# Detection of Light: Exercise 7

**Set:** Fri 23rd Mar 2018,
**Due:** Before the start of class Fri 6th Apr 2018
Questions, please e-mail to: dorval@strw.leidenuniv.nl

## 1 Design of a bolometer [10 marks]

Suppose you want to build a high performance bolometer based on a cubic thermo-element (0.45mm on a side) made of gallium-doped **germanium** connected to the heat sink via two cylindrical thin brass leads, each 1cm long. The bolometer is to be operated at $T = 2.7K$. Assume that the doping is such that the element resistance is $1.0 \times 10^6\ \Omega$ at the operating temperature, and that the temperature dependence of its resistance is given by:

$$R = R_0 \left(\frac{T}{T_0}\right)^{-4}$$

Assume also that the detector is blackened so that its quantum efficiency is $\eta = 0.55$ (all other detector properties are unaffected by this process).

a By analogy with the electrical case, derive an expression for the total thermal conductance $G$ between the bolometer pixel and heat sink, as a function of thermal conductivity $\kappa$, wire cross-section $A$ and length $L$.

[1 mark]

b To obtain a good performance, suppose that you want the detector to have a thermal-noise limited NEP of $4.5 \times 10^{-15}$ W Hz$^{-1/2}$. Calculate the conductance $G$ of the cylindrical leads, and hence their radius in order to achieve this performance. Take the electrical conductivity of brass to be $\sigma = 3.1 \times 10^5\ \Omega^{-1}\ \mathrm{cm}^{-1}$ at the operating temperature.

[Hint: The thermal conductivity $\kappa$ of a metal is related to its electrical conductivity $\sigma$ by the Wiedemann - Franz relation]

[2 marks]

c Starting from the definition of conductance as $G = P_0/T_1$, show that the load curve for the bolometer is given by

$$I = \frac{V}{R_0}\left(1 + \frac{IV}{GT_0}\right)^4$$

[1 mark]

d Fig.1 shows the electrical configuration of the bolometer, where $V_{\mathrm{bias}} = 1.5V$ and a load resistance $R_L = 10^7\Omega$. Iteratively solve the load curve expression in Part c to obtain the voltage across the bolometer pixel, and hence the power dissipated by the bolometer under this approximation.

[Hint: Assume the current through the bolometer is dominated by the load resistor.]

[2 marks]

e A more accurate definition of the Johnson NEP is:

$$\mathrm{NEP_J} = \left(\frac{4\mathrm{kT}}{\mathrm{P}}\right)^{1/2} \frac{\mathrm{G}}{\eta|\alpha(\mathrm{T})|}$$

where $P$ is the electrical power dissipated in the detector and $\alpha(T)$ is the temperature coefficient of resistance. Use this expression to calculate the Johnson-noise-limited NEP of our bolometer, using the connector lead radius calculated in part b.

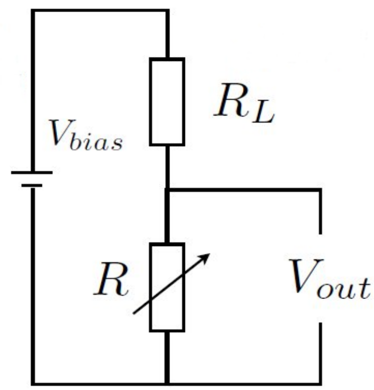


Figure 1: Bolometer circuit diagram

[2 marks]

f In semiconductors at low temperature, the specific heat capacity is dominated by the crystal lattice term. From Debye theory, it can be shown that this term is proportional to $T^3$. For the particular case of germanium:

$$c_v^{\mathrm{lat}} \approx 7 \times 10^{-6}\, T^3\ \mathrm{J\,K^{-4}\,cm^{-3}}$$

On the other hand, the contribution from the brass leads is dominated by the free electrons, and is proportional to T:

$$c_v^{e} \approx 1.3 \times 10^{-4}\, T\,\mathrm{J\,K^{-2}\,cm^{-3}}$$

Use the specified bolometer dimensions to calculate the ratio of **total** heat capacities $C_v^{\mathrm{lat}}/C_v^{e}$ of the thermo-element (pixel) to the brass connecting leads, as a function of temperature. Which contribution dominates at T=2.7K, and at what temperature are these two contributions equal?

[2 marks]