Detection of Light: Exercise 1

Set: Fri 3rd Feb 2017,
Due: Thurs 9th Feb 2017
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1 Fermi energy [10 marks]

Consider the Fermi distribution for the probability of finding an electron (or any other fermion) with an energy E in a given physical system:

$$f(E) = \frac{1}{1 + e^{(E-E_{\mathrm{F}})/kT}}$$

where E_F is the Fermi Energy, T is the system temperature and k is the Boltzmann constant.

a Sketch the shape of $f(E)$ for the cases of $T = 0K$ (absolute zero), $T > 0K$, and $T \gg 0K$. Describe the relative occupation of states with $E < E_F$ and $E > E_F$ in each case.

[2 marks]

b In an isolated system of electrons (i.e. a free electron gas), what is the probability of finding an electron with an energy equal to the Fermi energy? Does this depend on the temperature T of the system?

[2 marks]

c Show that the Fermi distribution is antisymmetric about the Fermi energy, E_{F}.

[2 marks]

d An intrinsic semiconductor may be modelled as a system of N electrons distributed between two narrow bands, with energies well approximated by E_{v} and E_{c} respectively, each with a bandwidth $\Delta E_{band} << E_{\mathrm{c}} - E_{\mathrm{v}}$. Show that under this approximation the Fermi level lies in the middle of the band gap[1].

[Hint: Consider the fraction of electrons occupying each band for some $T > 0K$]

[2 marks]

e Is it possible to find electrons in a semiconductor with energy E_{F}? Explain your answer in the context of the results of Part b & d.

[2 marks]

[1] This is not entirely true, since the position of the Fermi level in reality also depends on the ratio of the equivalent masses of electrons and holes. The difference is, however, very small and can be neglected for this exercise.

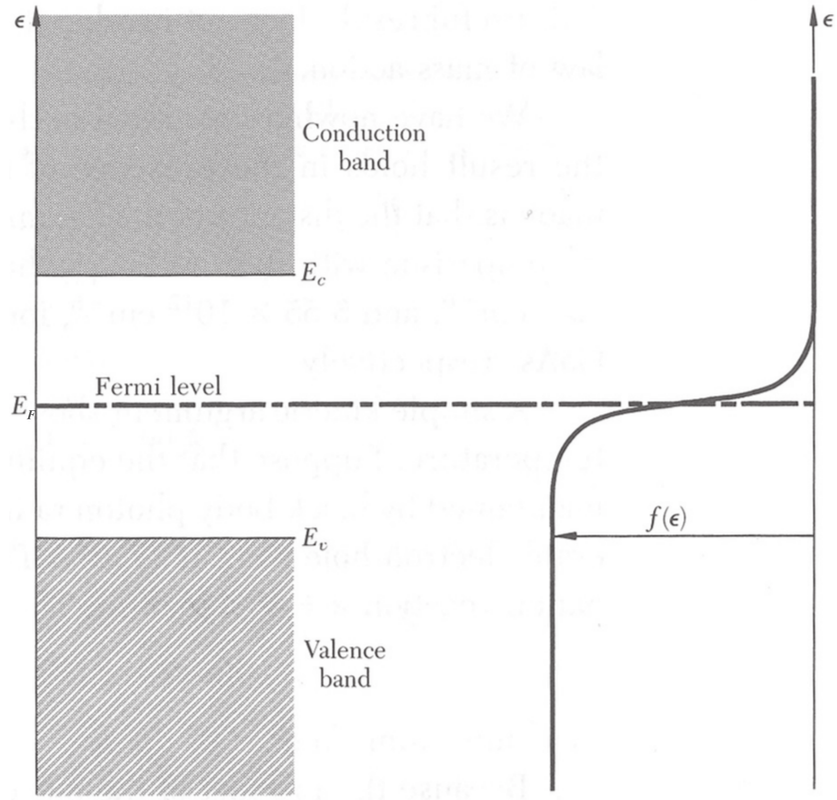

Figure 1: Schematic energy level diagram of a semiconductor. The Fermi distribution function is shown on the same scale for $kT \ll (E_\mathrm{c} - E_\mathrm{v})$.