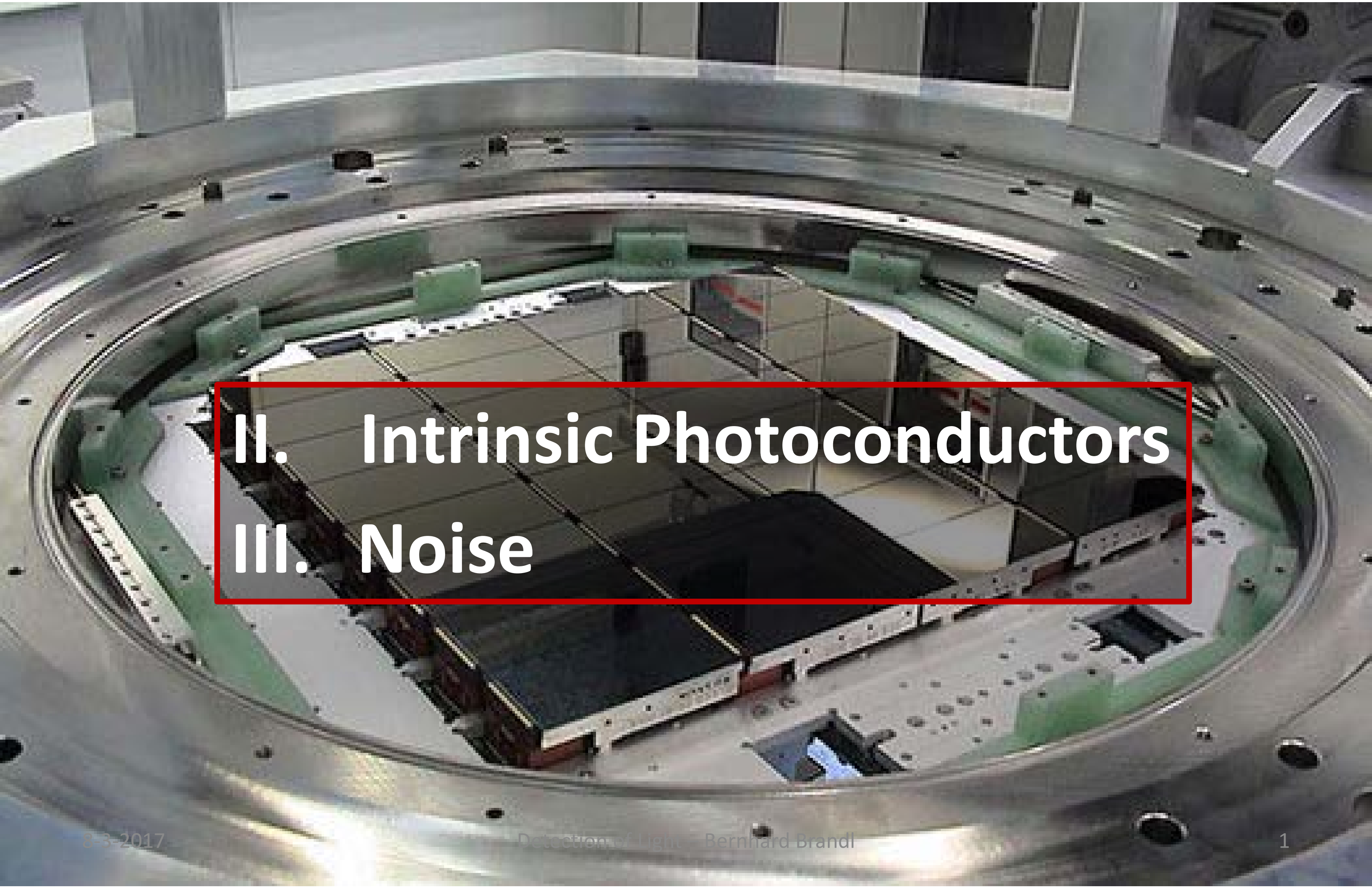
Detection of Light



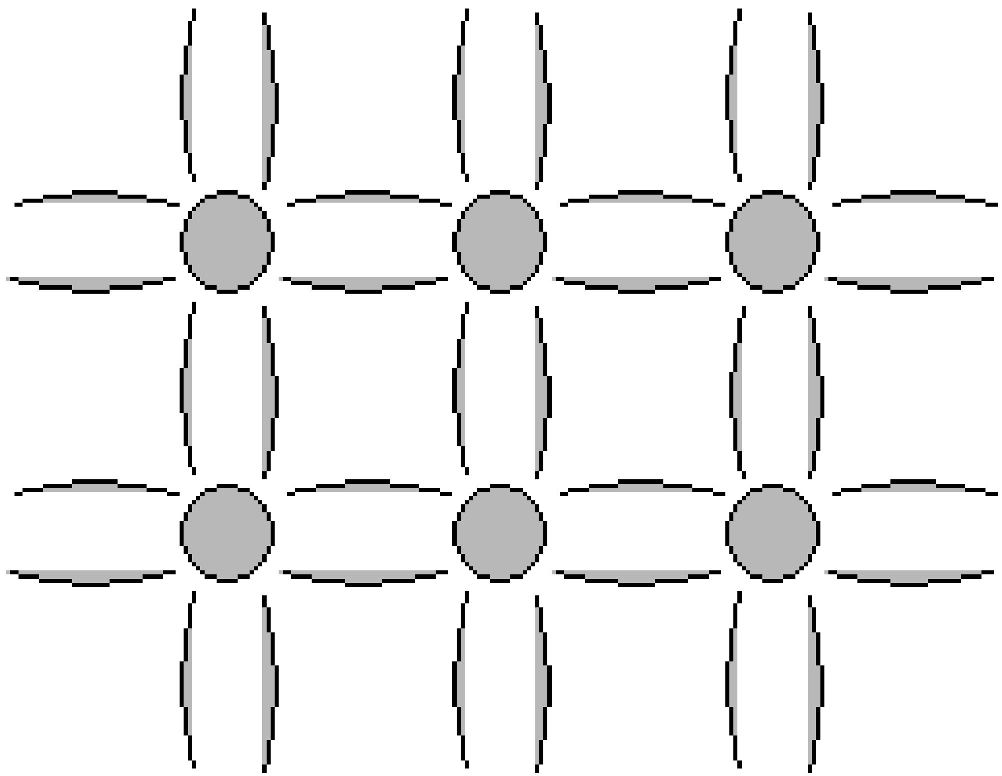
II. Intrinsic Photoconductors

III. Noise

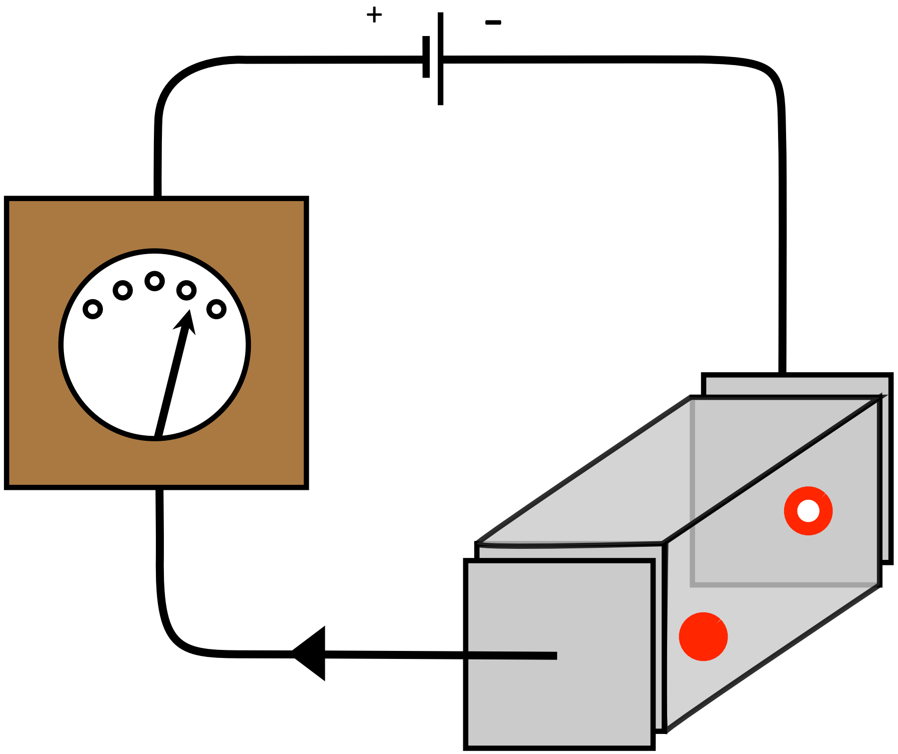
Fundamental Principle

Basic Principle – Physics

E_γ lifts e^- from valence into conduction band: $E_\gamma = \frac{hc}{\lambda} > E_{bandgap}$



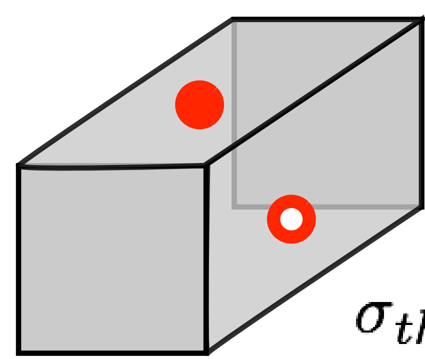
Basic Principle – Realization



Applying an electric field causes electric charges to move in the material and register a signal as an electric current

Practical Limitations

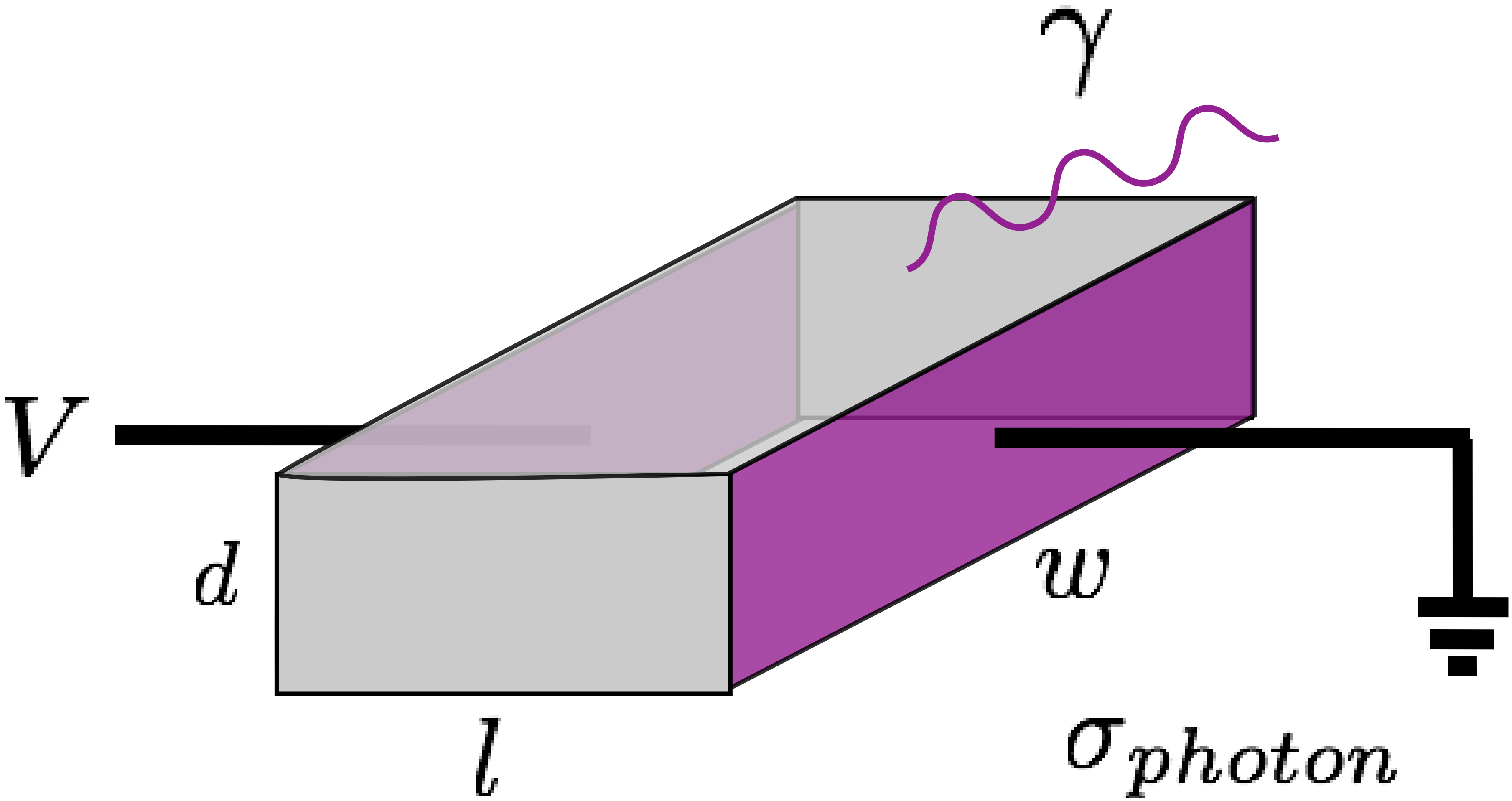
- Wavelength cutoffs: $$\lambda_c = \frac{hc}{E_g}$$
  - → Germanium: 1.85µm
  - → Silicon: 1.12µm
  - → GaAs: 0.87µm
- Cleanliness and non-uniformity of material
- Problems to make good electrical contacts to pure Si
- Charge carriers are generated with both photons and thermal excitation. We only measure the electrical conductivity!



Basic Electric Properties

Schematics of a Detector

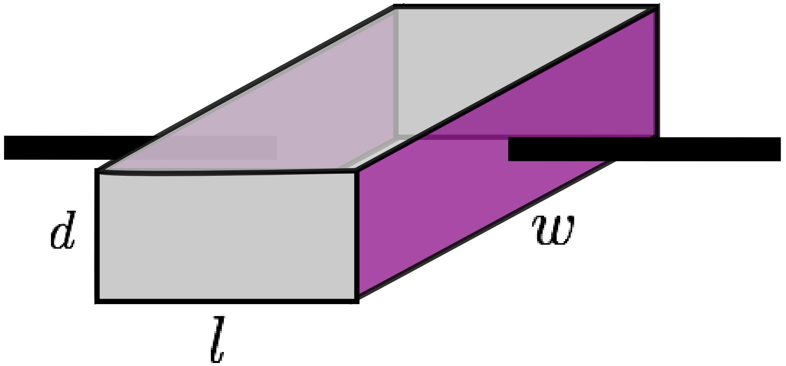
Consider a pixel with physical dimensions *d, l, w*:



Resistivity, Resistance, and Conductivity

Resistivity ρ $[\,\Omega\text{ m}\,] \equiv$ intrinsic material property to oppose the flow of an electric current.

Resistance R: $$R \equiv \frac{U}{I}; \quad R = \frac{\rho \cdot l}{A} = \frac{\rho \cdot l}{d \cdot w}$$

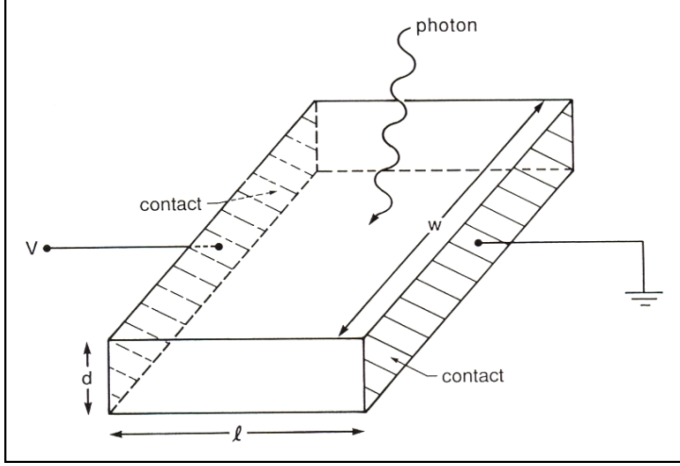


→ Resistivity ρ: $$\rho = R \cdot \frac{d \cdot w}{l}$$

Conductivity is the inverse of the resistivity: $$\sigma = \frac{1}{\rho}$$

Deriving the “Photo-Current” (1)

Conductivity σ ⇔ Photon Flux



Ohm's law: $R_d = \frac{V_b}{I_d}$

The conductivity σ [Ω^{-1} cm^{-1}] is related to R_d via: $\sigma = \frac{1}{R_d}\frac{l}{A} \Rightarrow R_d = \frac{1}{\sigma}\frac{l}{wd}$

where: $\sigma = \sigma_{th} + \sigma_{ph} \approx \sigma_{ph}$ (`th' denotes the thermal ("dark") current)

The current density is: $J_x = q_c n_0 \langle v_x \rangle$

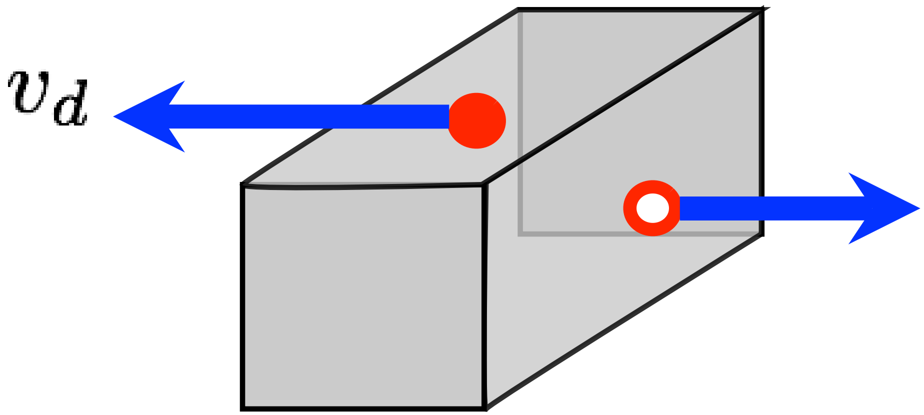
where q_c is the electrical charge and n_o the carrier density

But also: $J_x = \frac{I}{A} = \frac{I_d}{wd} = \frac{V_b}{R_d wd} = \frac{\sigma V_b}{l} = \sigma E_x$

and we get: $q_c n_0 \langle v_x \rangle = \sigma E_x$

Electron Mobility

Electron Mobility



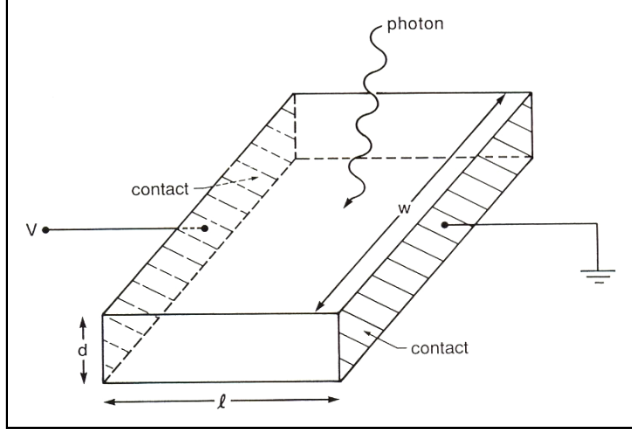
Mean drift velocity [m / s]

$$v_d = \mu E$$

Electric field [V / m]

Electron mobility [cm^2 / (V s)]

Electron Mobility = f{T}



Conductivity: $\sigma \overset{q_c=-q}{=} \frac{-qn_0\langle v_x\rangle}{E_x} \overset{-\mu_n=\frac{\langle v_x\rangle}{E_x}}{=} qn_0\mu_n$

where $-\mu_n$ is the electron mobility.

Mobility ~ mean time between collisions

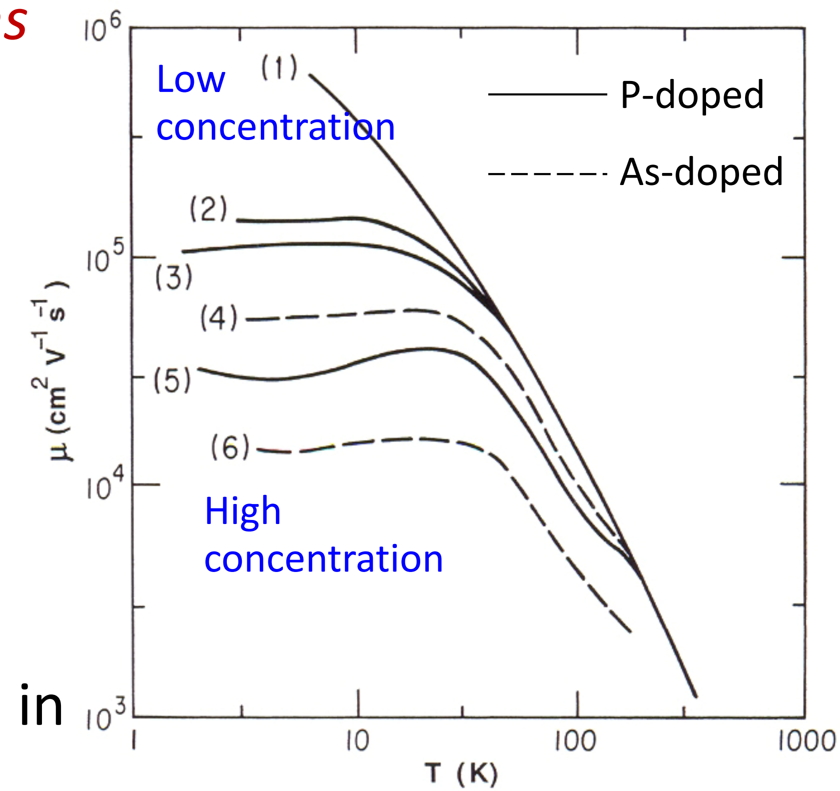
@low T → impurities dominate

ionized impurities: $\mu_n \sim T^{3/2}$

neutral impurities: $\mu_n \sim$ const

@high T → crystal lattice dominates

$\mu_n \sim T^{-3/2}$



Astronomical detectors usually operate in the low T regime.

Typical Electron Mobility Numbers

Table 3 Carrier mobilities at room temperature, in $cm^2/V\text{-}s$

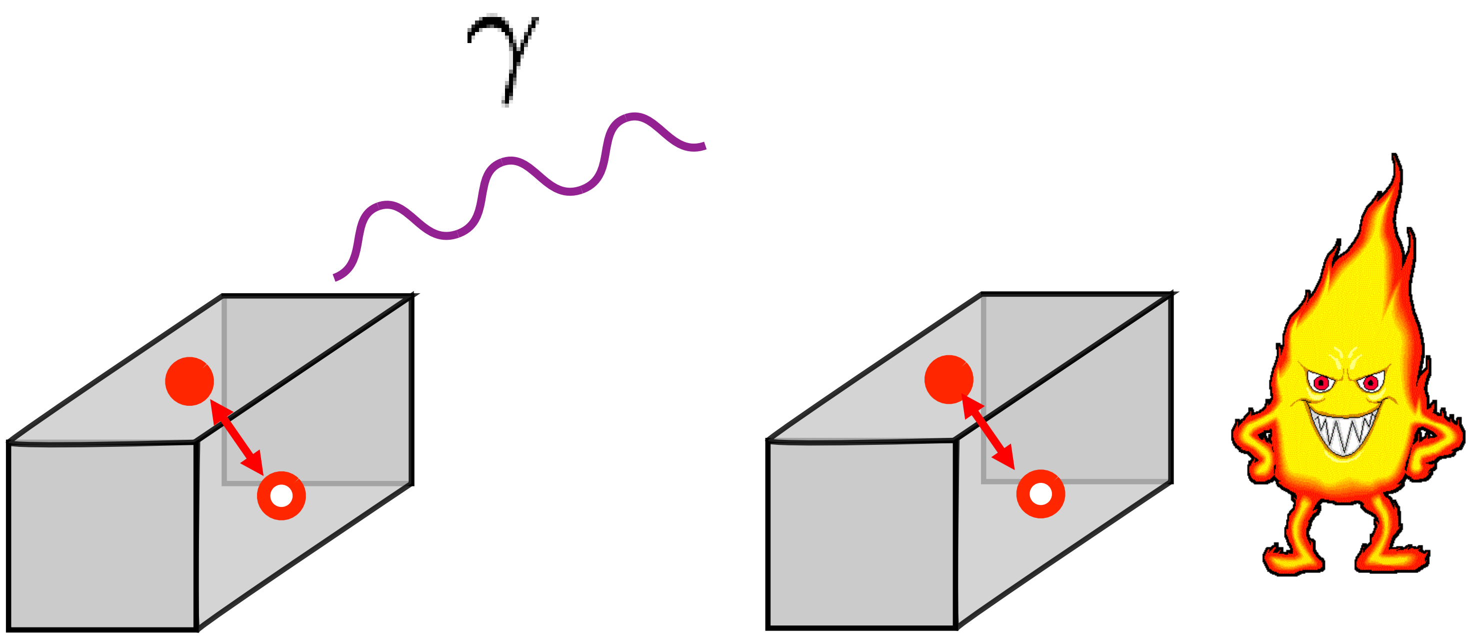
Crystal	Electrons	Holes	Crystal	Electrons	Holes
Diamond	1800	1200	GaAs	8000	300
Si	1350	480	GaSb	5000	1000
Ge	3600	1800	PbS	550	600
InSb	800	450	PbSe	1020	930
InAs	30000	450	PbTe	2500	1000
InP	4500	100	AgCl	50	—
AlAs	280	—	KBr (100 K)	100	—
AlSb	900	400	SiC	100	10–20

Generally, holes are less mobile than electrons

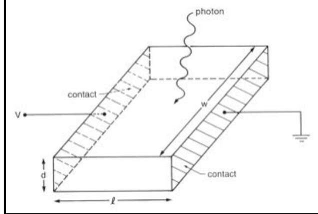
Deriving the “Photo-Current” (2)

Mean Lifetimes for the Charge Carriers

Eventually, the electrons and holes recombine after a mean lifetime τ, releasing the energy as heat or light.



Conductivity σ ⇔ Photon Flux



To the total conductivity, both electrons and holes contribute:

$$\sigma_{ph} = q\left(\mu_n n + \mu_p p\right)$$

(n and p are the negative and positive charge carrier concentrations)

Consider the incoming photon flux ϕ [γ/s]

→ number of charge carriers in equilibrium is $\phi\eta\tau$, where η is the quantum efficiency and τ is the mean lifetime before recombination. Typically, $\tau \sim$ (impurity concentration)$^{-1}$

Number of charge carriers per unit volume: $n = p = \dfrac{\varphi\eta\tau}{wdl}$

Hence, the resistance is:

$$R_d = \frac{1}{\sigma}\frac{l}{wd} = \frac{1}{q\left(\mu_n n + \mu_p p\right)}\frac{l}{wd} = \frac{1}{q\left(\mu_n + \mu_p\right)}\frac{wdl}{\varphi\eta\tau}\frac{l}{wd} = \frac{l^2}{q\left(\mu_n + \mu_p\right)\varphi\eta\tau}$$

Photoconductive Gain, Quantum Efficiency & Responsivity

The Photoconductive Gain (1)

(1) Time for an e⁻ to drift from one electrode to the other: $\tau_t = -\frac{l}{\langle v_x \rangle}$

(2) Recall the electron mobility: $\mu = -\frac{\langle v_x \rangle}{E_x}$

Combining (1) and (2) yields: $\tau_t = -\frac{l}{\mu E_x}$

→ Define a photoconductive gain: $\boxed{G \equiv \frac{\tau}{\tau_t} = \frac{\tau \cdot \mu E_x}{l}}$

where τ is the mean carrier lifetime before recombination.

➔ The photoconductive gain is the ratio of carrier lifetime to carrier transit time.

G quantifies the probability that a generated charge carrier will traverse the extent of the detector and reach an electrode.

The Photoconductive Gain (2)

The observed/detected photo current gets degraded by a factor:

$$G = \frac{\tau}{\tau_t}$$

$G << 1$ ⇔ $\tau_t >> \tau$ ⇔ charge carriers recombine before reaching an electrode

$G \sim 1$ ⇔ $\tau_t \sim \tau$ ⇔ all charge carriers are likely to reach an electrode

$G > 1$ is possible if charge multiplication occurs.

Options to optimize the gain G:

- make detector as this as possible
- increase the bias voltage (E_x)
- eliminate defects and impurities

The product ηG describes the probability that an incoming photon will produce an electric charge that will penetrate to an electrode.

Quantum Efficiency

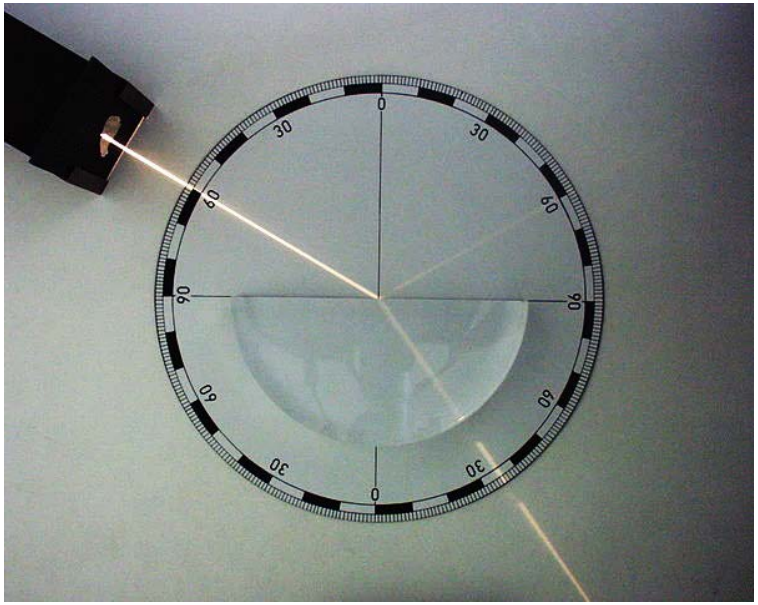
The quantum efficiency η is the percentage of photons hitting the detector surface that will produce an electron–hole pair.

Clear definition but difficult to measure.

η can be reduced by…:

1. reflection losses at the surface,

$$R = \frac{(n-1)^2}{(n+1)^2} \qquad R_{Ge} = \frac{(4-1)^2}{(4+1)^2} = \frac{9}{25} = 36\%$$



2. loss of photons that cross the crystal without interaction.

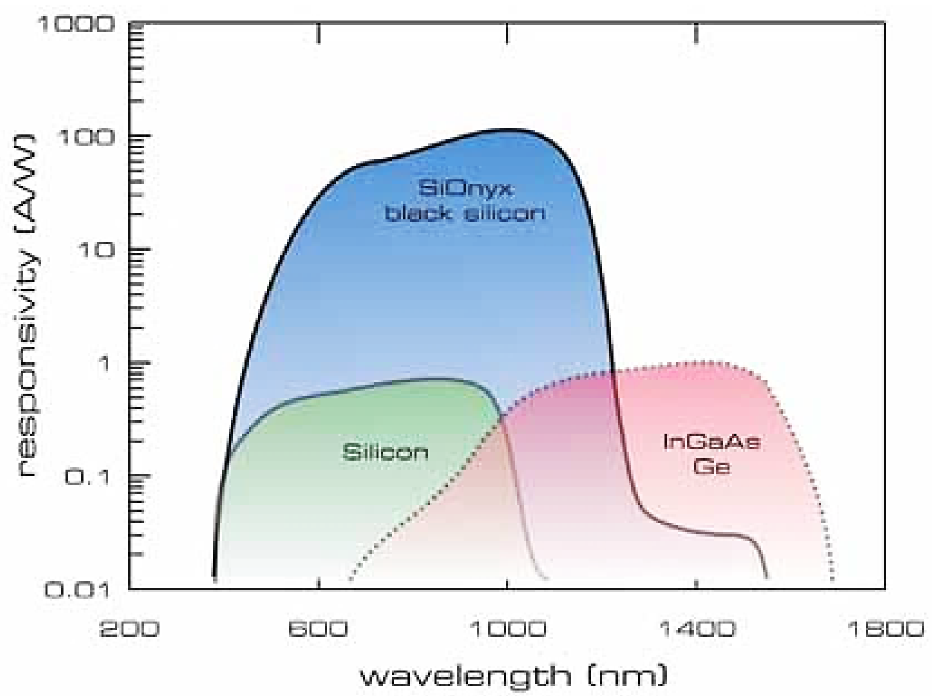
Responsivity

The responsivity S is the ratio between electrical signal at the detector output and incoming photon power.

Less elegant definition but easy accessible by measurement.

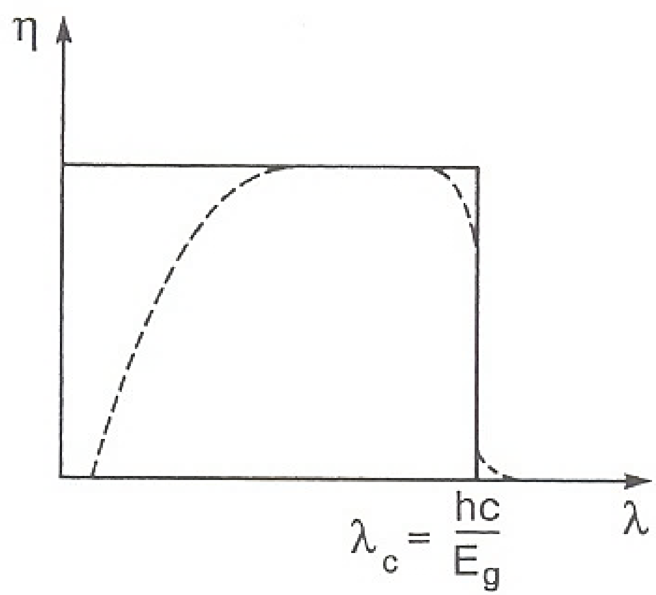
$$S = \frac{\text{electrical output signal}}{\text{input photon power}}$$

$$S = G\frac{q\eta\lambda}{hc}$$

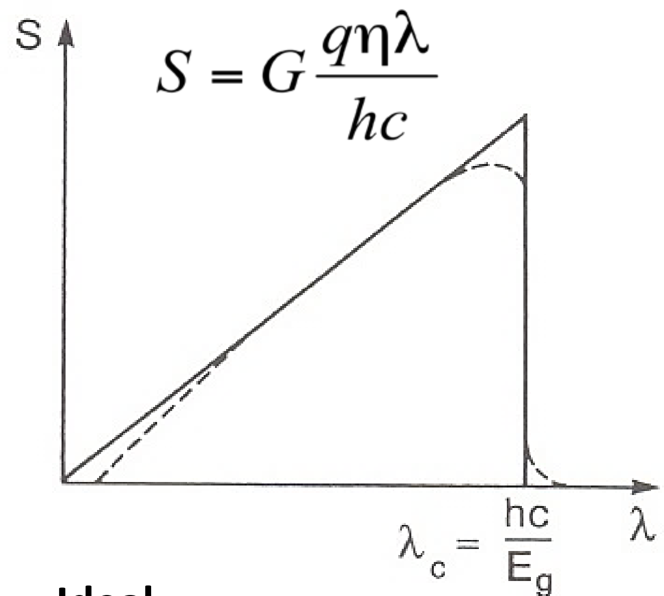


Quantum Efficiency ⇔ Responsivity

The quantum efficiency η is *independent of wavelength* up to the cutoff at λ_c:



The responsivity S increases proportionally to the wavelength:



Ideal

Typical

Deriving the “Photo-Current” (3)

Conductivity σ ⇔ Photon Flux

Responsivity S ≡ $\dfrac{\text{electrical output signal}}{\text{input photon power}}$

The "photon power" falling on the detector is: $P_{ph} = \varphi h\nu = \dfrac{\varphi hc}{\lambda}$

Photoconductive Gain $G \equiv \dfrac{\tau\mu E_x}{l}$

τ·µ/l = lifetime × mobility / pathlength

E_x = "amplifying" electric field

The responsivity S then becomes:

$$S = \frac{I_{ph}}{P_{ph}} \overset{Ohm}{=} \frac{V_b}{R_{ph}P_{ph}} \overset{E=V/l}{=} \frac{E_x l}{R_{ph}P_{ph}} = \frac{E_x l}{l^2} q\varphi\eta\tau\left(\mu_n + \mu_p\right)\frac{\lambda}{\varphi hc} = G\frac{q\eta\lambda}{hc}$$

$$R_d = \frac{l^2}{q\left(\mu_n + \mu_p\right)\varphi\eta\tau}$$

This yields the photo current: $\boxed{I_{ph} = \dfrac{\eta\lambda qG}{hc}P_{ph} = \eta q\varphi G}$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COFFEE
BREAK
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Refresher:
Noise Distributions

Gaussian Distribution (1)

Gaussian noise is the noise following a Gaussian (normal) distribution:

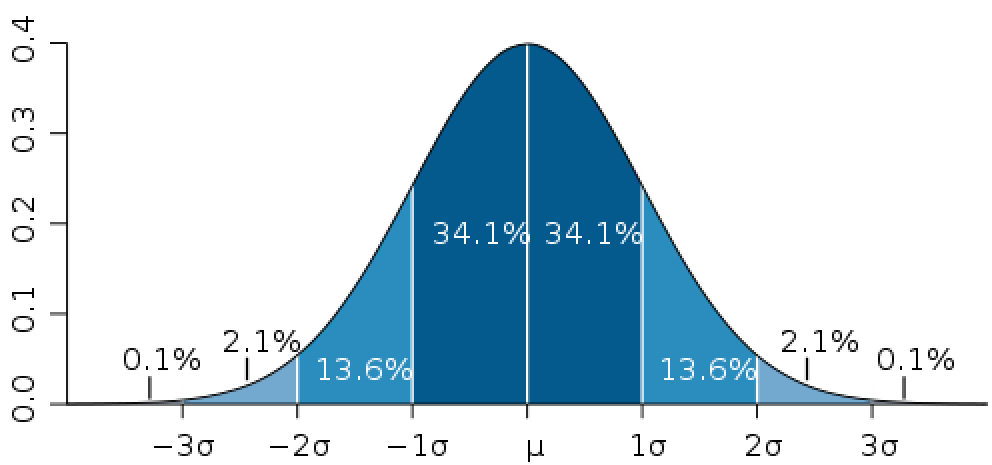
$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right]$$

It is often (incorrectly) called white noise, which refers to the uncorrelation of the noise.

x is the actual value

μ is the mean of the distribution

σ is the standard deviation of the distribution $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}$



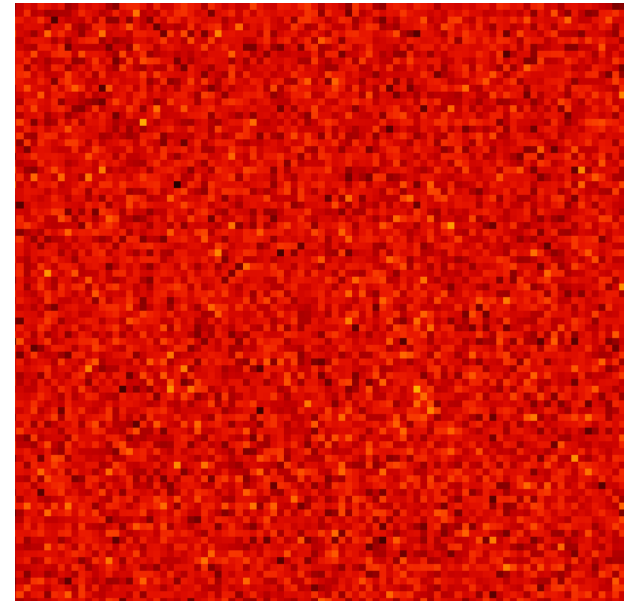
1-σ ~ 68%

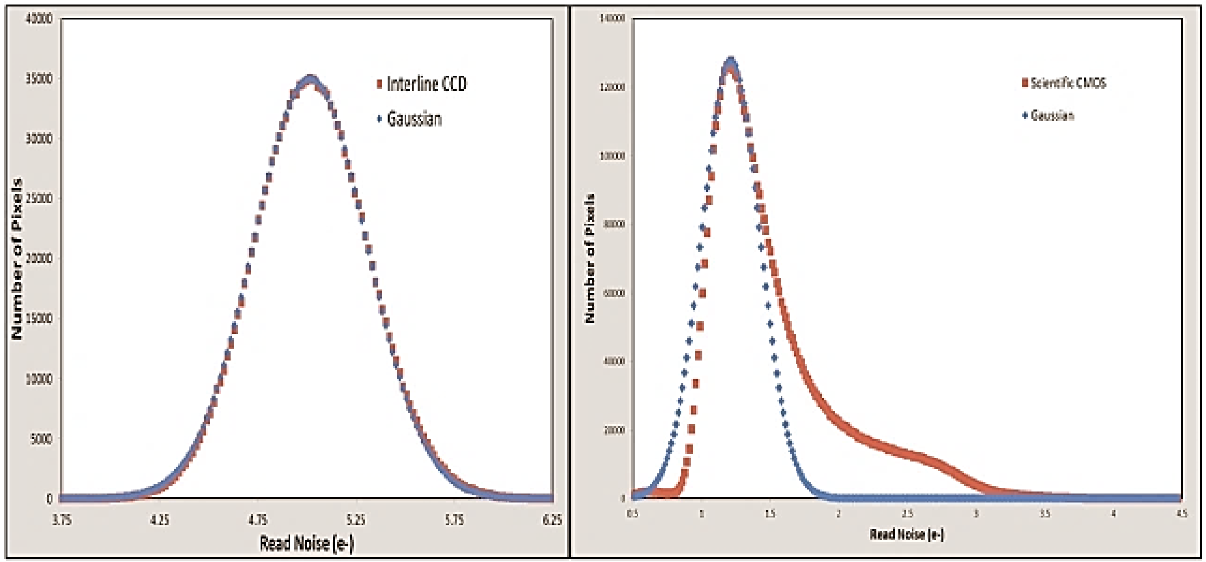
2-σ ~ 95%

3-σ ~ 99.7%

Gaussian Distribution (2)

Example: detector "dark frame" (readout without illumination) →





← from: `http://www.microscopy-analysis.com/editorials/editorial-listings/digital-camera-technologies-scientific-bio-imaging-part-3-noise-and`

Usually the dark frames show Gaussian behavior – but not always, in case of other, systematic noise sources (dead pixels, warm electrodes, etc.)

Poissonian Distribution (1)

Poisson noise is the noise following a Poissonian distribution:

$$P(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

k is the number of occurrences of an event (probability)
λ is the *expected* number of occurrences

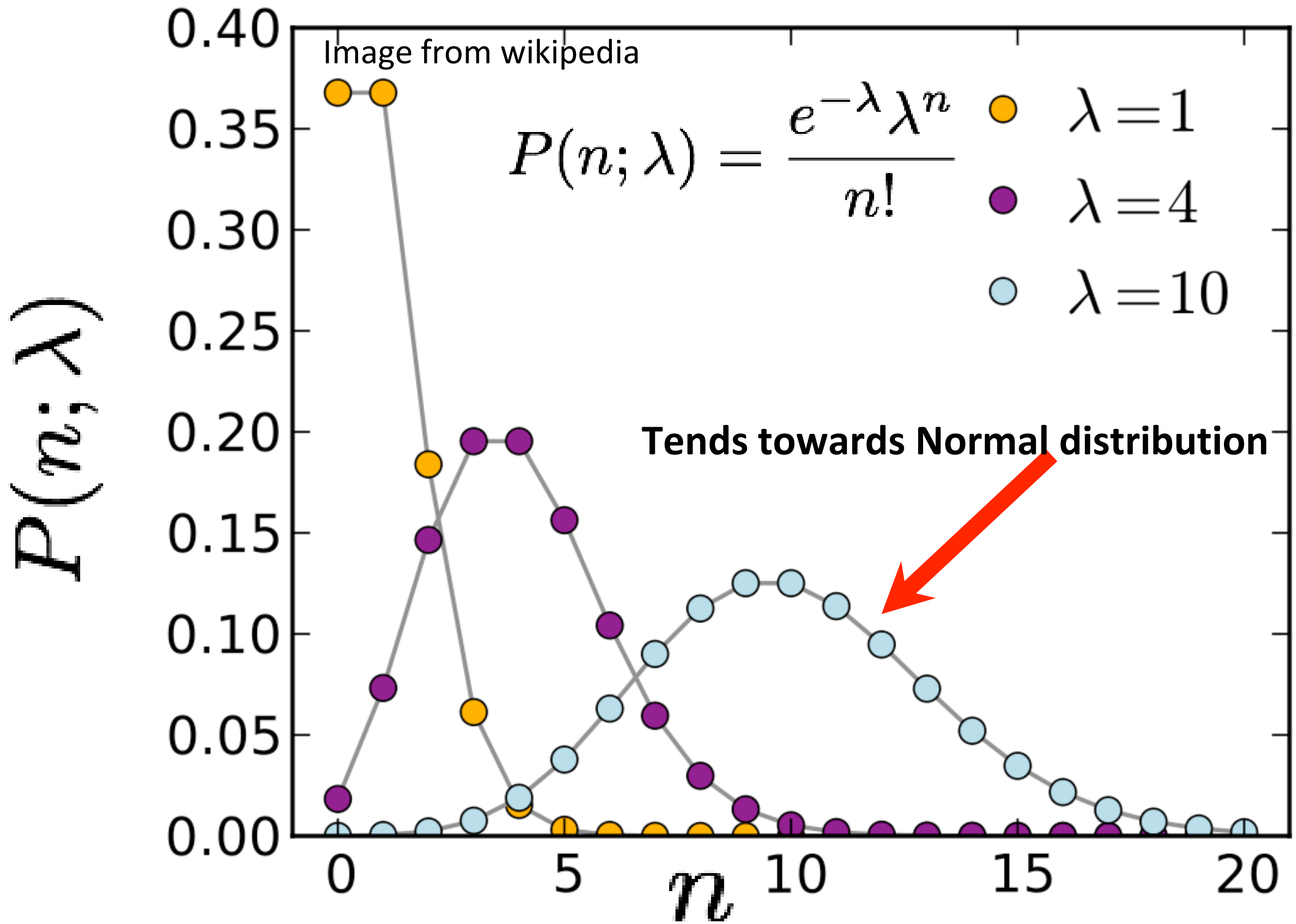
P(k,λ) expresses the probability of a number of events occurring in a fixed period of time, provided that:

- these events occur with a known average rate λ, and
- the arrival of one event is independent of the time since the last event

Properties:

- the mean (average) of *P(k,λ)* is *λ*.
- the standard deviation of *P(k,λ)* is *√λ*.

Poissonian Distribution (2)

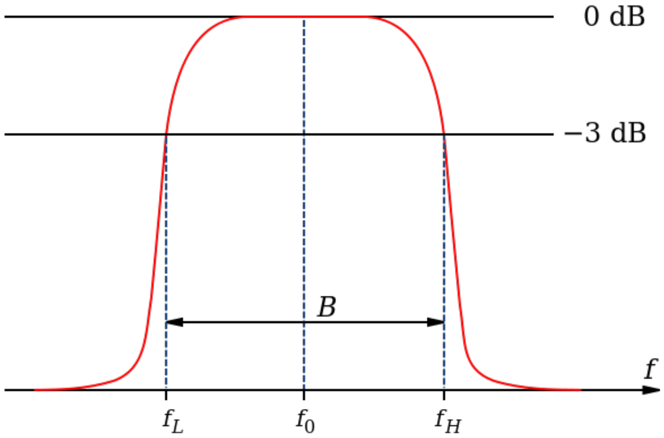


Example: fluctuations in the detected photon flux between time intervals Δt_i.

Detected are k photons, while expected are, on average, λ photons.

Noise Bandwidth

White noise has a wide frequency range, which we associate with an equivalent noise bandwidth B or $\Delta f = f_H - f_L$.



For a system – like our detector – with exponential response $U \sim e^{-t/\tau}$

we get $\Delta f = \dfrac{1}{4\tau}$

According to the Shannon Nyquist theorem, an output bandwidth of one hertz is equivalent to half a second of integration.

→ signal integrated over time Δt_{int}: $\boxed{\Delta f = \dfrac{1}{2\Delta t_{int}}}$

The Main Sources of Detector Noise

The G-R Noise Current

Photoconductor absorbs N photons: $N = \eta\phi\Delta t$

→ create N conduction electrons and N holes (but consider only e^- since $\mu_{e-} \gg \mu_p$)

Randomly **g**enerated e^- ***and*** randomly **r**ecombined e^- → two random processes

Hence: RMS noise ~ $(2N)^{1/2}$.

Now calculate the associated noise current: $$\left\langle I_{G-R}^2 \right\rangle^{1/2} = \frac{q\sqrt{2N}G}{\Delta t}$$

With the mean photo-current $I_{ph} = \eta q \varphi G$

one gets:

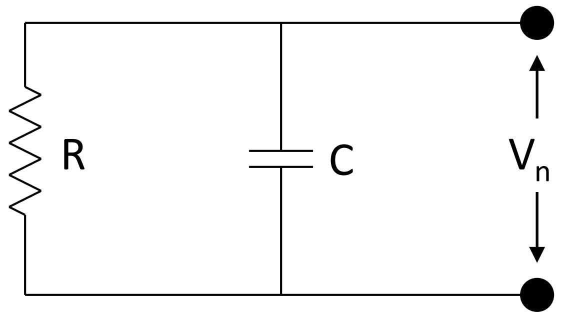
$$\left\langle I_{G-R}^2 \right\rangle = \frac{q^2 (2N) G^2}{(\Delta t)^2} = \left(\frac{2q}{\Delta t}\right)\left(\frac{qNG}{\Delta t}\right)G = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph} \right\rangle G$$

The noise current $\langle I^2_{G\text{-}R} \rangle$ can now be rewritten as:

$$\left\langle I_{G-R}^2 \right\rangle = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph} \right\rangle G = (2q2\Delta f)\langle \varphi q \eta G \rangle G = 4q^2 \varphi \eta G^2 \Delta f$$

Johnson (or kTC) Noise (1)

Consider a detector pixel as an RC circuit:



The energy stored in the capacitor is $E_{st} = ½CV^2$.

This system has *one* degree of freedom: V_n. Fluctuations in V_n are associated with an average energy of ½ kT:

$$\frac{1}{2}C\left\langle V_n^2\right\rangle = \frac{1}{2}kT$$

These fluctuations in E_{st} result in a Johnson noise current I_J.

The charge on the capacitor is Q = CV → $\langle Q^2\rangle = C^2V^2 = kTC$

Hence, this noise is also called kTC noise or reset noise.

Johnson (or kTC) Noise (2)

The power in I_J can thermodynamically also be expressed as:

$$\langle P \rangle t = \frac{1}{2} kT \quad \text{with time constant} \quad \tau = t = RC$$

$$\text{and} \quad P = U \cdot I = R \cdot I^2 \quad \xrightarrow{1\,\text{dof}} \quad \langle P \rangle = \frac{1}{2} \langle I^2 \rangle R$$

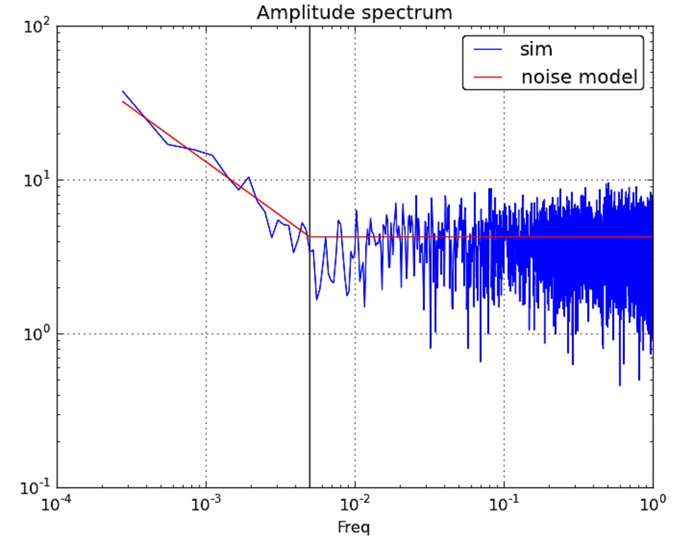
Hence, $$\langle I_J^2 \rangle = \frac{2\langle P \rangle}{R} = \frac{2\frac{1}{2}kT}{Rt} \overset{\Delta f = 1/4\tau}{=} \frac{4kT}{R} \Delta f$$

The Johnson (or kTC) noise is a fundamental thermodynamic noise due to the thermal (Brownian) motion of the charge carriers.

1/f Noise

Most electronic devices have increased noise at low frequencies, often dominating the system performance.

However, there is no general physical understanding of it.



Empirically, $\left\langle I_{1/f}^{2} \right\rangle \propto \frac{I^{a}}{f^{b}} \Delta f \quad \text{where} \quad a \approx 2, b \approx 1.$

May be caused by bad electrical contacts, temperature fluctuations, surface effects (damage), crystal defects, and junction field effect transistors (JFETs), etc.

This type of noise is empirically termed 1/f noise.

Noise Sources in Comparison

Total Noise

the G-R or noise current $\left\langle I_{G-R}^{2}\right\rangle = 4q^{2}\varphi\eta G^{2}\Delta f$

the Johnson noise $\left\langle I_{J}^{2}\right\rangle = \frac{4kT}{R}\Delta f$

the 1/f noise $\left\langle I_{1/f}^{2}\right\rangle \propto \frac{I^{a}}{f^{b}}\Delta f$

Note that *all* processes depend on the bandwidth $\Delta f = 1/(2\Delta t_{int})$

If the signal is Poisson distributed in time the relative error of the measurement is proportional to $1/\sqrt{t}$ or $(\Delta f)^{1/2}$ (longer t_{int} means smaller bandwidth means smaller relative errors)

The total noise in the system is $\left\langle I_{N}^{2}\right\rangle = \left\langle I_{G-R}^{2}\right\rangle + \left\langle I_{J}^{2}\right\rangle + \left\langle I_{1/f}^{2}\right\rangle$

Background-limited Performance

Ideally, we want the detector sensitivity not being limited by technical factors but by processes in nature (i.e., nothing we can do about).

That implies: $\langle I_{G-R}^2 \rangle >> \langle I_J^2 \rangle + \langle I_{1/f}^2 \rangle$

This is called background-limited performance **(BLIP)**

BLIP has significant impact on…:

- Detector design: e.g., MIR detectors for ground or space? (→ I_{dark})
- Instrument design: e.g., pixel scale (→ oversampling)
- Observation planning: e.g., exposure time (→ long or short?)

Noise Equivalent Flux Density (NEFD)

The noise equivalent flux density (NEFD) is the flux density that yields an RMS S/N of unity in a system of Δf = 1 Hz.

$$NEFD = \frac{E_S \sqrt{2\Delta t_{\mathrm{int}}}}{S / N}$$

...where E_S [W m^{-2} Hz^{-1}] is the measured flux density.

The NEFD usually refers to the entire system performance, including the camera optics.

Noise Equivalent Power (NEP)

The noise equivalent power (NEP) is the signal power that yields an RMS S/N of unity in a system of $\Delta f = 1$ Hz.

$$df = \frac{1}{2\Delta t_{int}}$$

$$\frac{S}{N} = \frac{P_s}{\mathrm{NEP}(df)^{1/2}} = \frac{P_s(2\Delta t_{int})^{1/2}}{\mathrm{NEP}} \longrightarrow \boxed{NEP = \frac{P_S\sqrt{2\Delta t_{\mathrm{int}}}}{S/N}}$$

An equivalent, more practical definition is:

$$\boxed{NEP = \frac{I_N}{S}}$$

...where I_N [W Hz$^{-1/2}$] is the total noise current in the system, and S [A W^{-1}] is the responsivity.

NEP for BLIP ⇔ kTC

(1) BLIP: $\left\langle I_{G-R}^{2}\right\rangle >> \left\langle I_{J}^{2}\right\rangle + \left\langle I_{1/f}^{2}\right\rangle$

With $S = G\frac{q\eta\lambda}{hc}$ and $\langle I_{G-R}^2 \rangle = 4q^2\varphi\eta G^2\Delta f$ one gets:

$$\mathrm{NEP}_{G-R} = \frac{I_{G-R}}{S} = \frac{(4q^2\varphi\eta G^2)^{1/2}hc}{Gq\eta\lambda} \boxed{= \frac{2hc}{\lambda}\left(\frac{\varphi}{\eta}\right)^{1/2}}$$

(The factor of Δf disappears from $<I^2_{G-R}>$ as we use a “normalized” noise current in units of [A Hz^{-1}].)

(2) kTC: $\left\langle I_{J}^{2}\right\rangle >> \left\langle I_{G-R}^{2}\right\rangle + \left\langle I_{1/f}^{2}\right\rangle$

With $S = G\frac{q\eta\lambda}{hc}$ and $\langle I_J^2 \rangle = \frac{4kT\Delta f}{R}$ one gets:

$$\mathrm{NEP}_{J} = \frac{I_J}{S} = \frac{(4kT)^{1/2}hc}{R^{1/2}Gq\eta\lambda} \boxed{= \frac{2hc}{Gq\eta\lambda}\left(\frac{kT}{R}\right)^{1/2}}$$

Here, the NEP can be improved by increasing η, G, R or reducing T.