Detection of Light 2020
Homework 3
Detector Noise and Extrinsic Photoconductors
17 marks total

28 February 2020

Due: Before the start of class Fri 06 Mar 2020
Questions, please e-mail to: dorval@strw.leidenuniv.nl

1. Detector Noise [4]

   (a) [2 points] The Noise-Equivalent Power (NEP) is defined as "The signal power yielding an RMS signal-to-noise ratio of unity for a frequency bandwidth of 1Hz".

   i) From this, derive an expression for NEP as a function of **only** the RMS noise current $\sqrt{\langle {I_N}^2 \rangle}$ (in Amps), sensitivity S and frequency bandwidth Δf.

   ii) Hence show that for a system where 1/f noise sources may be neglected, the total system NEP is given by

$$NEP = \frac{2hc}{\lambda}\sqrt{\frac{\phi}{\eta} + \frac{kT}{(q\eta G)^2 R}}$$

   (b) [2 points] Consider now our intrinsic detector from Ex.2 part 2, operated under the same conditions (i.e. $\phi = 10^5 \gamma/s$, $t_{exp} = 0.5s$, take $\eta = 0.44$, $G = 0.68$, $R = 7.9 \times 10^8 \Omega$). Calculate:

   i) The critical temperature beyond which the detector must be cooled, in order to obtain photon-limited performance, and

   ii) The NEP in this limit and the maximum signal-to-noise ratio attainable with this detector for the source and exposure time in question. Take $\lambda = 0.7$ microns

2. Thermal Excitation of Extrinsic Photoconductors [3]

   By analogy with intrinsic semiconductors, it can be shown that the concentration of negative carriers in a doped n-type semiconductor is given by:

$$n = \left(\frac{N_D - N_A}{N_A + n}\right)\left(\frac{2}{\delta}\right)\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} e^{-E_i/kT}$$

	Ge		Si	
Impurity	Cutoff wavelength λ_c (μm)	Photoionization cross section σ_i (cm^2)	Cutoff wavelength λ_c (μm)	Photoionization cross section σ_i (cm^2)
Al			18.5	8×10^{-16}
B	119	1.0×10^{-14}	28	1.4×10^{-15}
Be	52		8.3	5×10^{-18}
Ga	115	1.0×10^{-14}	17.2	5×10^{-16}
In	111		7.9	3.3×10^{-17}
As	98	1.1×10^{-14}	23	2.2×10^{-15}
Cu	31	1.0×10^{-15}	5.2	5×10^{-18}
P	103	1.5×10^{-14}	27	1.7×10^{-15}
Sb	129	1.6×10^{-14}	29	6.2×10^{-15}

Figure 1: Properties of some extrinsic photoconductors.

where N_D and N_A are the concentrations of donors and acceptor respectively, δ is the ground state degeneracy of the donor impurity (very close to unity) and E_i is in this case not the gap energy but the ionization energy for the n-type impurity. Similarly, for p-type materials:

$$p = \left(\frac{N_A - N_D}{N_D + p}\right)\left(\frac{2}{\delta}\right)\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} e^{-E_i/kT}$$

where it has been assumed that $E_i >> kT$.

(a) [2 points] Consider a Si:B material. Is this a n-type, or p-type material? Use the value of the cutoff wavelength λ_c from Fig. 1 to calculate the ionization energy of the impurity atoms (in meV). Is the stated assumption valid at room temperature (T = 300K)? Is it valid at 77K, the temperature of liquid nitrogen?

(b) [1 points] Use Fig. 1 to calculate the linear absorption coefficient of a silicon crystal doped with 1.3×10^{13}cm^{-3} boron atoms (Assume that the photoionization cross-section does not significantly change with energy). How does this compare to the absorption coefficient of intrinsic Si from Ex.2? What would be a natural solution to improve the quantum efficiency of the extrinsic photoconductor?

3. BIB Detectors [10]

Consider the Blocked Impurity Band (BIB) photoconductor shown in Fig. 2, consisting of a highly-doped Si:As (n-type) infrared-active layer with a small but non-negligible fraction of p-type impurities, which is overlaid with a high-purity Si blocking layer to which a positive bias voltage V_b is applied. A depletion region of width w is formed in which photons may be absorbed and subsequently detected with high efficiency.

For this question take the blocking and IR-active layer widths to be $t_B = 5.0\,\mu$m and $t_A = 15.0\mu$m respectively, and the densities of of n-type dopants and p-type impurities are $N_D = 5 \times 10^{17}$ cm^{-3}, $N_A = 10^{13}$ cm^{-3}. The dielectric constant $\kappa_0 = 11.8$ for Si at room temperature, and the permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}.

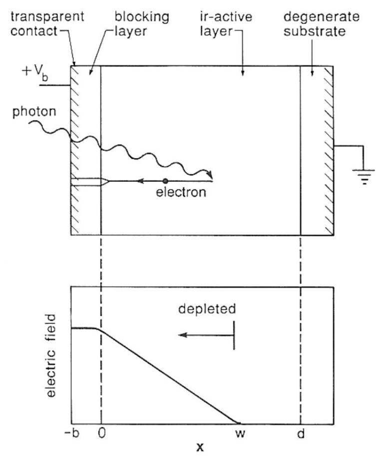

Figure 2: Diagram of a BIB depletion region.

(a) [3 points] Sketch an energy-level diagram for this BIB detector as a function of position x along the detector. With reference to this explain:

i) What is meant by "hopping" in over-doped extrinsic semiconductors, and how does the BIB band structure help overcome this problem?

ii) What is meant by the depletion region of the detector, and why does it have a finite extent within the IR-active layer?

(b) [4 points] i) By solving the one-dimensional Poisson equation

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{\rho}{\epsilon_0 \kappa_0},$$

obtain an expression for the electric field E as a function of distance x from the inner edge of the blocking layer, depletion layer width w and acceptor density N_A.

ii) Hence show that the width of the depletion region as a function of the bias voltage V_b and thickness of the blocking layer t_B is given by:

$$w = \left(\frac{2\kappa_0 \epsilon_0}{q N_A} |V_b| + t_B^2 \right)^{\frac{1}{2}} - t_B$$

[Hint: Use the boundary condition $V(x = -t_B) = |V_b|$ at the outer edge of the blocking layer.]

(c) [1 points] What is the value of the critical bias voltage in this detector for which the thickness of the depletion region is the same as the thickness of the IR-active layer, t_A?

(d) [2 points] Consider photo-electrons accelerated near the boundary of the blocking layer, where the mean-free path length is $\langle l \rangle \sim 0.3$ μm.

i) On average how much energy will these electrons gain in between collisions, when the detector is operated under the critical bias voltaqe?

ii) Given that the ionisation energy of arsenic in Si:As is $E_i = 54$ meV, explain briefly how gains of $G > 1$ may be obtained in BIB detectors. In what part of the detector does this occur, and why?