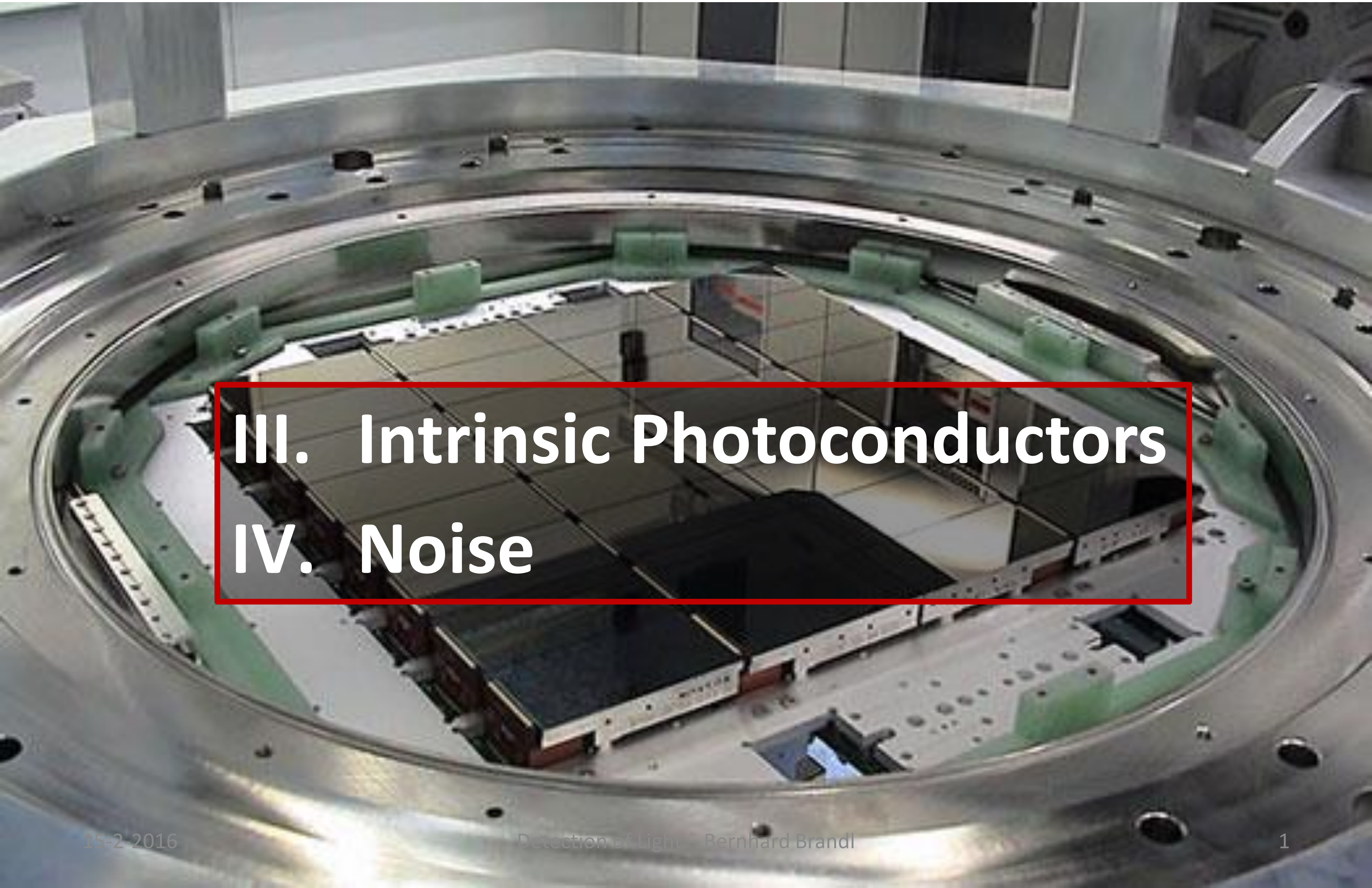
# Detection of Light

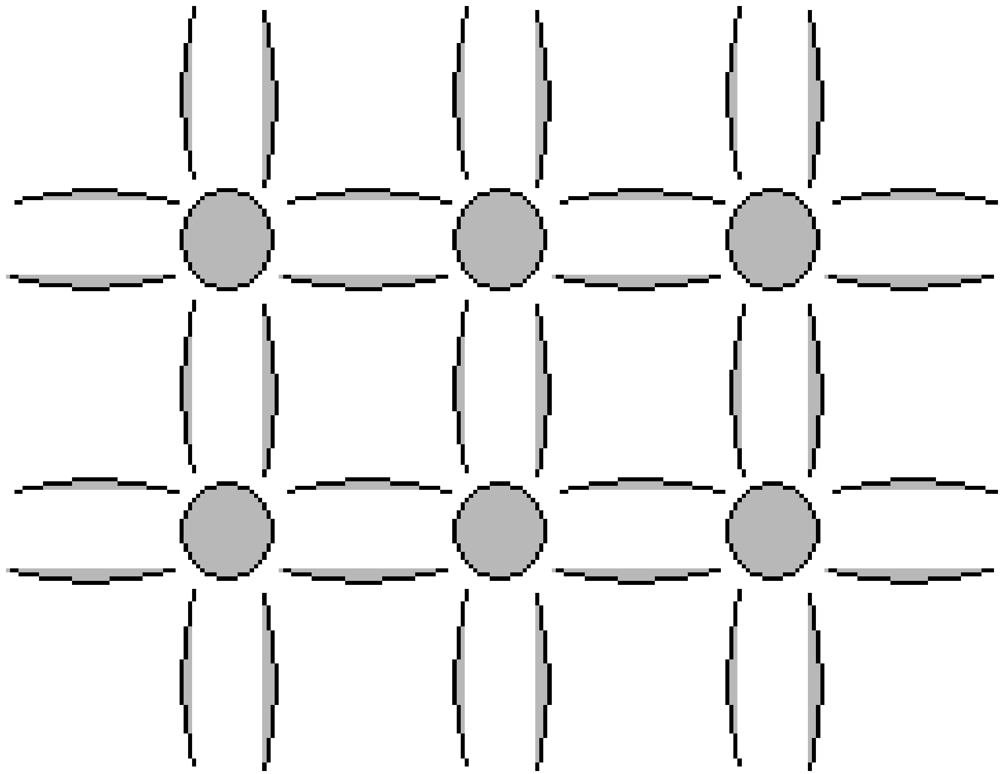


III. Intrinsic Photoconductors
IV. Noise

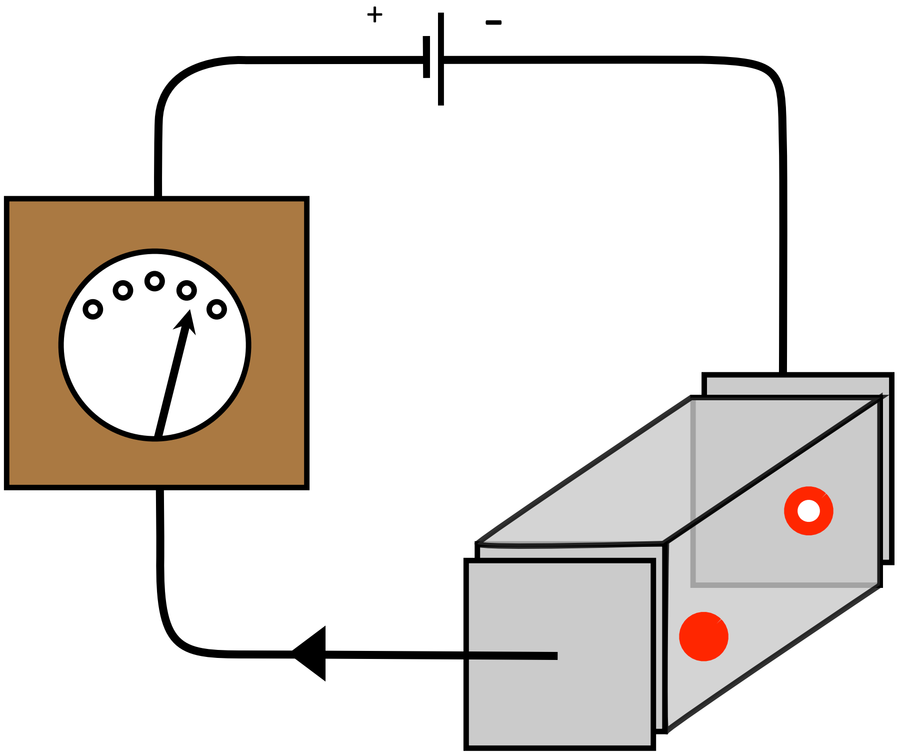
# Fundamental Principle and Limitations

# Basic Principle – Physics

$E_\gamma$ lifts $e^-$ from valence into conduction band: $E_\gamma = \frac{hc}{\lambda} > E_{bandgap}$



# Basic Principle – Realization




Applying an electric field causes electric charges to move in the material and register a signal as an electric current

# Practical Limitations

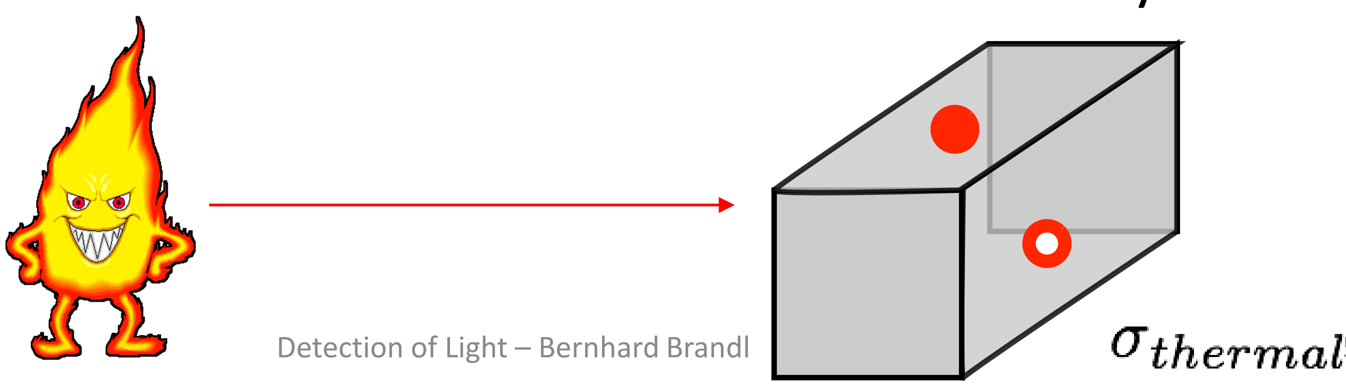
- Wavelength cutoffs: $$\lambda_c = \frac{hc}{E_g}$$

  → Germanium: 1.85µm

  → Silicon: 1.12µm

  → GaAs: 0.87µm

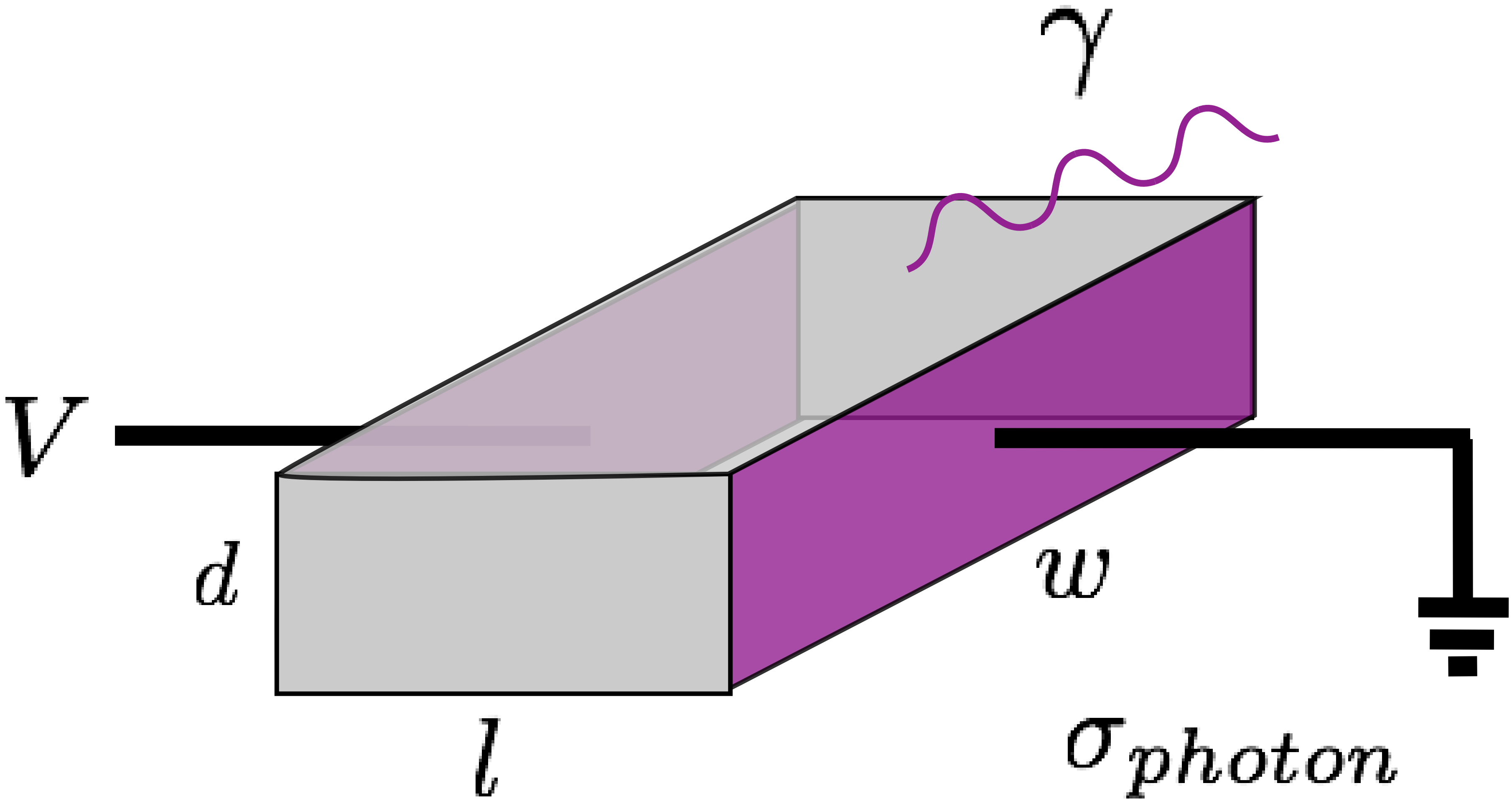
- Cleanliness and non-uniformity of material
- Problems to make good electrical contacts to pure Si
- Charge carriers are generated with both photons and thermal excitation. We only measure the electrical conductivity!



# Basic Electric Properties

# Schematics of a Detector

Consider a pixel with physical dimensions *d, l, w*:



# Resistivity, Resistance, and Conductivity

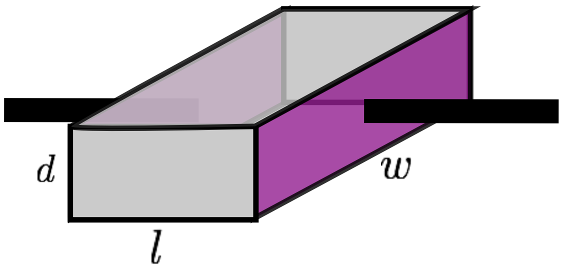
Resistivity [ Ω m ] $\rho = \frac{E}{J}$

Electric field [ V / m ]

Current density [ A / m$^2$ ]

Resistivity and resistance:

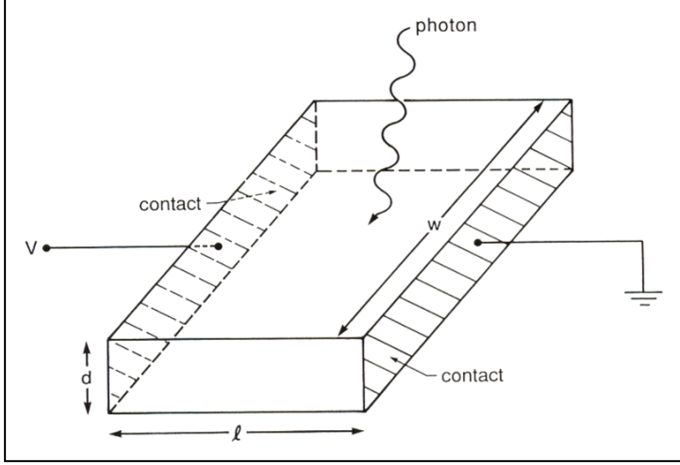
$$\rho = R\frac{Area}{l} \qquad \rho = R\frac{dw}{l}$$




Conductivity is the inverse of the resistivity: $\sigma = \frac{1}{\rho}$

# Deriving the “Photo-Current” (1)

# Conductivity σ ⇔ Photon Flux




Ohm's law: $R_d = \frac{V_b}{I_d}$

The conductivity σ [$\Omega^{-1}$ $cm^{-1}$] is related to $R_d$ via: $\sigma = \frac{1}{R_d}\frac{l}{A} \Rightarrow R_d = \frac{1}{\sigma}\frac{l}{wd}$

where: $\sigma = \sigma_{th} + \sigma_{ph} \approx \sigma_{ph}$ (`th' denotes the thermal ("dark") current)

The current density is: $J_x = q_c n_0 \langle v_x \rangle$

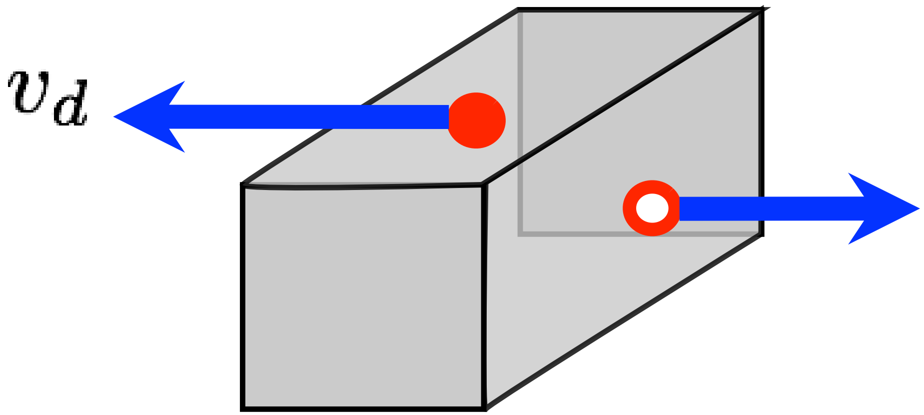
where $q_c$ is the electrical charge and $n_o$ the carrier density

But also: $J_x = \frac{I}{A} = \frac{I_d}{wd} = \frac{V_b}{R_d wd} = \frac{\sigma V_b}{l} = \sigma E_x$

and we get: $q_c n_0 \langle v_x \rangle = \sigma E_x$

# Electron Mobility

# Electron Mobility



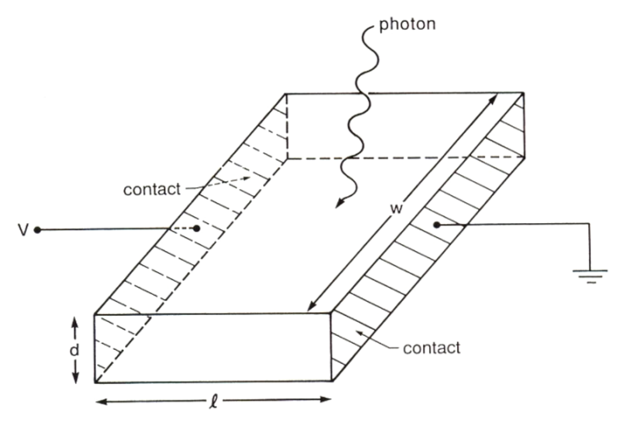

Mean drift velocity [ m / s ]

$$v_d = \mu E$$

Electric field [ V / m ]

Electron mobility [ cm$^2$ / (V s) ]

# Electron Mobility = f{T}




Conductivity: $\sigma \overset{q_c=-q}{=} \frac{-qn_0\langle v_x\rangle}{E_x} \overset{-\mu_n=\frac{\langle v_x\rangle}{E_x}}{=} qn_0\mu_n$

where $-\mu_n$ is the electron mobility.

*Mobility ~ mean time between collisions*

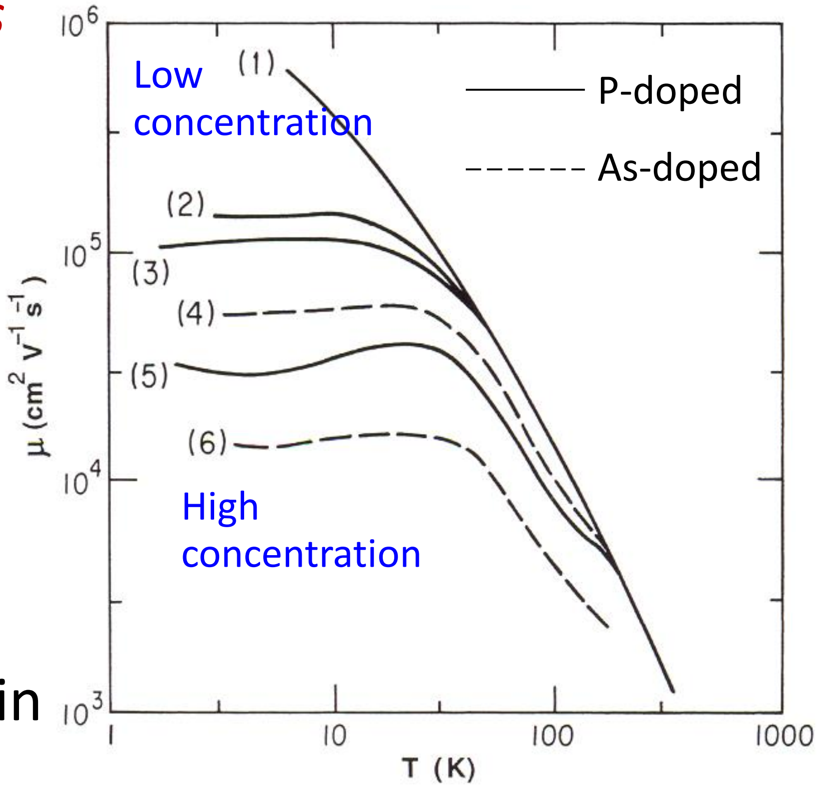
@low T → impurities dominate

ionized impurities: $\mu_n \sim T^{3/2}$

neutral impurities: $\mu_n \sim$ const

@high T → crystal lattice dominates

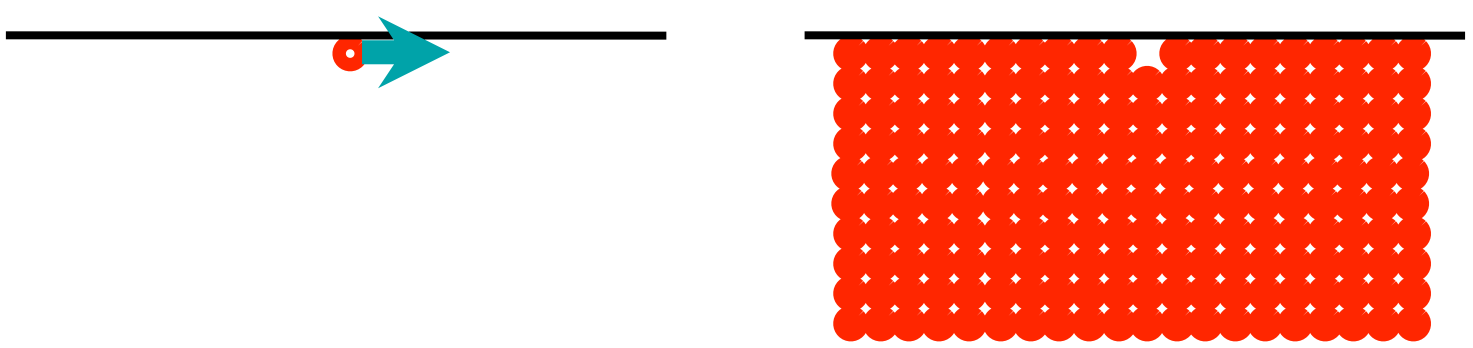
$\mu_n \sim T^{-3/2}$




Astronomical detectors usually operate in the low T regime.

# Holes are less mobile than Electrons

$$\mu_p << \mu_e$$



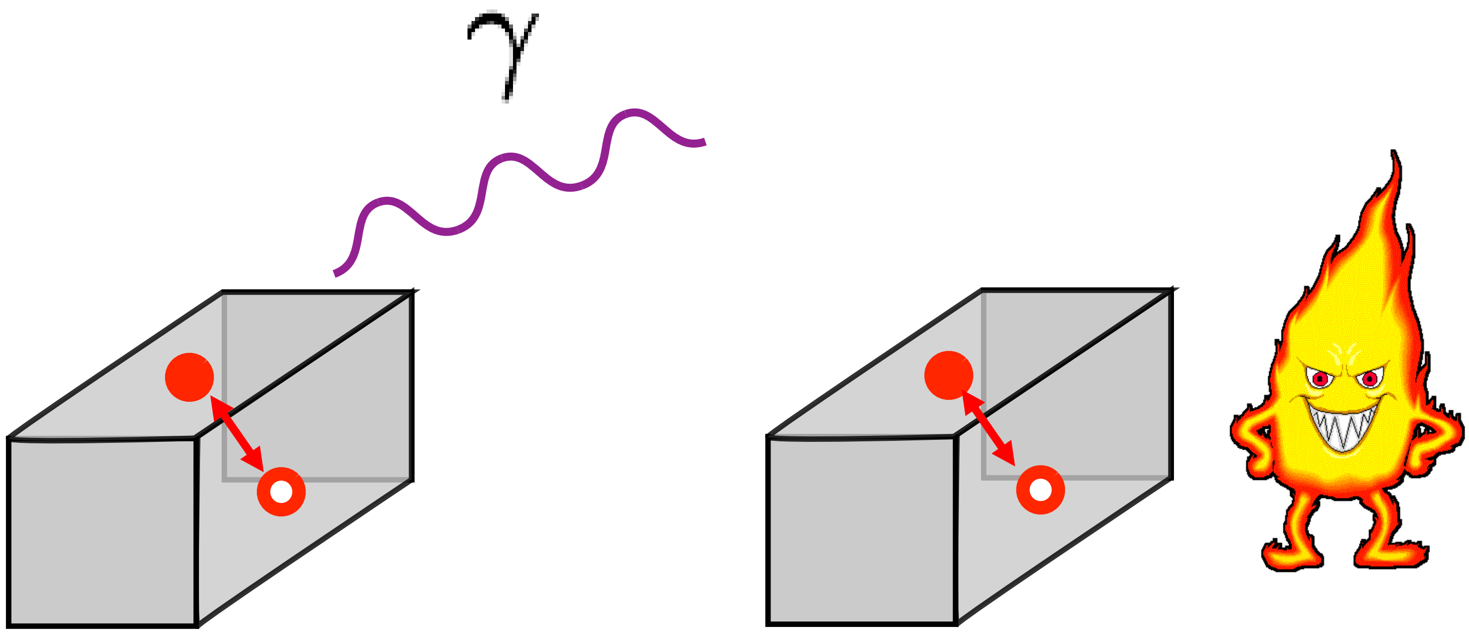
# Typical Electron Mobility Numbers

**Table 3 Carrier mobilities at room temperature, in $cm^2/V\text{-}s$**

| Crystal | Electrons | Holes | Crystal | Electrons | Holes |
|---|---|---|---|---|---|
| Diamond | 1800 | 1200 | GaAs | 8000 | 300 |
| Si | 1350 | 480 | GaSb | 5000 | 1000 |
| Ge | 3600 | 1800 | PbS | 550 | 600 |
| InSb | 800 | 450 | PbSe | 1020 | 930 |
| InAs | 30000 | 450 | PbTe | 2500 | 1000 |
| InP | 4500 | 100 | AgCl | 50 | — |
| AlAs | 280 | — | KBr (100 K) | 100 | — |
| AlSb | 900 | 400 | SiC | 100 | 10–20 |

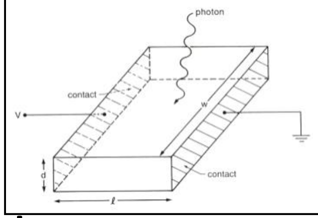
# Mean Lifetimes for the Charge Carriers

Eventually, the electrons and holes recombine after a mean lifetime $\tau$, releasing the energy as heat or light.



# Deriving the “Photo-Current” (2)

# Conductivity σ ⇔ Photon Flux




To the total conductivity, both electrons and holes contribute:

$$\sigma_{ph} = q(\mu_n n + \mu_p p)$$

(n and p are the negative and positive charge carrier concentrations)

Consider the incoming photon flux ϕ [γ/s]

→ number of charge carriers in equilibrium is $\phi\eta\tau$, where η is the quantum efficiency and $\tau$ is the mean lifetime before recombination. Typically, $\tau \sim$ (impurity concentration)$^{-1}$

Number of charge carriers per unit volume: $n = p = \dfrac{\varphi\eta\tau}{wdl}$

Hence, the resistance is:

$$R_d = \frac{1}{\sigma}\frac{l}{wd} = \frac{1}{q(\mu_n n + \mu_p p)}\frac{l}{wd} = \frac{1}{q(\mu_n + \mu_p)}\frac{wdl}{\varphi\eta\tau}\frac{l}{wd} = \frac{l^2}{q(\mu_n + \mu_p)\varphi\eta\tau}$$

# Photoconductive Gain & Responsivity

# The Photoconductive Gain

(1) Time for an e⁻ to drift from one electrode to the other: $\tau_t = -\frac{l}{\langle v_x \rangle}$

(2) Recall the electron mobility: $\mu = -\frac{\langle v_x \rangle}{E_x}$

Combining (1) and (2) yields: $\tau_t = -\frac{l}{\mu E_x}$

→ Define a photoconductive gain: $G = \frac{\tau \mu E_x}{l} = \frac{\tau}{\tau_t}$

where $\tau$ is the mean carrier lifetime before recombination.

➔ The photoconductive gain is the ratio of carrier lifetime to carrier transit time.

*G quantifies the probability that a generated charge carrier will traverse the extent of the detector and reach an electrode.*

# The Photoconductive Gain (2)

Since the photoconductive gain is determined via: $I_{ph} = q\varphi\eta G$

the observed current is degraded by $G = \dfrac{\tau}{\tau_t}$ (for G <1).

G << 1 ⇔ $\tau_t >> \tau$ ⇔ charge carriers recombine before reaching an electrode

G ~ 1 ⇔ $\tau_t \sim \tau$ ⇔ all charge carriers have a good chance of reaching an electrode

G > 1 is possible if charge multiplication occurs.

Options to optimize the gain G:

- make detector as this as possible
- increase the bias voltage ($E_x$)
- eliminate defects and impurities

The product ηG describes the probability that an incoming photon will produce an electric charge that will penetrate to an electrode.

# Quantum Efficiency & Responsivity

# Quantum Efficiency & Responsivity

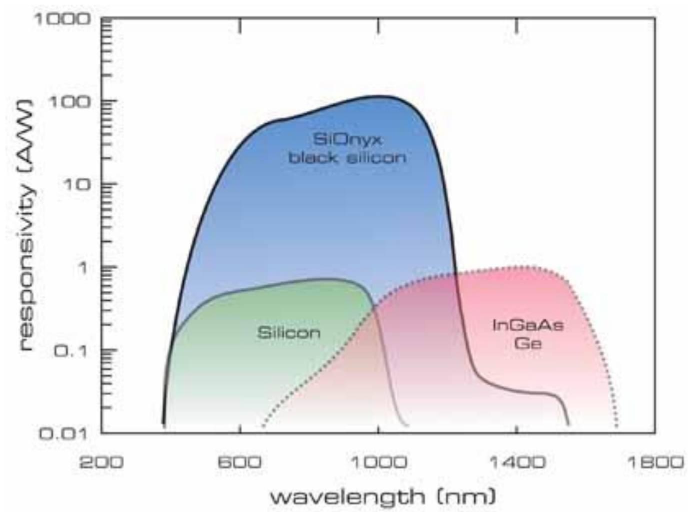
The quantum efficiency $\eta$ is the percentage of photons hitting the detector surface that will produce an electron–hole pair.

Clear definition but difficult to measure.

The responsivity S is the ratio between electrical signal at the detector output and incoming photon power.

Less elegant definition but easy accessible by measurement.

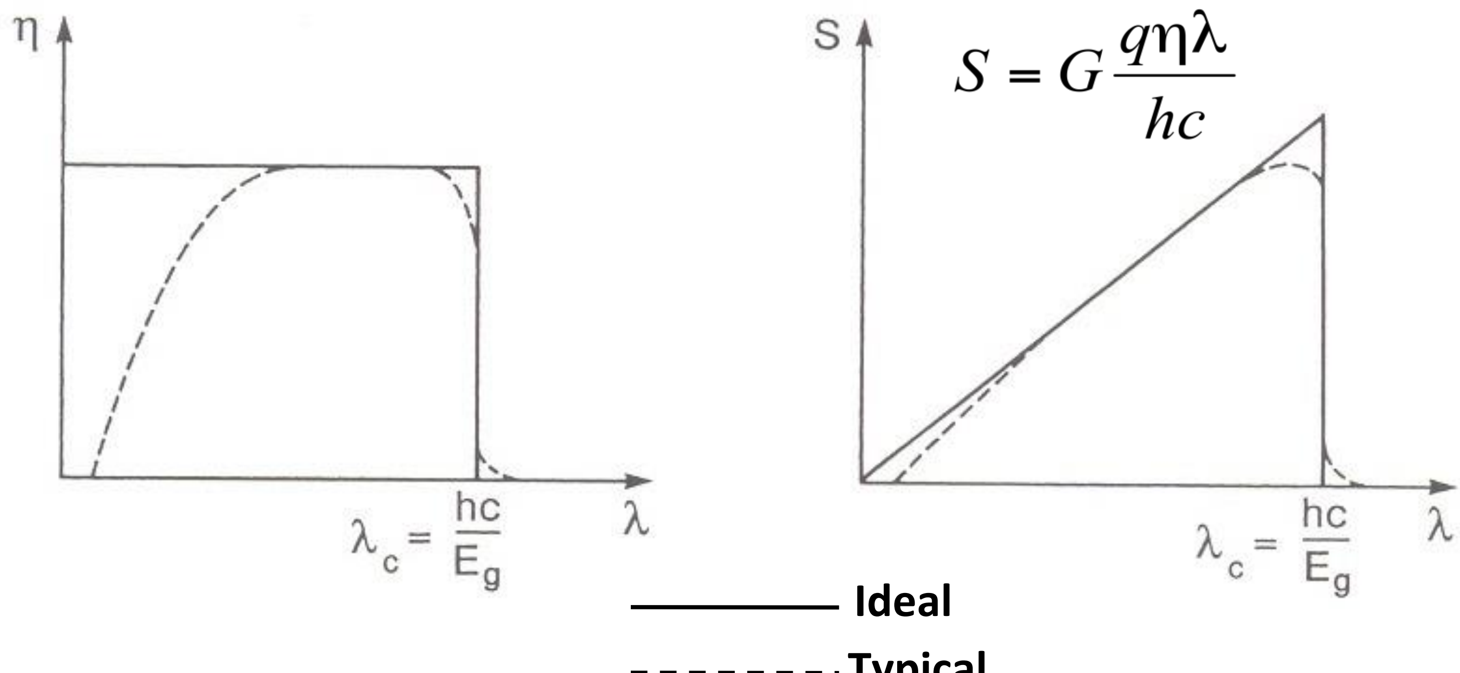
$$S = \frac{\text{electrical output signal}}{\text{input photon power}}$$



# Quantum Efficiency & Responsivity (2)

The quantum efficiency $\eta$ is the percentage of photons producing an electron–hole pair. It is thus *independent of wavelength* up to the cutoff at $\lambda_c$.

The responsivity S increases proportional with wavelength:



# Deriving the "Photo-Current" (3)

# Conductivity σ ⇔ Photon Flux

Responsivity S $\equiv \dfrac{\text{electrical output signal}}{\text{input photon power}}$

The "photon power" falling on the detector is: $P_{ph} = \varphi h\nu = \dfrac{\varphi hc}{\lambda}$

Photoconductive Gain $G \equiv \dfrac{\tau\mu E_x}{l}$

τ·μ/l = lifetime × mobility / pathlength

$E_x$ = "amplifying" electric field

The responsivity S then becomes:

$$S = \frac{I_{ph}}{P_{ph}} \overset{Ohm}{=} \frac{V_b}{R_{ph}P_{ph}} \overset{E=V/l}{=} \frac{E_x l}{R_{ph}P_{ph}} = \frac{E_x l}{l^2} q\varphi\eta\tau\left(\mu_n + \mu_p\right)\frac{\lambda}{\varphi hc} = G\frac{q\eta\lambda}{hc}$$

This yields the photo current: $\boxed{I_{ph} = \dfrac{\eta\lambda q G}{hc} P_{ph} = \eta q \varphi G}$

$$R_d = \frac{l^2}{q\left(\mu_n + \mu_p\right)\varphi\eta\tau}$$

You can watch the live simulcast of the National Press Club event at:
https://scontent.webcaster4.com/web/nsfligo
Following is a YouTube link to use as a back-up:
https://www.youtube.com/user/VideosatNSF/live

# Refresher:
# Noise Distributions

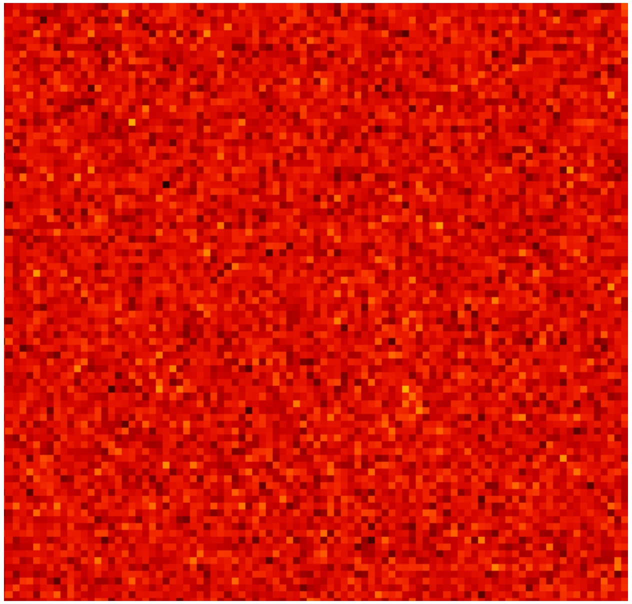
*<u>Goal:</u> Estimate the total noise in the system*

*<u>Assumption:</u> All statistical noise sources are Poisson distributed*

# I. Gaussian Distribution

Gaussian noise is the noise following a Gaussian (normal) distribution:

$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$



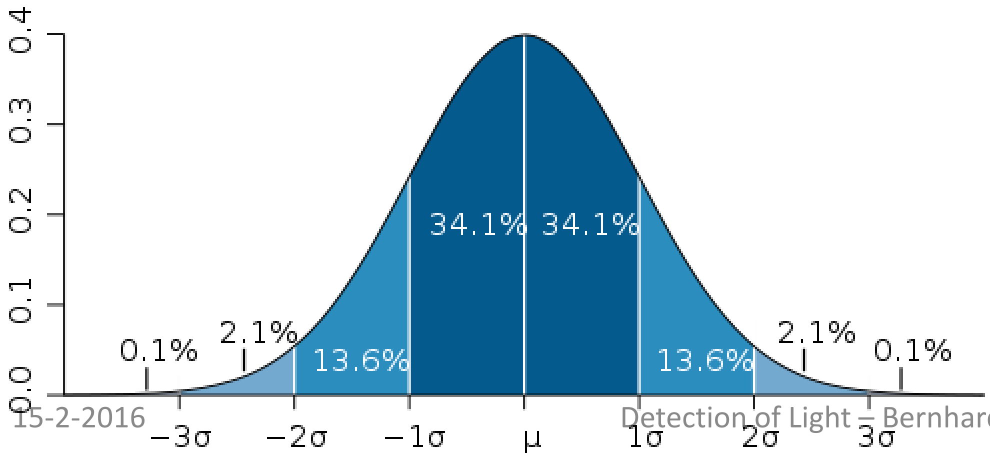
*It is often (incorrectly) called white noise, which refers to the (un-)correlation of the noise.*

$x$ is the actual value

$\mu$ is the mean of the distribution

$\sigma$ is the standard deviation of the distribution

*Example: detector "dark frame"*




1-σ ~ 68%

2-σ ~ 95%

3-σ ~ 99.7%

# II. Poissonian Distribution

Poisson noise is the noise following a Poissonian distribution:

$$P(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

*k* is the number of occurrences of an event (probability)
*λ* is the *expected* number of occurrences

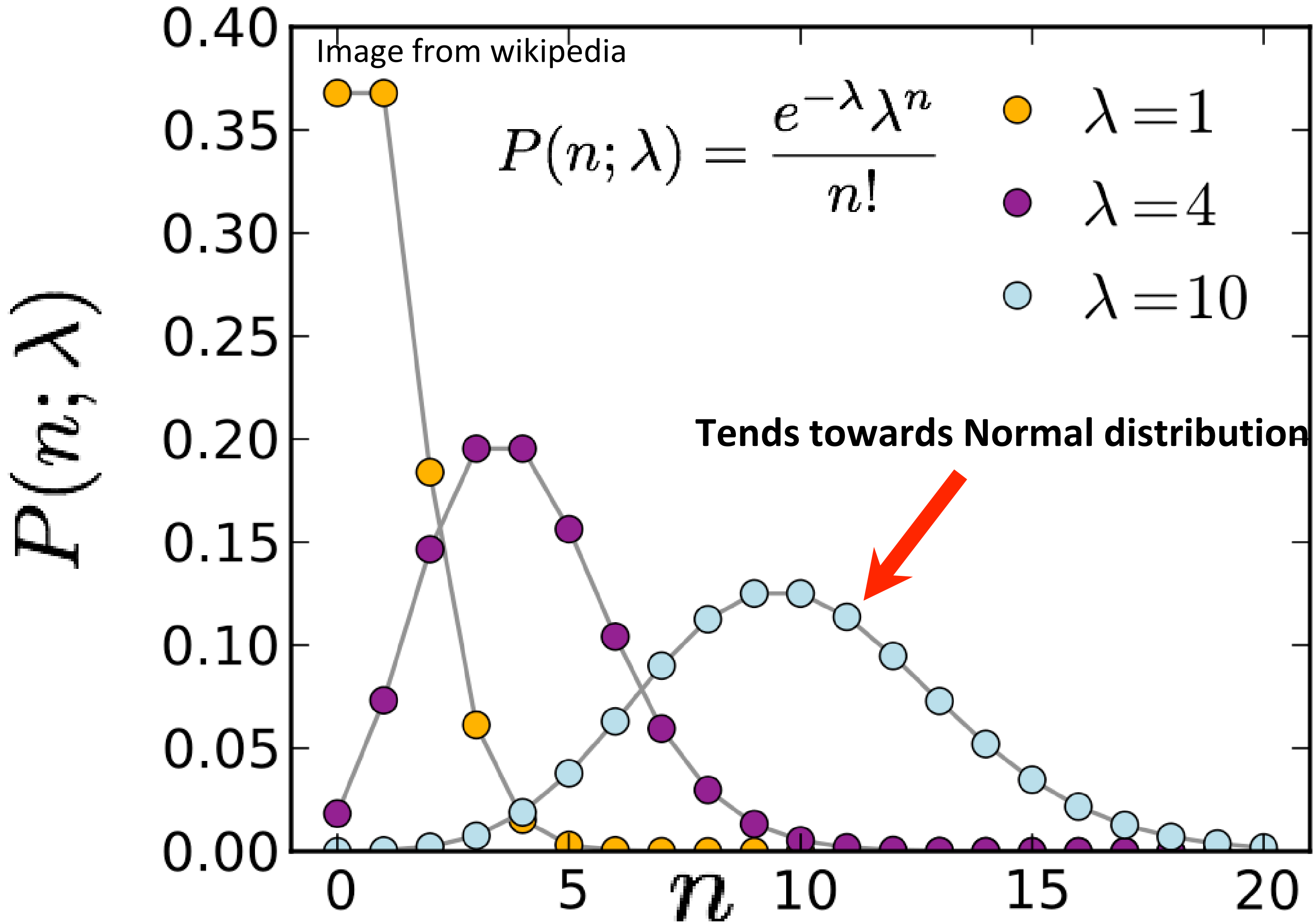
P(k,λ) expresses the probability of a number of events occurring in a fixed period of time, provided that:

- these events occur with a known average rate λ, and
- the arrival of one event is independent of the time since the last event

Properties:

- the mean (average) of *P(k,λ)* is *λ.*
- the standard deviation of *P(k,λ)* is *√λ.*

# II. Poissonian Distribution (2)

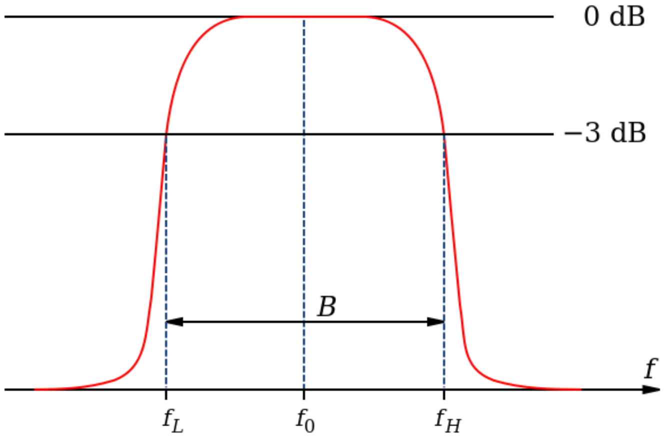



Example: fluctuations in the detected photon flux between time intervals $\Delta t_i$.

Detected are k photons, while expected are, on average, λ photons.

# Noise Bandwidth

White noise has a wide frequency range, which we associate with an equivalent noise bandwidth $B$ or $\Delta f = f_H - f_L$.




For a system – like our detector – with exponential response $U \sim e^{-t/\tau}$

we get $\Delta f = \dfrac{1}{4\tau}$

*According to the Shannon Nyquist theorem, an output bandwidth of one hertz is equivalent to half a second of integration.*

→ signal integrated over time $\Delta t_{int}$: $\boxed{\Delta f = \dfrac{1}{2\Delta t_{int}}}$

# The Main Sources of Detector Noise

# The G-R Noise Current

Photoconductor absorbs N photons: $N = \eta\phi\Delta t$

→ create N conduction electrons and N holes (but consider only $e^-$ since $\mu_{e-} \gg \mu_p$)

Randomly **g**enerated $e^-$ ***and*** randomly **r**ecombined $e^-$ → two random processes

Hence: RMS noise ~ $(2N)^{1/2}$.

Now calculate the associated noise current: $\left\langle I_{G-R}^2 \right\rangle^{1/2} = \dfrac{q\sqrt{2N}G}{\Delta t}$

With the mean photo-current $I_{ph} = \eta q \varphi G$

one gets:

$$\left\langle I_{G-R}^2 \right\rangle = \frac{q^2(2N)G^2}{(\Delta t)^2} = \left(\frac{2q}{\Delta t}\right)\left(\frac{qNG}{\Delta t}\right)G = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph} \right\rangle G$$

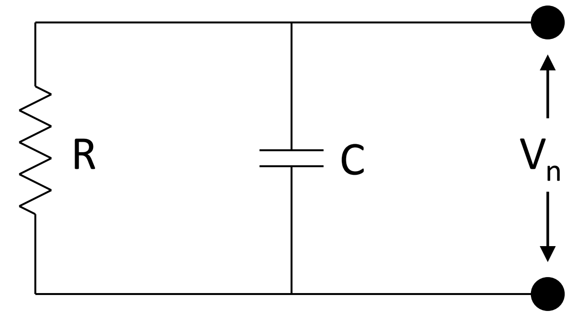
The noise current $\langle I^2_{G\text{-}R} \rangle$ can now be rewritten as:

$$\left\langle I_{G-R}^2 \right\rangle = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph} \right\rangle G = (2q2\Delta f)\left\langle \varphi q \eta G \right\rangle G = 4q^2\varphi\eta G^2\Delta f$$

# Johnson Noise

The Johnson or Nyquist noise is a fundamental thermodynamic noise due to the thermal motion of the charge carriers.

Consider a photoconductor as a RC circuit:




This system has *one* degree of freedom: $V_{n,}$ which is associated with an average energy of ½ kT

The energy $E_{st}$ stored in a capacitor is $E_{st}$ = ½$CV^2$, hence:

$$\frac{1}{2}C\left\langle V_n^2\right\rangle = \frac{1}{2}kT$$

The fluctuations in $E_{st}$ are associated with the Johnson noise current $I_J$.

# kTC Noise

The power in $I_J$ can also be expressed thermodynamically:

$$\langle P \rangle t = \frac{1}{2} kT \quad \text{where} \quad t = RC \quad \text{and} \quad \langle P \rangle = \frac{1}{2} \langle I^2 \rangle R$$

Hence, $$\langle I_J^2 \rangle = \frac{2\langle P \rangle}{R} = \frac{2 \frac{1}{2} kT}{Rt} \overset{\Delta f = 1/4\tau}{=} \frac{4kT}{R} \Delta f$$

The charge on the capacitor is Q = CV.

Above we derived C<$V^2$> = kT → hence: <$Q^2$> = $C^2V^2$ = kTC

This charge noise is also called kTC noise or reset noise.

Johnson noise and kTC noise are equivalent, and due to the Brownian motion of the charge carriers.

# 1/f Noise

Most electronic devices have increased noise at low frequencies, often dominating the system performance.

*However, there is no general physical understanding of it.*

Empirically, $\left\langle I_{1/f}^{2} \right\rangle \propto \frac{I^{a}}{f^{b}} \Delta f \quad \text{where} \quad a \approx 2, b \approx 1.$

Bad electrical contacts, temperature fluctuations, surface effects (damage), crystal defects, and junction field effect transistors (JFETs) may contribute to this noise.

Due to the lack of solid physical understanding of this type of noise it is simply termed 1/f noise.

# Noise Sources in Comparison

# Noise Sources – Overview

So far we discussed:

the G-R or noise current $$\left\langle I_{G-R}^{2} \right\rangle = 4q^{2}\varphi\eta G^{2}\Delta f$$

the Johnson noise $$\left\langle I_{J}^{2} \right\rangle = \frac{4kT}{R}\Delta f$$

the 1/f noise $$\left\langle I_{1/f}^{2} \right\rangle \propto \frac{I^{a}}{f^{b}}\Delta f$$

# Background-limited Performance

Note that *all* processes depend on the bandwidth $\Delta f = 1/(2\Delta t_{int})$

If the signal is Poisson distributed in time the relative error of the measurement is proportional to $1/\sqrt{t}$ or $(\Delta f)^{1/2}$ (longer $t_{int}$ means smaller bandwidth means smaller relative errors)

The total noise in the system is $\langle I_N^2 \rangle = \langle I_{G-R}^2 \rangle + \langle I_J^2 \rangle + \langle I_{1/f}^2 \rangle$

Operationally, background-limited performance **(BLIP)** is always preferred: $\langle I_{G-R}^2 \rangle >> \langle I_J^2 \rangle + \langle I_{1/f}^2 \rangle$

*In other words: the system sensitivity is not limited by technical issues but by natural statistical processes.*

# Noise Equivalent Flux Density (NEFD)

The noise equivalent flux density (NEFD) is the flux density that yields an RMS S/N of unity in a system of $\Delta f = 1$ Hz.

$$NEFD = \frac{E_S \sqrt{2\Delta t_{\text{int}}}}{S / N}$$

...where $E_S$ [W m$^{-2}$ Hz$^{-1}$] is the measured flux density.

*The NEFD usually refers to the entire system performance, including the camera optics.*

# Noise Equivalent Power (NEP)

The noise equivalent power (NEP) is the signal power that yields an RMS S/N of unity in a system of $\Delta f = 1$ Hz.

$$df = \frac{1}{2\Delta t_{int}}$$

$$\frac{S}{N} = \frac{P_s}{\mathrm{NEP}(df)^{1/2}} = \frac{P_s(2\Delta t_{int})^{1/2}}{\mathrm{NEP}} \rightarrow \boxed{NEP = \frac{P_S\sqrt{2\Delta t_{\mathrm{int}}}}{S/N}}$$

An equivalent, more practical definition is:

$$\boxed{NEP = \frac{I_N}{S}}$$

...where $I_N$ [W Hz$^{-1/2}$] is the total noise current in the system, and $S$ [A W$^{-1}$] is the responsivity.

# NEP for BLIP ⇔ kTC

(1) BLIP: $\left\langle I_{G-R}^2 \right\rangle >> \left\langle I_J^2 \right\rangle + \left\langle I_{1/f}^2 \right\rangle$

With $S = G\dfrac{q\eta\lambda}{hc}$ and $\langle I_{G-R}^2 \rangle = 4q^2\varphi\eta G^2 \Delta f$ one gets:

$$\mathrm{NEP}_{G-R} = \frac{I_{G-R}}{S} = \frac{(4q^2\varphi\eta G^2)^{1/2} hc}{Gq\eta\lambda} \boxed{= \frac{2hc}{\lambda}\left(\frac{\varphi}{\eta}\right)^{1/2}}$$

(The factor of $\Delta f$ disappears from $<I^2_{G-R}>$ as we use a "normalized" noise current in units of [A Hz$^{-1}$].)

(2) kTC: $\left\langle I_J^2 \right\rangle >> \left\langle I_{G-R}^2 \right\rangle + \left\langle I_{1/f}^2 \right\rangle$

With $S = G\dfrac{q\eta\lambda}{hc}$ and $\langle I_J^2 \rangle = \dfrac{4kT\Delta f}{R}$ one gets:

$$\mathrm{NEP}_J = \frac{I_J}{S} = \frac{(4kT)^{1/2} hc}{R^{1/2} Gq\eta\lambda} \boxed{= \frac{2hc}{Gq\eta\lambda}\left(\frac{kT}{R}\right)^{1/2}}$$

Here, the NEP can be improved by increasing η, G, R or reducing T.