Detection of Light 2019
Homework 2
Carrier Concentration in Intrinsic Semiconductors
15 marks total

21 February 2020

Due: Before the start of class Fri 28 Feb 2020
Questions, please e-mail to: dorval@strw.leidenuniv.nl

1. Fermi distributions [7]

   The Fermi distribution gives us only the probability of finding an electron with certain energy E. But in a given physical system, each energy has several states associated with it. Hence, to account for the actual number of electrons within an infinitesimal energy range dE, we need to know the *density of states* (cm^{-3}) of the system, $N(E)$. We can then calculate the concentration of electrons n_0 in an energy range (E_1, E_2) as follows:

   $$n_e = \int_{E_1}^{E_2} f(E) N(E) \,\mathrm{dE}$$

   For a semiconductor material, $N(E)$ can be calculated by solving the Schrödinger equation for the electron in the potential energy of the the lattice. For the conduction band, it can be shown that the density of states adopts the form:

   $$N(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \,\mathrm{dE}$$

   where m^* is the effective mass of the carrier, associated with the potential energy in the lattice, E_c is the lowest energy state of the conduction band, and $\hbar = h/2\pi$ is the reduced Planck constant.

   (a) [2 points] Consider the concentration of carriers (electrons) in the conduction band of a semiconductor. Show that in the limit where $E - E_\mathrm{F} >> kT$, the concentration of electrons is given by:

   $$n_e = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} e^{-(E_\mathrm{c} - E_\mathrm{F})/kT}$$

where m_e^* is the effective mass of electrons in the conduction band, E_F is the Fermi energy of the system, k is the Boltzmann constant and T is the system temperature.

$$\left[\mathrm{Hint}: \int_0^\infty \mathrm{x}^{1/2}\mathrm{e}^{-\mathrm{ax}}\,\mathrm{dx} = \frac{\sqrt{\pi}}{2\mathrm{a}\sqrt{\mathrm{a}}}\right]$$

(b) [2 points] In a similar way, it can be shown that the concentration of holes in the valence band can be written as:

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{-(E_\mathrm{F}-E_\mathrm{v})/kT}$$

where E_v is the highest energy state of the valence band and m_h^* is the effective mass of holes in the valence band.

Show that for a **fixed material and temperature T**, the product of n_e and n_h is a constant that depends only on the band gap energy E_g.

(c) [3 points] In the case of an intrinsic semiconductor, the effective carrier concentration (due to both electrons and holes) is given by:

$$n_\mathrm{i} = \sqrt{N_\mathrm{c}N_\mathrm{v}}e^{-E_\mathrm{g}/2kT}$$

where $N_\mathrm{c} = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$ and $N_\mathrm{v} = 2\left(\frac{2\pi m_\mathrm{h}^* kT}{h^2}\right)^{3/2}$ are the number densities of carriers in the conduction and valence bands respectively.

In general $m_\mathrm{e}^* \neq m_\mathrm{h}^*$. In this case, use the results above to derive a new expression for the position of the Fermi level with respect to E_v and E_c, as a function of $m_\mathrm{h}^*/m_\mathrm{e}^*$. How does this compare to the result in Question 1d) from Exercise 1? In which direction does E_F shift for any given electron-hole effective mass ratio?

[Hint: Consider the relation between n_e and n_h in an intrinsic semiconductor]

2. Design of an Intrinsic Semiconductor [8]

Consider an intrinsic silicon photoconductor operating at 0.7 μm and constructed as shown in Fig. 1. Let its surface area $w = l = 1$ mm in a square pixel configuration, operating at a temperature of 300K. Assume the detector breaks down when the bias voltage, V_b, exceeds 50 mV.

(a) [2 points] The minimum detector thickness required for good quantum efficiency corresponds to one absorption length (photon mean free path length) in the material.

i) Use the data in Fig. 2 to estimate the minimum detector thickness d for this detector. What is the corresponding quantum efficiency η of this detector, if reflection is neglected?

ii) What is a more realistic value of η, if reflection is now taken into account? Assume normal photon incidence on the detector surface and a refractive index for Si of $n = 3.4$.

(b) [2 points] Calculate the responsivity S using your value of η in part a.ii, explicitly stating the units of your answer (If you do not have the value of η, keep it as a variable). The recombination time for Si under the given conditions is $\tau = 1 \times 10^{-4}$ s. Assume the detector is operated at 11.5 mV below the breakdown voltage. What is the probability of any given photon reaching the detector contact?

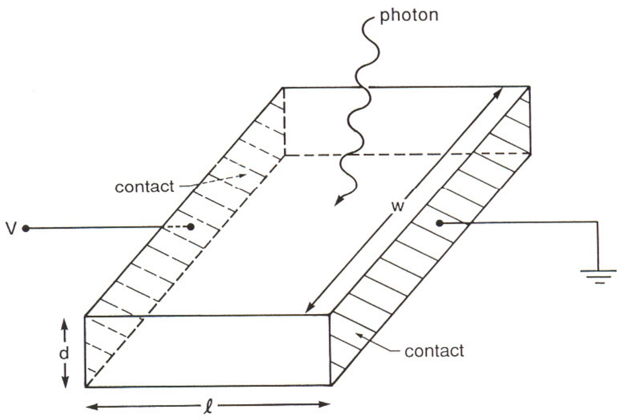

Figure 1: Photoconductor with transverse contacts.

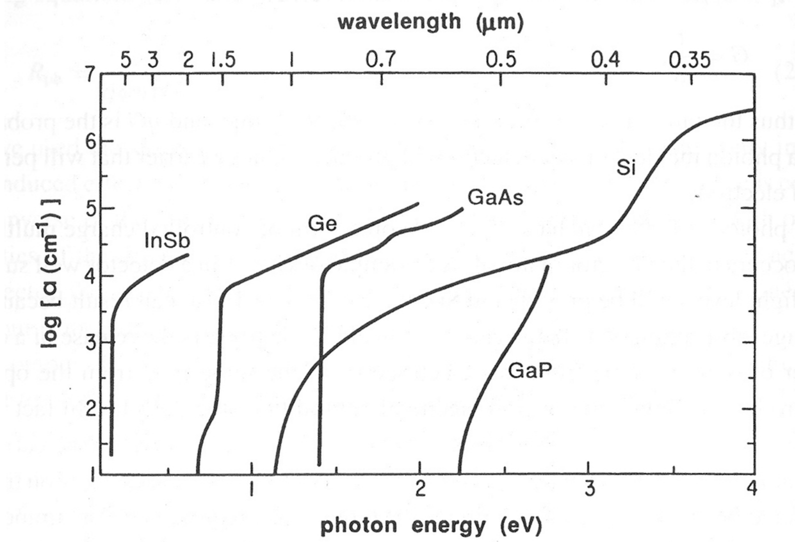

Figure 2: Linear absorption coefficients a for various semiconductors (on $\log_{10}$ scale).

(c) [2 points] Calculate the dark resistance, and hence the dark current of our detector. Compare this to the photo-current obtained when illuminated by an astrophysical source with a photon flux of $\phi = 10^5\ \gamma/s$.

What do you notice? Why does your result *not* rule out this photoconductor for use as a detector?

(Again, take the electron concentration in Si to be $n_i = 1.5 \times 10^{10}$ cm^{-3}.)

(d) [2 points] Calculate the signal-to-noise ratio our photoconductor obtains for a 0.5s exposure of the source in part c), assuming only thermal Johnson (kTC) and G-R noise sources. State the factor limiting detector performance in this scenario, and suggest how this may be improved.