Observational Cosmology
Problem Set 2
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Here is the exercise set for lectures 3 through 6 to verify your comprehension of the material. The problems will be due on March 5, 2025.

1. Surface brightness fluctuations. Assume that the surface brightness of galaxies with older stellar populations ("galaxy full of old stars") is dominated by light from red giant branch stars and assume that the surface density of these stars has a fixed value F (units of kpc^{-2}), i.e., it does not vary as a function of position within a galaxy. Furthermore, assume the size of pixels on a detector is 0.4 arcsec and no smearing of light from one pixel into adjacent ones. For a Euclidean geometry (angular size = physical size / distance), derive an expression for the distance of an object based on fluctuations in the surface brightness. (Hint assume that the number of stars in a given pixel has a Poissonian distribution.)

2. This problem is to derive the distance to a Cepheid variable star via the Baade-Wesselink method. The period of the star is 1.0 days. Let's assume that the star is spending half of its time in the expansion phase and the other half of its time in the contraction phase. During expansion, the line-of-sight velocity for the photosphere of the star has an average value of 16.8 km/s (measured from the Doppler-shifted light). Let's furthermore assume that the Cepheid is 1.8× brighter when it reaches its maximum brightness than when it is at its minimum brightness. The temperature we measure for the Cepheid (from its spectrum) at its maximum brightness is 6000 Kelvin, while the temperature we measure at its minimum brightness is 5500 Kelvin. The total flux we measure coming from the Cepheid when it reaches its maximum brightness is 1.9×10^{-9} ergs s^{-1} cm^{-2} (integrated over the entire SED). What is its distance? Assume the Cepheid has a perfect blackbody spectrum. Notice that this distance estimate is derived from first principles (no calibration needed).

3. Consider a universe with $\Omega_{m,0} = 4$, $\Omega_{\Lambda,0} = 0$, and $\Omega_{r,0} \sim 0$.
(a) Consider the propagation of a pulse of light from four sources with coordinate $(r, \theta, \phi) = (0, 0, 0)$ at $z = 0.3$, $z = 1$, $z = 10$, and $z = 1000$ (4 cases) to $z = 0$. By $z = 0$, the light will have propagated to different coordinates

$r_{0.3}$, r_1, r_{10}, r_{1000}. What are those coordinates?

(b) If we represent the position of the light cone as a circle on the surface of a sphere (with coordinates θ and r), illustrate on the surface of a sphere (drawn or plotted) the coordinates where light from those sources would propagate by $z = 0$.

(c) How would you answer to part (a) change if you considered a universe with $\Omega_{m,0} = 1.5$ or 20? Are there values of $\Omega_{m,0}$ where light emitted by a source at $z = 3000$ would be reobserved by the same source at $z = 0$? Are there values of $\Omega_{m,0}$ where light emitted by a source at $z = 3000$ would reach the extreme opposite end of the universe by $z = 0$? If yes, what are they?

4. In the supplementary reading on SNe (February 12), there is an estimate of how many $z = 0.8$ SNe are necessary to determine the value of Ω_M to within an uncertainty of 0.04 (equation 12.8) – assuming a flat $\Omega_M + \Omega_\Lambda = 1$ universe and 10% distance errors for the SNe. The answer given in the supplemental reading is 100 $z = 0.8$ SNe. Repeat this exercise assuming all the SNe you find are at $z = 2$. What does this say about the relative value of SNe's discovered at $z = 0.8$ vs. $z = 2$? You are welcome to use Ned Wright's cosmology calculator to estimate $d(\ln D_L)/\ d\Omega_M$:
http://www.astro.ucla.edu/~wright/CosmoCalc.html

5. (a) Examine Figure 2.4 in your book (or the equivalent figure from the lecture on Big Bang Nucleosynthesis). If the relative primordial abundances of H^2 to H^1 were 2× lower than is found in the current universe (blue box in the figure), estimate $\Omega_{b,0}$ (read the number off of the figure). Assume h = 0.7 (i.e., $H_0 = 70$ km s^{-1} Mpc^{-3}). How would the primordial abundances of He^3, He^4, and Li^7 change? (very approximately)

(b) Despite the optimistic view given in class, reconciling the predicted and observed primordial abundance for He^3 and Li^7 remain a huge challenge. Briefly summarize some of the suggested issues in reconciling the observed and predicted He^3 abundances by consulting with Olive+1994: "https://arxiv.org/abs/astro-ph/9410058".

6. Please derive the formula giving the redshift of matter-radiation equality z_{eq}. Express it in terms of the temperature of the microwave background T_B

and Ω_M.

7. Use the online CAMB web simulator "http://lambda.gsfc.nasa.gov/toolbox/camb_online.html" to show what effects changes in various cosmological parameters have on the TT power spectrum. By copying and pasting the results into an electronic document and printing out the results, show the effects of changing Ω_M (0.05,0.2,0.8), Ω_b (0.01,0.03,0.07), Ω_k (-0.2,0.0,0.2), primordial power-law slope n_s (0.93, 0.96, 1.0: "scalar spectrum index"), optical depth τ (0.05, 0.1, 0.15), and Hubble constant (65, 70, 75) on the TT power spectrum. While varying individual parameters, take $\Omega_M = 0.25$, $\Omega_b = 0.05$, $\Omega_k = 0$, primorial power-law slope n_s (0.96: "scalar spectrum index"), redshift of reionization $z_{reion} = 11$, and $H_0 = 70$ km s^{-1} Mpc^{-3} as the defaults for the other parameters. Please describe in words how the TT spectrum changes for each change in parameters. Note that $\Omega_c = \Omega_m$ (c = cold dark matter).