

Observational Cosmology
Problem Set #3
Instructor: Dr. Bouwens

Here is a problem set for lectures 5-9 to verify your comprehension of the material. The problem set will be due on March 25, 2025.

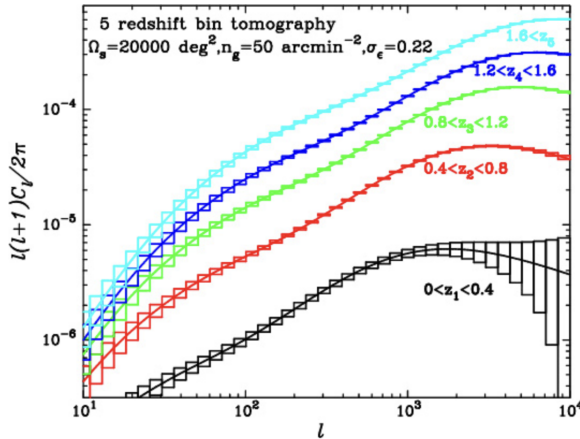
1. In the linear regime ($\delta \ll 1$), by what factor does an overdensity grow from $z \sim 5$ to $z \sim 0$ in a $\Omega_m = 0.2$, $\Omega_\Lambda = 0$ universe? What about in a $\Omega_m = 1$, $\Omega_\Lambda = 0$ universe? Note that to solve this problem, you will want to use the formula for the growth factor D_+ .
2. The power spectrum of galaxies in the local universe and the cosmic microwave background both provide us with probes of the same underlying set of primordial density fluctuations. Determine the k mode in the matter power spectrum that corresponds to the equivalent multipole l in the CMB power spectrum. Determine the k mode (and hence comoving length scale in the present day universe) that corresponds to an l of 100, 200, 400, and 1000. Draw approximate diagram with matter power spectrum and the CMB, one over top the other. Assume $\Omega_m = 0.26$, $H_0 = 69 \text{ km/s/Mpc}$, and $\Omega_\Lambda = 0.74$, and feel free to use public cosmological calculators to compute distances as a function of redshift. Use slide 9 in lecture 8 for inspiration.
3. Assume we do a survey for galaxies at $z \sim 0.9$ and we find an excess in the power spectrum at a comoving distance of 150 Mpc. For two different cosmologies ($\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $H_0 = 70 \text{ km/s/Mpc}$, $w = -1$ and $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 74 \text{ km/s/Mpc}$, $w = -1.2$), how far apart in arcsec will this 150 Mpc peak be in the plane on the sky and how far apart will sources be for this 150 Mpc peak in redshift (Δz), e.g., along the line of sight? Also answer this exercise for $z = 0.6$ and $z = 1.8$.
4. Consider efforts to measure both Ω_m and σ_8 with galaxy clusters. Measures of the cluster mass function in the nearby universe allow for constraints on $\sigma_8 \Omega_m^{1/2}$ and the evolution of clusters allow for constraints on Ω_m and Ω_Λ . Consider a universe where the true values of $\sigma_8 = 0.81$ and $\Omega_m = 0.3$. Given the degeneracy in deriving σ_8 and Ω_m based on measures of the cluster mass function, what would Ω_m equal if $\sigma_8 = 0.95$ or $\sigma_8 = 0.65$? Calculate the ratio of the apparent volume at $z \sim 1.1$ for these values of Ω_m vs. the true volume

at $z \sim 1.1$, the ratio of the apparent growth factors at $z \sim 1.1$ vs. the true growth factor at $z \sim 1.1$, and the relative luminosity distances at $z \sim 1.1$ vs. the true luminosity distance at $z \sim 1.1$. All of these would impact the apparent evolution of cluster abundance at $z \sim 1.1$. Do this calculation for both Ω_m 's calculated for the first part of this problem.

5. Write a formula for the “sound horizon”, $d_H(z)$, which is the distance travelled by a sound wave starting at time $t = 0$ and finishing at a particular redshift z . Assume a sound speed $c_S = c/\sqrt{3}$ in the photon-baryon plasma prior to recombination. Evaluate the size in Mpc of the sound horizon at $z_R = 1080$. Using the angular diameter distance (feel free to use the on-line cosmological calculator), calculate the angular size of this peak on the sky and the equivalent l mode number. Consider both the case $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.1$, $\Omega_\Lambda = 0.9$. To what size does this region expand at the present epoch ($z = 0$)? To simplify the problem, you can assume that the universe is matter-dominated throughout.

6. The cosmic microwave background has a lot of information about the process by which the universe became reionized (likely from the first stars + galaxies). Using the same CAMB calculator as in the previous problem set, show how the TT, TE, and EE power spectra change as a function of the Thomson optical depth (assume $\tau = 0.17, 0.1, 0.08, 0.05$ as was inferred in cosmic microwave background measurements from 2003+) and also if you ignore reionization altogether (set include “Include Reionization” to “No”). Describe the impact reionization has on the TT, TE, and EE power spectra both at lower l values (e.g., <20) and higher l values (e.g., >20). Feel free to look at <https://arxiv.org/pdf/1511.01117.pdf> to help you in answering this question. In particular, you can look at Figure 1 and 2 from that paper. You may want to set l_{max} equal to 70 or 100 to examine the power on large scales. Using results from the previous problem, show that an l value on order of 20 is of the same order of magnitude as the horizon at $z \sim 15$ when cosmic reionization is starting. As the horizon scale provides the limiting scale to which information can have traveled since inflation, how many independent views of the $z \sim 15$ universe do we have over the sky (if we take the size of independent regions to the horizon scale at $z \sim 15$)? How does this compare with the number of independent views we have of the universe at the time of decoupling ($z \sim 1080$)?

7. Examine the following figure on weak lensing showing $l(l+1)C_l/2\pi$ vs. l credited as being part of a Refregier et al. study. At which angular scale in arcminutes does this figure suggest the largest weak lensing signal occurs (you should be able to read it off the figure)?



Does this agree with the value given in the supplemental reading for March 5 (page 107 in the Bartelmann notes)?

See <https://home.strw.leidenuniv.nl/~bouwens/obscosmo/schedule.html>

(a) At which angular scale in arcminutes does the above figure suggest the largest weak lensing signal occurs? Why do you suspect the peaks occur at smaller angular scales at higher redshifts? Why do you think the amplitudes of the power spectra (at higher redshifts) are larger? Feel free to use the internet to help you figure out the answer.

(b) There is a modest degeneracy between the impact of σ_8 and Ω_m on the measured shear. It is worthwhile noting however that quantifying the cosmic shear of sources as a function of redshift (e.g. comparing $z \sim 0.5$ and $z \sim 1.0$) can allow us to break this degeneracy by allowing for a measurement of Ω_m . Explain in words how this would work and make a reference to the impact of the growth factor on the result.