Detection of Light

Heterodyne Receivers

(part II)

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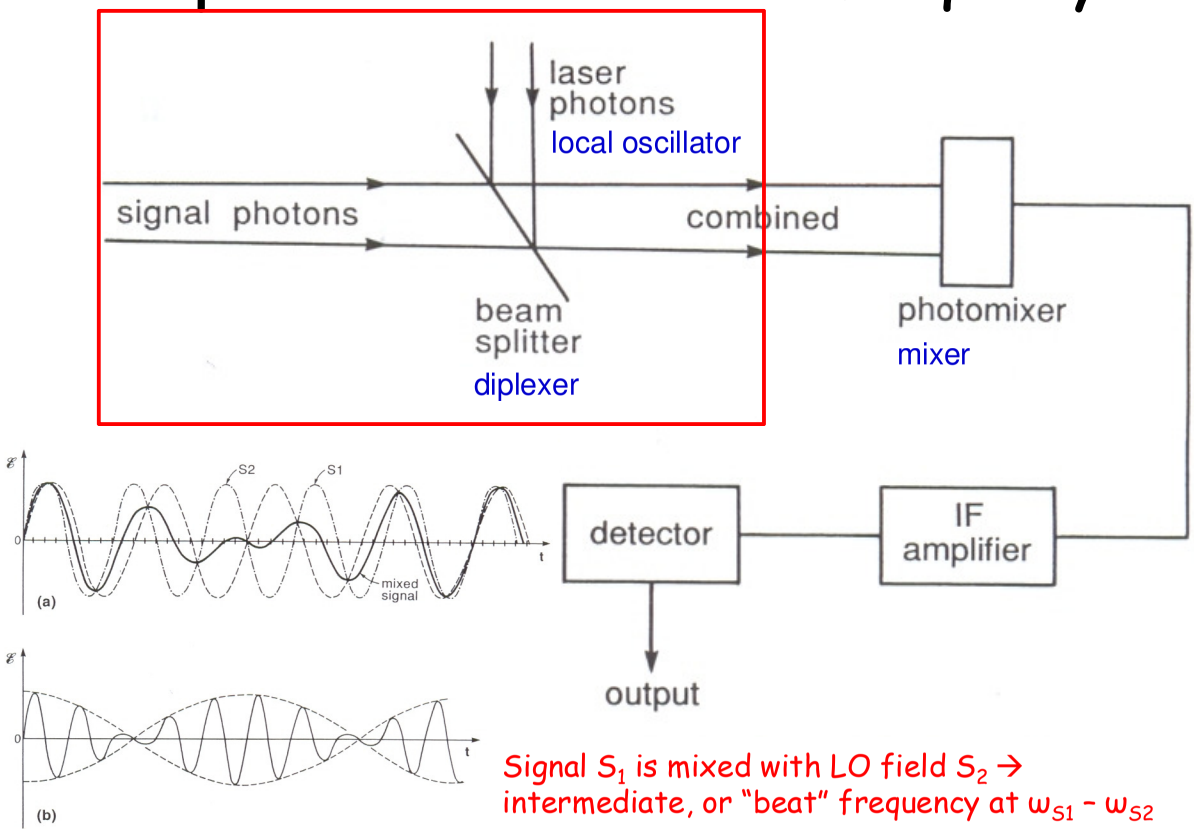
See http://www.strw.leidenuniv.nl/~brandl/DOL/Detection_of_Light.html for more info

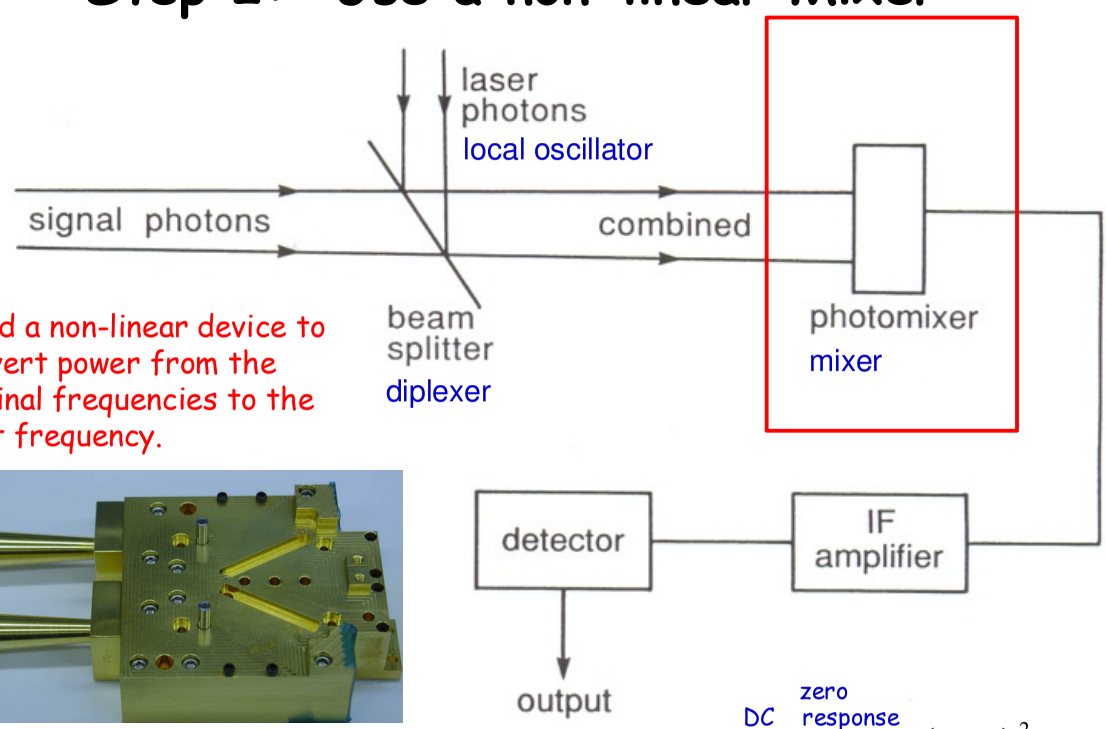
Repetition from last week: Heterodyne Receivers

Step 1: Down-convert the Frequency

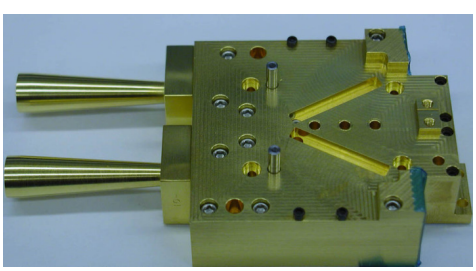


Signal S_1 is mixed with LO field S_2 → intermediate, or "beat" frequency at $\omega_{S1} - \omega_{S2}$

Step 2: Use a non-linear Mixer

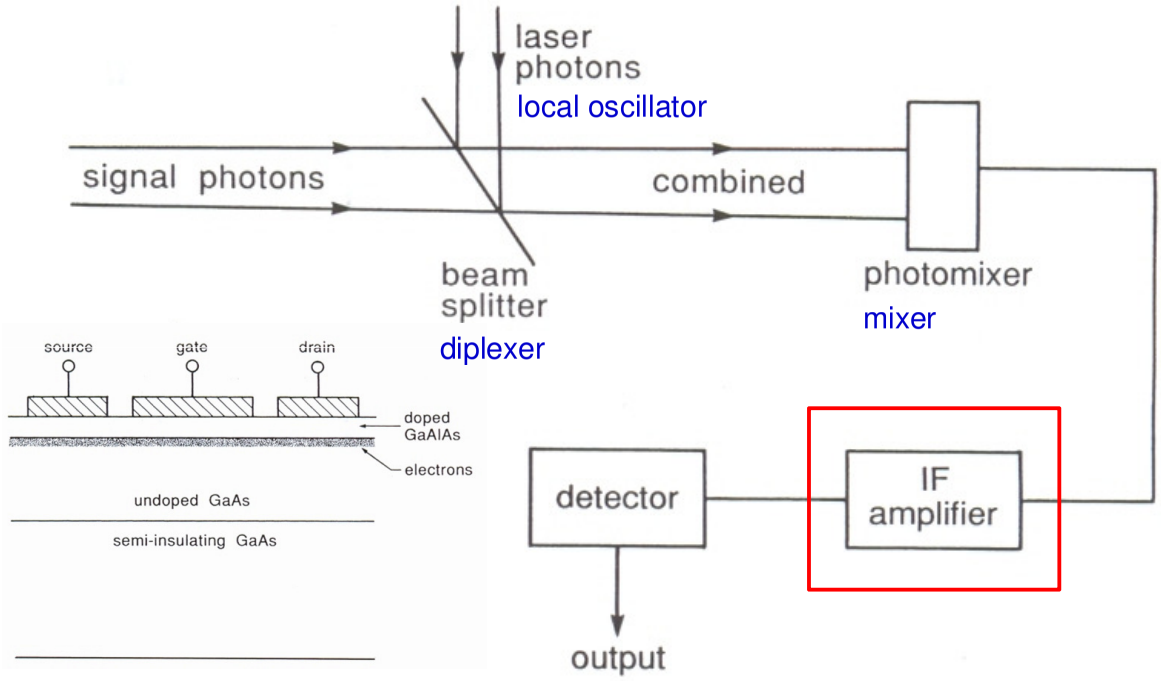


Need a non-linear device to convert power from the original frequencies to the beat frequency.



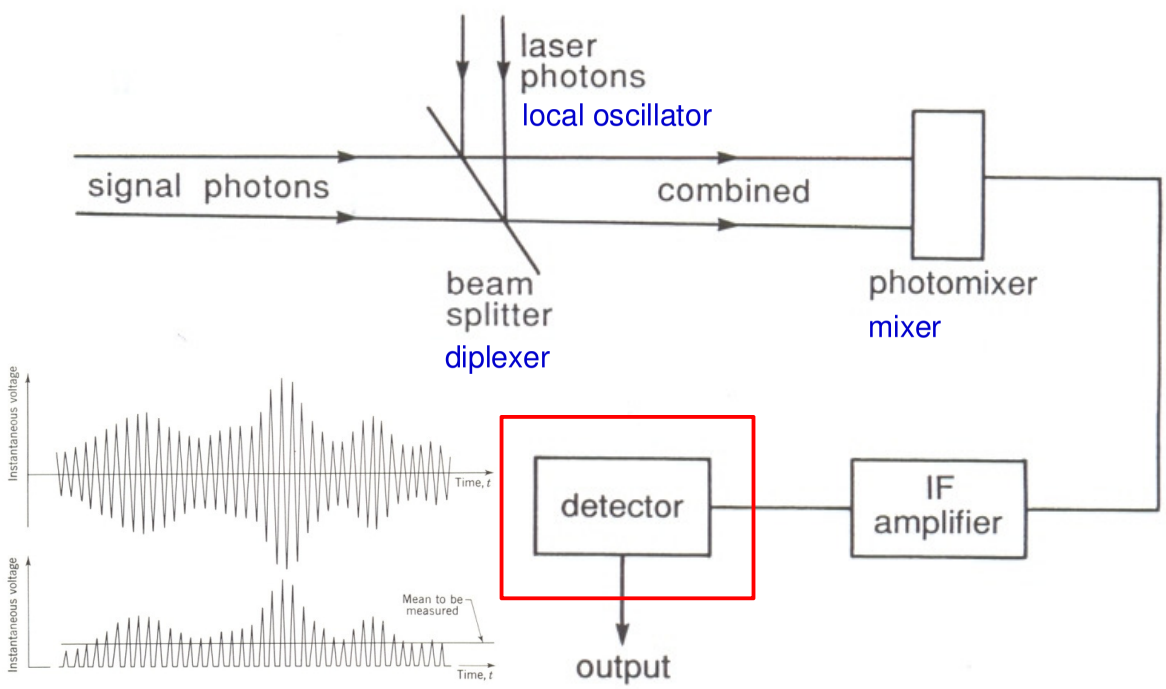
Use diode as mixer: $I = I_0\left(e^{qV_B/kT} - 1\right)$ and: $e^{qV_B/kT} - 1 \approx \underbrace{\frac{qV_B}{kT}}_{\text{DC}} + \underbrace{\frac{1}{2}\left(\frac{qV_B}{kT}\right)^2}_{\text{zero response}} + \ldots$

Step 3: Use a fast IF Amplifier



Need fast amplifiers: Best performance with high electron mobility transistors (HEMTs) on GaAs or InP up to ~ 10^{11} Hz.

Step 4: The Detector delivers the Signal

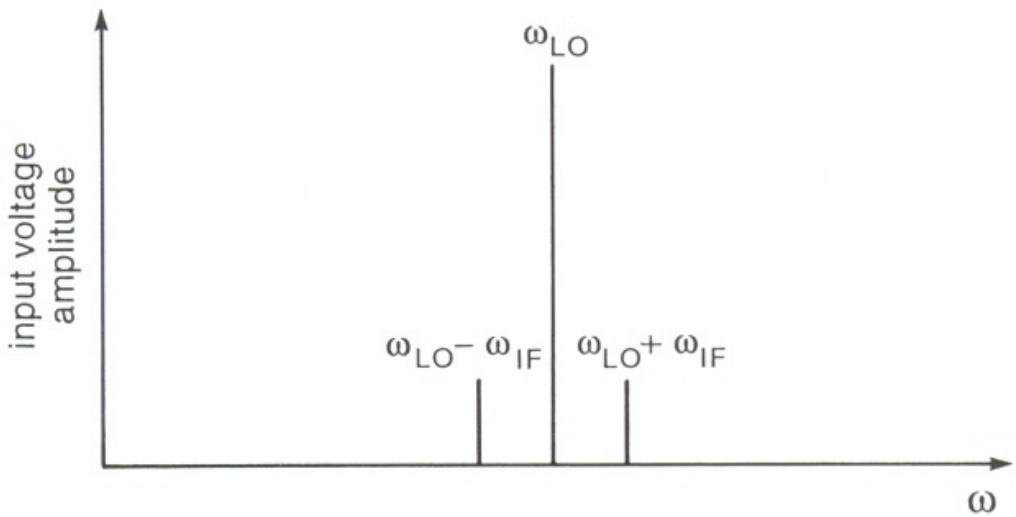


The detector rectifies the signal and sends it through a low-pass filter. Hence it converts the signal to a *slowly* varying output

Note 1: We get two Sidebands !

Unfortunately, from the measured IF signal one cannot tell whether the signal frequency ω_S is slightly lower or higher than ω_{LO}.

Hence, one assumes that the signal contains two components of equal strength, one at $\omega_{LO} + \omega_{IF}$, and one at $\omega_{LO} - \omega_{IF}$



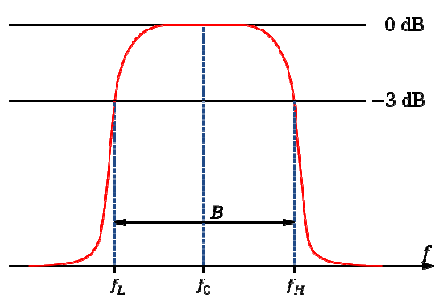
Note 2: The Bandwidth

The bandwidth is the range of frequencies [Hz] that can be processed or detected by a given system.

$\Delta f = f_{upper} - f_{lower}$

$f_{u/l}$ refers to the -3dB point ($1/\sqrt{2}$)

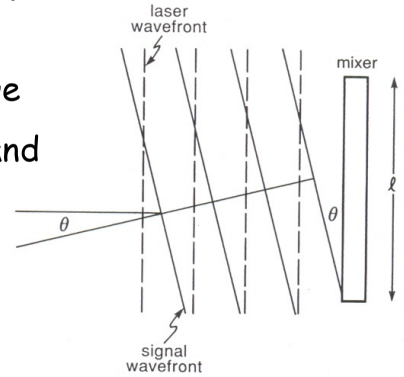
Note that: $\nu = \dfrac{c}{\lambda} \quad \rightarrow df = \dfrac{c}{\lambda^2} d\lambda$



The bandwidth Δf_{IF} (even of the best photodiode mixers) is usually small compared to the signal frequency – a fraction of a percent to a few percents.

Heterodyne receivers operating at short wavelengths have poor S/N on continuum sources but are good for line detections at high spectral resolution.

Note 3: Throughput



1. The signal beam strikes the mixer in a range of angles relative to the LO / laser beam, and the conditions for interference limit the maximum angular displacement.

2. Since the LO / laser field is polarized only one polarization component of the source can interfere and produce a signal. Heterodyne receivers are single-mode detectors.

3. The 'Etendue' $A\Omega$ limits the beam that can be accepted by a telescope of diameter D. A coherent receiver should operate at the diffraction limit of the telescope.

S/N and the Quantum/thermal Limit

S/N and Fundamental Limits (1)

Two types of noise:

1. Noise independent of the LO-generated current $I_{LO.}$
2. Noise that depends on $I_{LO.}$

Component 1. can be eliminated using sufficient LO power, but 2. contains the fundamental noise limits for heterodyne receivers.

The fundamental noise can be subdivided into:

A. Noise in the mixer from the generation of charge carriers by the LO power.

B. Noise from the thermal background detected by the system.

Assumptions: $I_{LO} \gg I_S$ and $I_{LO} \gg I_B$, where I_B is the signal from the background.

S/N and Fundamental Limits (2)

Remember, the photocurrent in a photoconductor is:

$$I(t) = \frac{\eta q G P(t)}{h\nu}$$

Hence, the current generated by the LO in a photoconductive mixer is:

$$I_{LO} = \frac{\eta q G P_{LO}}{h\nu}$$

From the discussion of photo-conductors we know that the generation-recombination noise is:

$$\left\langle I_{G-R}^2 \right\rangle = \left(\frac{2q}{\Delta t}\right)\left(\frac{qNG}{\Delta t}\right)G = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph}\right\rangle G \overset{1/\Delta t = 2\Delta f}{=} 4qI_{LO}G\Delta f_{IF}$$

To account for the fact that photodiodes have no recombination noise, we parametrize:

$$\left\langle I_{G-R}^2 \right\rangle = 2aqI_{LO}G\Delta f_{IF} = \frac{2aq^2\eta G^2 P_{LO}\Delta f_{IF}}{h\nu}$$

with a = 2 for a photoconductor, a = 1 for a photodiode mixer.

S/N and Fundamental Limits (3)

Last week we have seen that the IF current has a mean-square amplitude of: $\left\langle I_{IF}^2 \right\rangle_t = 2 I_{LO} I_S$

Analogously we define the noise in the thermal background as:

$$\left\langle I_B^2 \right\rangle = 2 I_{LO} I_B$$

with : $I_{LO} = \dfrac{\eta q G P_{LO}}{h\nu}$; $I_B = \dfrac{\eta q G P_B}{h\nu}$

From radiometry we know that

the background power is: $P_B = \dfrac{1}{2} L_\nu(\varepsilon, T_B) A\Omega(2\Delta f_{IF})$

one polarization only

spectral radiance

emissivity

temperature

double sideband receiver

S/N and Fundamental Limits (4)

Hence the noise current from the thermal background can be calculated as:

$$\left\langle I_B^2 \right\rangle = 2 I_{LO} I_B = 2 \frac{\eta q G P_{LO}}{h\nu} \frac{\eta q G P_B}{h\nu} =$$

$$= \frac{2\eta^2 q^2 G^2 P_{LO}}{h^2 \nu^2} \frac{1}{2} L_\nu A\Omega(2\Delta f_{IF}) =$$

$$\overset{A\Omega \approx \lambda^2 = \frac{c^2}{\nu^2}}{=} \frac{\eta^2 q^2 G^2 P_{LO}}{h^2 \nu^2} \left(\frac{\varepsilon \frac{2h\nu^3}{c^2}}{e^{h\nu/kT_B} - 1} \right) \frac{c^2}{\nu^2} 2\Delta f_{IF}$$

$$= \frac{4 q^2 \eta^2 G^2 \varepsilon P_{LO} \Delta f_{IF}}{h\nu \left(e^{h\nu/kT_B} - 1 \right)}$$

S/N and Fundamental Limits (5)

Now we want to see how $\langle I^2_{G\text{-}R}\rangle$ and $\langle I^2_B\rangle$ affect the (S/N) ratio of the receiver. Since the output *power* goes as as I^2 the S/N at the IF output is:

$$\left(\frac{S}{N}\right)_{IF} = \frac{\left\langle I_{IF}^2\right\rangle}{\left\langle I_{G-R}^2\right\rangle + \left\langle I_B^2\right\rangle} = \frac{2I_{LO}I_S}{\dfrac{2aq^2\eta G^2 P_{LO}\Delta f_{IF}}{h\nu} + \dfrac{4q^2\eta^2 G^2 \varepsilon P_{LO}\Delta f_{IF}}{h\nu\left(e^{h\nu/kT_B}-1\right)}} =$$

$$\overset{P_{LO}=\frac{I_{LO}h\nu}{\eta qG}}{=} \frac{2I_{LO}I_S}{2aqI_{LO}\Delta f_{IF} + \dfrac{4q\eta G\varepsilon I_{LO}\Delta f_{IF}}{\left(e^{h\nu/kT_B}-1\right)}} \overset{I_S=\frac{P_S\eta qG}{h\nu}}{=} \frac{\dfrac{P_S\eta qG}{h\nu}}{aq\Delta f_{IF} + \dfrac{2q\eta G\varepsilon\Delta f_{IF}}{\left(e^{h\nu/kT_B}-1\right)}} =$$

$$= \frac{\eta P_S}{h\nu\Delta f_{IF}\left[\dfrac{a}{G} + \dfrac{2\eta\varepsilon}{e^{h\nu/kT_B}-1}\right]}$$

Here is a problem: the factor G^2 should cancel out completely but it does not (see Rieke book p.292). There you'll only find a linear factor G in the first term, but it should be quadratic in $\langle I^2_{G\text{-}R}\rangle$.

S/N and Fundamental Limits (6)

The S/N at the IF output has two limiting cases:

1. The "quantum limit" when G-R noise from the LO power dominates
2. The "thermal limit" when noise from the background emission dominates

The dividing point between these cases is where both terms in

$$\frac{\eta P_S}{h\nu\Delta f_{IF}\left[\dfrac{a}{G} + \dfrac{2\eta\varepsilon}{e^{h\nu/kT_B}-1}\right]}$$

are equally large:

$$\frac{a}{G} = \frac{2\eta\varepsilon}{e^{h\nu/kT_B}-1} \Rightarrow e^{h\nu/kT_B} = 1 + \frac{2\eta\varepsilon G}{a} \Rightarrow \frac{h\nu}{kT_B} = \ln\left(1 + \frac{2\eta G\varepsilon}{a}\right)$$

Since $\eta \approx G \approx \varepsilon \approx a \approx 1 \Rightarrow \ln\left(1 + \dfrac{2\eta G\varepsilon}{a}\right) \approx 1$

Hence one simplifies:

If $h\nu \gg kT_B$ → quantum limit
If $h\nu \ll kT_B$ → thermal limit

S/N and Fundamental Limits (7)

Now we need to include the detector stage (which rectifies and smoothes the IF signal) in our (S/N) estimates.

Generally: (S/N) ~ $\sqrt{t_{int}}$.

Smoothing: (S/N) ~ $\sqrt{\{\text{effective } t_{int} \text{ of the IF / } t_{int} \text{ of the smoothing circuit}\}}$

Or: $$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{IF}\left(\frac{\tau_{RC}}{\tau_{IF}}\right)^{1/2}$$

We convert the integration times to frequency bandwidths. Note that:

$\tau_{IF} = \dfrac{1}{2\Delta f_{IF}}$, for the uniformly sampled IF stage, and

$\tau_{RC} = \dfrac{1}{4\Delta f_{RC}}$ for a system with exponential response, like a single-stage RC circuit

S/N and Fundamental Limits (8)

Remember, the noise equivalent power (NEP) is the signal that can be detected at (S/N)~1 in unity frequency bandwidth Δf.

The NEP in the quantum limit is:

$$NEP_{ql} \equiv \frac{P}{\left(\frac{S}{N}\right)_{out}(\Delta f)^{1/2}} = \frac{P}{\frac{\eta P}{h\nu\Delta f_{IF}\frac{a}{G}}\left(\frac{\tau_{RC}}{\tau_{IF}}\right)^{1/2}(\Delta f)^{1/2}} =$$

$$= \frac{h\nu a}{\eta G}\Delta f_{IF}\left[\frac{\tau_{IF}}{\tau_{RC}}\right]^{1/2} = \frac{h\nu a}{\eta G}\Delta f_{IF}\left[\frac{2\Delta f_{RC}}{\Delta f_{IF}}\right]^{1/2} \overset{\Delta f_{RC}=1Hz}{=}$$

$$= \frac{h\nu a}{\eta G}(2\Delta f_{IF})^{1/2}$$

S/N and Fundamental Limits (9)

Similarly, the NEP in the thermal limit is:

$$NEP_{th} \equiv \frac{P}{\left(\frac{S}{N}\right)_{out} (\Delta f)^{1/2}} = \frac{P}{\frac{\eta P}{h\nu \Delta f_{IF} \frac{2\eta\varepsilon}{e^{h\nu/kT_B}-1}} \left(\frac{\tau_{RC}}{\tau_{IF}}\right)^{1/2} (\Delta f)^{1/2}} =$$

$$= \frac{2h\nu\varepsilon}{e^{h\nu/kT_B}-1} \Delta f_{IF} \left[\frac{\tau_{IF}}{\tau_{RC}}\right]^{1/2} = \frac{2h\nu\varepsilon}{e^{h\nu/kT_B}-1} (2\Delta f_{IF})^{1/2}$$

Noise and Antenna Temperatures

Noise Temperature (1)

Our goal: to compare the theoretical performance of heterodyne detectors with that of incoherent receivers (e.g. bolometers).

So far: $NEP_H \propto \sqrt{2\Delta f_{IF}}$ implies that the NEP decreases (i.e., the S/N increases) when narrowing down the bandwidth!

However, this is only correct if all power falls within an interval, which is smaller than Δf_{IF}

This is given for narrow emission lines but *not* for continuum sources.

Solution: we introduce the noise temperature T_N *defined such that a matched blackbody at the receiver input at a temperature T_N produces a S/N = 1.*

Obviously, the lower T_N the better the S/N.

Noise Temperature (2)

Now we want to estimate the noise temperature in the thermal limit.

If the BB emissivity ε = 1 then $T_N = T_B$.

If the BB emissivity ε < 1 then (...) :

$$T_N = \frac{h\nu}{k} \frac{1}{\ln\left(\varepsilon - 1 + e^{h\nu/kT_B}\right) - \ln\varepsilon}$$

Similarly, the noise temperature in the quantum limit (double sideband) is:

$$T_N = \frac{h\nu}{k\ln\left(1 + \frac{2G\eta}{a}\right)} \underset{G=\eta=a=1}{\overset{ideal\ case}{\approx}} \frac{h\nu}{k}$$

Antenna Temperature

Just like the noise temperature describes the strength of the *noise background* (S/N=1) we can assign the *source flux* an antenna temperature T_S.

In the Rayleigh-Jeans approximation ($h\nu \ll kT$) we can write:

$$P_S = L_\nu(T_S) A\Omega \Delta f_{IF} \overset{\substack{Rayleigh \\ JeansBB}}{=} \frac{2kT_S\nu^2}{c^2} A\Omega\Delta f_{IF} \overset{A\Omega=\lambda^2=\frac{c^2}{\nu^2}}{=} 2\Delta f_{IF} kT_S$$

where $2\Delta f_{IF}$ is the frequency bandpass for a double sideband receiver.

Hence, in the R-J case, the antenna temperature is linearly related to the input flux density: $P_S \sim T_S$

Noise Temperature (3)

The concept of noise temperatures offers a convenient means to quantify the LO-independent components, such as amplifier noise.

The amplifier noise is usually Johnson noise (see earlier lectures) and is: $\left\langle I_A^2 \right\rangle = \frac{4kT_A\Delta f_{IF}}{R_A}$

where R_A and T_A are the amplifier input resistance and noise temperature.

The lower limit for the noise temperature is given by $T_N \approx \frac{h\nu}{k}$

For an amplifier operating at 32 GHz $T_N \sim 1.5$K. For a good HEMT amplifier $T_A \sim 10$K.

Noise Temperature (4)

Let's assume that:

- the amplifier noise is dominating the LO-independent noise types
- we operate in the quantum limit: $\langle I_B^2 \rangle \to 0$

What is the (minimum) LO power, corresponding to the quantum limit?

$$\langle I_A^2 \rangle < \langle I_{G-R}^2 \rangle \;\Rightarrow\; \langle I_{G-R}^2 \rangle = \frac{2aq^2\eta G^2 P_{LO}\Delta f_{IF}}{h\nu} > \langle I_A^2 \rangle = \frac{4kT_A\Delta f_{IF}}{R_A}$$

$$\Rightarrow P_{LO} > \frac{4kT_A\Delta f_{IF}h\nu}{2aq^2\eta G^2\Delta f_{IF}R_A} = \frac{2kT_A h\nu}{q^2\eta R_A a G^2}$$

Losses and Gains

Losses and Gains (1)

However, the above equation is not quite correct as we need to account for losses ζ in transferring the output of the mixer to the amplifier:

$$P_{LO} > \frac{2kT_A \zeta h\nu}{q^2 \eta R_A a G^2}$$

The system noise temperature T_N contains contributions from the mixer T_M, the amplifier T_A, the thermal background T_B, and other sources.

However, not all components do fully contribute to the total system temperature T_N as noise can also suffer losses. The total is:

$$T_N^{sys} = T_M + \zeta_1 T_A + \zeta_1 \zeta_2 T_3 + \ldots$$

Losses and Gains (2)

Example: Consider the loss between mixer and first amplifier stage ζ_1.

Remember: the conversion gain is: $\Gamma \equiv \frac{\text{deliverable IF signal power}}{\text{input signal power}} = \left(\frac{\eta q}{h\nu}\right) G V_b$

The loss ζ is the inverse of the conversion gain Γ_{CS}:

$$\zeta_S = \frac{1}{\Gamma_{CS}} = \frac{P_S}{P_{IF}}$$

where P_S is the (single sideband) signal power and P_{IF} the IF power from the mixer.

The conversion loss is usually expressed in decibels (db)*:

$$\zeta_S(dB) = 10\log(\zeta_S) = 10\log\left(\frac{P_S}{P_{IF}}\right)$$

$$*X_{db} \equiv 10\log_{10}\left(\frac{X}{X_0}\right)$$