spatial filters, which limit the FOV

**Sensitivity**

Signals \( V_1 \) and \( V_2 \) from telescopes 1 and 2 correspond to their respective areas \( A_1 \) and \( A_2 \) as:

\[ \langle V_s^2 \rangle \approx \langle V_1 \cdot V_2 \rangle \propto \sqrt{A_1 A_2} = A_{\text{effective}} \]

Thus, the effective area of an interferometer with two identical elements is that of one of the elements.

However, the noise from the two elements is independent and hence the noise output from the correlator is reduced by \( \sqrt{2} \)

\( \rightarrow S/N \) of a two-element interferometer is \( \sqrt{2} \) higher.

Since an array of \( N \) identical telescopes can be seen as \( N(N-1)/2 \) two-element interferometers, their net sensitivity is given by:

\[ \left( \frac{S}{N} \right)_c \approx \frac{T_S}{T_N} \left[ N(N-1)(\Delta f_{IF} \Delta t) \right]^{1/2} \]
Dirty Images & CLEAN

Ideally we would like to measure visibilities with a densely covered \((u,v)\) plane, just as for a filled aperture telescope, to get:

\[
I(x, y) = \iint V(u, v)e^{2\pi i (ux + vy)} \, du \, dv
\]

However, interferometer arrays leave gaps in the \((u,v)\) plane, resulting in a "dirty" image:

\[
I_D = \iint V(u, v)S(u, v)e^{2\pi i (ux + vy)} \, du \, dv
\]

where \(S(u, v)\) describes the sampling of the \((u,v)\) plane. \(S(u, v) = 1\) where measurements exist, and is zero elsewhere.

Hence, our image has artifacts and can be described by:

\[
I_D(x, y) = I(x, y) \otimes B(x, y)
\]

i.e., the true distribution is convolved with the PSF

\[
B(x, y) = \iint S(u, v)e^{2\pi i (ux + vy)} \, du \, dv
\]

Use the CLEAN (Högbom 1974) algorithm to iteratively remove the "dirty beam".

Radio Interferometry Projects
**Very Large Array VLA**

- Y-shaped array of 27 telescopes moved on railroad tracks
- Telescope diameter 25-m each
- Located: high Plains of San Augustin in New Mexico
- "D", "C", "B", and "A" configurations, spanning 1.0, 3.4, 11, and 36 km, respectively

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**Australia Telescope Compact Array ATCA**

*Six 22 m telescopes on an east-west baseline*
**Westerbork**

- *Westerbork Synthesis Radio Telescope (WSRT)*
- 14 telescopes
- 25-meter each
- East-west baseline
- 3 km in length
- Effective collecting area of a 92 m dish

**LOFAR in the Netherlands**

The LOw Frequency ARray uses two types of low-cost antennas:
- Low Band Antenna (10-90 MHz)
- High Band Antenna (110-250 MHz).

Antennae are organized in 36 stations over ~100 km. Each station contains 96 LBAs and 48 HBAs.

Baselines: 100m - 1500km

Main LOFAR subsystems:
- sensor fields
- wide area networks
- central processing systems
- user interfaces
The Very Long Baseline Interferometer (VLBI) Technique

VLBI in the US: The Very Long Baseline Array VLBA

Ten 25 m antennas form an array of 8000 km in size.

Recently it has become possible to connect the VLBI radio telescopes in real-time → e-VLBI.

In Europe, six radio telescopes of the EVN are now connected with Gbit/s-links.

Data processing in real time at the European Data Processing centre at JIVE (Astron/Dwingeloo)
Plateau de Bure

Interferometer of six 15 m antennas

Combined Array for Research in Millimeter-wave Astronomy (CARMA)

$\text{CARMA = six 10-meter telescopes from Caltech's Owens Valley Radio Observatory + nine 6-meter telescopes from the Berkeley-Illinois-Maryland Association \rightarrow Cedar (CA)}$
Sub-Millimeter Array (SMA)

The SMA consists of eight 6 m antennas on Mauna Kea (HI)

ALMA

- Giant array of 50 antennas (12m each)
- Baselines from 160m – 16 km.
- Additional compact array of twelve 7m and four 12m antennas
- Located on the Chajnantor plain at 5000m altitude
- Wavelength range 3 mm – 400 µm (84 to 720 GHz)
Observing frequencies: Band 3 (>84 GHz) to band 9 (<720 GHz).

Field of view ~ size of individual antennas & frequency (but independent of array configuration!). FWHM of beam is 21" at 300 GHz. To achieve uniform sensitivity over a larger field mosaicking is required.

Spatial resolution ~ frequency & maximum baseline. Most extended configuration (~16 km): 6 mas at 675 GHz. Structures > 0.6»/b_{min} (b_{min} = shortest baseline) are not well reproduced in reconstructed images. Measure with the ALMA Compact Array (ACA) using the 7-m antennae (come closer).

Spectral resolution: up to 8192 frequency channels (spectral resolution elements). At 110 GHz, R=30,000,000 or 10m/s velocity resolution.

Sensitivity: use ALMA Sensitivity Calculator to estimate noise levels or required integration times at http://almascience.eso.org/call-for-proposals/sensitivity-calculator