1. Atmospheric Layers

Assumption: atmosphere is in local radiative equilibrium and the composition is approximately constant.

The structure can be described by three parameters:

- altitude $z$
- temperature $T(z)$
- density $\rho(z)$

The pressure $P(z)$ can be described by:  $P(z) = P_0 e^{-\frac{z}{H}}$

where $H =$ scale height ($H \sim 8\text{km near ground}$).
**Constituents of the Atmosphere**

- **Main constituents**: $O_2$ and $N_2$
  - relative constant proportions (78.1% $N_2$, 20.9% $O_2$) up to 100 km

- **Ozone** - absorbs mainly in the UV
  - distribution depends on latitude and season
  - maximum concentration around 16 km height

- **$CO_2$** - important component for (mid)IR absorption
  - mixing independent of altitude (similar to $N_2$, $O_2$)

- **Ions** - varies strongly with altitude and solar activity
  - relevant above 60km where reactions with UV photons occur:
    
    $O_2 + h\nu \rightarrow O_2^+ + e^-$ \quad \text{and} \quad O_2 + h\nu \rightarrow O^+ + O + e^-$
  
  - electron showers along magnetic fields cause Aurora
  - at 100 - 300 km height: $n_e \sim 10^5 - 10^6 \text{ cm}^{-3}$

- **Water vapour** - causes very strong absorption bands
2. Absorption of Radiation

Atomic and molecular transitions that cause absorption features:
- pure rotational molecular transitions: $H_2O$, $CO_2$, $O_3$,
- rotation-vibrational molecular transitions: $CO_2$, $NO$, $CO$
- electronic molecular transitions: $CH_4$, $CO$, $H_2O$, $O_2$, $O_3$, $OH$
- electronic atomic transitions: $O$, $N$, ...

The attenuation at altitude $z_0$ is given by:

$$I(z_0) = I_0(\infty) \cdot \exp \left[ -\frac{1}{\cos \theta} \sum_i \tau_i(\lambda, z_0) \right]$$

for $i$ absorbing species with an optical depth of $\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} r_i(z) \rho_0(z) \kappa_i(\lambda) dz$

($\theta$ is the zenith distance; $\kappa$ is the absorption coefficient; $\rho_0$ is the mass density of air, and $r(z)$ the mixing ratio).

Atmospheric Bands

Two cases of absorption:
- total absorption $\Rightarrow$ atmospheric transmission windows
- partial absorption $\Rightarrow$ reduced transmission due to narrow telluric* absorption features

*Telluric = related to the Earth; of terrestrial origin

The atmospheric opacity defines the atmospheric transmission bands - and thus the wavelengths that are accessible to observations
Ground based astronomy is limited to visible, near/mid-IR and radio wavelengths.
Space astronomy provides access to γ-rays, X-rays, UV, FIR, sub-mm

Absorption by Water Vapour

The water vapor is a strong function of $T$ and $z$.

The precipitable water vapor (PWV) is the depth of the amount of water in a column of the atmosphere if all the water in that column were precipitated as rain.

The amount of PWV above an altitude $z_0$ is:

$$w(z_0) = \int_{z_0}^{\infty} N_{H_2O} \, dz ,$$

where $N_{H_2O} \, [m^{-3}] = 4.3 \times 10^{25} \frac{P}{P_0} \frac{T}{T_0} r(z)$

Scale height for PWV is only ~3 km $\rightarrow$ observatories on high altitudes

0.1 mm PWV
0.4 mm PWV
1.0 mm PWV
3.0 mm PWV
3. Atmospheric Emission

A. Fluorescent Emission

Fluorescence = recombination of electrons with ions.

The recombination probability is low; takes several hours → night time

- Produces both continuum + line emission = airglow
- Occurs mainly at ~ 100 km height
- Main sources of emission are: O I, Na I, O₂, OH (←NIR), H

The emission intensity is measured in Rayleigh:

$$1 \text{ Rayleigh} = 10^6 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} = \frac{1.58 \times 10^{-11}}{\lambda [\text{nm}]} \text{ W cm}^{-2} \text{ sr}^{-1}$$
B. Thermal Emission

Up to 60 km is the atmosphere in local thermodynamic equilibrium (LTE), i.e., the excitation levels are thermally populated.

Calculating the specific energy received requires a full radiative transfer calculation (see below), but for \( \tau \ll 1 \) one can use the approximation:

\[
I_\lambda(z) = \tau_\lambda B_\lambda(\bar{T}) \frac{1}{\cos \theta}
\]

where \( B(\bar{T}) \) is the Planck function at the mean temperature of the atmosphere.

<table>
<thead>
<tr>
<th>Spectral band</th>
<th>( L )</th>
<th>( M )</th>
<th>( N )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wavelength [( \mu \text{m} )]</td>
<td>3.4</td>
<td>5.0</td>
<td>10.2</td>
<td>21.0</td>
</tr>
<tr>
<td>Mean optical depth ( \tau )</td>
<td>0.15</td>
<td>0.3</td>
<td>0.08</td>
<td>0.3</td>
</tr>
<tr>
<td>Magnitude [arcsec(^{-2})]</td>
<td>8.1</td>
<td>2.0</td>
<td>-2.1</td>
<td>-5.8</td>
</tr>
<tr>
<td>Intensity [Jy arcsec(^{-2})]</td>
<td>0.16</td>
<td>22.5</td>
<td>250</td>
<td>2100</td>
</tr>
</tbody>
</table>

For \( \bar{T} = 250 \text{ K} \) and \( \theta = 0 \):

Fluorescent and Thermal Emission

Sky surface brightness.

Important as even an unresolved point source has a finite angular diameter when viewed through a telescope.
4. Scattering, Refraction and Dispersion

A. Scattering by Air Molecules

Molecular scattering in the visible and NIR is Rayleigh scattering given by:

$$\sigma_R(\lambda) = \frac{8\pi^3}{3} \frac{(n^2 - 1)^2}{N^2\lambda^4}$$

where $N$ is the number of molecules per unit volume and $n$ is the refractive index of air ($n - 1 \approx 8 \times 10^{-5} P/T$).

Remember, Rayleigh scattering is not isotropic: $I_{\text{scattered}} = I_0 \frac{3}{16\pi} \sigma_R(1 + \cos^2 \theta) d\omega$
B. Aerosol Scattering

Aerosols (like sea salts, hydrocarbons, volcanic dust) are much bigger than air molecules → Rayleigh scattering does not apply.

Instead, scattering is described by Mie’s theory (from classical electrodynamics, using a “scattering efficiency factor” Q):

\[ Q_{\text{scattering}} = \frac{\sigma_M}{\pi a^2} = \frac{\text{scattering cross section}}{\text{geometrical cross section}} \]

If \( a \gg \lambda \) then \( Q_{\text{scattering}} \sim Q_{\text{absorption}} \) and:

- the scattered power is equal to the absorbed power
- the effective cross section is twice the geometrical size

If \( a \sim \lambda \) then \( Q_s \propto 1/\lambda \) (for dielectric spheres):

- the scattered intensity goes with \( 1/\lambda \)

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Atmospheric Refraction

Due to atmospheric refraction, the apparent location of a source is significantly altered (up to half a degree near the horizon) → telescope pointing model.

Refraction \( R = (n(\lambda) - 1)\tan \theta \)

Note that the refractive index of air depends on the wavelength \( \lambda \):

\[ [n(\lambda) - 1] \times 10^6 = 64.328 + \frac{29498.1}{\lambda^2} + \frac{255.4}{41 - \frac{1}{\lambda^2}} \]

(valid for dry air, 1 atm pressure, \( T \sim 290K \) and \( \lambda_0 \) in [\( \mu m \)]).
Atmospheric Dispersion

Dispersion: The elongation of points in broadband filters due to $n(\lambda)$ $\rightarrow$ “rainbow”.
The magnitude of the dispersion is a strong function of airmass and wavelength.

No problem if dispersion $< \lambda/D \leftrightarrow$ o.k. for small or seeing limited telescopes, but big problem for large diffraction limited telescopes.

Atmospheric Dispersion Corrector

To counterbalance atmospheric dispersion use:
1. a refractive element (e.g., prism)
2. a second prism (different material with different dispersion) to maintain the optical axis
3. use a second (identical) double prism assembly to adjust the strength of the correction for different zenith angles.
5. Atmospheric Turbulence

The scales $L_1, L_2, L_3$ are characteristic of the outer (external) scales of turbulence caused by the wind around the obstacles 1, 2, 3.

At $Re \sim 2200$ the transition from laminar to turbulent flow occurs.

\[ Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \]

Example: wind speed $\sim 1 \text{ m/s}$, $L = 15 \text{ m} \Rightarrow Re = 10^6 \Rightarrow$ turbulent!
The kinetic energy of large scale (~L) movements is gradually transferred to smaller and smaller scales, down to a minimum scale length $l_0$, at which the energy is dissipated by viscous friction.

The local velocity field can be decomposed into spatial harmonics of the wave vector $\kappa$. The reciprocal value $1/\kappa$ represents the scale under consideration.

The mean 1D spectrum of the kinetic energy, or Kolmogorov spectrum, is:

$$E(\kappa) \propto \kappa^{-5/3}$$

where $l_0$ is the inner scale, $L_0$ the outer scale of the turbulence, and $L_0^{-1} < \kappa < l_0^{-1}$.
Air Refractive Index Fluctuations

Winds mix layers of different temperature $\rightarrow$ fluctuations of temperature $T \rightarrow$ fluctuations of density $\rho \rightarrow$ fluctuations of refractive index $n$.

Of interest: difference between $n(r)$ at point $r$ and $n(r+\rho)$ at a nearby point $r+\rho$. The variance of the two values is given by:

$$D_n(\rho) = \left\langle (n(r) - n(r + \rho))^2 \right\rangle = C_n^2 \rho^{2/3}$$

where $D_n(\rho)$ is the index structure function and $C_n^2$ is the index structure coefficient or structure constant of the refractive index.

Air Refractive Index Fluctuations (2)

Usually, one is only interested in the integral of fluctuations along the line of sight: $C_n^2 \Delta h$.

Typical value: $C_n^2 \Delta h \sim 4 \cdot 10^{-13} \text{ cm}^{1/3}$ for a 3 km altitude layer

But: there are always several layers of turbulence

Median seeing conditions on Mauna Kea are taken to be $r_o \sim 0.23$ meters at 0.55 microns. The 10% best seeing conditions are taken to be $r_o \sim 0.40$ meters. Figure taken from a paper by Ellerbroek and Tyler (1997).
Figure 2. Median $C_n^2$ profile obtained with the complete sample of 43 nights, the summer [April-June] and winter [October-March] time samples. Results are obtained with the standard GS technique.

Image Degradation by the Atmosphere
...and why we care ...!

PHARO LGS Ks image
500s integ., 40" FOV, 150 mas FWHM

WIRO H image
Kobulnicky et al. 2005, AJ 129, 239-250
Aspects of Image Degradation

Scintillation - the energy received by the pupil varies in time

Image motion - the average slope of the wavefront at the pupil varies ("tip-tilt")

Image blurring - the spatial coherence of the wavefront is reduced ("seeing")

The Fried Parameter $r_0$

The radius of the spatial coherence area is given by the so-called Fried parameter $r_0$:

$$r_0(\lambda) = 0.185 \lambda^{6/5} \left[ \int_0^\infty C_n^2(z)dz \right]^{-3/5}$$

Note that $r_0$ increases as the 6/5 power of the wavelength and decreases as the -3/5 power of the air mass.

Another "definition" is that $r_0$ is the average turbulent scale over which the RMS optical phase distortion is 1 radian.

The angle $\Delta \theta = \frac{\lambda}{r_0}$ is often called the atmospheric seeing.
Short Exposures through Turbulence

Random intensity distribution of speckles in the focal plane:

The observed image from some source is given by the convolution of $I_0$ with the MTF or pupil transfer function $T(\omega)$:

$$I(\theta) = I_0(\theta) * T(\theta) \quad \text{or} \quad \langle |I(\omega)|^2 \rangle = |I_0(\omega)|^2 \langle |T(\omega)|^2 \rangle$$

If a point source is observed as reference through the same $r_0$ we can calculate:

$$|I_0(\omega)| = \left( \frac{\langle |I(\omega)|^2 \rangle_{\text{obs}}}{\langle |T(\omega)|^2 \rangle_{\text{obs}}} \right)^{1/2}$$

This is called speckle interferometry.

Speckle Interferometry

Example: Real-time bispectrum speckle interferometry: 76 mas resolution.
http://www.mpifr-bonn.mpg.de/div/ir-interferometry/movie/speckle/specklemovie.html

Several related techniques do exist, e.g., Shift-and-add, Lucky Imaging, bispectrum analysis, Aperture masking, Triple correlation, ...
Turbulence Correlation Time $\tau_c$

Time series of a patch of atmosphere above the 3.6m telescope aperture (Gendron 1994)

Two effects:
1. The turbulence does not change arbitrarily fast but with a correlation time or coherence time $\tau_c$.
2. Often, the turbulent time scales $\gg$ time for the turbulent medium to pass the telescope aperture (wind speed) $\Rightarrow$ “frozen turbulence”

Summary: $r_0$, seeing, $\tau_0$, $\Theta_0$

The Fried parameter $r_0(\lambda) = 0.185\lambda^{6/5}\left[\int_0^\infty C_n^2(z)dz\right]^{-3/5}$ is the radius of the spatial coherence area. It is the average turbulent scale over which the RMS optical phase distortion is 1 radian. Note that $r_0$ increases as $\lambda^{6/5}$.

$$\Delta \theta = \frac{\lambda}{r_0} \sim \lambda^{-1/5}$$ is called the seeing. At good sites $r_0$ (0.5 $\mu$m) $\sim$ 10 - 30 cm.

The atmospheric coherence (or Greenwood delay) time is: $\tau_0 = 0.314 \frac{r_0}{V}$
It is the maximum time delay for the RMS wavefront error to be less than 1 rad (where $V$ is the mean propagation velocity).

The isoplanatic angle $\Theta_0 = 0.314 \cos \zeta \frac{r_0}{h}$ is the angle over which the RMS wavefront error is smaller than 1 rad.
Long Exposures through Turbulence

When $t_{int} \gg \tau_c$ the image is the mean of the instantaneous intensity:

$$I(\theta) = \langle I_0(\theta) \star T(\theta, t) \rangle$$

With the mean modulation transfer function (MTF):

$$\langle \tilde{T}(\omega) \rangle \approx \exp \left[ -1.45 k^2 C_n^2 \Delta h \left( \frac{\lambda \omega}{\lambda} \right)^{5/3} \right]$$

The image is smeared or spatially filtered (loss of high spatial frequencies).

The angular dimension now has order of $\lambda/r_0$ rather than $\lambda/D$.

In other words:

As long as $D > r_0$, bigger telescopes will not provide sharper images.