Detection of Light: Exercise 2

Set: Fri 9th Feb 2018,
Due: Fri 16th Feb 2018
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1 Carrier Concentration in Intrinsic Semiconductors [10 marks]

The Fermi distribution gives us only the probability of finding an electron with certain energy E. But in a given physical system, each energy has several states associated with it. Hence, to account for the actual number of electrons within an infinitesimal energy range dE, we need to know the *density of states* (cm^{-3}) of the system, $N(E)$. We can then calculate the concentration of electrons n_0 in an energy range (E_1, E_2) as follows:

$$n_e = \int_{E_1}^{E_2} f(E)N(E)\,\mathrm{dE}$$

For a semiconductor material, $N(E)$ can be calculated by solving the Schrödinger equation for the electron in the potential energy of the the lattice. For the conduction band, it can be shown that the density of states adopts the form:

$$N(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E - E_c)^{1/2}\,\mathrm{dE}$$

where m^* is the effective mass of the carrier, associated with the potential energy in the lattice, E_c is the lowest energy state of the conduction band, and $\hbar = h/2\pi$ is the reduced Planck constant.

a Consider the concentration of carriers (electrons) in the conduction band of a semiconductor. Show that in the limit where $E - E_\mathrm{F} >> kT$, the concentration of electrons is given by:

$$n_e = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{-(E_\mathrm{c}-E_\mathrm{F})/kT}$$

where m_e^* is the effective mass of electrons in the conduction band, E_F is the Fermi energy of the system, k is the Boltzmann constant and T is the system temperature.

$$\left[\mathrm{Hint}: \int_0^\infty \mathrm{x}^{1/2}\mathrm{e}^{-\mathrm{ax}}\,\mathrm{dx} = \frac{\sqrt{\pi}}{2\mathrm{a}\sqrt{\mathrm{a}}}\right]$$

[2 marks]

b In a similar way, it can be shown that the concentration of holes in the valence band can be written as:

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{-(E_\mathrm{F}-E_\mathrm{v})/kT}$$

where E_v is the highest energy state of the valence band and m_h^* is the effective mass of holes in the valence band.

i) Show that for a fixed material and temperature T, the product of n_e and n_h is a constant that depends only on the band gap energy E_g.

ii) Hence show that in the case of an intrinsic semiconductor, the effective carrier concentration (due to both electrons and holes) is given by:

$$n_\mathrm{i} = \sqrt{N_\mathrm{c}N_\mathrm{v}}e^{-E_\mathrm{g}/2kT}$$

where $N_\mathrm{c} = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$ and $N_\mathrm{v} = 2\left(\frac{2\pi m_\mathrm{h}^* kT}{h^2}\right)^{3/2}$ are the number densities of carriers in the conduction and valence bands respectively.

[3 marks, 2 for i) and 1 for ii)]

c In general $m_\mathrm{e}^* \neq m_\mathrm{h}^*$. In this case, use the results above to derive a new expression for the position of the Fermi level with respect to E_v and E_c, as a function of $m_\mathrm{h}^*/m_\mathrm{e}^*$. How does this compare to the result in Question 1d)? In which direction does E_F shift for any given electron-hole effective mass ratio?

[Hint: Consider the relation between n_e and n_h in an intrinsic semiconductor]

[3 marks]

d i) Now let's plug in some numbers. Calculate the effective carrier concentration of pure intrinsic Silicon (E_g = 1.11eV), both at room temperature (293 K) and the temperature of liquid nitrogen (77 K) using the previous expression. Assume $m_\mathrm{e}^* = 1.1m_0$ and $m_\mathrm{h}^* = 0.56m_0$, where m_0 is the rest mass of the electron.

ii) What can you say from this about the relative conductivity of the material at these two temperatures, in the absence of any other sources of excitation?

[2 marks, 1 each]