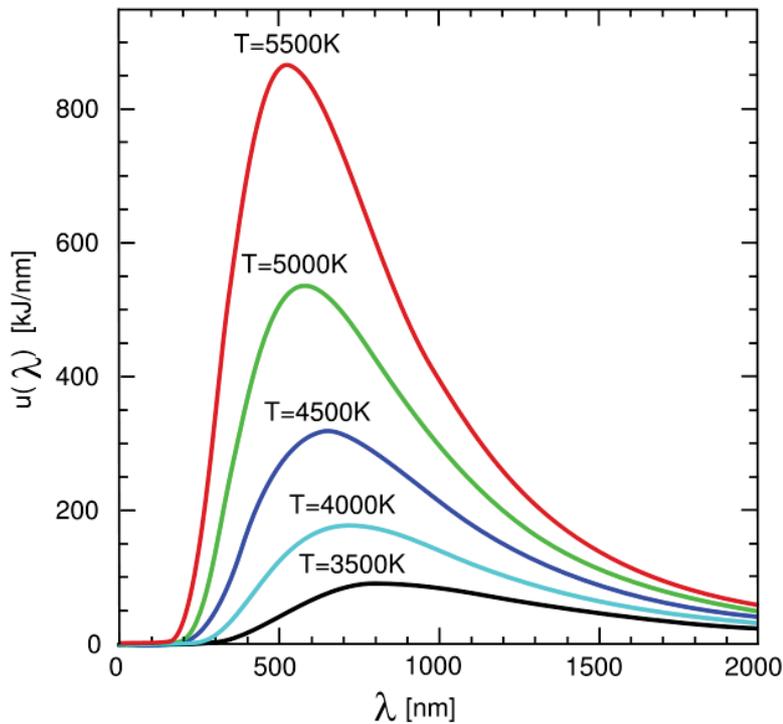


Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

1st Lecture: 3 September 2012



This lecture:

- Black body radiation
- Astronomical magnitudes
- Point \leftrightarrow extended sources

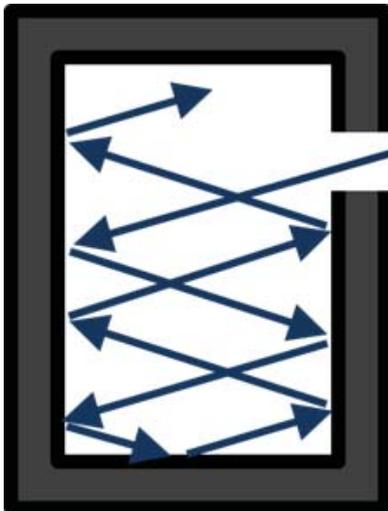
Black Body Radiation

Introduction

(from Wikipedia)

Kirchhoff (1860): "...imagine that bodies (...) completely absorb all incident rays, and neither reflect nor transmit any. I shall call such bodies perfectly black, or, more briefly, black bodies."

This shall be true of radiation for all wavelengths and for all angles of incidence.



Suppose cavity at fixed temperature T and at thermal equilibrium

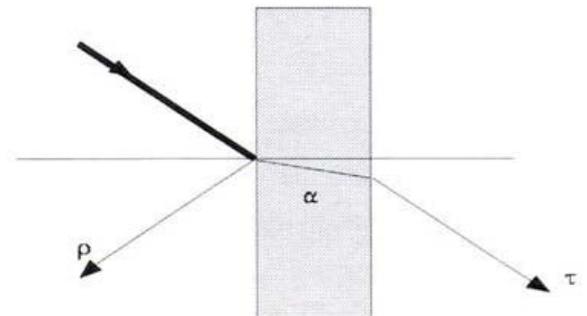
Radiation entering the cavity will be "thermalized" by continual absorption and re-emission of radiation by material in the cavity or its walls.

Hole is small \rightarrow escaping radiation will approximate black-body radiation and does not depend upon the properties of the cavity or the hole.

Kirchhoff's Law

Conservation of power requires that:

$$\alpha + \rho + \tau = 1$$



with α = absorptivity, ρ = reflectivity, τ = transmissivity

Consider a cavity in thermal equilibrium with completely opaque sides:

$$\left. \begin{array}{l} \varepsilon = 1 - \rho \\ \alpha + \rho + \tau = 1 \\ \tau = 0 \end{array} \right\} \alpha = \varepsilon$$

This is Kirchhoff's law, which applies to a perfect black body

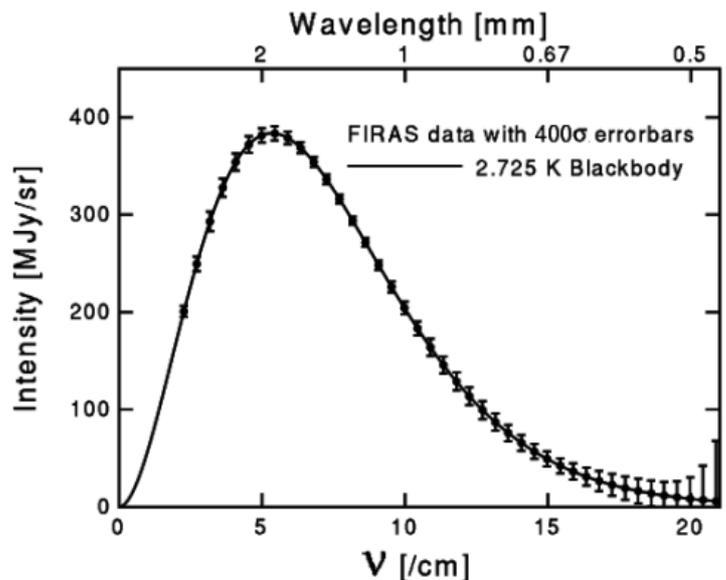
A radiator with $\varepsilon = \varepsilon(\lambda) < 1$ is often called a grey body

Definition of a Black Body

- A black body (BB) is an idealized object that absorbs all EM radiation
- Cold ($T \sim 0\text{K}$) BBs are black (no emitted or reflected light)
- At $T > 0\text{K}$ BBs absorb and re-emit a characteristic EM spectrum

Many astronomical sources emit close to a **black body**.

Example: COBE measurement of the cosmic microwave background



Black Body Emission

The specific intensity I_ν of a blackbody is given by **Planck's law** as:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

in units of $[\text{W m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}]$
(see course on Radiative Processes)

In terms of **wavelength units** this corresponds to:

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

in units of $[\text{W m}^{-3} \text{sr}^{-1}]$

Note for the **conversion** of frequency \leftrightarrow wavelength units:

$$d\nu = \frac{c}{\lambda^2} d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2} d\nu$$

Useful Approximations

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

At high frequencies ($h\nu \gg kT$) we get **Wien's** approximation:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

At low frequencies ($h\nu \ll kT$) we get **Rayleigh-Jeans'** approximation:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

Emission \Leftrightarrow Power \Leftrightarrow Temperature

The total radiated power per unit surface (radiant exitance) is proportional to the **fourth power of the temperature**:

$$\int_{\Omega} \int_{\nu} I_\nu(T) d\nu d\Omega = M = \sigma T^4$$

where $\sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the **Stefan-Boltzmann constant**.

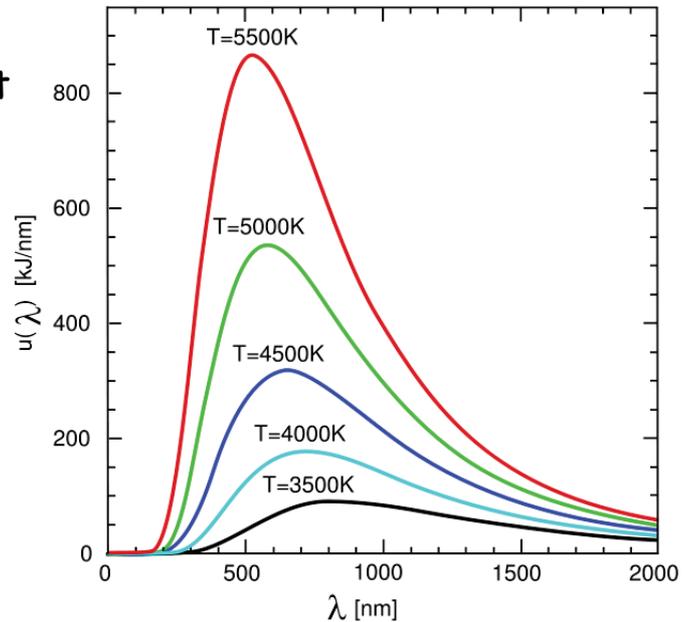
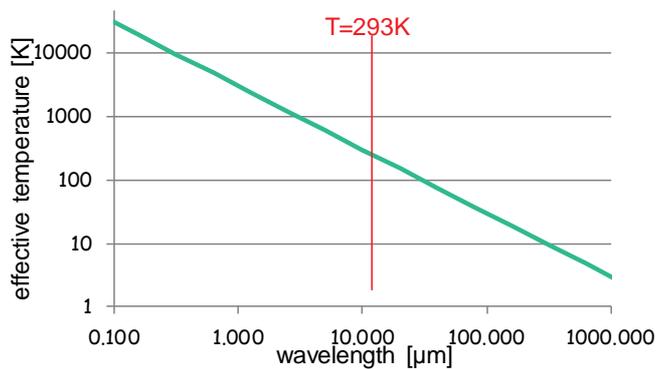
Assuming BB radiations, astronomers often specify the emission from objects via their effective temperature.

Effective Temperatures

The temperature corresponding to the maximum specific intensity is given by **Wien's displacement law**:

$$\frac{c}{\nu_{\max}} T = 5.096 \cdot 10^{-3} \text{ mK} \quad \text{or} \quad \lambda_{\max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

Hence, cooler BBs have their peak emission (**effective temperatures**) at longer wavelengths and at lower intensities:



$$\left. \begin{aligned} \varepsilon &= 1 - \rho \\ \alpha + \rho + \tau &= 1 \\ \tau &= 0 \end{aligned} \right\} \alpha = \varepsilon$$

Gustav Kirchhoff (1824 – 1887)



$$M = \sigma T^4$$

Josef Stefan (1835 – 1893)



Ludwig Eduard Boltzmann (1844 – 1906)



$$I_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

Max Planck (1858 – 1947)



$$I_{\nu}(T) \approx \frac{2\nu^2}{c^2} kT$$

John William Strutt, 3rd Baron Rayleigh (1842 – 1919)



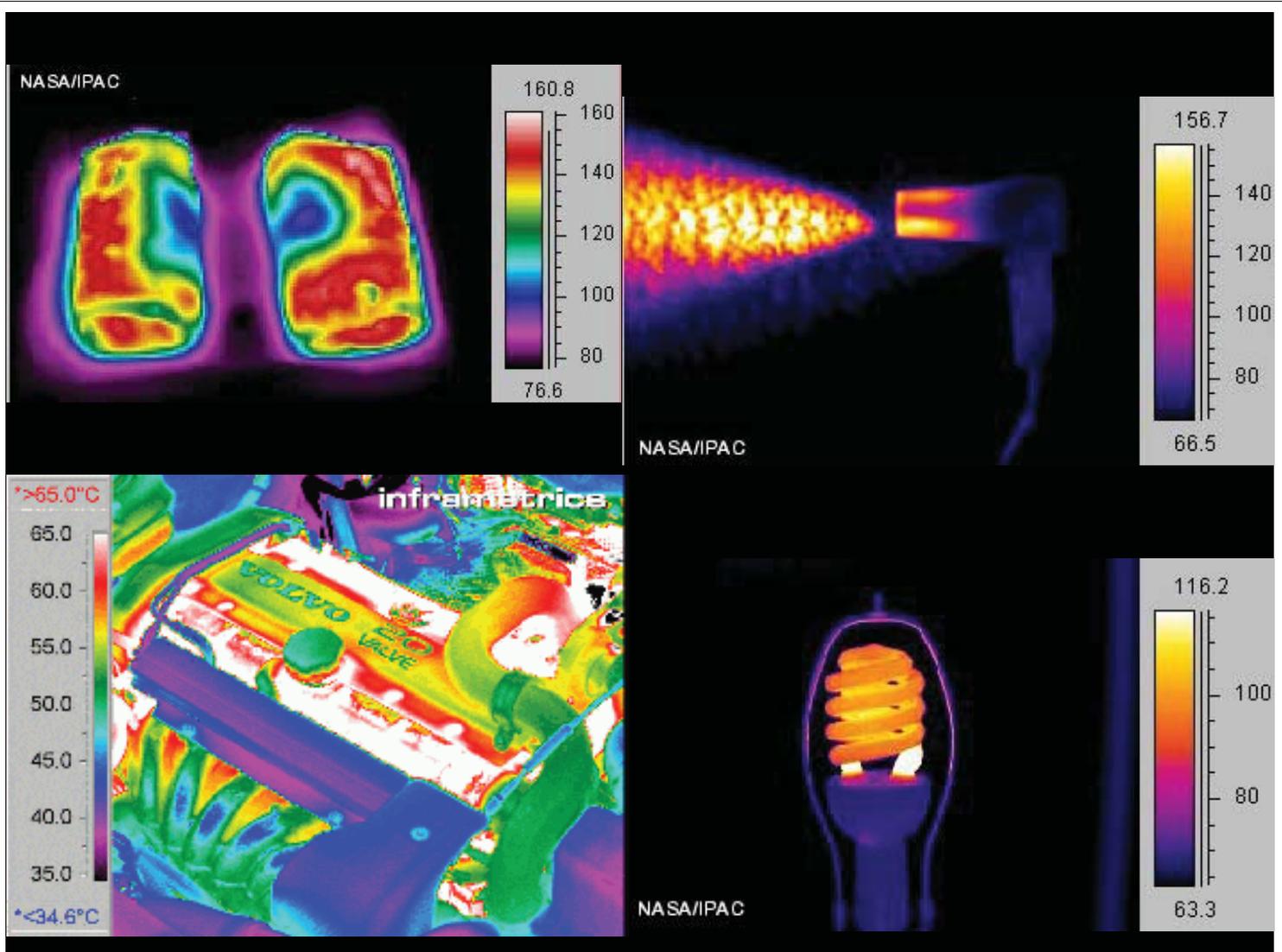
Sir James Hopwood Jeans (1877 – 1946)



$$I_{\nu}(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

$$\lambda_{\max} = \frac{2.98 \cdot 10^{-3} \text{ mK}}{T}$$

Wilhelm Wien (1864 – 1928)

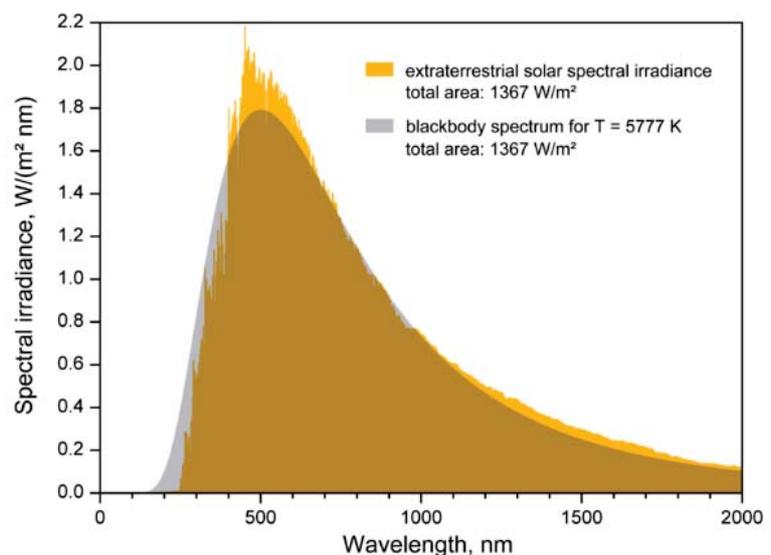


Grey Bodies

Many emitters are close to but not perfect black bodies, with a wavelength-dependent emissivity $\epsilon < 1$:

$$I_{\lambda}(T) = \epsilon(\lambda) \cdot \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Example: the Sun
(like many stars)



Brightness Temperature

Brightness temperature is the temperature a perfect black body would have to be at to duplicate the observed intensity of a grey body object at a frequency ν .

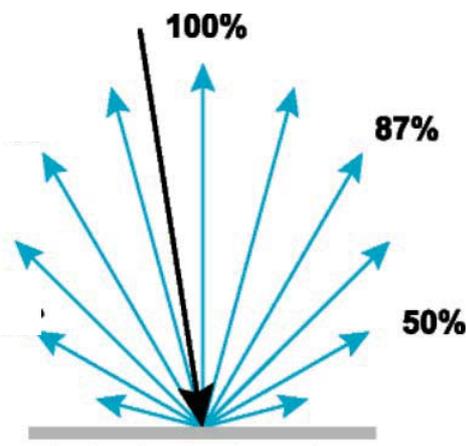
For low frequencies ($h\nu \ll kT$) we can write:

$$T_b = \varepsilon(\nu) \cdot T \stackrel{\text{Rayleigh-}}{\underset{\text{Jeans}}{=}} \varepsilon(\nu) \cdot \frac{c^2}{2k\nu^2} I_\nu$$

Only for perfect BBs is T_b the same for all frequencies.

Lambert's Cosine Law

(Wikipedia:) **Lambert's cosine law** states that the radiant intensity from an ideal **diffusively reflecting surface** is directly proportional to the cosine of the angle θ between the surface normal and the observer.



If the Moon were a Lambertian scatterer, its scattered brightness would decrease towards its terminator (i.e. the Moon surface scatters more light at large angles than a Lambertian surface would.



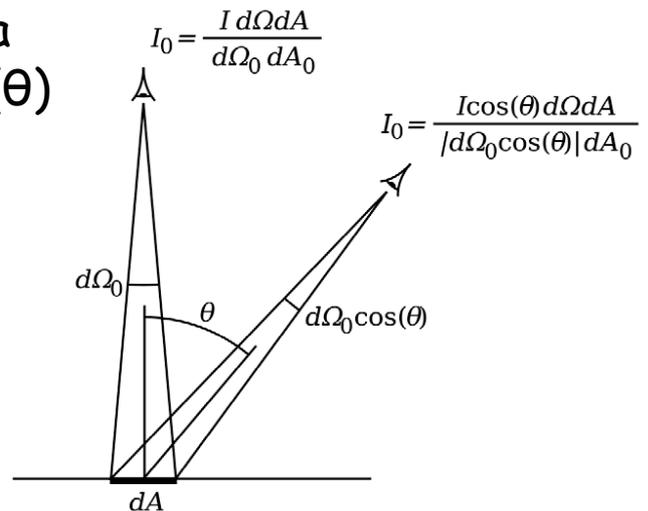
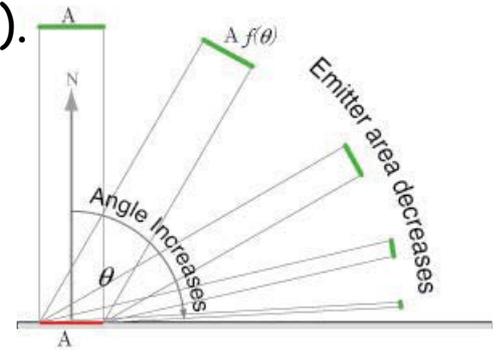
Johann Heinrich Lambert
(1728 – 1777)

Lambertian Emitters

The radiance of Lambertian emitters is independent of the direction θ of observation (i.e., isotropic).

Two effects:

1. Lambert's cosine law \rightarrow radiant intensity and $d\Omega$ are reduced by $\cos(\theta)$
 2. Emitting surface area dA for a given $d\Omega$ is increased by $\cos^{-1}(\theta)$
- \rightarrow Both effects cancel out



*Perfect black bodies are Lambertian emitters!
(The Sun is almost ...)*

Astronomical Magnitudes

Summary of Radiometric Quantities

(see course on Radiative Processes!)

Name	Symbol	Unit	Definition	Equation
Spectral radiance or specific intensity	L_ν, I_ν	$W m^{-2} Hz^{-1} sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\nu$	
Spectral radiance or specific intensity	L_λ, I_λ	$W m^{-3} sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\lambda$	
Radiance or Intensity	L, I	$W m^{-2} sr^{-1}$	Spectral radiance integrated over spectral bandwidth	$L = \int L_\nu d\nu$
Radiant exitance	M	$W m^{-2}$	Total power emitted per unit surface area	$M = \int L(\theta) d\Omega$
Flux or luminosity	Φ, L	W	Total power emitted by a source of surface area A	$\Phi = \int M dA$
Spectral irradiance or flux density	L_ν, F_ν, I_ν	$W m^{-2} Hz^{-1} *$	Power received at a unit surface element per unit $\Delta\nu$	
Spectral irradiance or flux density	$L_\lambda, F_\lambda, I_\lambda$	$W m^{-3} *$	Power received at a unit surface element per unit $\Delta\lambda$	
Irradiance	E	$W m^{-2}$	Power received at a unit surface element	$E = \frac{\int M dA}{4\pi r^2}$

Karl Guthe Jansky (1905 – 1950)



* $10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}$ is called 1 **Jansky**

Optical Astronomers use 'Magnitudes'

This system has its origins in the Greek classification of stars according to their visual brightness. The brightest stars were $m = 1$, the faintest detected with the bare eye were $m = 6$.

Later formalized by Pogson (1856): $1^{st} \text{ mag} \sim 100 \times 6^{th} \text{ mag}$

Magnitude	Example	#stars brighter
-27	Sun	
-13	Full moon	
-5	Venus	
0	Vega	4
2	Polaris	48
3.4	Andromeda	250
6	Limit of naked eye	4800
10	Limit of good binoculars	
14	Pluto	
27	Visible light limit of 8m telescopes	

Apparent Magnitude

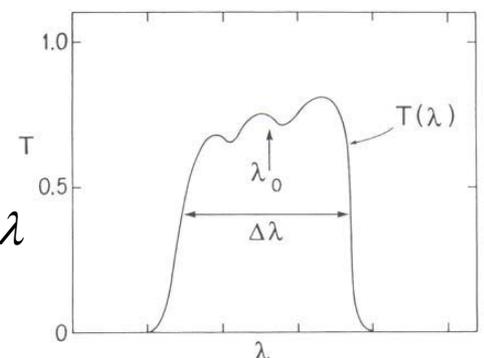
The **apparent magnitude** is a *relative* measure of the monochromatic flux density F_λ of a source:

$$m_\lambda - M_0 = -2.5 \cdot \log\left(\frac{F_\lambda}{F_0}\right)$$

M_0 defines the reference point (usually **magnitude zero**).

In practice, measurements are done through a **transmission filter** $\mathcal{T}(\lambda)$ that defines the bandwidth:

$$m_\lambda - M = -2.5 \log \int_0^\infty T(\lambda) F_\lambda d\lambda + 2.5 \log \int_0^\infty T(\lambda) d\lambda$$



Photometric Systems

Filters are usually matched to the atmospheric transmission
→ different observatories = different filters

→ many **photometric systems**:

- **Johnson UBV system** →
- Gunn griz
- USNO
- SDSS
- 2MASS JHK
- HST filter system (STMAG)
- AB magnitude system
- ...

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]
U	0.36	0.068
B	0.44	0.098
V	0.55	0.089
R	0.70	0.22
I	0.90	0.24
J	1.25	0.30
H	1.65	0.35
K	2.20	0.40
L	3.40	0.55
M	5.0	0.3
N	10.2	5
Q	21.0	8

AB and STMAG Systems

For a given flux density F_ν , the **AB magnitude** is defined as:

$$m(AB) = -2.5 \cdot \log F_\nu - 48.60$$

- an object with constant flux per unit **frequency** interval has zero color.
- zero points are defined to match the zero point of the Johnson V-band
- used by SDSS and GALEX
- F_ν is given in units of $[\text{erg s}^{-1} \text{cm}^2 \text{Hz}^{-1}]$

Similarly, the **STMAG system** is defined such that an object with constant flux per unit **wavelength** interval has zero color.

- STMAGs are used by the HST photometry packages.

Color Indices

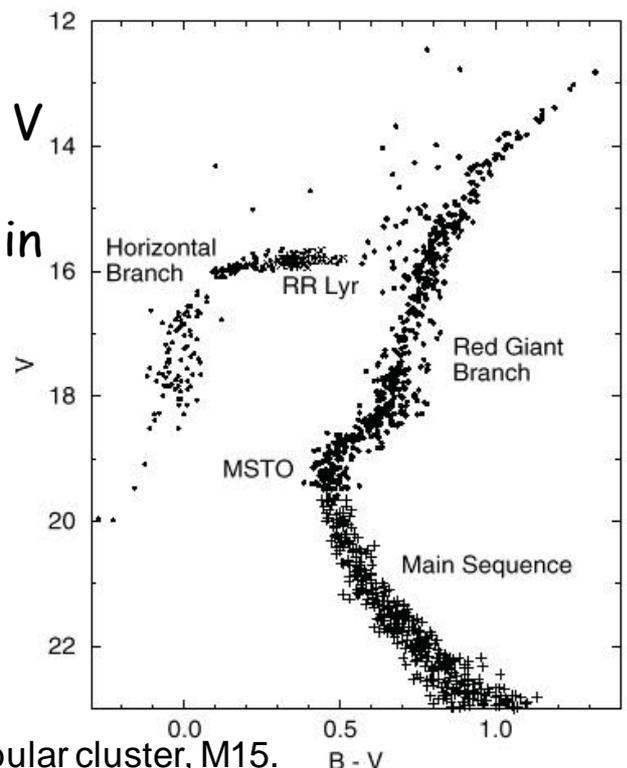
Color index = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths.

- The color indices of an AOV star (**Vega**) are about zero longward of V
- The color indices of a blackbody in the **Rayleigh-Jeans tail** are:

$$B-V = -0.46$$

$$U-B = -1.33$$

$$V-R = V-I = \dots = V-N = 0.0$$



Color-magnitude diagram for a typical globular cluster, M15.

Absolute Magnitude

Absolute magnitude = apparent magnitude of the source if it were **at a distance of $D = 10$ parsecs**: $M = m + 5 - 5 \log D$

$$M_{\text{Sun}} = 4.83 (V); M_{\text{Milky Way}} = -20.5 \rightarrow \Delta \text{mag} = 25.3 \rightarrow \Delta \text{lumi} = 14 \text{ billion } L_{\odot}$$

However, **interstellar extinction E** or **absorption A** affects the apparent magnitudes

$$E(B - V) = A(B) - A(V) = (B - V)_{\text{observed}} - (B - V)_{\text{intrinsic}}$$

Hence we need to include **absorption** to get the correct absolute magnitude:

$$M = m + 5 - 5 \log D - A$$

Bolometric Magnitude

Bolometric magnitude is the luminosity expressed in magnitude units = **integral of the monochromatic flux over all wavelengths**:

$$M_{bol} = -2.5 \cdot \log \frac{\int_0^{\infty} F(\lambda) d\lambda}{F_{bol}} \quad ; F_{bol} = 2.52 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2}$$

If the source radiates isotropically one can simplify:

$$M_{bol} = -0.25 + 5 \cdot \log D - 2.5 \cdot \log \frac{L}{L_{\odot}} \quad ; L_{\odot} = 3.827 \cdot 10^{26} \text{ W}$$

The bolometric magnitude can also be derived from the **visual magnitude plus a bolometric correction BC**:

$$M_{bol} = M_V + BC$$

BC is large for stars that have a peak emission very different from the Sun's.

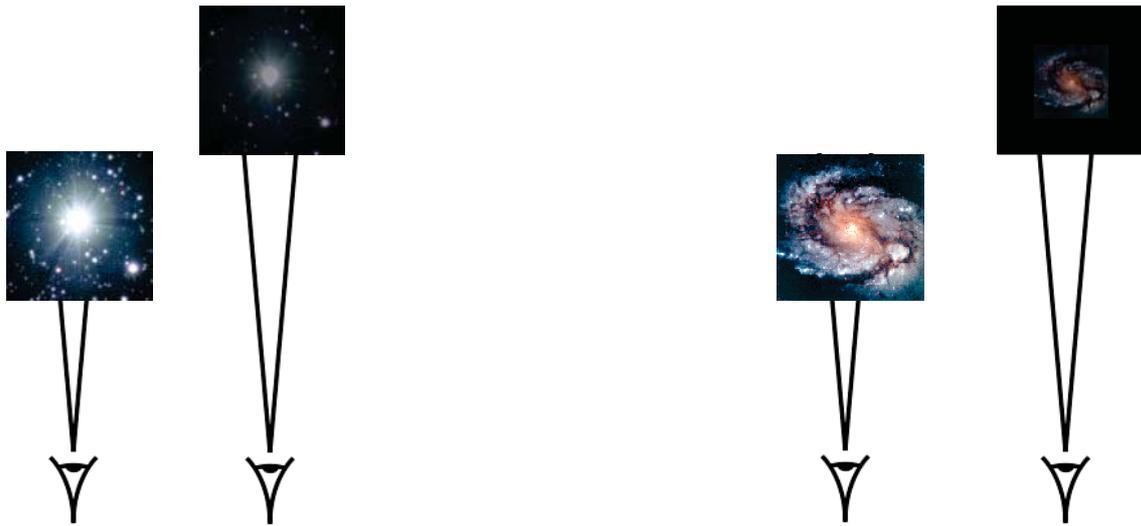
Photometric Systems and Conversions

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [$\text{W m}^{-2} \mu\text{m}^{-1}$]	F_ν [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m⁻² Hz⁻¹.

Point Sources and Surface Brightness

Point Sources and Extended Sources



Point sources = spatially unresolved

Extended sources = well resolved

Brightness $\sim 1 / \text{distance}^2$

Surface brightness $\sim \text{const}(\text{distance})$

Size given by observation

Brightness $\sim 1/d^2$ and size $\sim 1/d^2$

Surface brightness [mag/arcsec²] is constant with distance!

Calculating Surface Brightness

To describe the **surface brightness** of extended objects one uses units of **mag/sr** or **mag/arcsec²**.

Magnitudes are logarithmic units; to get the surface brightness of an area A :

$$S = m + 2.5 \cdot \log_{10} A$$

The observed surface brightness [mag/arcsec²] can be converted into physical surface brightness units via

$$S[\text{mag/arcsec}^2] = M_{\odot} + 21.572 - 2.5 \cdot \log_{10} S[L_{\odot}/\text{pc}^2]$$

with $L_{\odot} = 3.839 \times 10^{26} \text{ W} = 3.839 \times 10^{33} \text{ erg s}^{-1}$