Large Scale Structure +

Baryonic Acoustic Oscillations
(as seen in galaxy large scale structure)

Searching for an Understanding of Dark Energy

Layout of the Course

Feb 7: Introduction / Overview / General Concepts
Feb 14: Age of Universe / Distance Ladder
Feb 21: Distance Ladder / Hubble Constant
Feb 28: Distant Measures / SNe science / Baryonic Content
Mar 7: Dark Matter Content of Universe / Cosmic Microwave Background
Mar 14: Cosmic Microwave Background / Large-Scale Structure
Mar 21: No class
Mar 28: Large Scale Structure / Baryon Acoustic Oscillations
Apr 4: Baryon Acoustic Oscillations / Dark Energy / Clusters
Apr 11: Clusters / Cosmic Shear
Apr 18: No class
Apr 25: Dark Energy Missions / Review for Final Exam

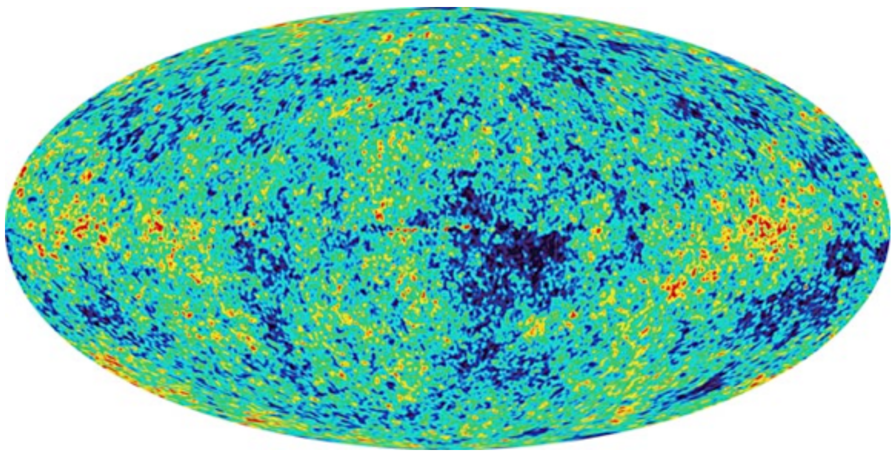
May 23: Final Exam

This Week

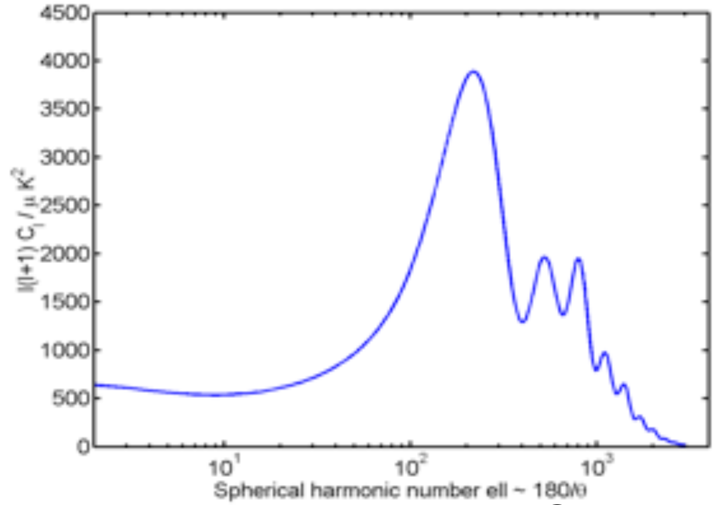
Review Material from Last Week

Power Spectra Derived from Fluctuations in CMB

-- Use the spherical harmonic expansion to construct a power spectrum to describe anisotropies of the CMB on the sky



Power Spectrum



l = 180 / θ

Expansion:

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^{\ell}(\theta, \phi)$$

After deriving the a_{lm} coefficients from the data, determine the statistical average

$$c_\ell = \langle |a_{\ell m}| \rangle^2$$

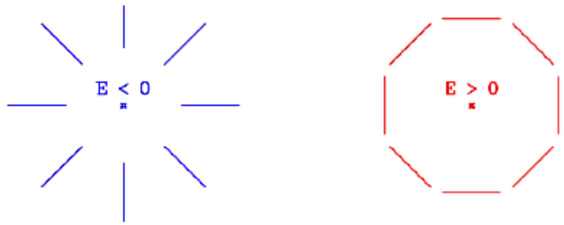
How CMB light can be broken down?

Measure Temperature and Polarization of Light

One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

90% of the photons in the CMB are unpolarized; this leaves 10% which is polarized.

E-modes

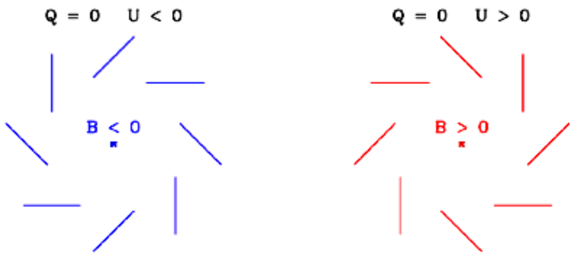


hot spots   cold spots

E-modes are curl free and can be written as the gradient of a potential

$$\nabla \times E = 0$$

B-modes



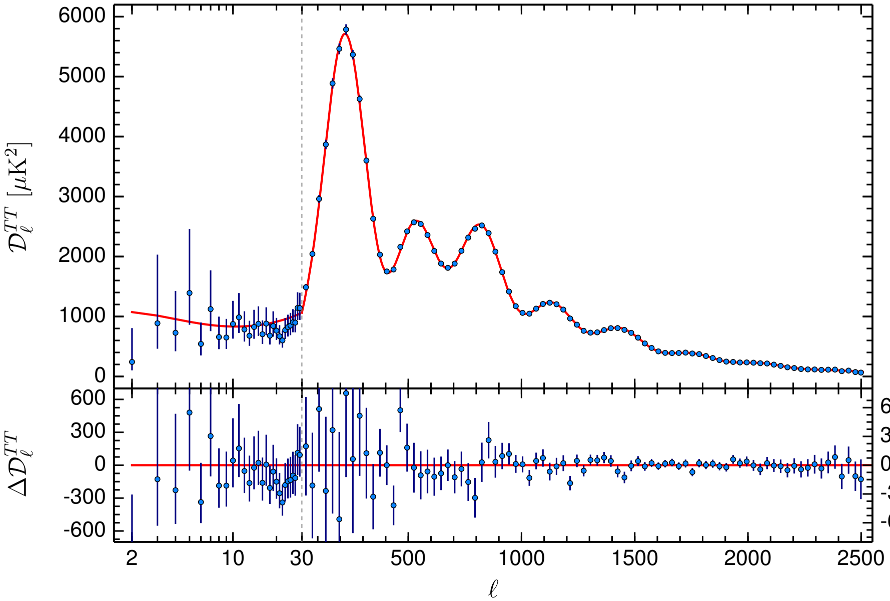
B-modes have no divergence.

$$\nabla \cdot B = 0$$

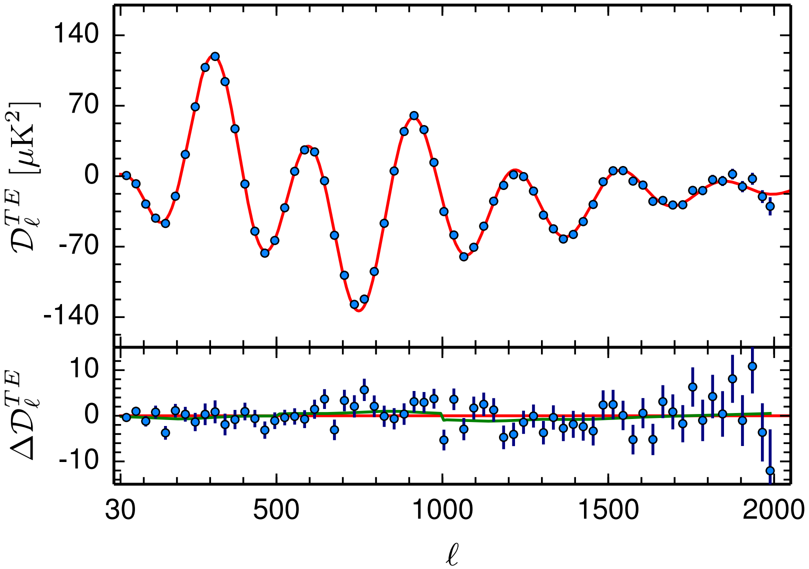
The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

Cosmic Microwave Background

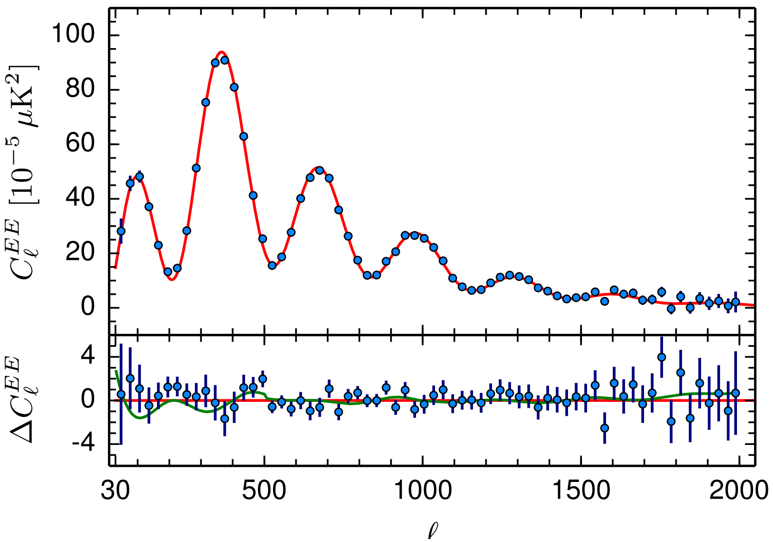
TT spectrum



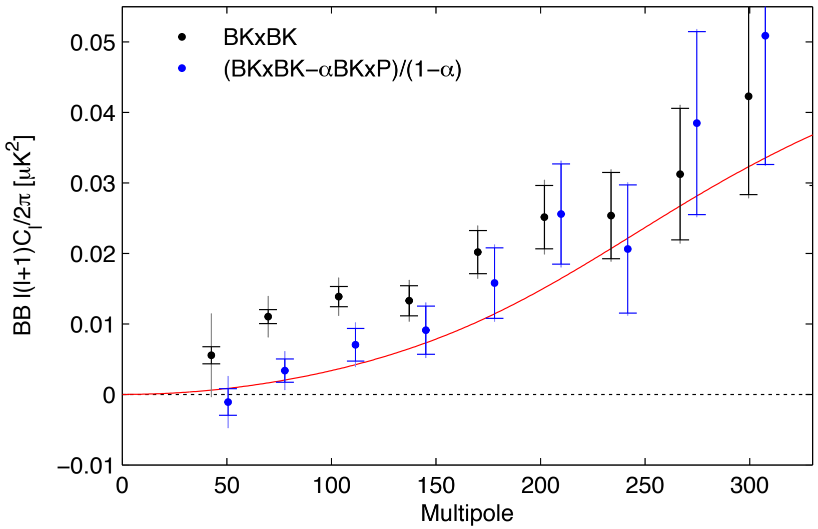
TE spectrum



EE spectrum

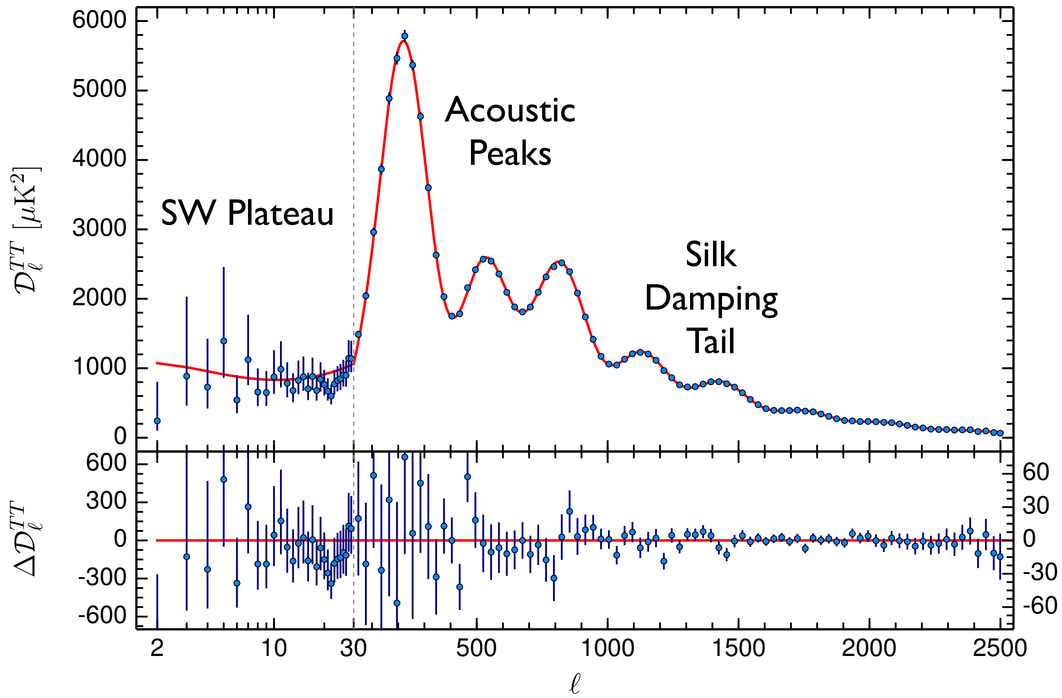


BB spectrum



Cosmic Microwave Background

TT spectrum



Sachs-Wolfe Plateau: Constrain normalization of primordial power spectrum

1st acoustic peak: Measure Angular Diameter Distance to Last Scattering Surface

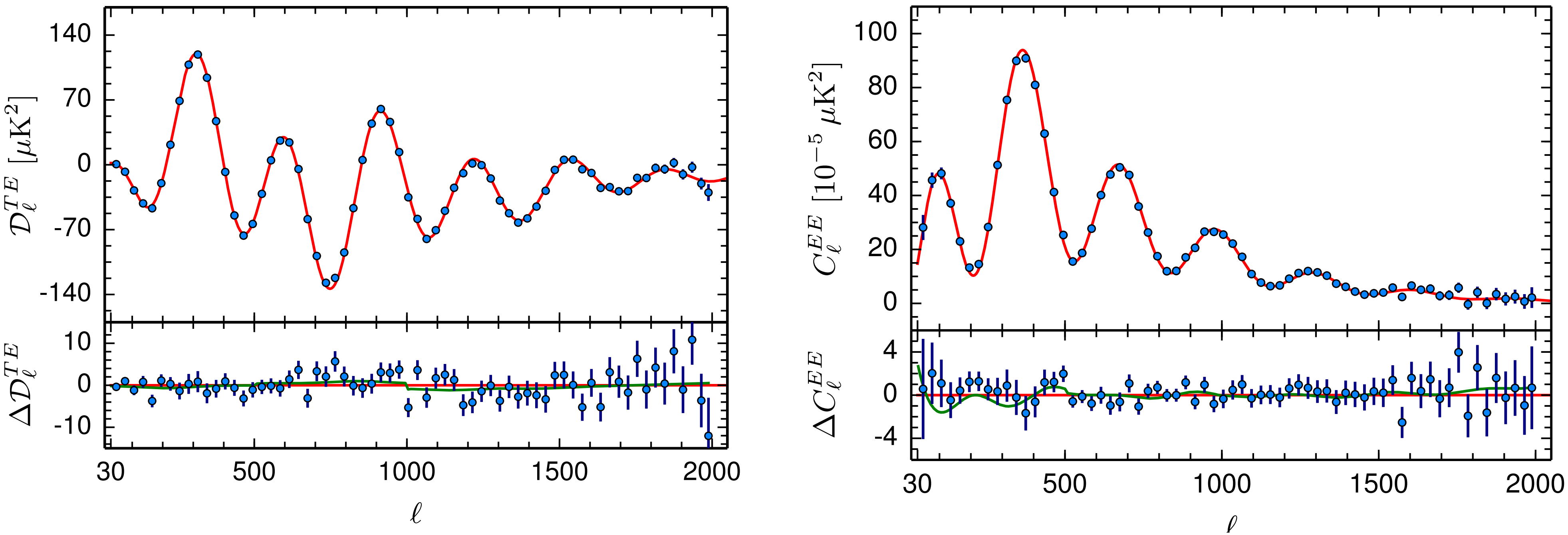
Ratio of Even and Odd Acoustic Peaks: Probe Baryon Content

Ratio of Amplitude of 3rd to 1st Acoustic Peak: Matter Content

High Frequency Modes: Silk Damping...

Cosmic Microwave Background

TE / EE spectra



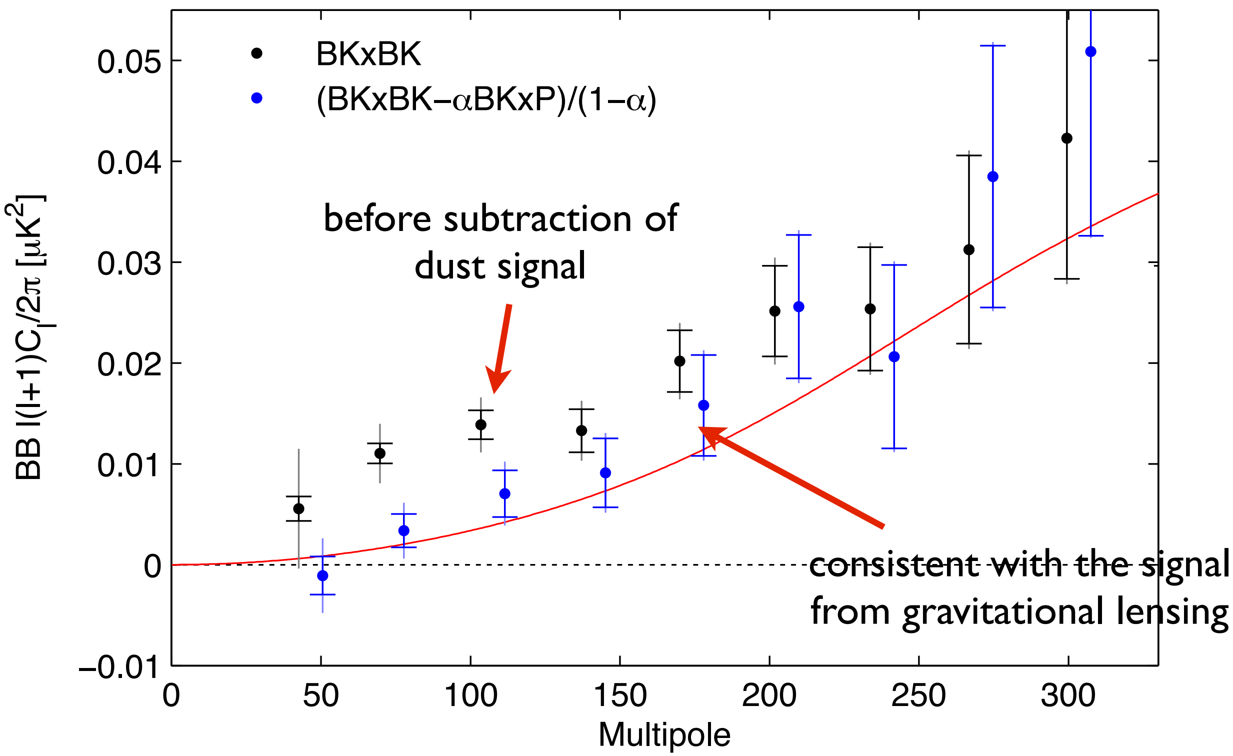
Contains Very Similar Information to that Present in TT Power Spectrum...

Allows us to verify that we understand the physics correctly...

Expect some difference from TT power spectrum -- depending on the ionization history of universe

Cosmic Microwave Background

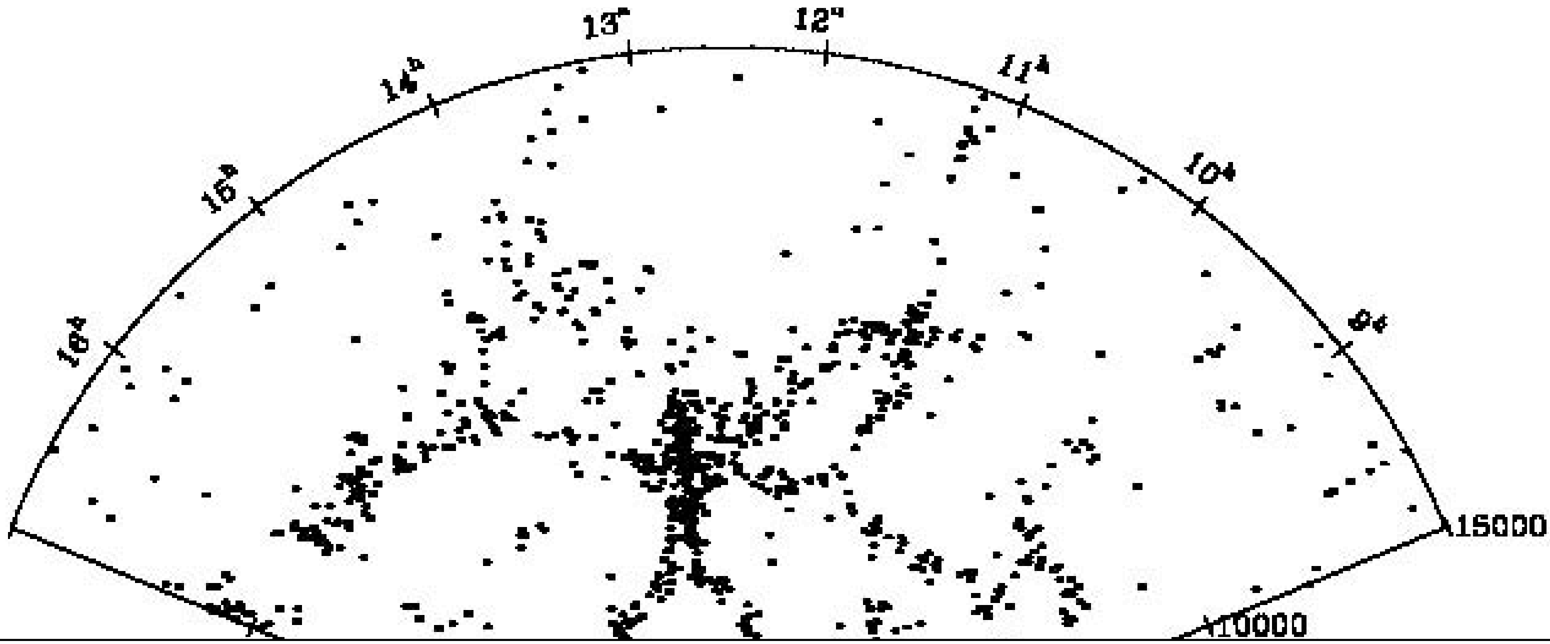
BB spectra



Signal arises from (1) gravity waves from inflation and (2) the impact of gravitational lensing on CMB...

Detection first reported in 2014 by BICEP II, but most of the signal likely from dust emission in our own galaxy

So what can we learn from the spatial distribution of galaxies on the sky?



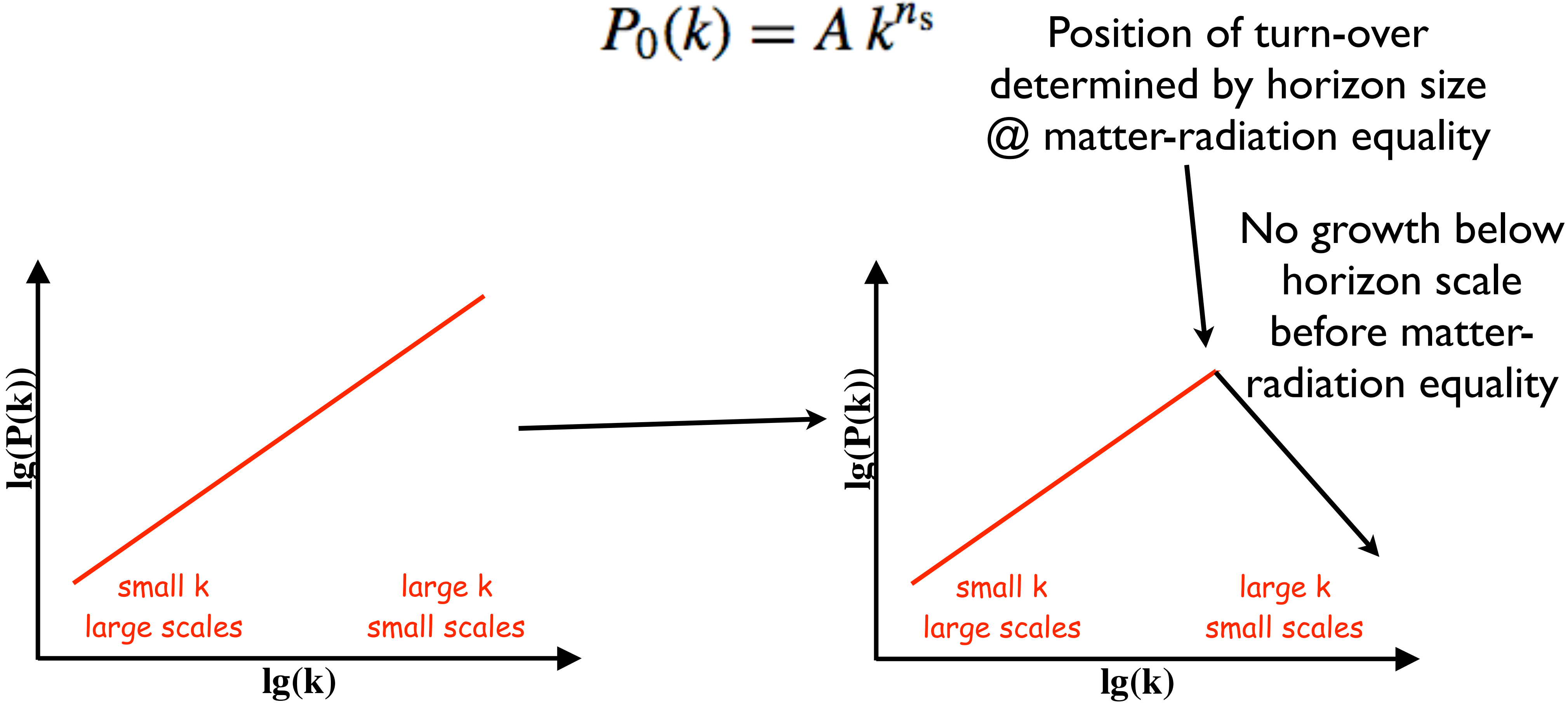
Spatial Distribution of Galaxies on some part of sky

→ We can derive the matter power spectrum

How does the matter power spectrum take on its shape?

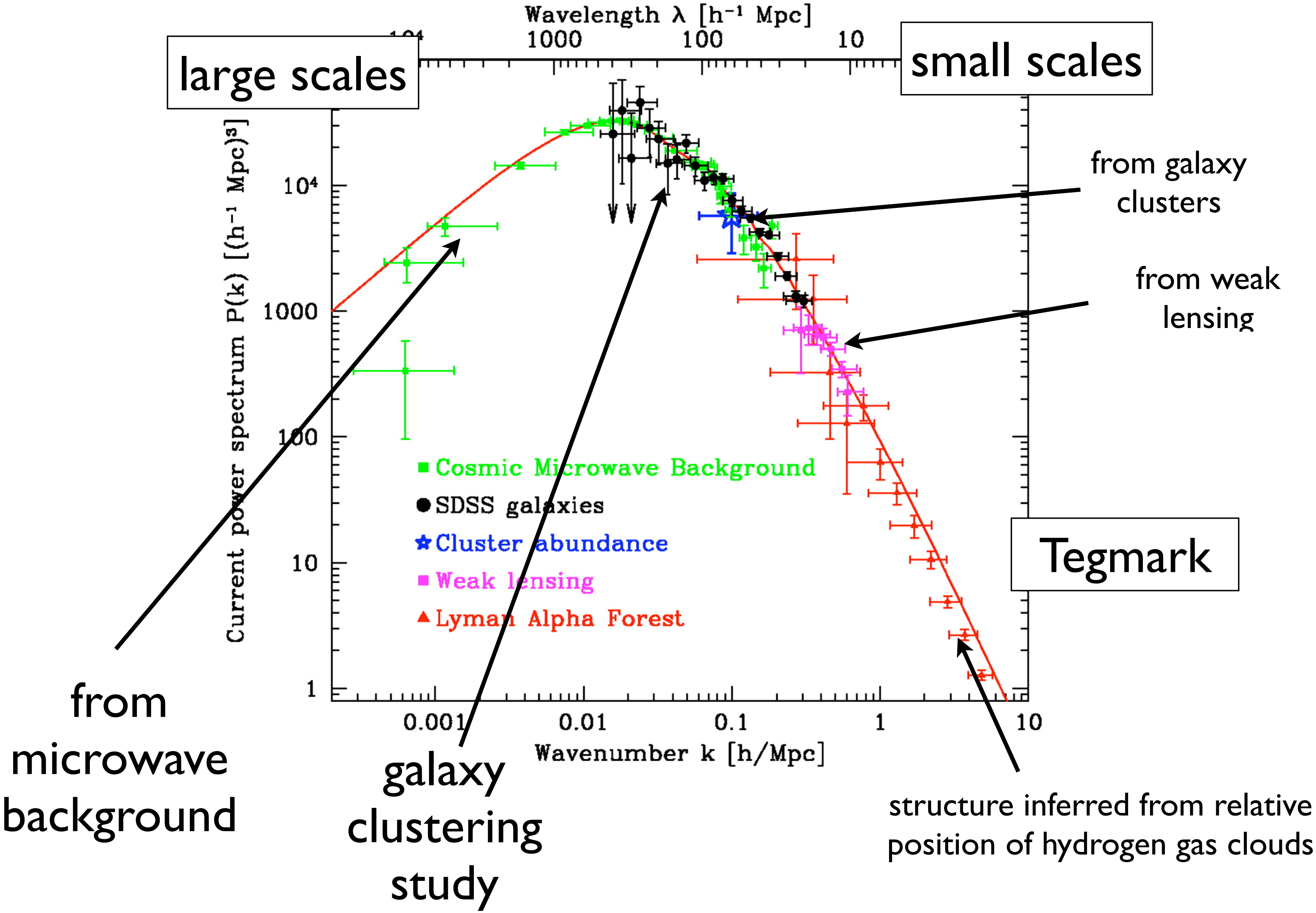
The initial power spectrum of fluctuations is the following:

$$P_0(k) = A\,k^{n_s}$$



The matter power spectrum is one of the most important quantities to derive in observational cosmology.

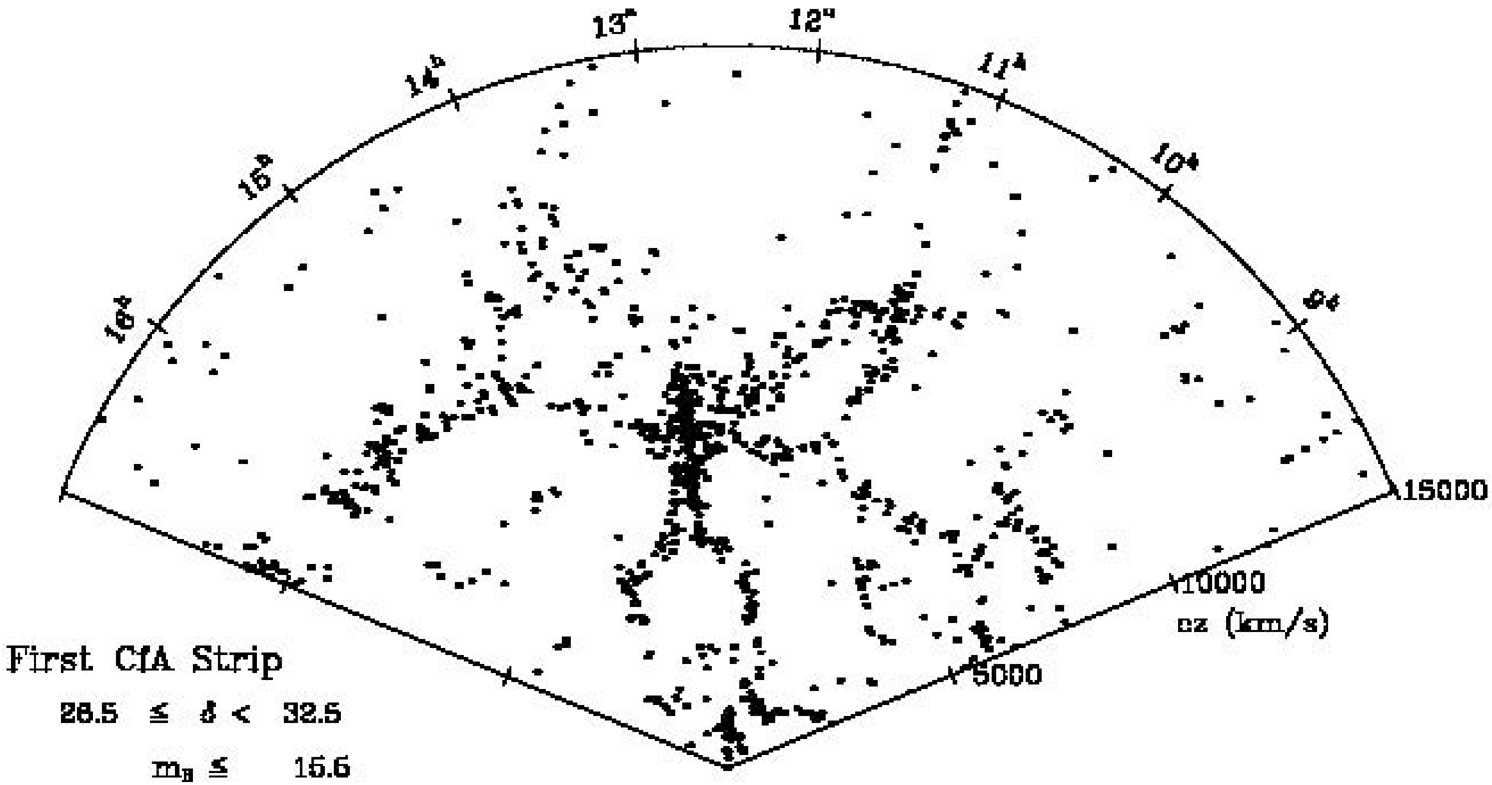
Different techniques/sources probe different regimes in matter power spectrum



New Material

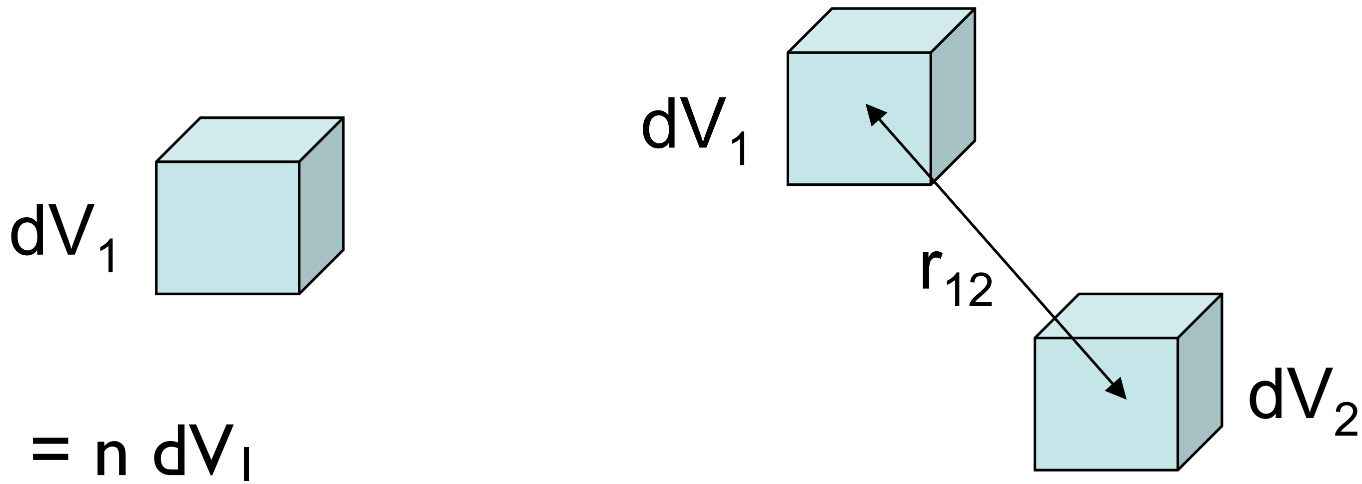
How do we derive the matter power spectrum?

We can derive the matter power spectrum from the clustering of galaxies on sky



We quantify clustering in terms of correlation functions

The Correlation function ξ tells us -- given the existence of a galaxy in some volume dV_1 -- how much more likely we are to find a nearby galaxy at some distance r_{12}



$$dP_1 = n\, dV_1$$

$$dP_{12} = n^2\,(1 + \xi(r_{12}))\, dV_1\, dV_2$$

n = average density of galaxies

Why do we care about correlation functions?

The power spectrum is the Fourier transform of the correlation function ξ

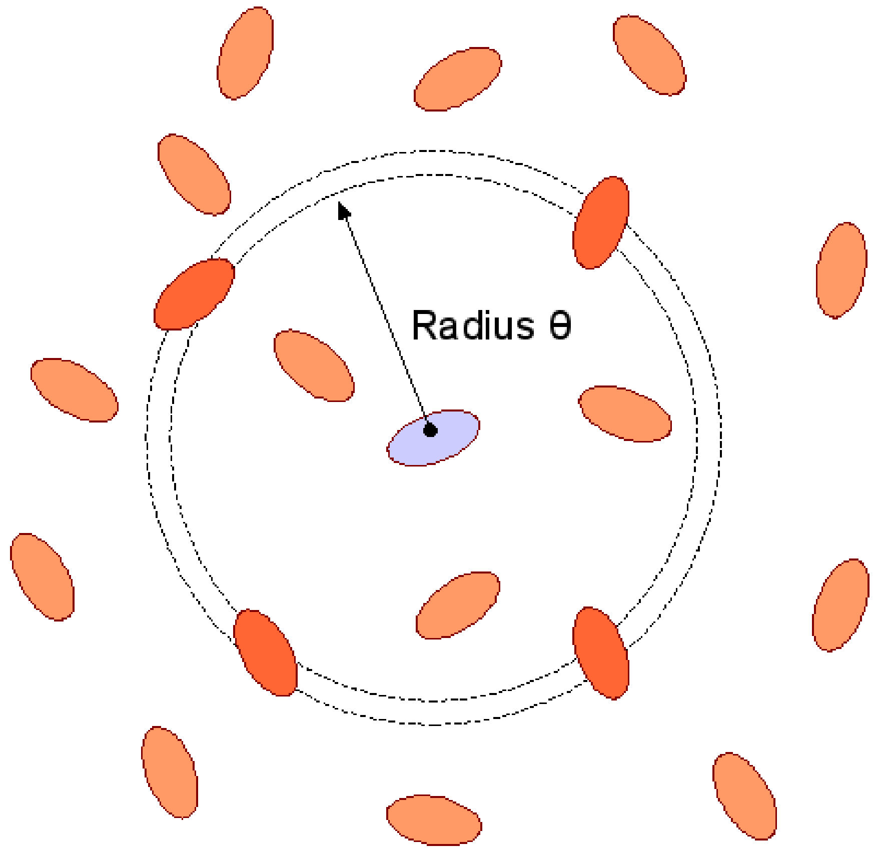
$$P(k) = \int \xi(r) e^{ik \bullet r} d^3 r \equiv \int \xi(r) \frac{\sin(kr)}{r} dr$$

We quantify clustering in terms of correlation functions

The Correlation function ξ is calculated by examining the distances between every pair of galaxies in a survey and comparing it to a random distribution

$\xi(r) > -1$ since probability always positive

$\xi(r) \to 0$ at $r \to \infty$



Correlations between points can be determined by counting pairs.

We quantify clustering in terms of correlation functions

One estimates the correlation function by comparing the number of sources seen at a given distance r and angle in the data D and comparing it to a purely random distribution R:

Standard Estimator : w(r) $= (2DD/DR) - 1$

Landy & Szalay- SL Estimator : w(r) $= (DD - 2DR + RR)/RR$

Smaller uncertainties on large scales

Hamilton Estimator : w(r) $= 4(DD\text{x}DR)/(DR^2 - 1)$

We quantify clustering in terms of correlation functions

The Correlation function ξ is typically parametrized as a power-law in radius:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Typical values for γ are 1.8. r_0 is known as the correlation length and it tells us the typical distance from a source we can expect a large enhancement in neighboring sources

We quantify clustering in terms of correlation functions

The Correlation function ξ is typically parametrized as a power-law in radius:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Red galaxies have a larger correlation length than blue galaxies. Typical r_0's for red galaxies at 5 h^{-1}Mpc and for blue galaxies 3 h^{-1}Mpc.

What are the typical properties of the correlation function?

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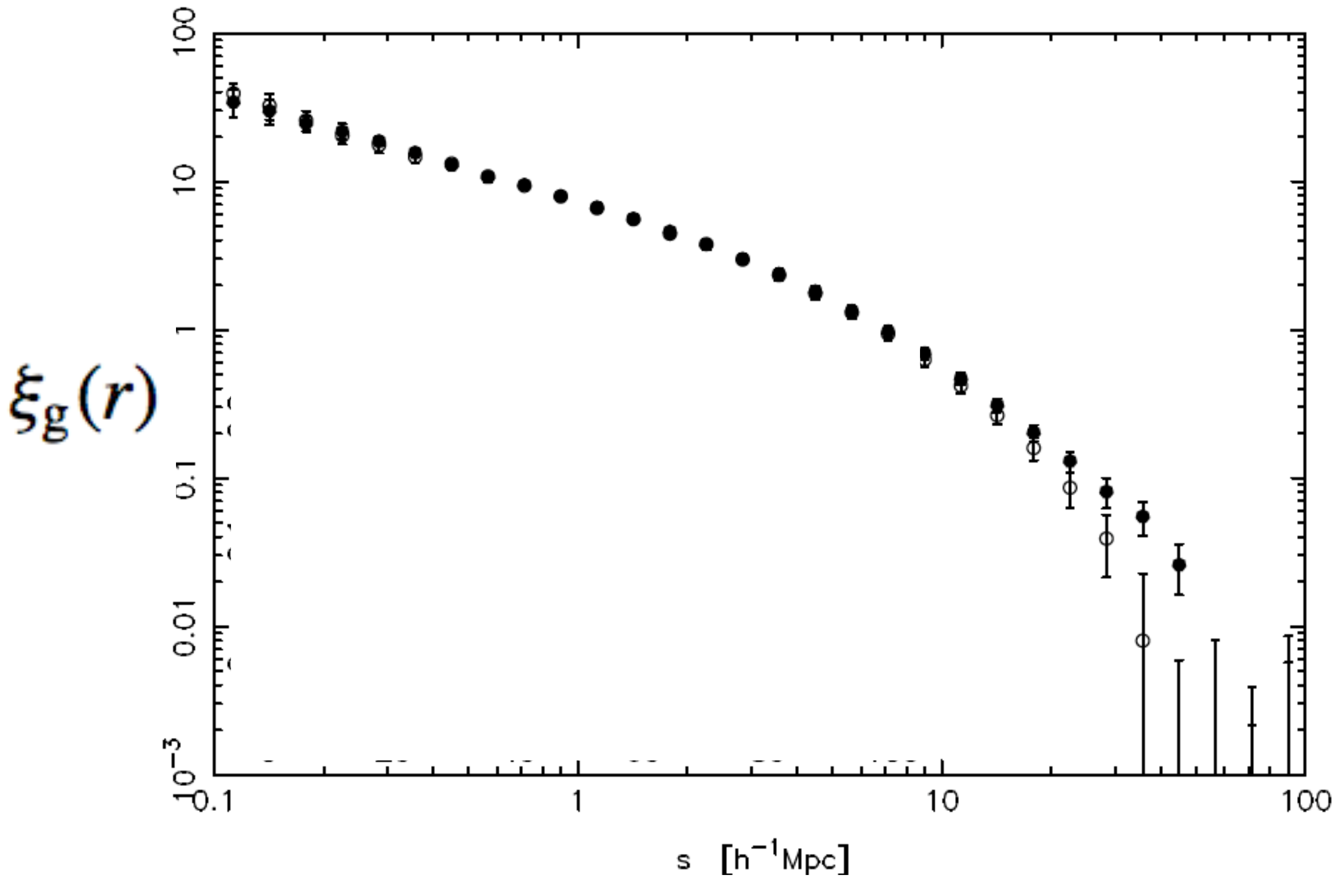
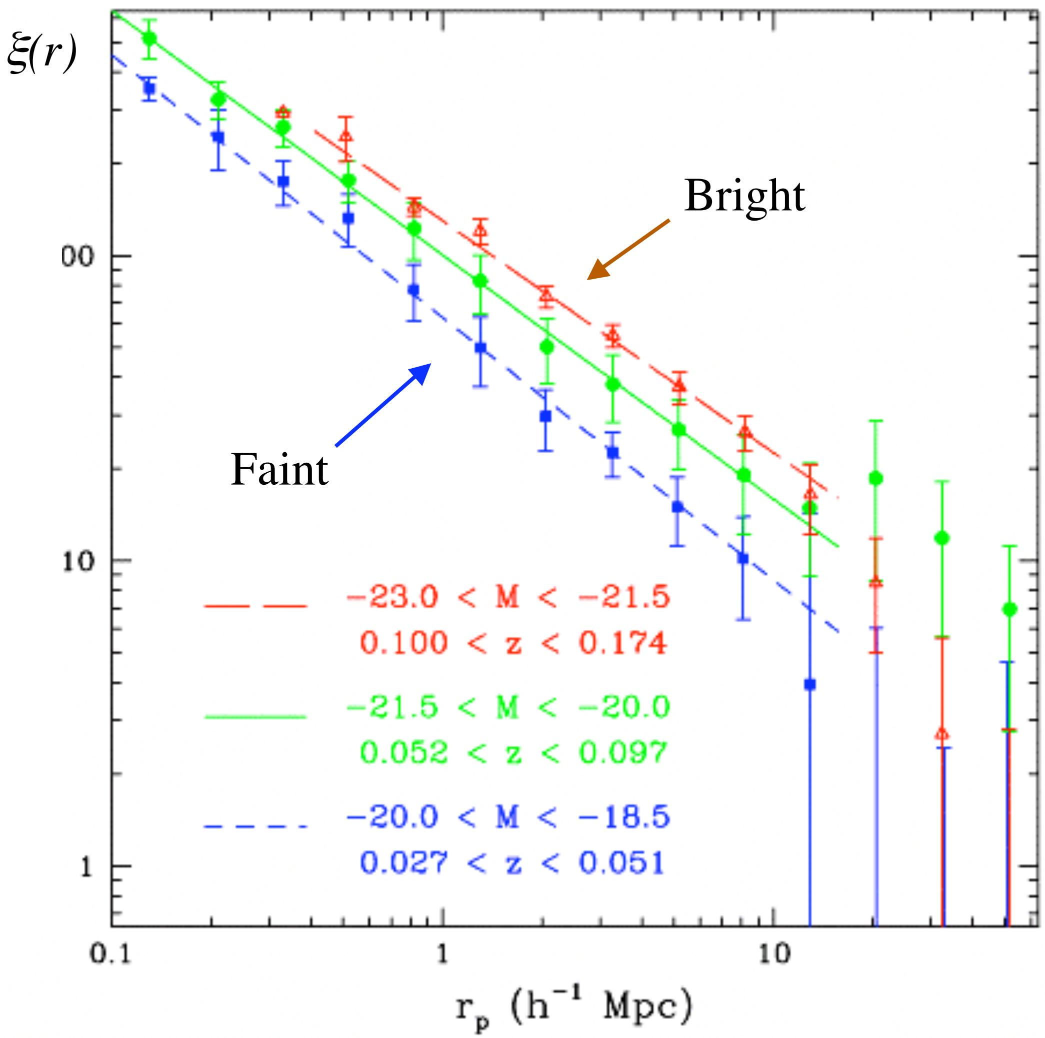


What are the typical properties of the correlation function?

Brighter, more massive galaxies have a larger correlation length than fainter, lower mass galaxies:



Red galaxies have a larger correlation length than blue galaxies. Typical r_0's for red galaxies at 5 h^{-1}Mpc and for blue galaxies 3 h^{-1}Mpc.

The different clustering properties of these galaxies tell us something about how they form

Angular Correlation Function

The Correlation function ξ that I've described thus far is the spatial correlation function.

There's also an angular correlation function $w(\theta)$ that one can measure -- when one knows the position of the sources on the sky and does not know their redshift

It makes sense that you could use the position of sources on the sky -- even without redshift information -- to measure clustering since if sources are close to each other in 3-dimensional space -- they will be close to each other in 2-dimensional space

Angular Correlation Function

It makes sense that you could use the position of sources on the sky -- even without redshift information -- to measure clustering since if sources are close to each other in 3-dimensional space -- they will be close to each

Of course, the observed clustering will be diluted depending on how large the redshift dimension is to one's samples

Angular Correlation Function

Therefore we can go from the angular correlation function to a spatial correlation function, but there's not *if* we know the distribution of galaxies in redshift.

There's a well-known equation called Limber's equation that relates the angular correlation function w(θ) to the spatial correlation function ξ(r)

$$w(\theta) = \int \mathrm{d}z\, p^2(z) \int \mathrm{d}(\Delta z)\ \times \xi_\mathrm{g} \left(\sqrt{[D_\mathrm{A}(z)\theta]^2 + \left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2 (\Delta z)^2} \right)$$

where $D_A(z)$ is the angular diameter distance and p(z) is the redshift distribution of the sources.

Other Techniques for Quantifying Clustering

Of course, there are other techniques as well for quantifying clustering... counts in cells, void probability functions

Counts In Cell -- Divide the Space into Discrete Grid Points "Cells" and Calculate the Variation in the # of Sources per Grid Point

Void Probability Function -- Probability of Finding Zero Galaxies in a Volume of Radius R

We quantify clustering in terms of correlation functions

But we need samples of galaxies to derive these correlation functions...

How do we compile these samples?

How do we compile these samples?

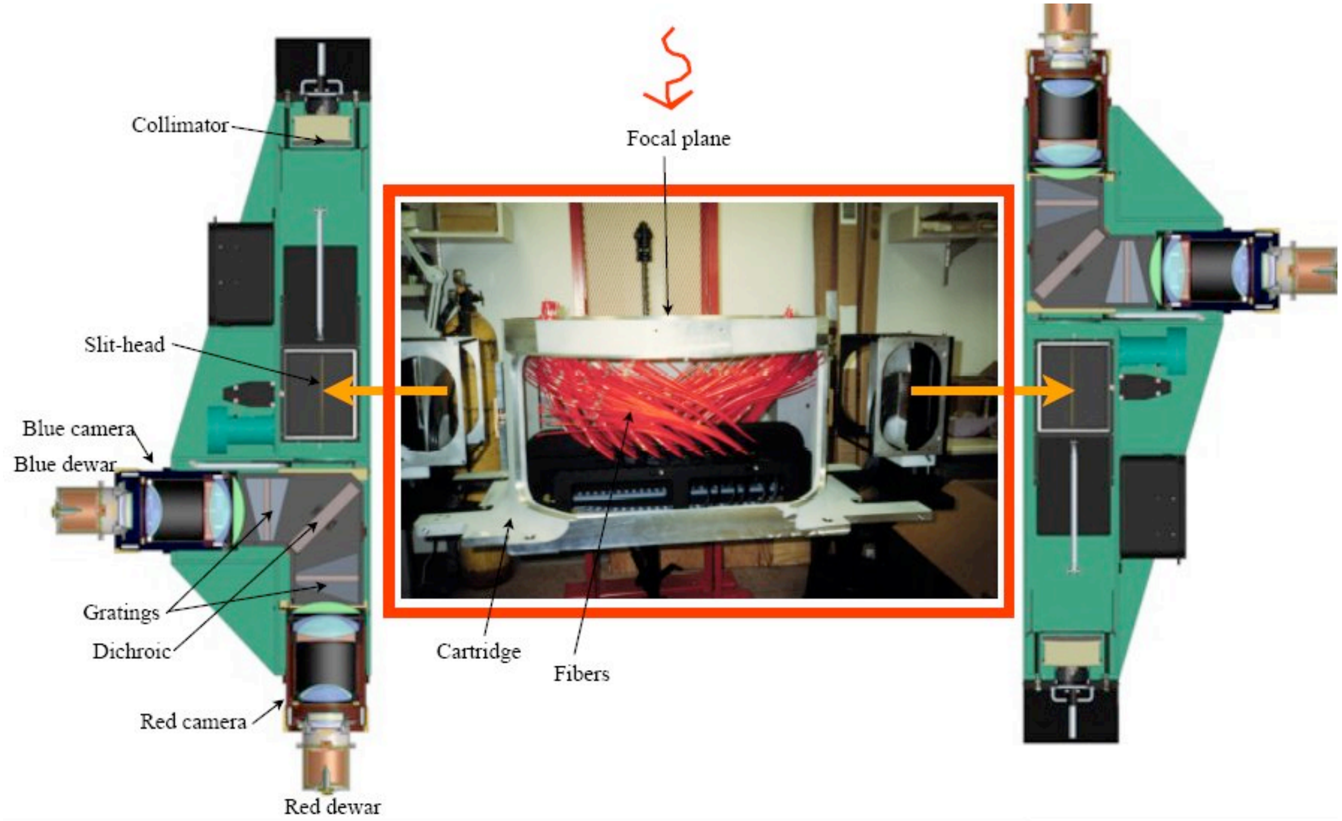
I. Obtaining multi-colour images of a large area of the sky

II. Create a catalogue and then select the sources over some range of brightness (and perhaps using some other criteria)

III. Measure redshifts for sources (to add third dimension)

How do we obtain redshifts for large numbers of sources?

(using a fiber)

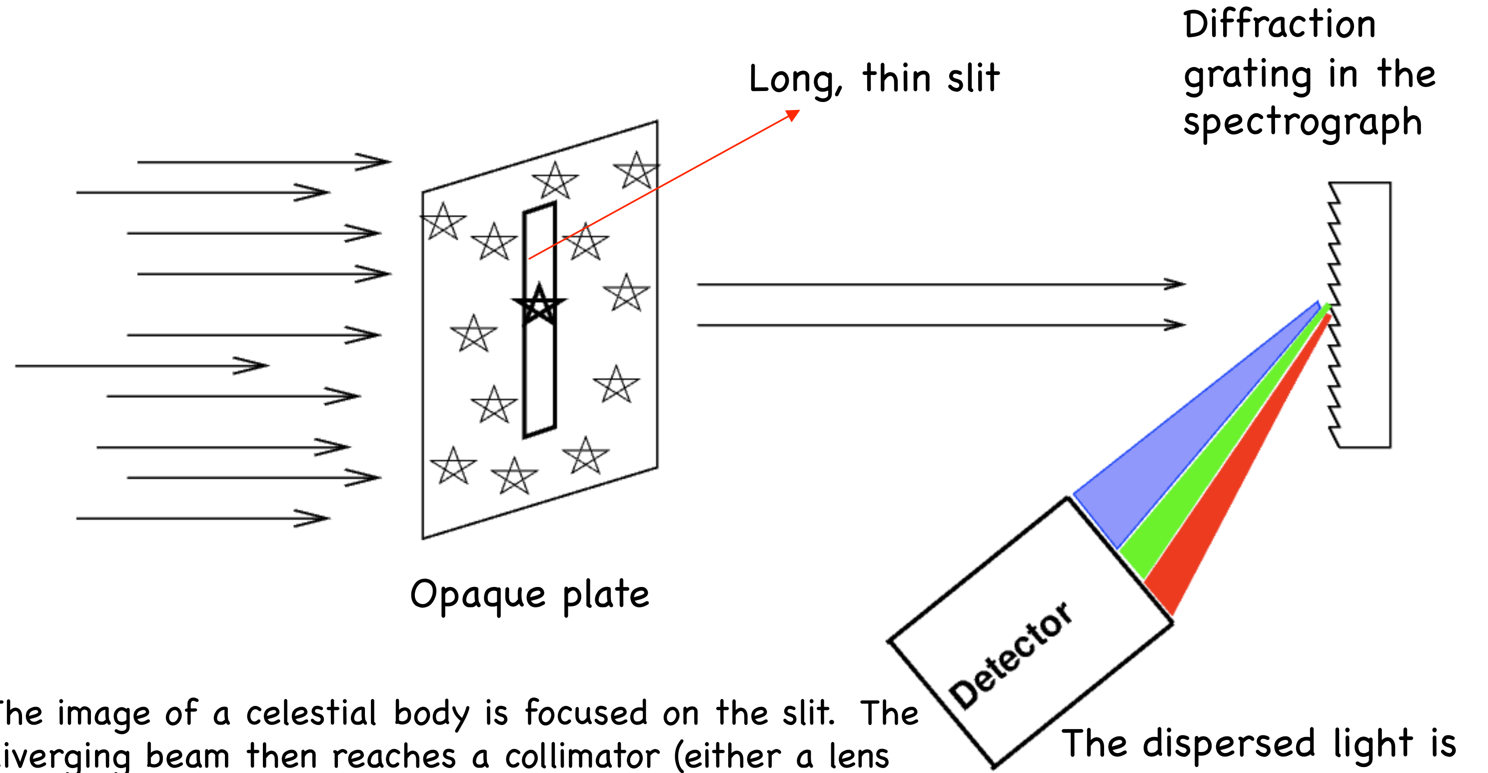


Why use fibers?

Can obtain spectra of > 100s of galaxies or stars per field by packing fibers closely

How do we obtain redshifts for large numbers of sources?

(using a slit)



The image of a celestial body is focused on the slit. The diverging beam then reaches a collimator (either a lens or a mirror – not shown –). This produces parallel light which is then dispersed by a diffraction grating.

The dispersed light is imaged on a detector (typically a CCD)

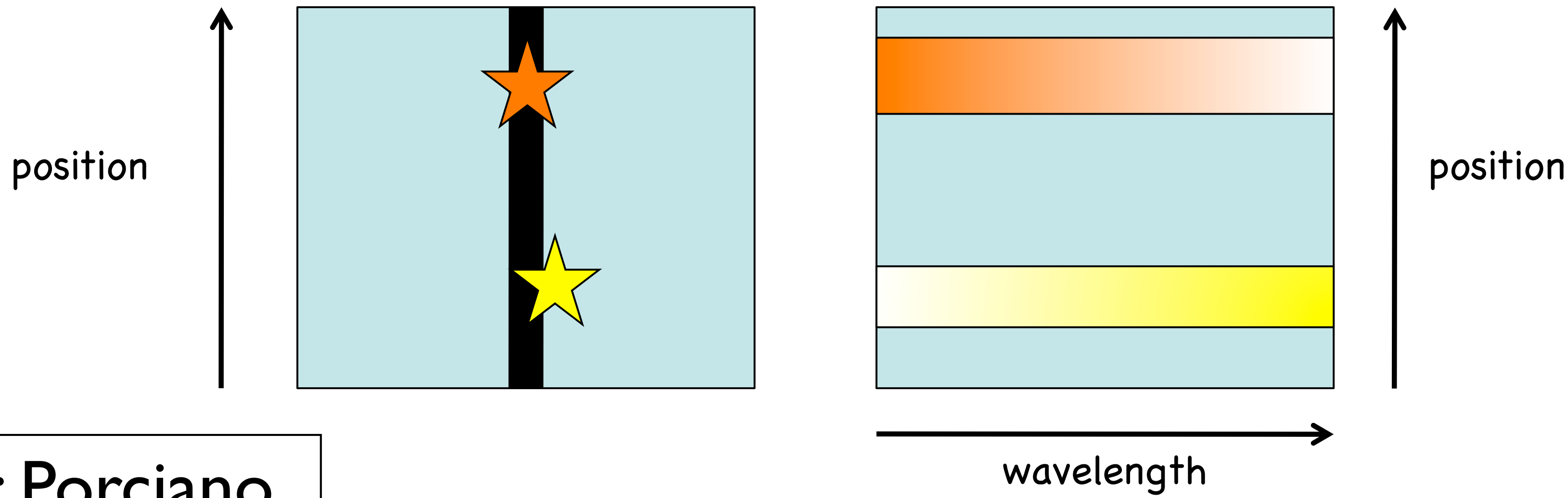
credit: Porciano

How do we obtain redshifts for large numbers of sources?

(using a slit)

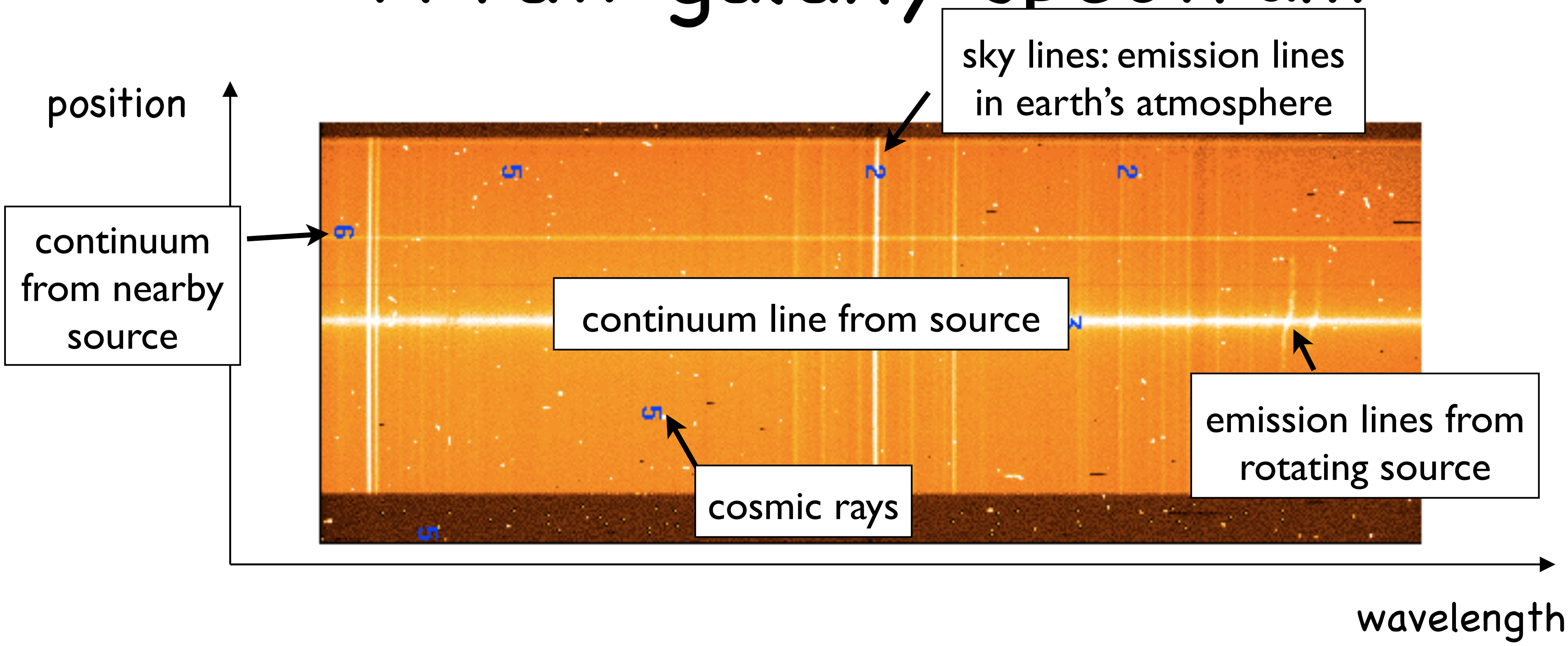
- Why using a slit? To keep out as much as background light as possible

- How does the output look like? 2D spectrum



credit: Porciano

A raw galaxy spectrum



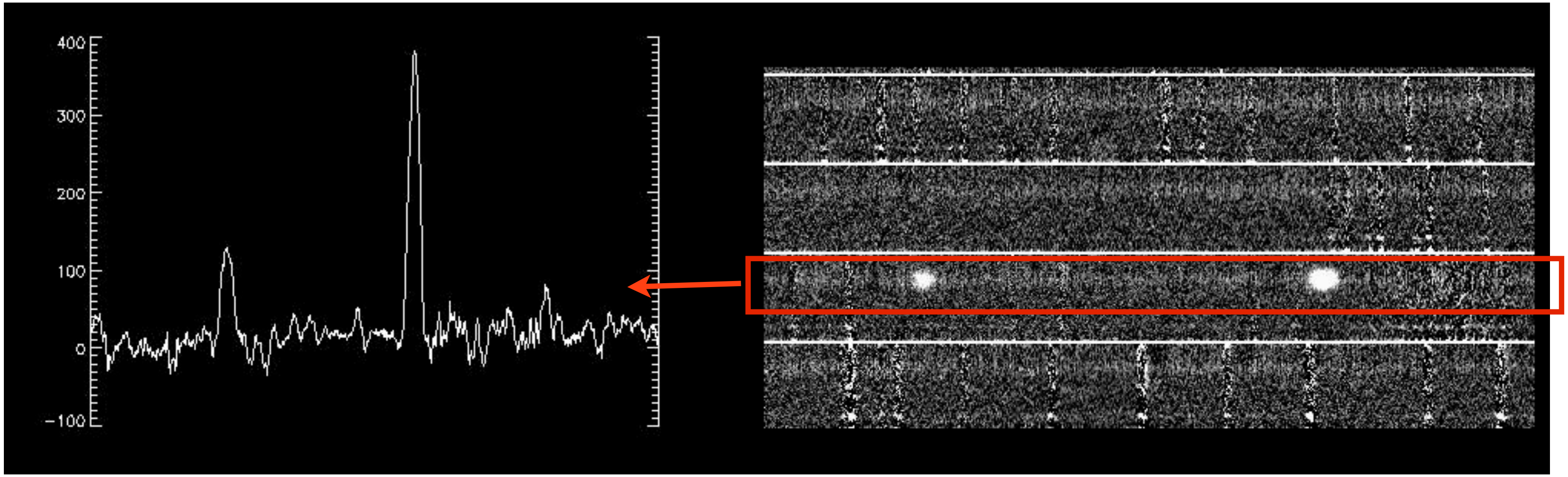
- This is the typical output of a spectrograph mounted on a telescope.
- Can you identify the origin of the different features?

credit: Porciano

Typically these two-dimensional spectra are converted into one-dimensional spectra

1D spectrum

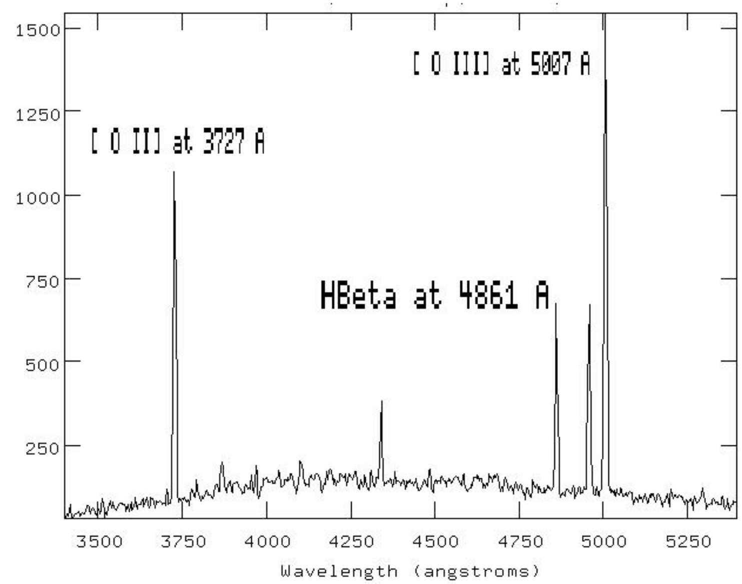
2D spectrum



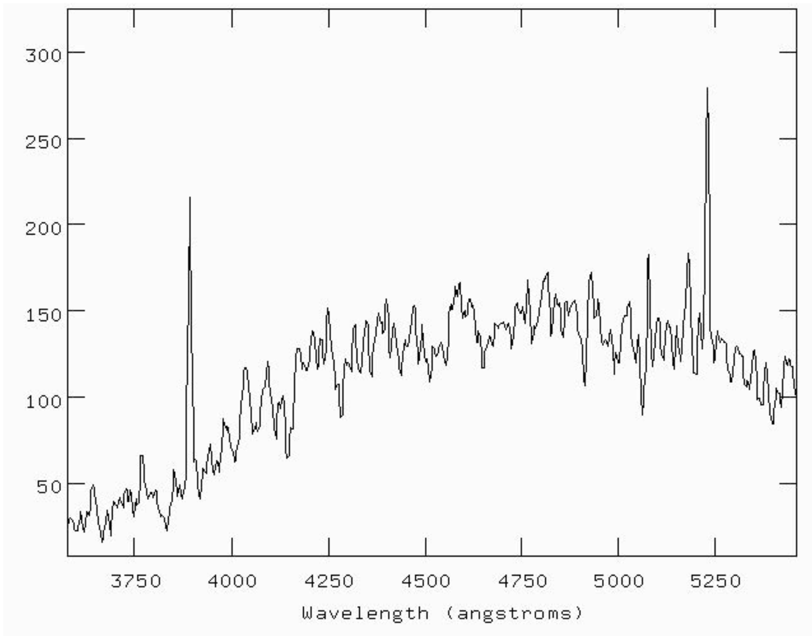
wavelength

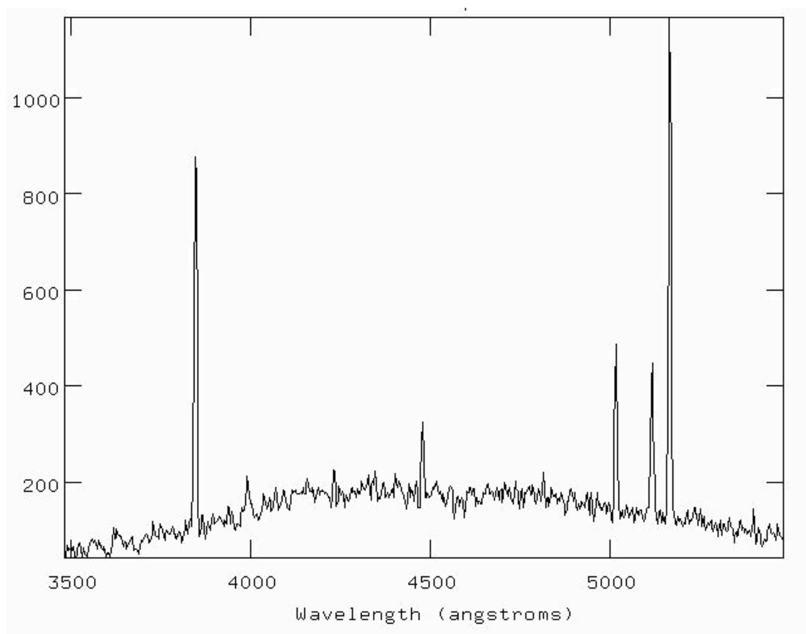
credit: Porciano

Measuring galaxy redshifts



Template spectrum at z=0





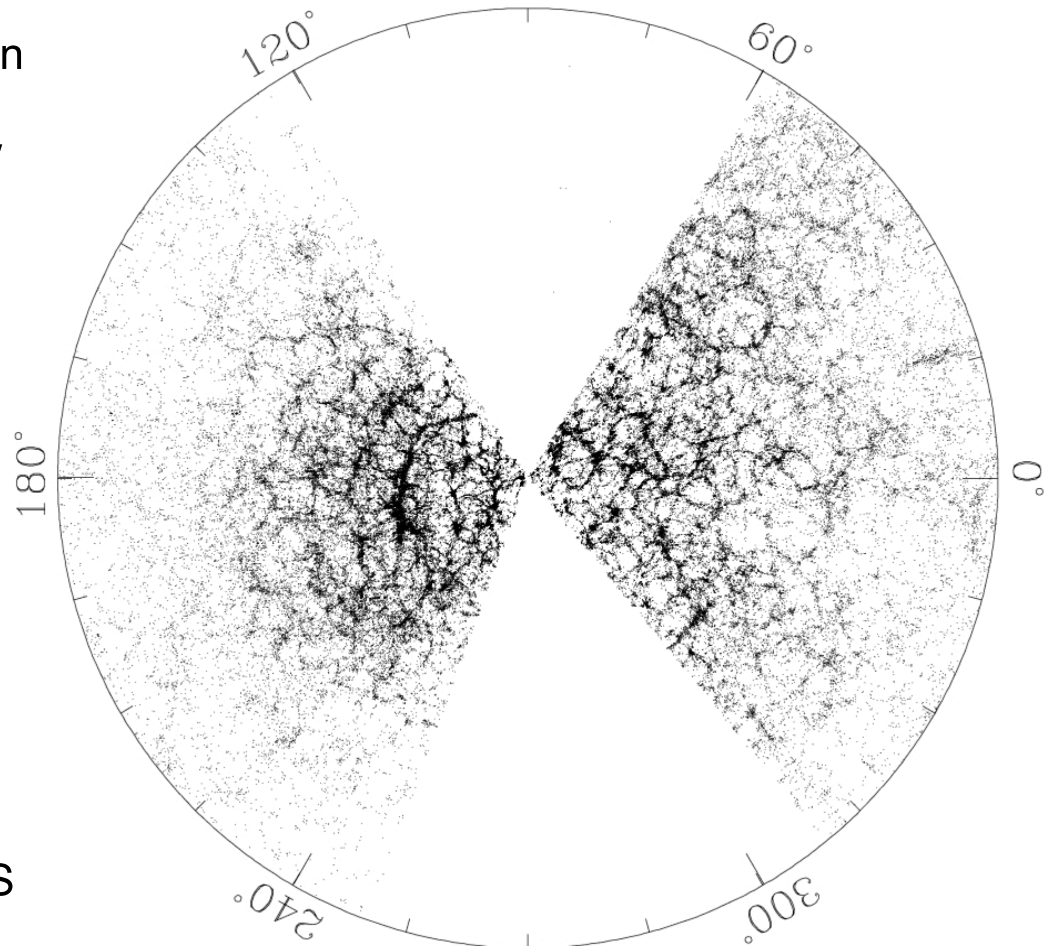
Measure redshifts by comparing with unredshifted template spectrum

credit: Porciano

Massive redshift surveys

- Multifibre technology, digitalization and multiobject spectrographs now allow us to measure redshift of millions of galaxies on a time scale of a few years.

- Recently completed or ongoing surveys: (local) 2dF, SDSS, 6dF (high-z) VVDS, DEEP2, zCOSMOS



credit: Porciano

The Sloan Digital Sky Survey

- Over eight years of operation (SDSS I, 2000-2005; SDSS II, 2005-2008; SDSS III, 2008-2014)

- It used a dedicated 2.5m telescope at Apache Point Observatory (New Mexico) equipped with 2 special purpose instruments: a 120 Mpixel camera imaging 1.5 sq. deg. of the sky at a time (8 times the area of the full moon); a pair of spectrographs fed by optical fibers (640 objects per pointing)

- It obtained deep multi-color images (u,g,r,i,z) covering more than a quarter of the sky (8,400 square degrees)

- Created 3D maps containing more than 930,000 galaxies and more than 120,000 quasars (in 5,700 square degrees)

credit: Porciano

What science do astronomers do with these big surveys?

measure many, many things...

- Luminosity function and number densities
- Group and cluster catalogs (FoF, Voronoi, BCG)

- The density field
- Reconstruct the linear density field (time machine)

- Counts in cells
- Measure 2-point, 3-point correlation function
- Measure power spectrum, bispectrum
- Topological invariants: Minkowski functionals (mean genus, void probability function)

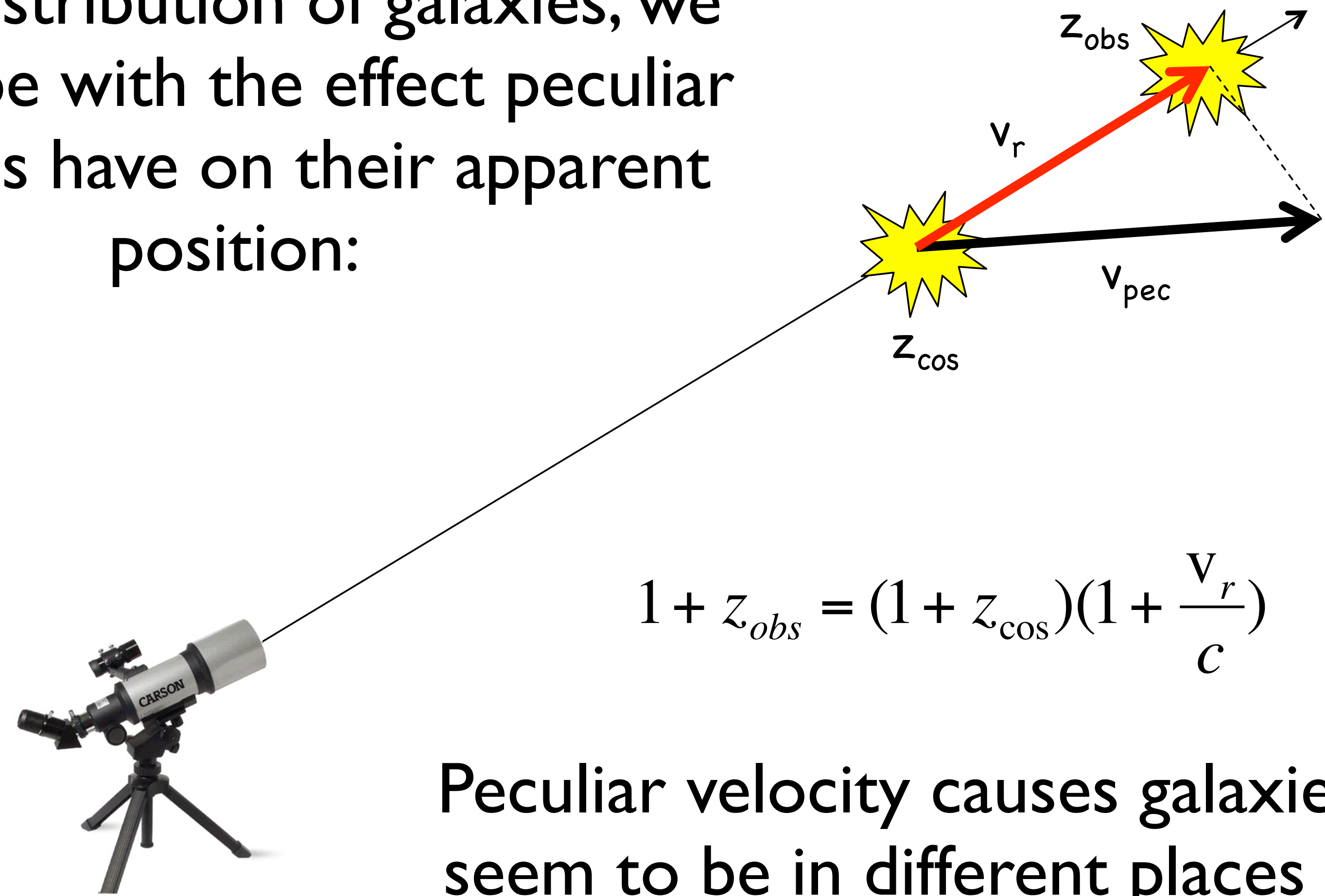
credit: Porciano

However there are a number of important complications...

Complication #1: Redshift Distortions

Redshift Space Distortions

When mapping the three-dimensional spatial distribution of galaxies, we must cope with the effect peculiar velocities have on their apparent position:



$$1 + z_{obs} = (1 + z_{\cos})(1 + \frac{v_r}{c})$$

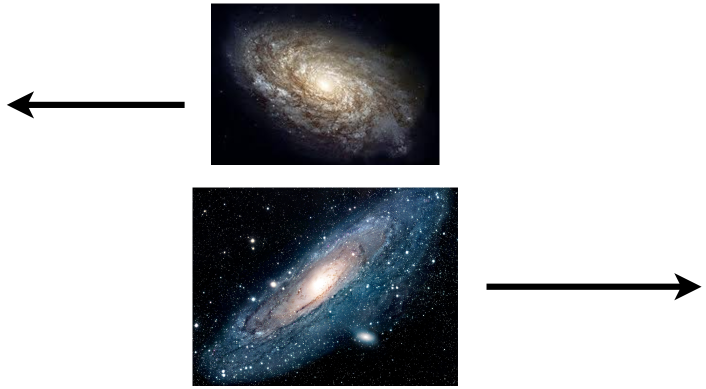
Peculiar velocity causes galaxies to seem to be in different places that they really are

credit: Porciano

Redshift Space Distortions

For example, let's say we're looking at two galaxies which are the same distance from us, but one is moving towards us and the other is moving away

REALITY



APPEARANCE

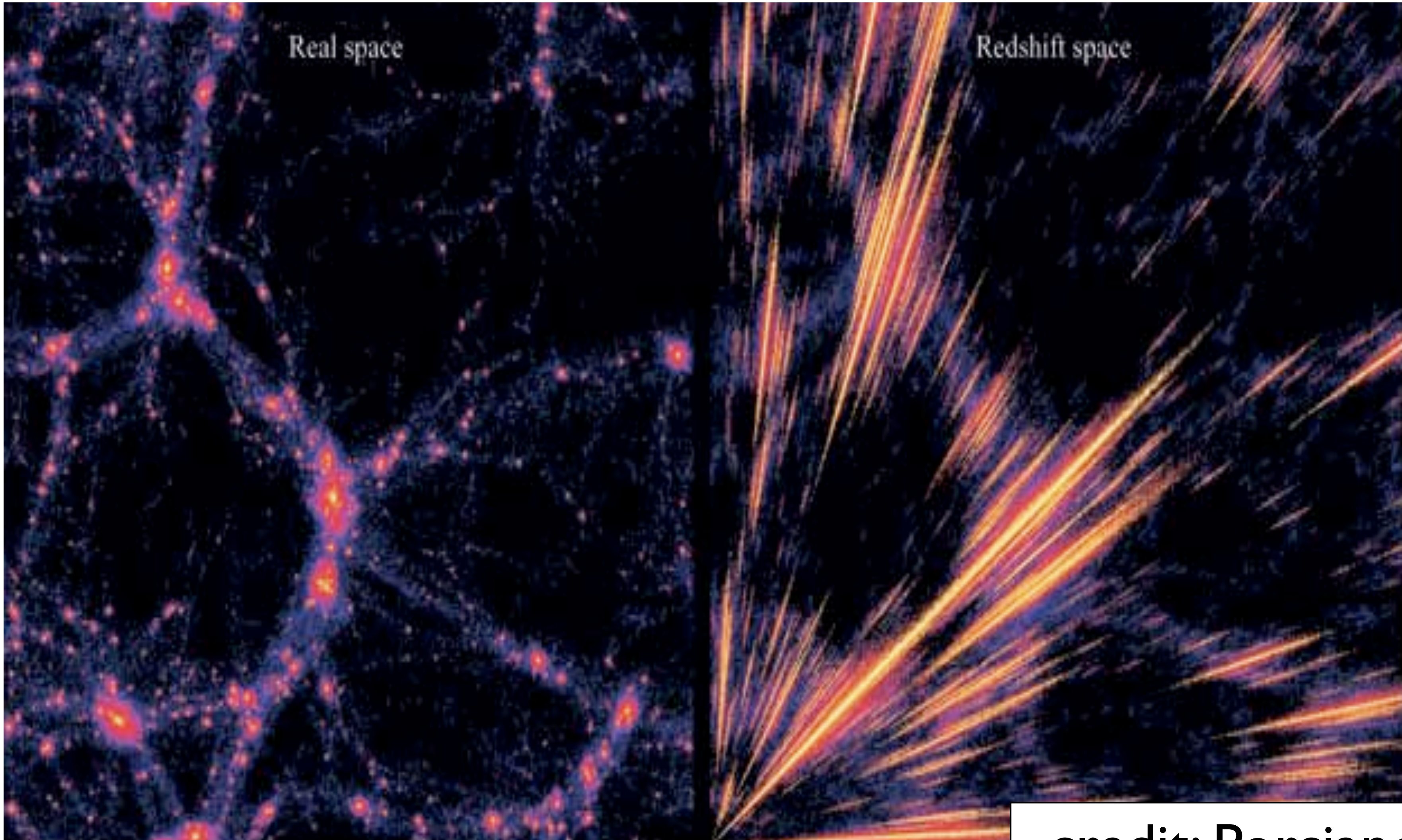
doppler shift changes the apparent position of galaxies along line of sight

==> cannot infer exact distribution of galaxies in real space (peculiar velocity and distance along line of sight degenerate)

Illustration of Redshift Space Distortions

Real Space

Redshift Space

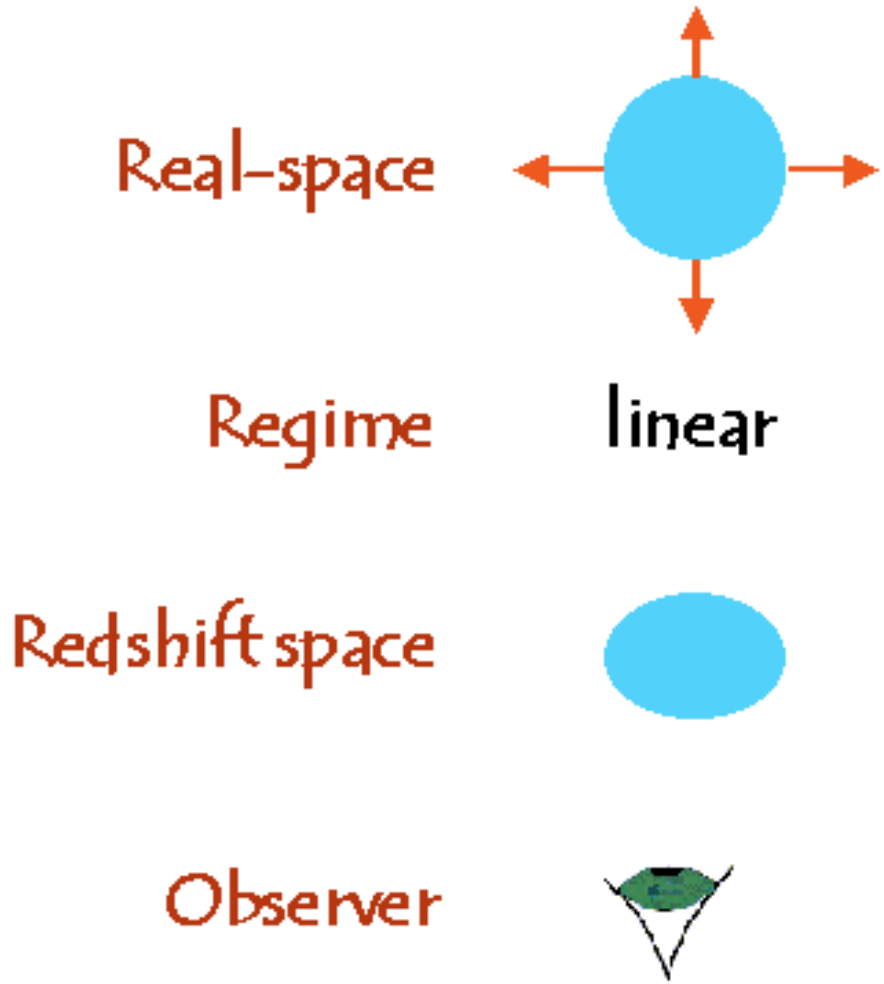


credit: Porciano

Redshift Space Distortions

So what sort of effects would we expect the peculiar velocities of galaxies to have on the spatial positioning of galaxies?

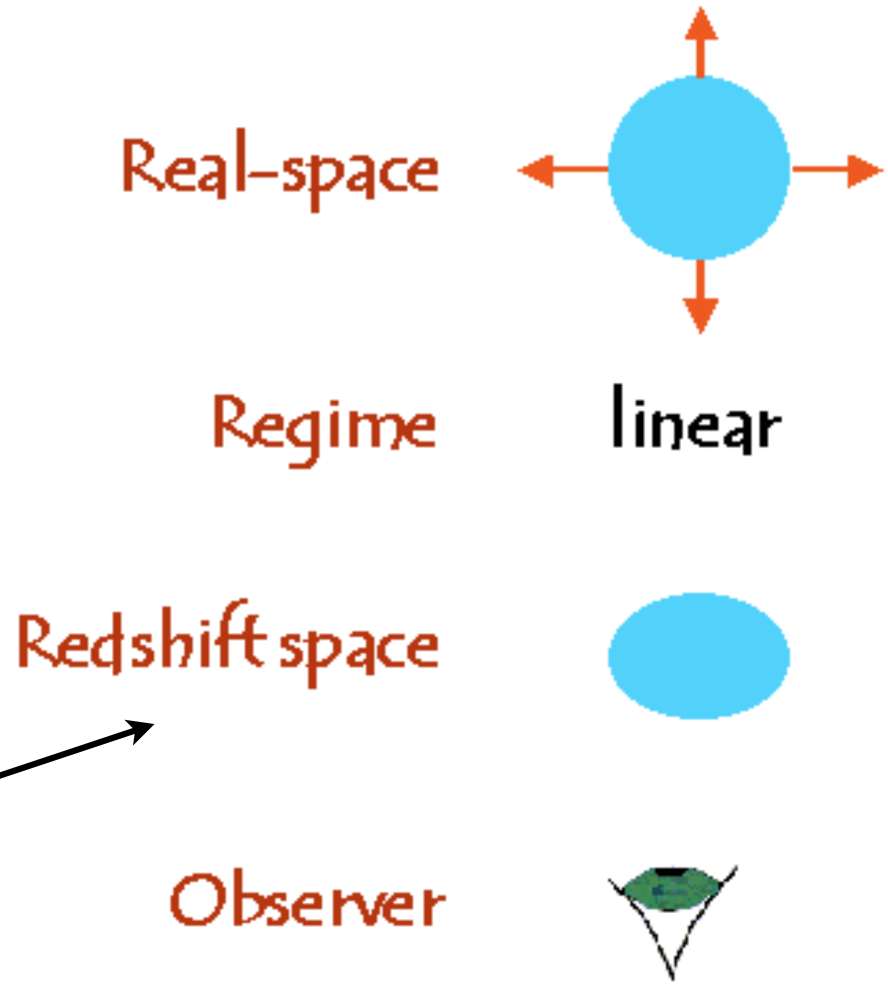
Let's say we have a group of galaxies which are still expanding with the Hubble flow... and have not quite started to turn around...



credit: Porciano

Redshift Space Distortions

What would the appearance of this structure appear to be in redshift and angular space?   It is shown to the right.

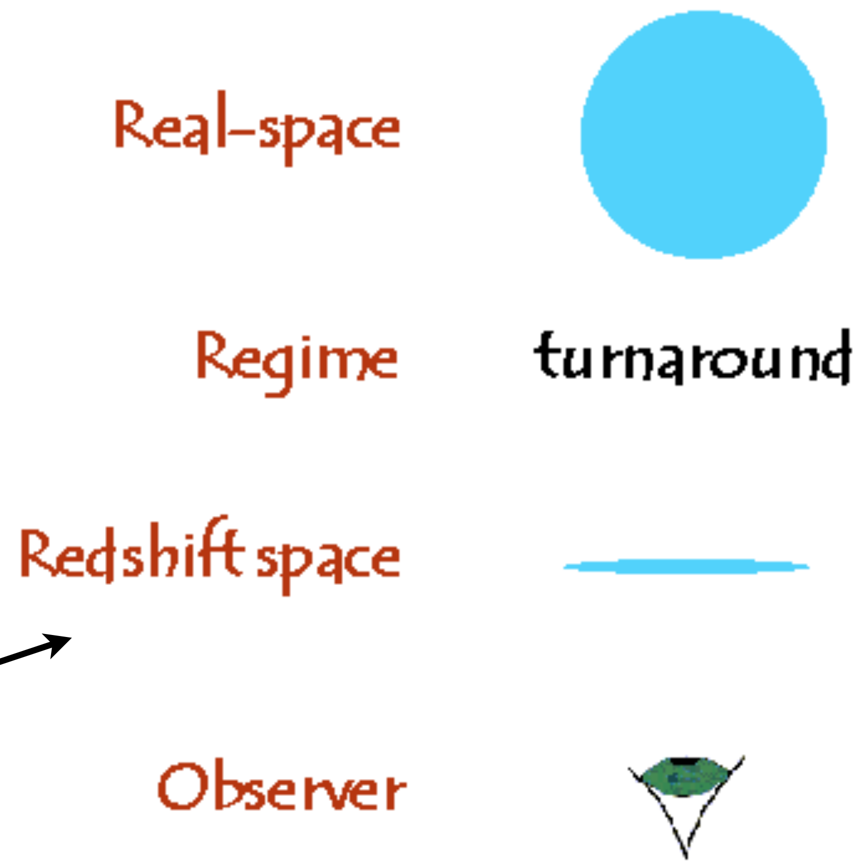


Gravity has slowed Hubble flow slightly.... This is why compressed in redshift space...

credit: Porciano

Redshift Space Distortions

What about a little later on in the evolution of a structure when it starts to turn around (from gravity)?



Doppler shift cancels cosmological redshift

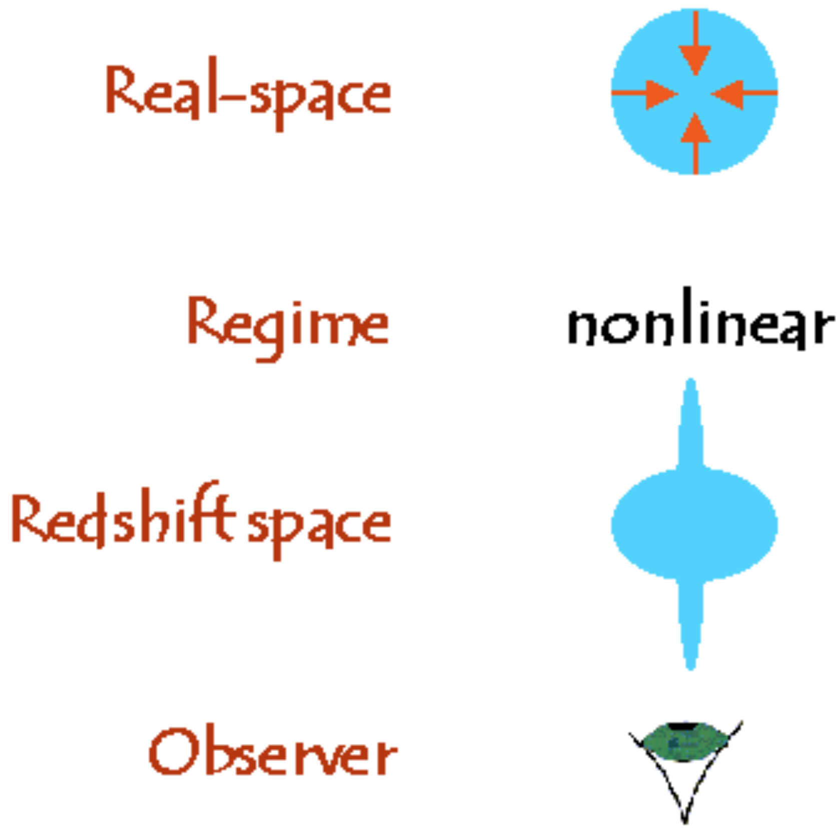
credit: Porciano

Redshift Space Distortions

And a little later on in the evolution when the structure has collapsed?

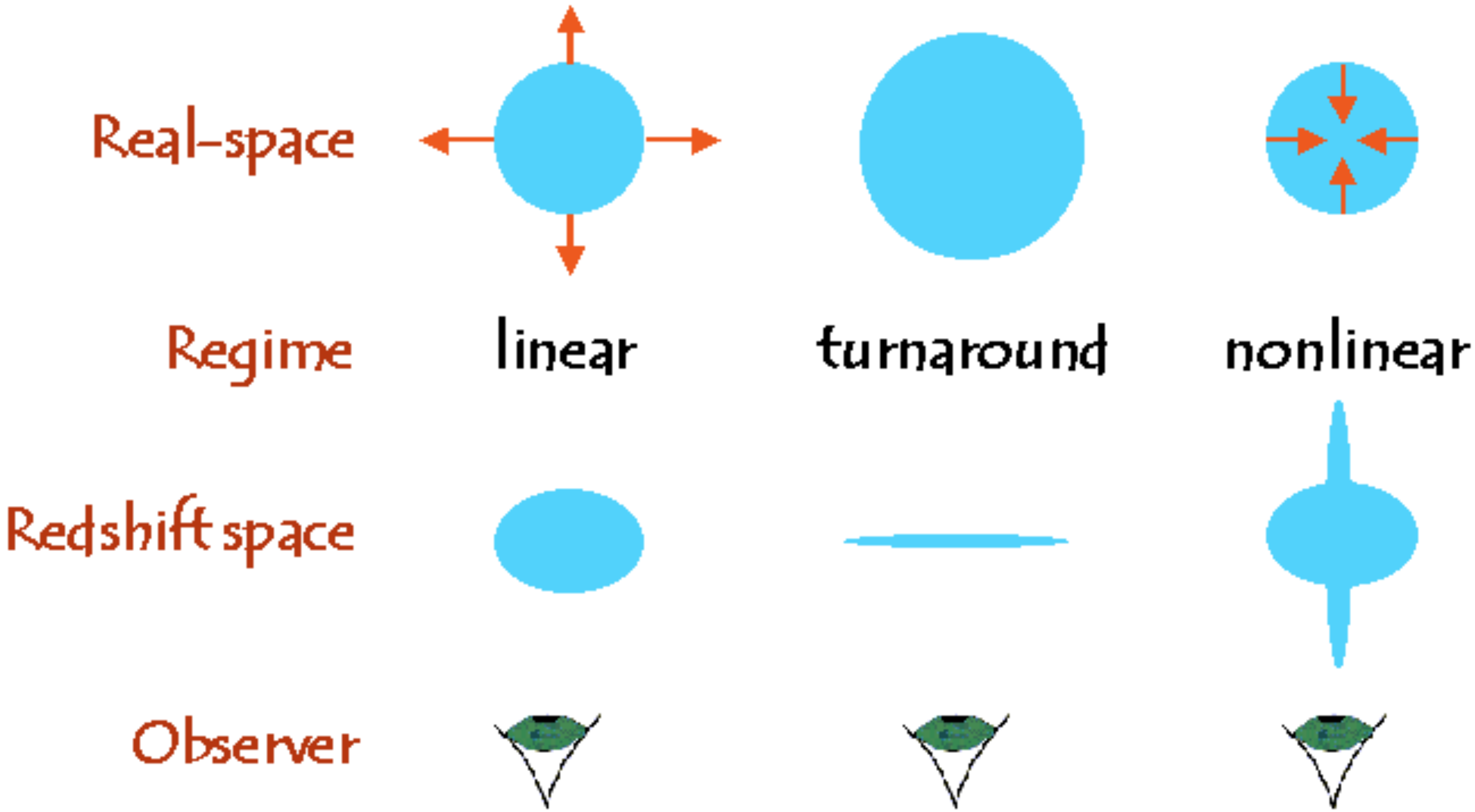
This apparent structure is called a finger of God... and is evident whenever one has a galaxy cluster

This finger of God is due to the substantial speeds with galaxies within clusters move around (often ~700-1000 km/s)



credit: Porciano

Redshift Space Distortions



credit: Porciano

Redshift Space Distortions

Typically, one looks at the spatial positioning of galaxies in this angle - redshift space as shown to the right:

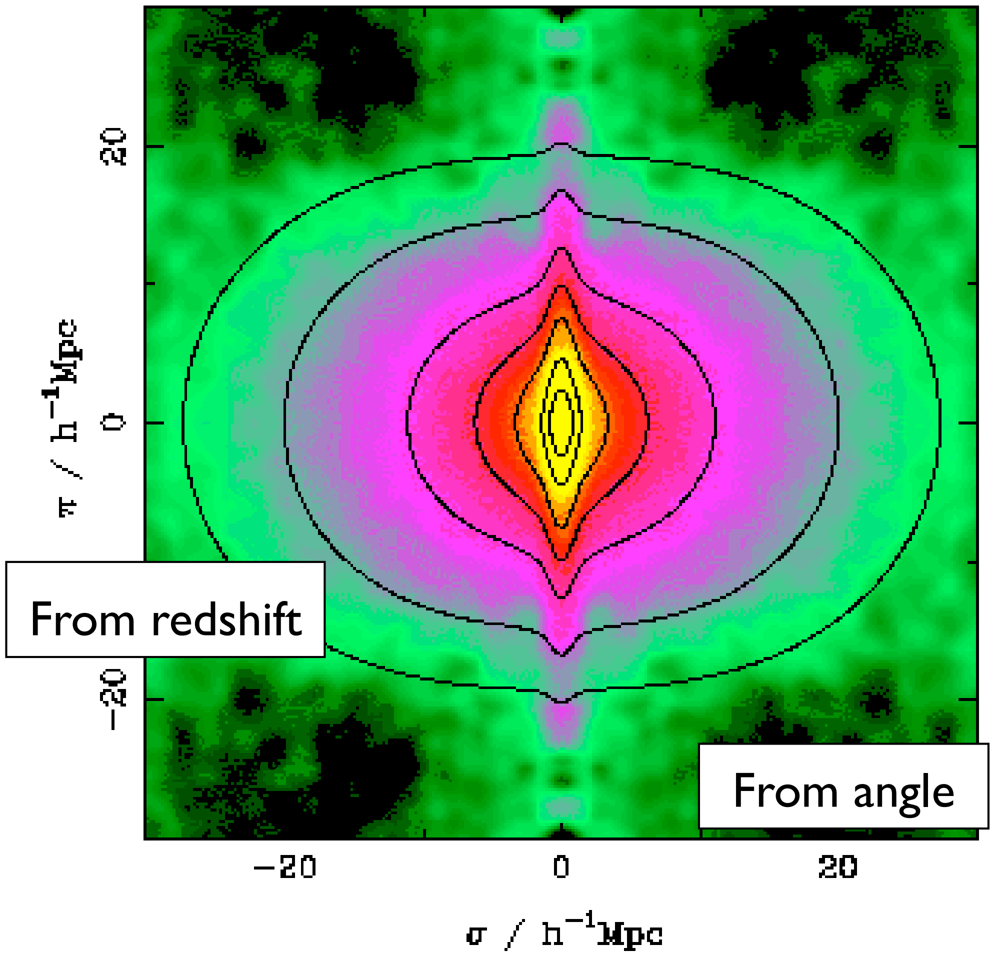
σ is the apparent position of the galaxy in physical space based on the observed angle

$$\sigma = \Delta\theta \, (D_A)$$

π is the apparent position of the galaxy in physical space based on the observed redshift

$$\pi \equiv \frac{c \, \Delta z}{H_0}$$

$\xi(\sigma,\pi)$

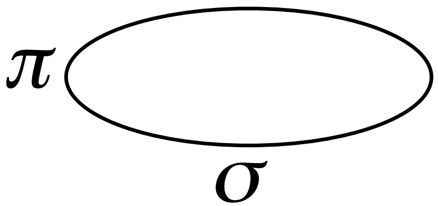


Hawkins et al. 2002

Redshift Space Distortions

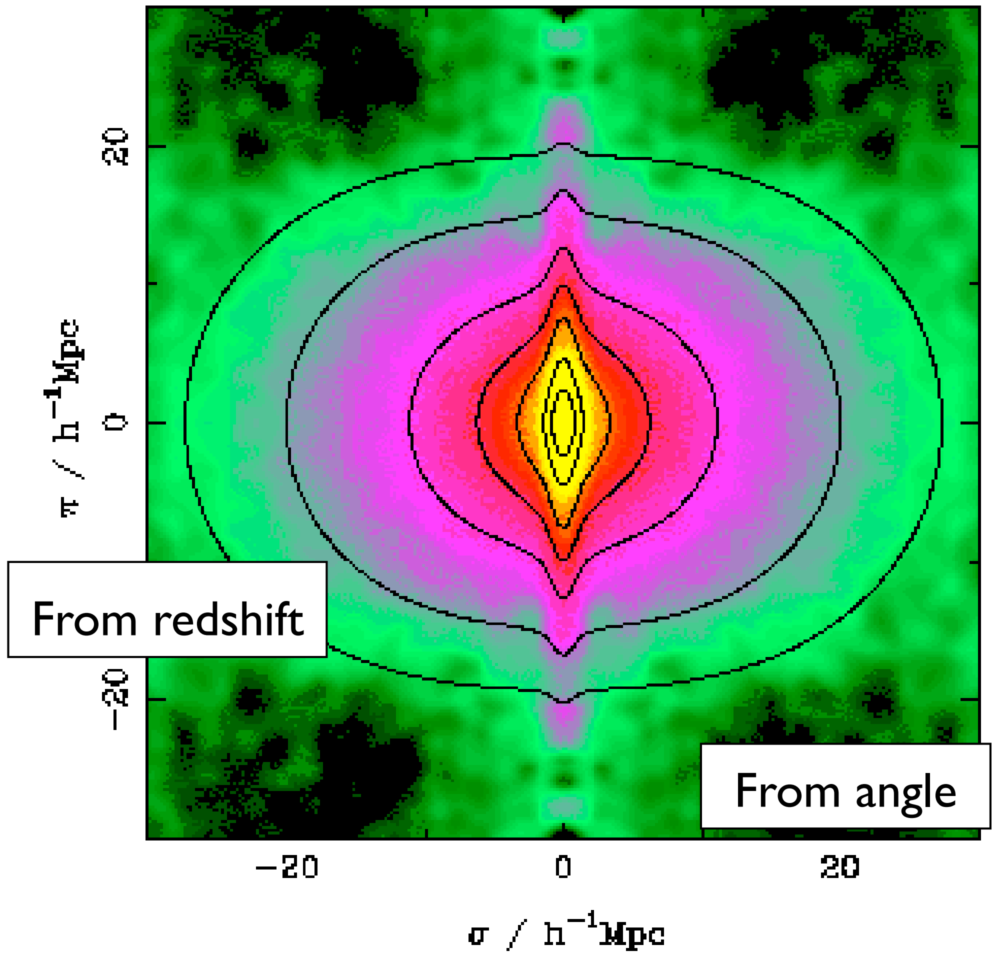
Why is this distribution not circularly symmetric?

The flattening along the π (redshift) direction is due to galaxies on the near and far sides of an overdensity falling back towards that overdensity



Infall velocity squashes π
"Kaiser effect"

ξ(σ,π)

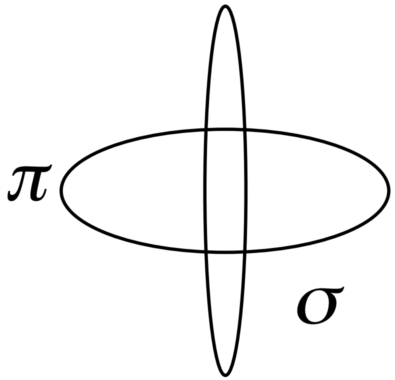


Hawkins et al. 2002

Redshift Space Distortions

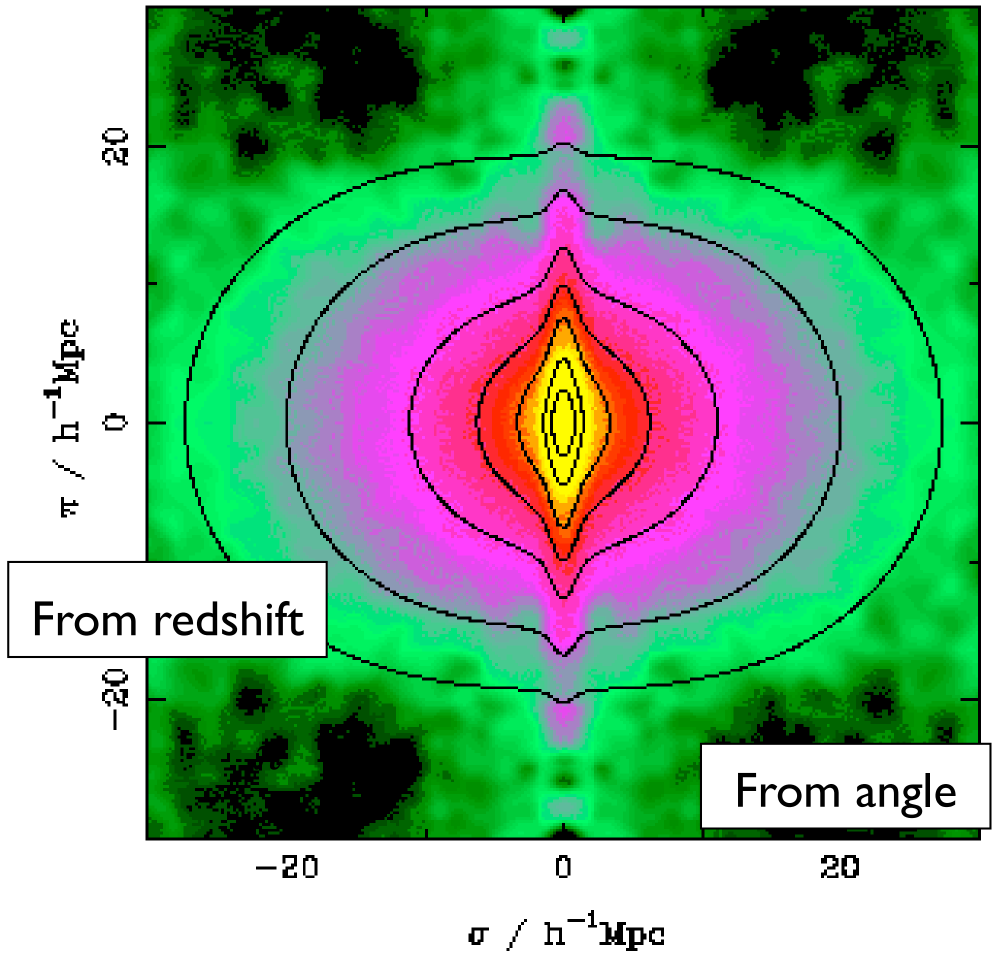
Why is this distribution not circularly symmetric?

Fingers of God results from the high internal motions within massive cluster type regions of universe



$V \sim (GM/r)^{1/2}$

$\xi(\sigma,\pi)$



Hawkins et al. 2002

Redshift Space Distortions

From the Kaiser effect, it is actually possible to learn about the total amount of dark matter in universe

Basically this is because a higher dark matter mass density means that galaxies will be falling faster towards each other

The predicted two point correlation function -- shown in the previous plots -- can be shown to be dependent on a parameter known as β

$$\beta \equiv \frac{\Omega_M^{0.6}}{b} = 0.43 \pm 0.07$$

b = bias parameter > 1

(equivalent to $\Omega_M = 0.24 \pm 0.07$)

Looking at Kaiser effect is just another way of getting at the dark matter content of universe

In a previous lecture, I discussed deriving this based on a velocity flow model of local universe

There, the dark matter content was inferred by looking at the overall convergence in the velocity flow (i.e., the strength of the "attractors" in flow provide measure of the total mass density in universe)

Inferences for the dark matter content from velocity flows

One can also use the peculiar velocities (bulk flows) of galaxies in the nearby universe to estimate the amount of dark matter in the universe

This is because the peculiar velocities are set by the matter within the universe -- which causes galaxies to fall towards each other.

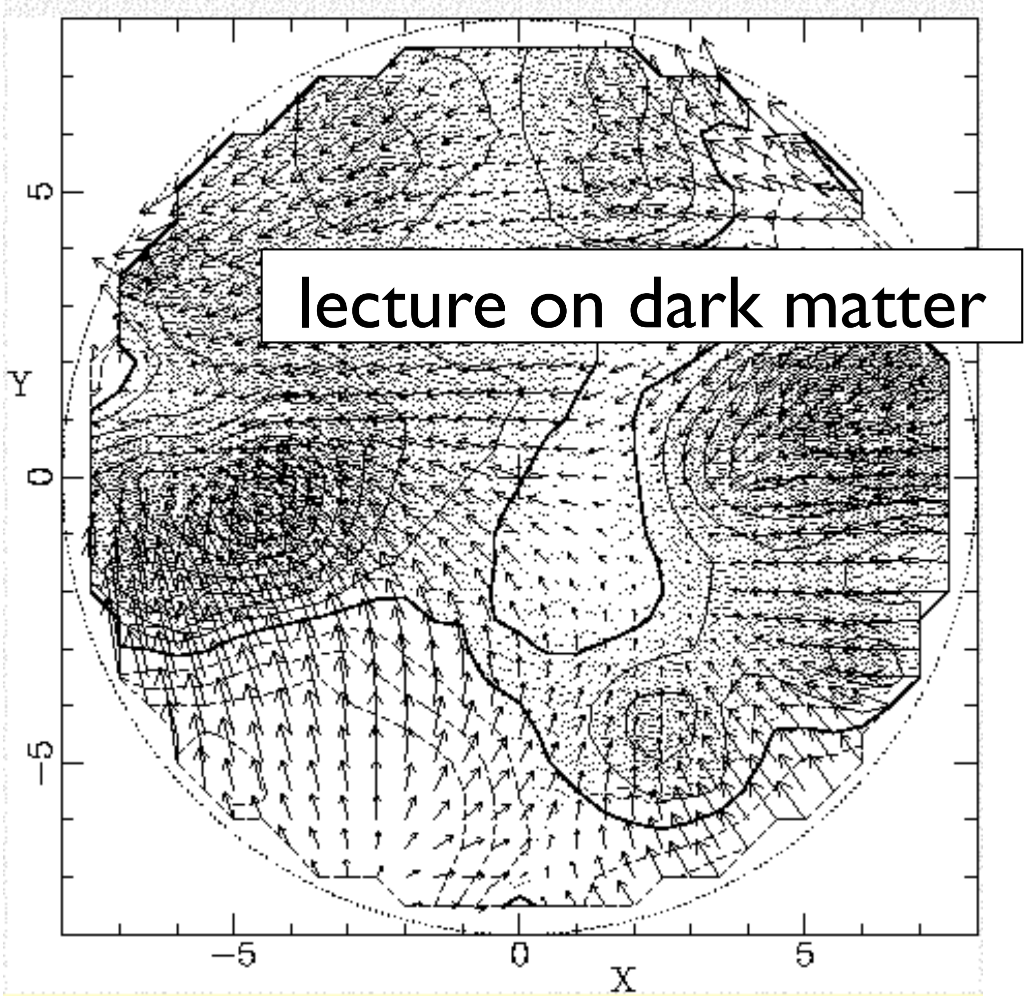
An approximate equation to describe this is the following:

$$\nabla \cdot v = -\Omega_M^{0.6}\, \delta_M$$

convergence points for fluid flow

gravitational mass

Velocity flow model in the nearby universe



Complication #2

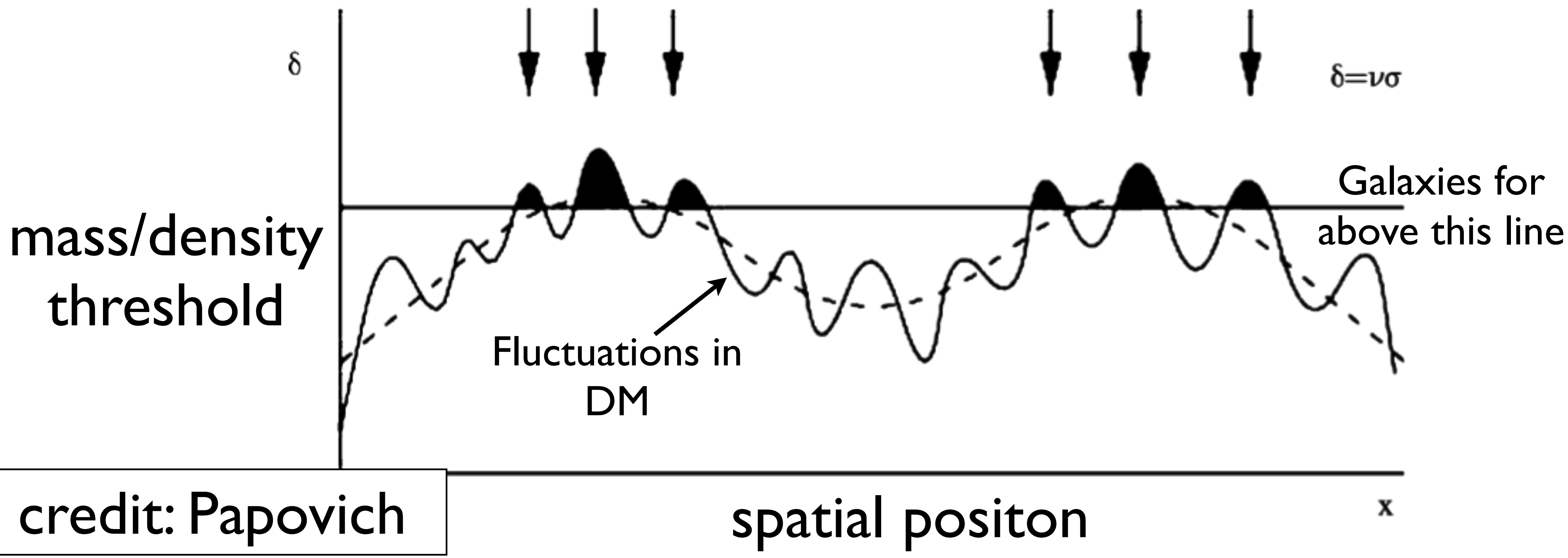
Galaxy Bias

(how well do galaxies trace the underlying perturbations in the matter?)

Galaxy Bias

-- The galaxies we observe do not perfectly trace the underlying mass distribution in the universe (i.e., light does not trace mass)

-- Expect galaxies to be found preferentially in the most prominent high-mass peaks



credit: Papovich

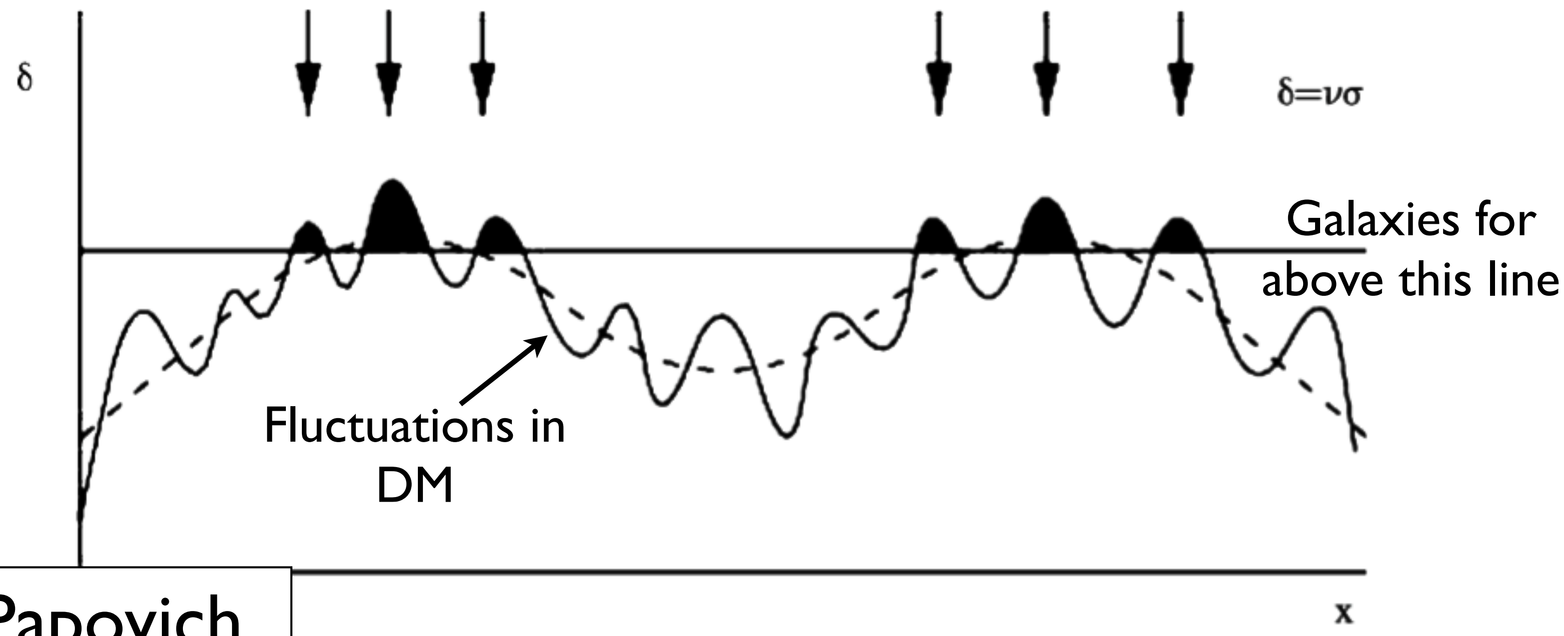
Galaxy Bias

-- Express fluctuations in the number of observed galaxies in terms of fluctuations in the mass density times biasing factor:

$$\delta_g := \frac{\Delta n}{\bar{n}} = b \frac{\Delta \rho}{\bar{\rho}} = b\,\delta$$

(linear bias, in general, more complicated)

In general, bias b >= 1



credit: Papovich

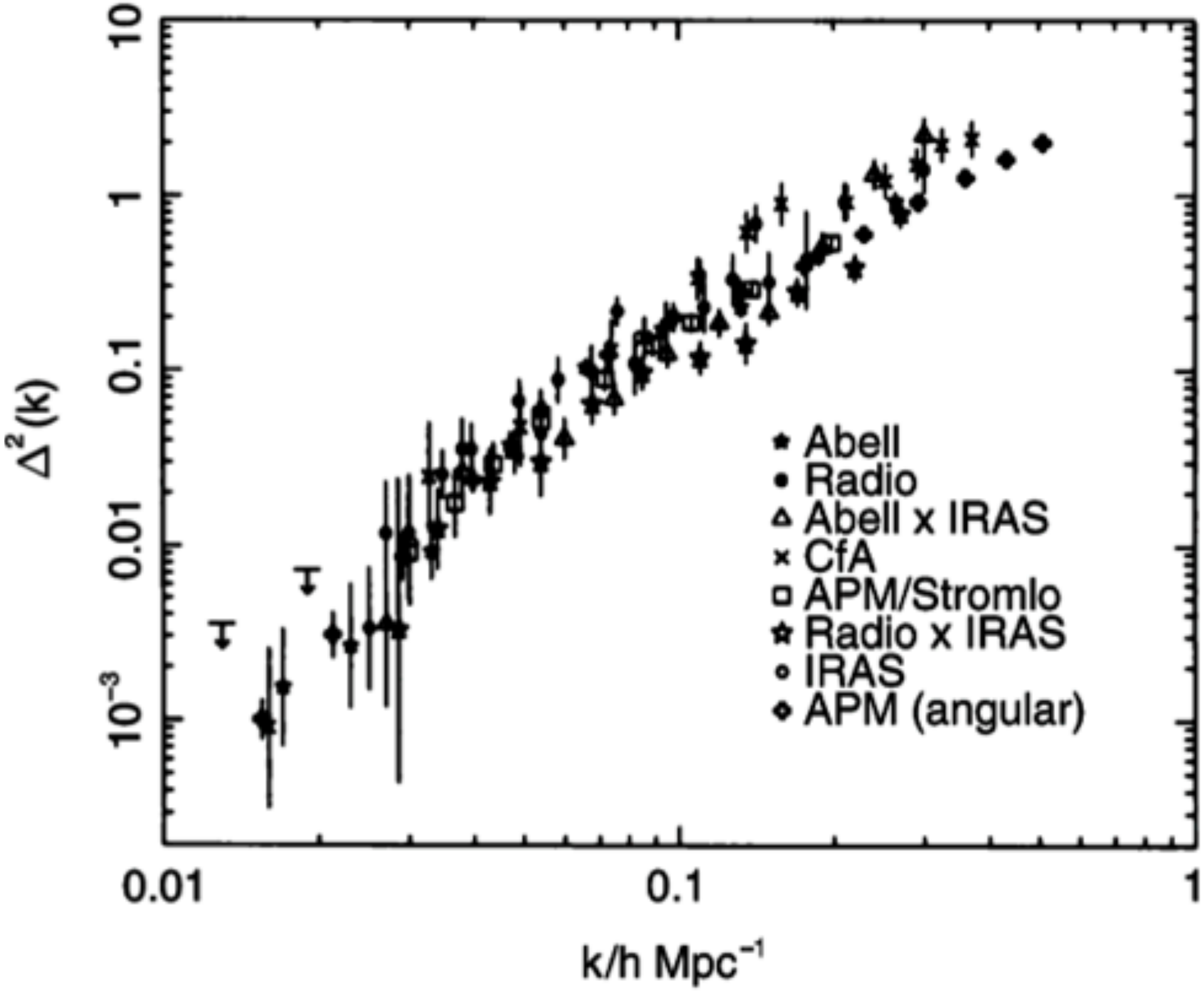
We’re discussed how to measure the matter power spectrum from the correlation function

We’re also discussed two complications in recovering the matter power spectrum from the correlation function

1. Redshift Space Distortions
2. Galaxy Biasing

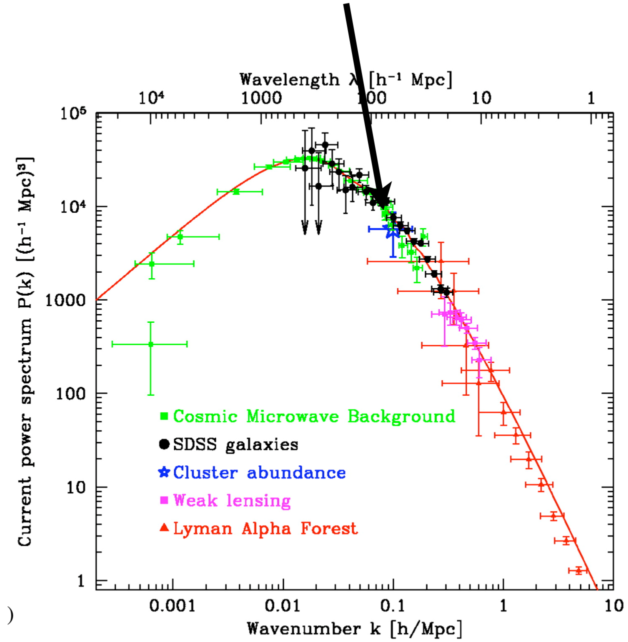
After coping with the above effects, we can measure the matter power spectrum

What does an extracted power spectrum look like?



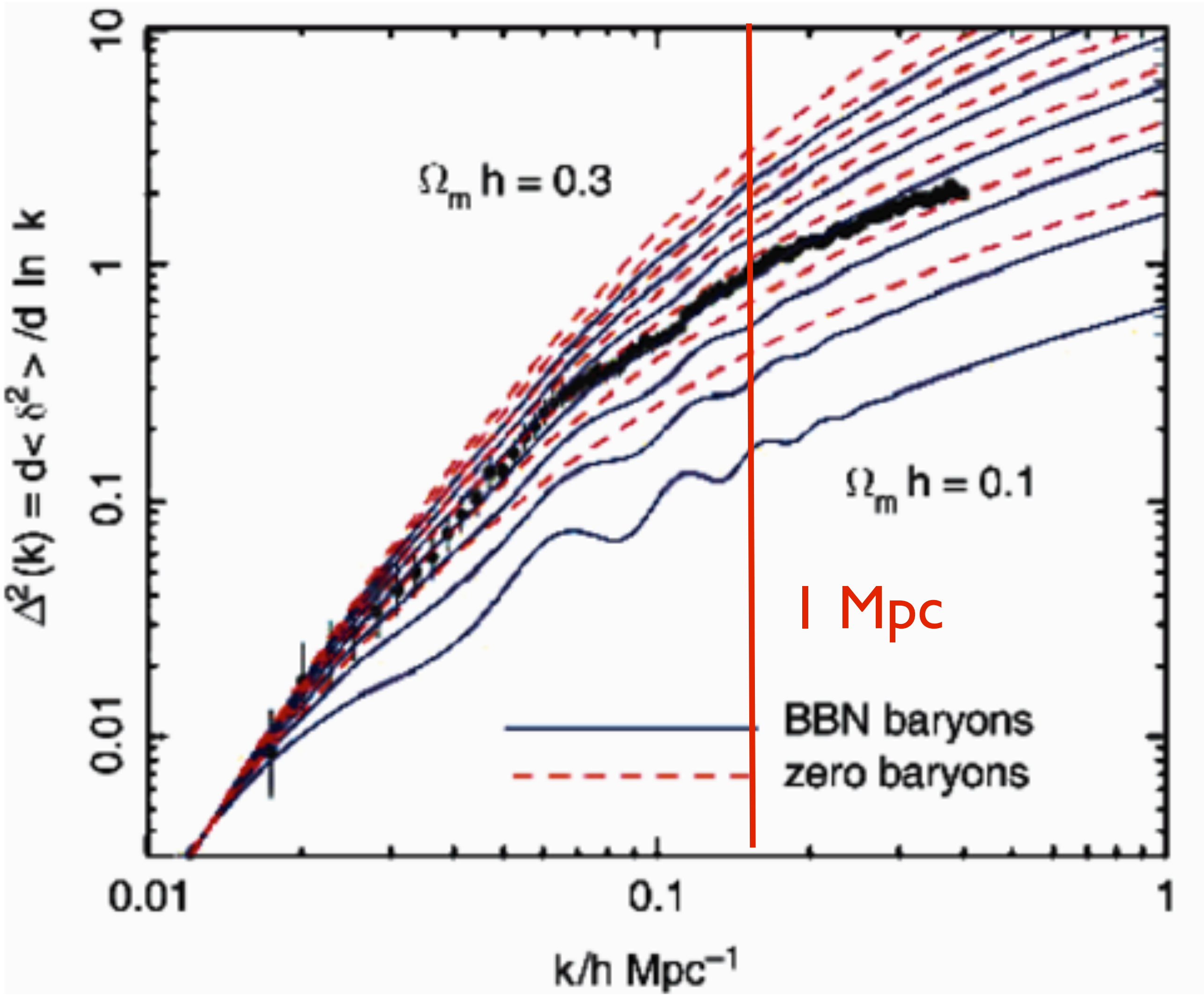
Measurements of $\Delta^2 = k^3\,P(k)$

Differs from the matter power spectrum shown earlier due to k^3 term



What does the power spectrum teach us about the cosmological parameters?

Compare observed power spectrum with that found in simulations



Biggest constraint is on Ω_m h

due to its role in determining time of matter-radiation equality

also can constrain Ω_b

Peacock 2003, astro-ph/0309240 and Peacock 2001, astro-ph/0105450.

Implications for Cosmological Parameters

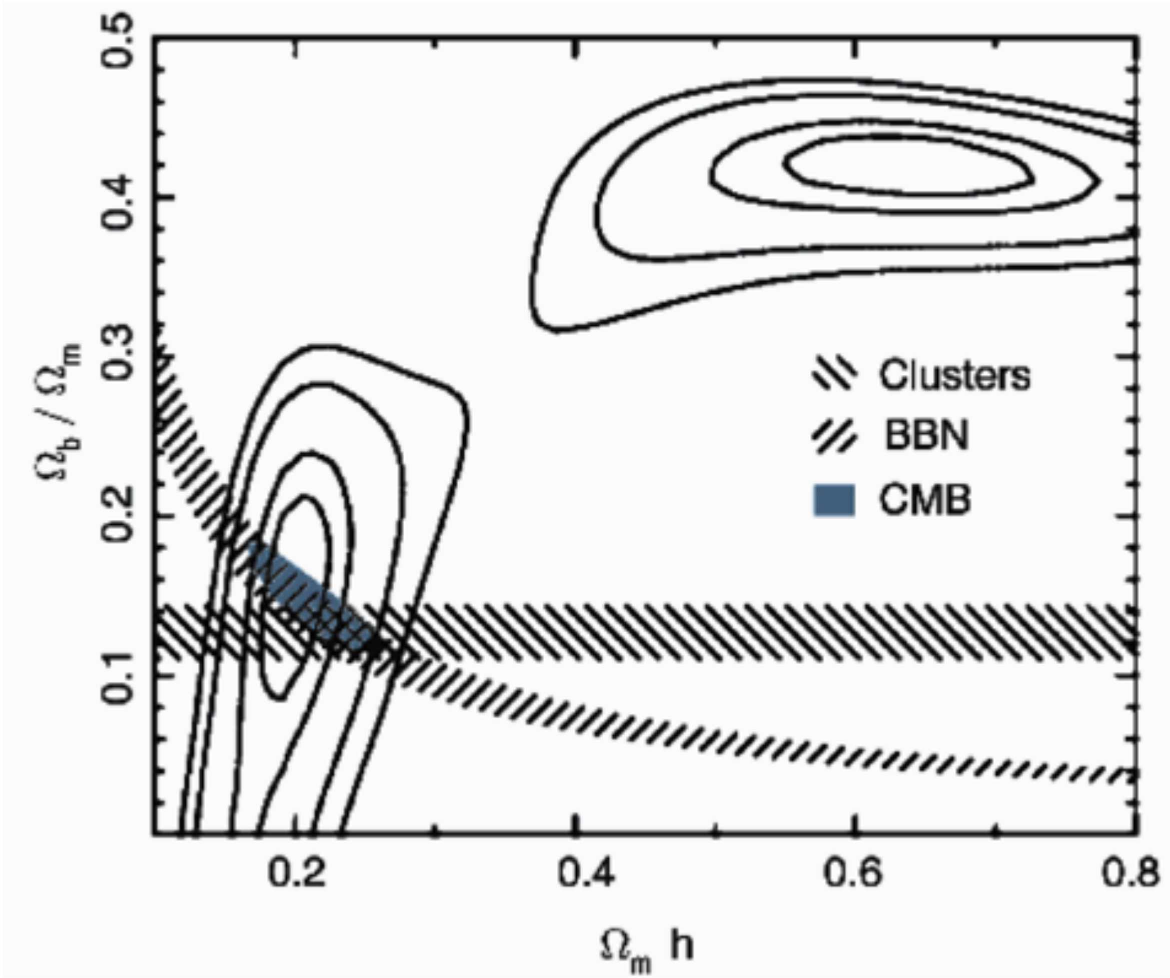
Can use comparison to constrain cosmological parameters!

Allowed solutions are bimodal, but we can eliminate one of solutions using other constraints

$\Omega_m h = 0.2$
$\Rightarrow \Omega_m = 0.3$

$\Omega_b = 0.04$

Yet another constraint on the baryonic density of universe!



Peacock 2003, astro-ph/0309240 and Peacock 2001, astro-ph/0105450.

How do we normalize the power spectrum?

We parameterize this using the σ_8 parameter

While deriving correlation function and Power spectrum from galaxy survey, one thing we are particularly interested in is the normalization of the power spectrum

$$P_0(k) = A\,k^{n_s}$$

(related to the A parameter here)
($n_s = 1$)

This is defined using this parameter σ_8 (intended to represent the root-mean-squared fluctuations in a 8 h^{-1}Mpc volume):

$$\sigma_{8,g}^2 := \left\langle \left(\frac{\Delta n}{\bar{n}} \right)^2 \right\rangle_8 \approx 1$$

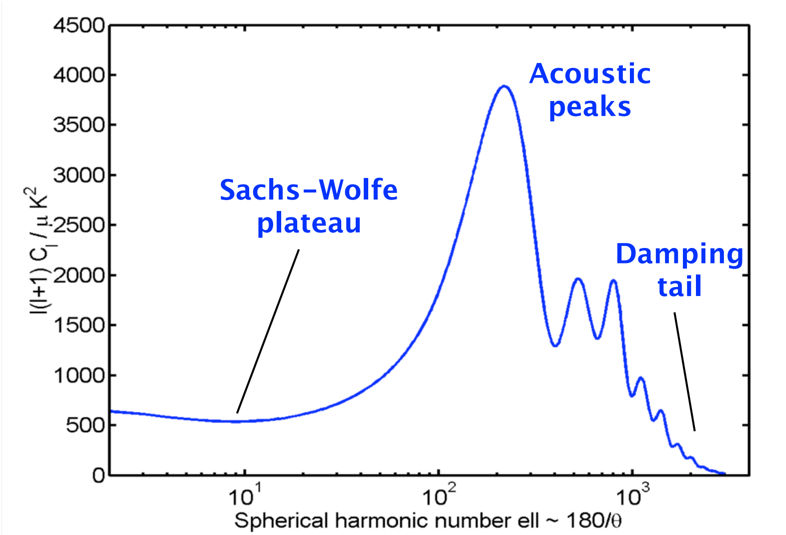
(8 h^{-1} Mpc was chosen because appeared close to 1)

Size of density fluctuations in a volume really defines the amplitude of power spectrum

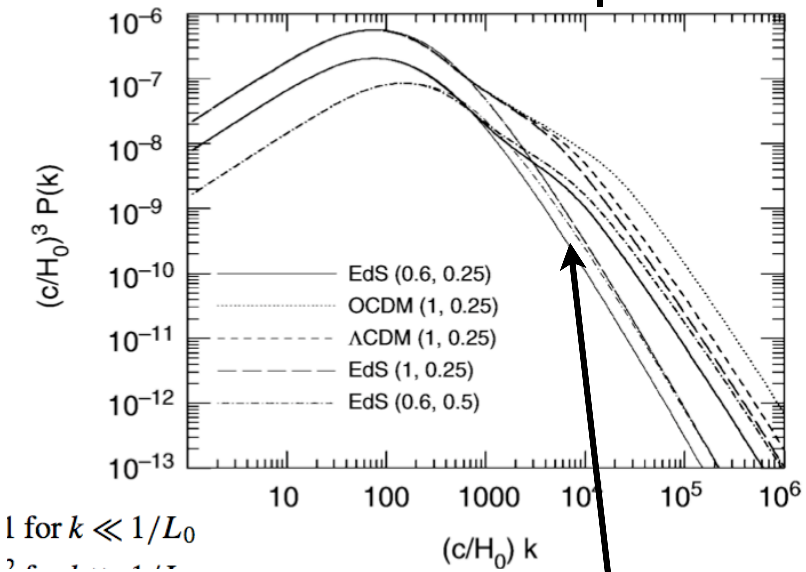
What effect do baryons have on the matter power spectrum?

Just like in the CMB, baryons impart acoustic oscillations on matter power spectrum

CMB Power Spectrum

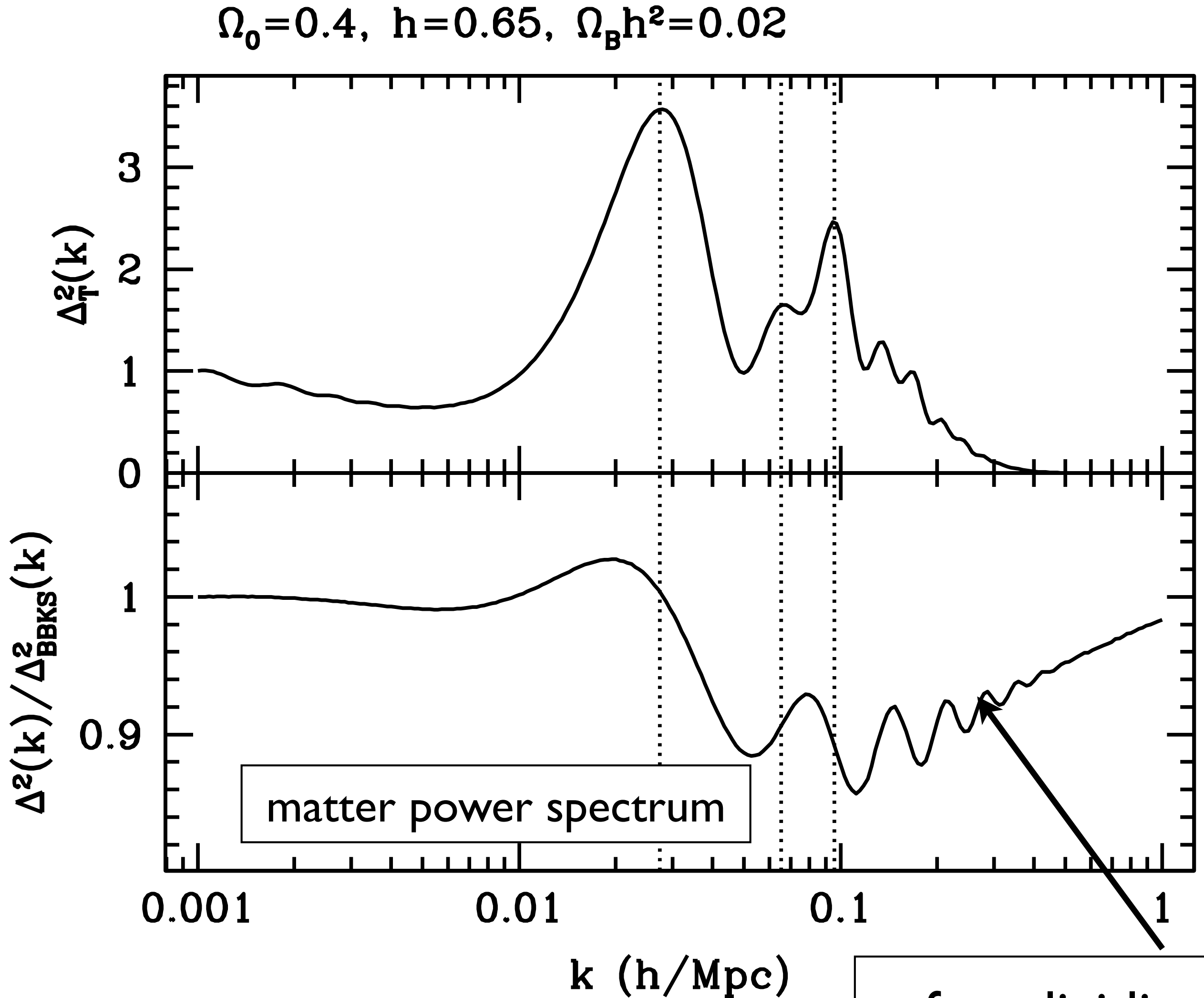


Matter Power Spectrum



Has small acoustic oscillations in the matter power spectrum

Here are the CMB and matter power spectrum overlaid one over top of the other



note k = 2π/length (a wavenumber)

CMB and LSS peaks are out of phase

Acoustic peaks in CMB are much more prominent

after dividing out overall shape

This makes the acoustic peaks more obvious

Meiksin et al. 1999

But where do these acoustic peaks come from?

Between z = 3500 (when universe became matter dominated) and z = 1080 (photons and baryons decoupled):

Perturbations in baryonic material cannot grow (being coupled to radiation) and will just oscillate

baryonic material ⇒ no growth

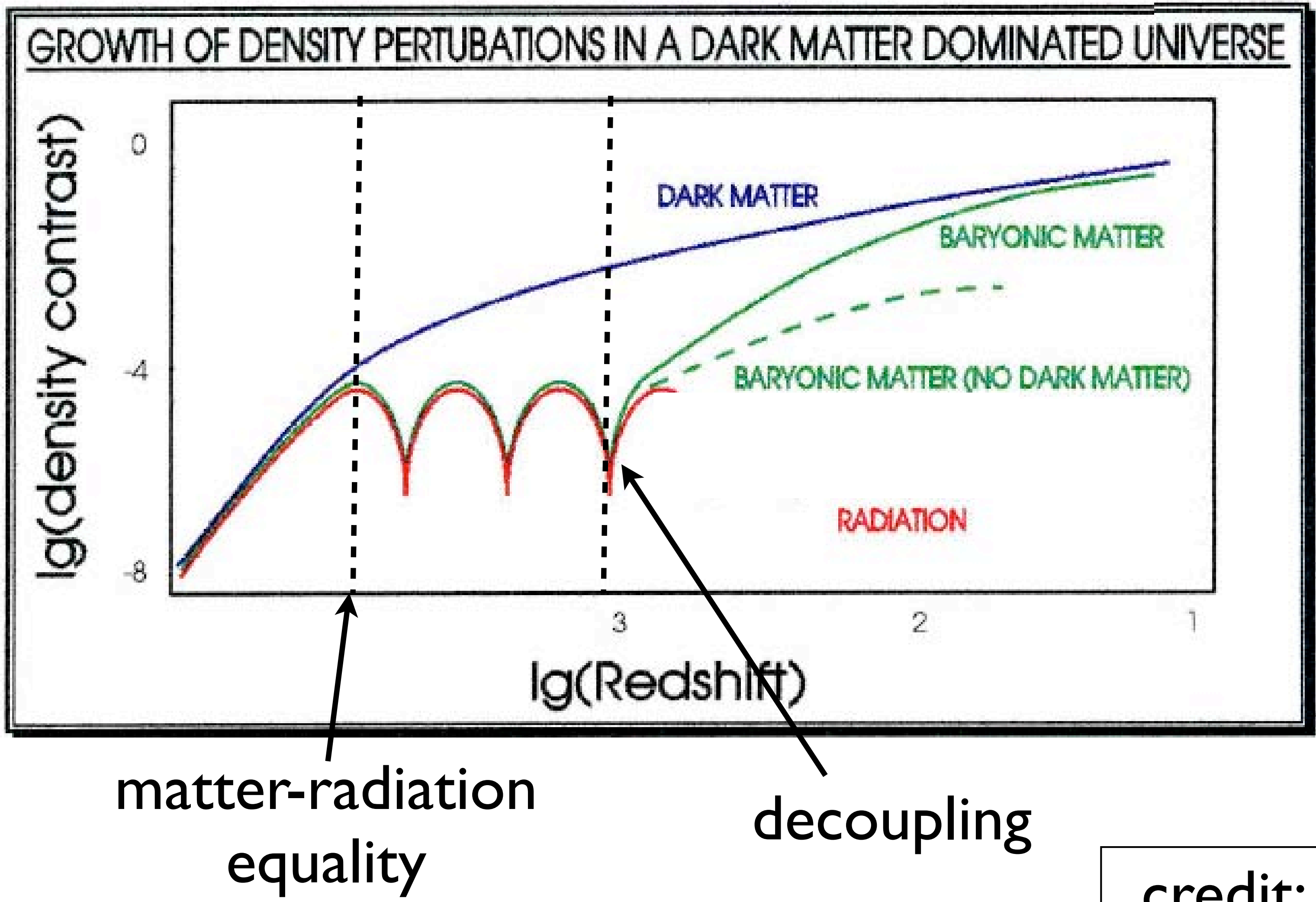
Perturbations in dark material can grow (not being coupled to the radiation)

dark matter ⇒ growth

As a result, perturbations in dark matter get a head start

But where do these acoustic peaks come from?

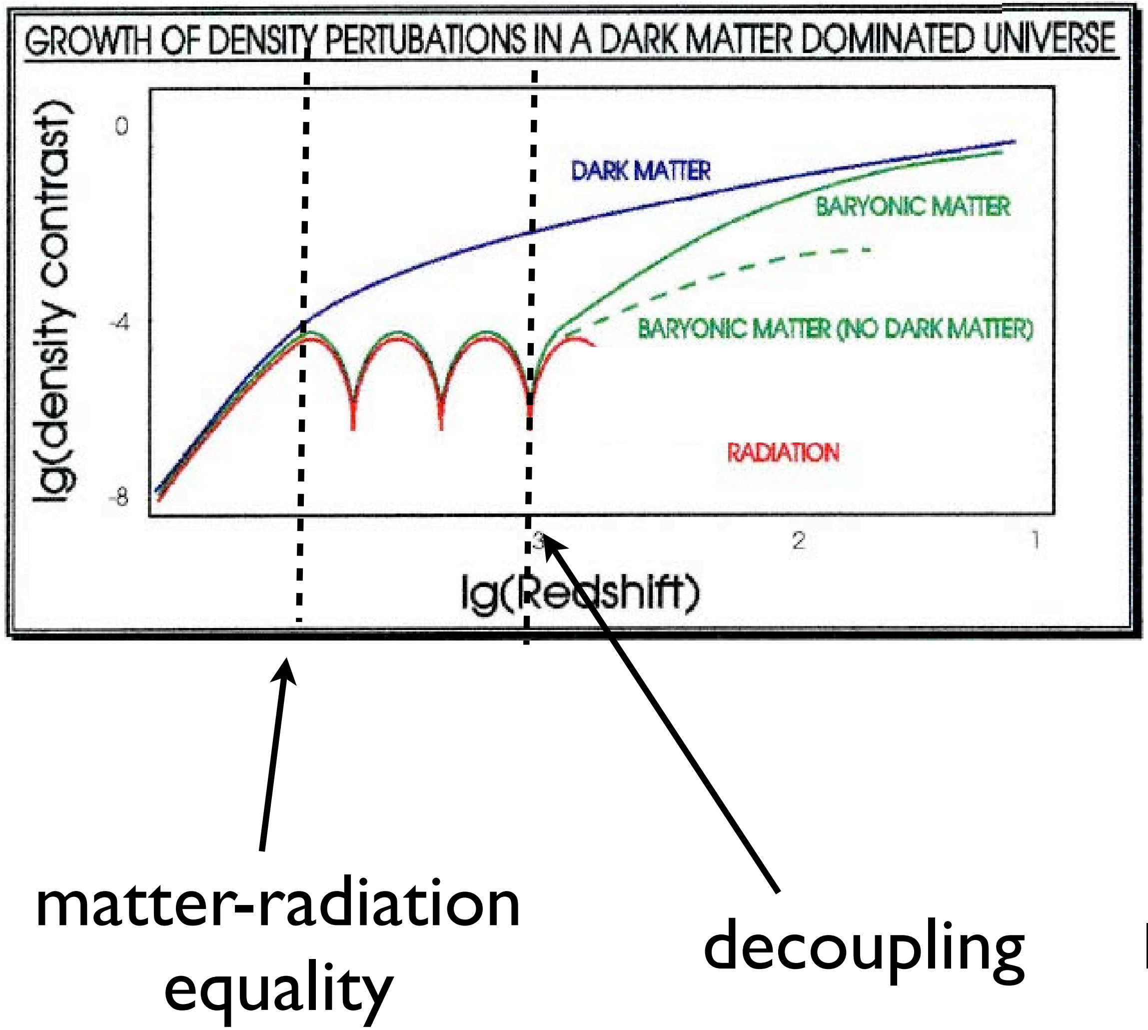
here's an illustration (notice difference between dark matter and baryonic matter)



credit: Pearson

But where do these acoustic peaks come from?

here's an illustration (notice difference between dark matter and baryonic matter)



Before decoupling, perturbations in dark matter are able to grow, but perturbations in baryons are not.

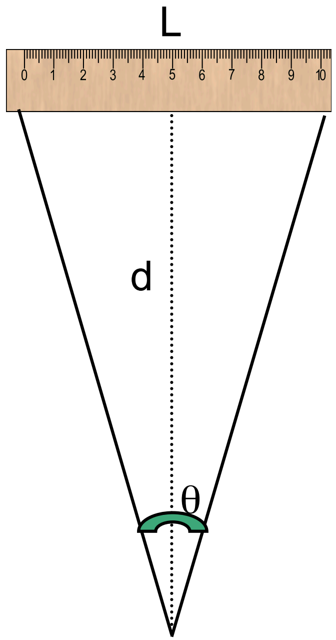
After decoupling, baryons fall into overdensities from dark matter.

But, in doing so, they affect the dark matter; they add the oscillatory ringing structure to larger perturbations defined by dark matter

Why do we care about these small oscillations in matter power spectrum caused by baryons?

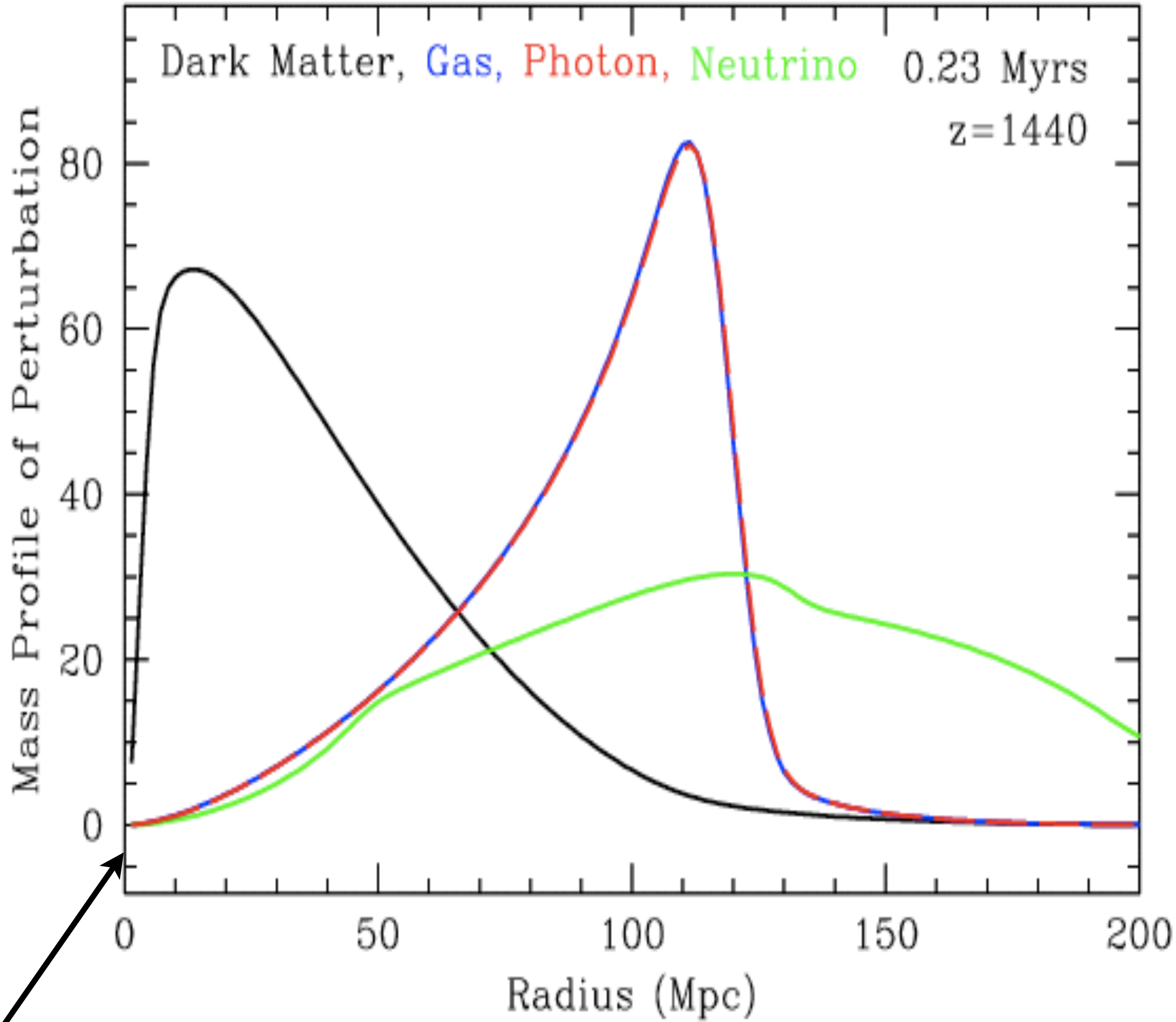
These oscillations cause there to be preferential structure at a certain comoving physical scale!

It provides us with a standard rod again that we can use to learn about the universe!



We can therefore do a galaxy survey at any epoch or redshift, measure the power spectrum, and look for the acoustic peak from baryons!

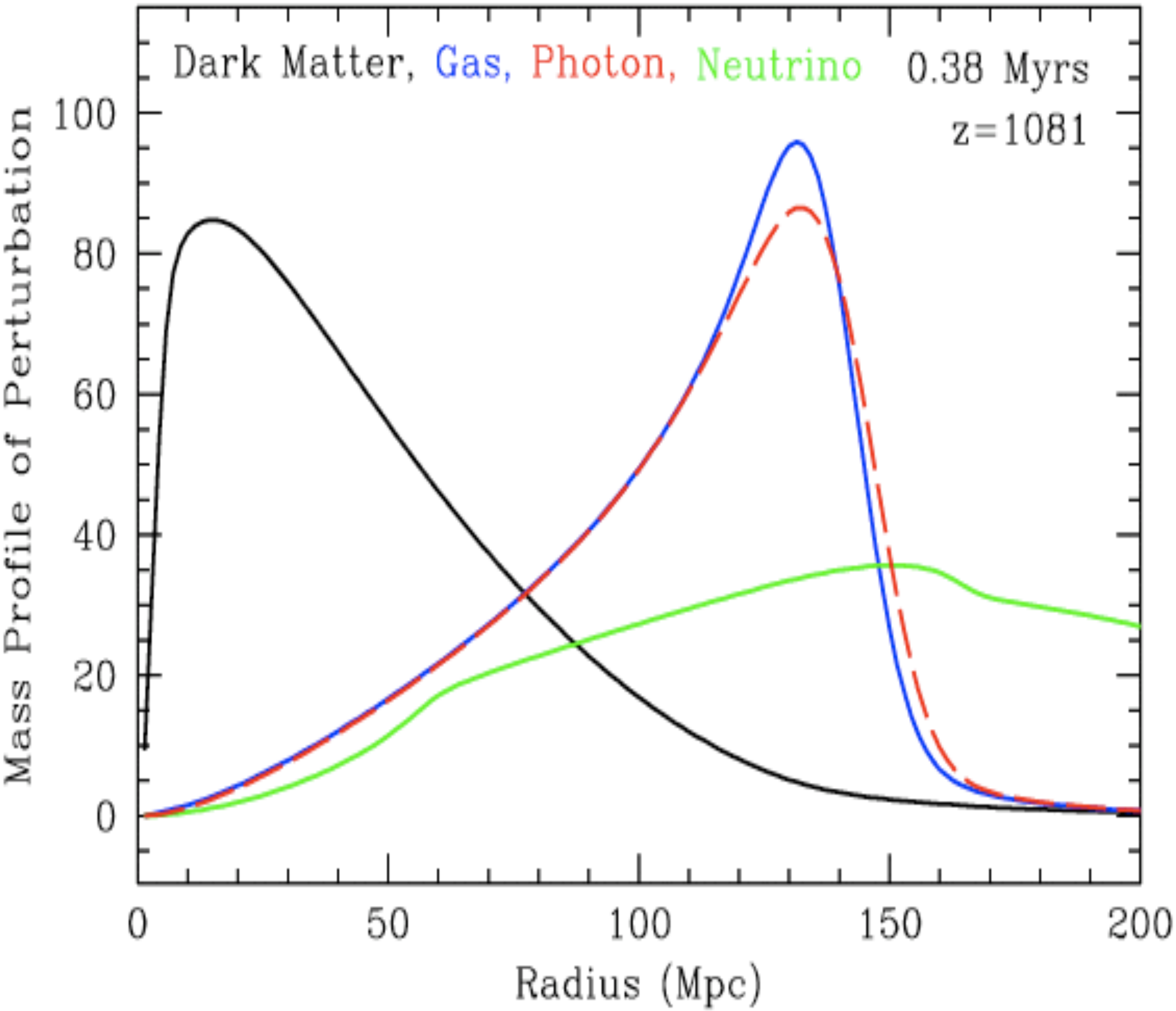
It will define same comoving scale at all epochs!



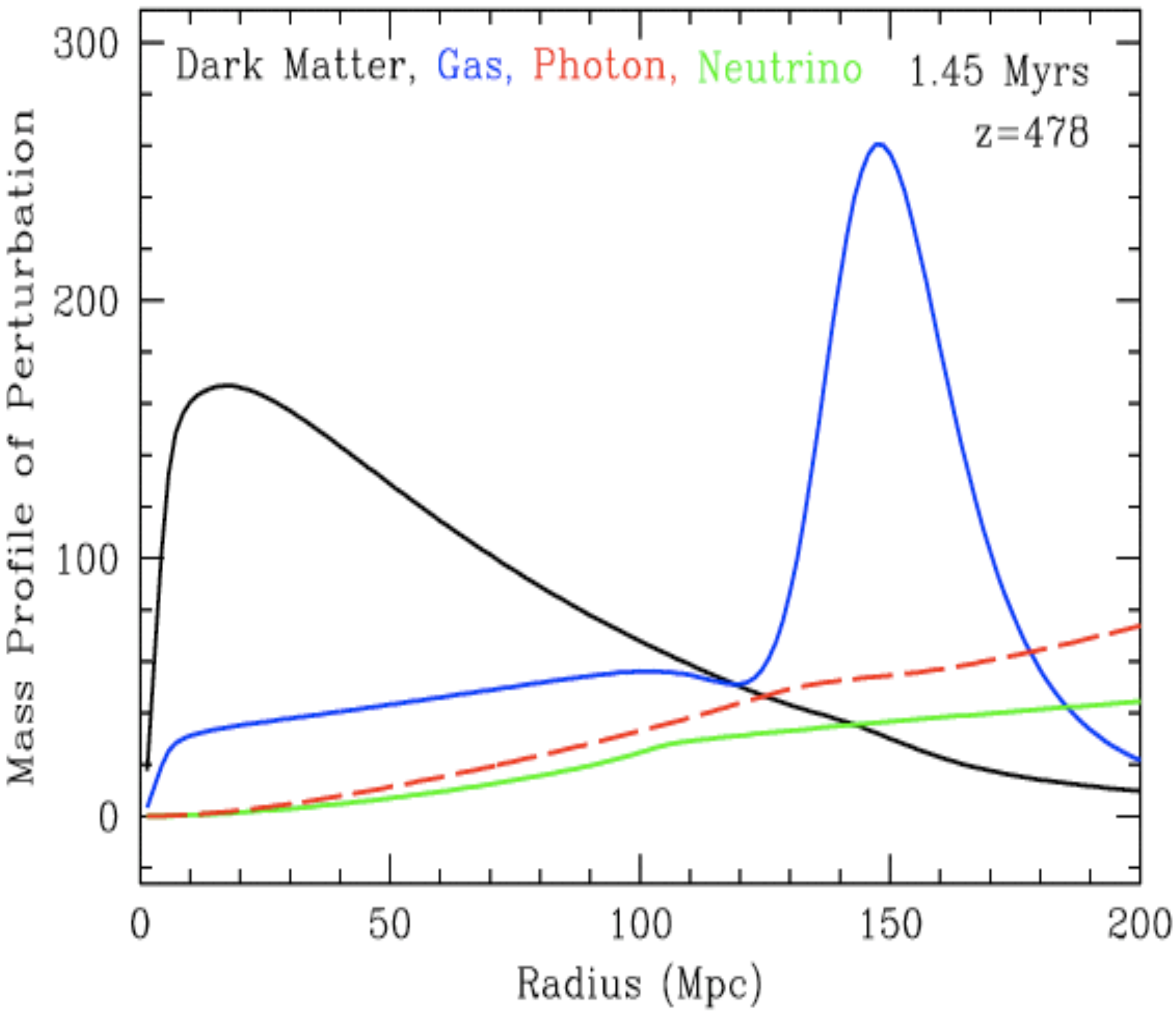
imagine we have a overdensity here at time t = 0

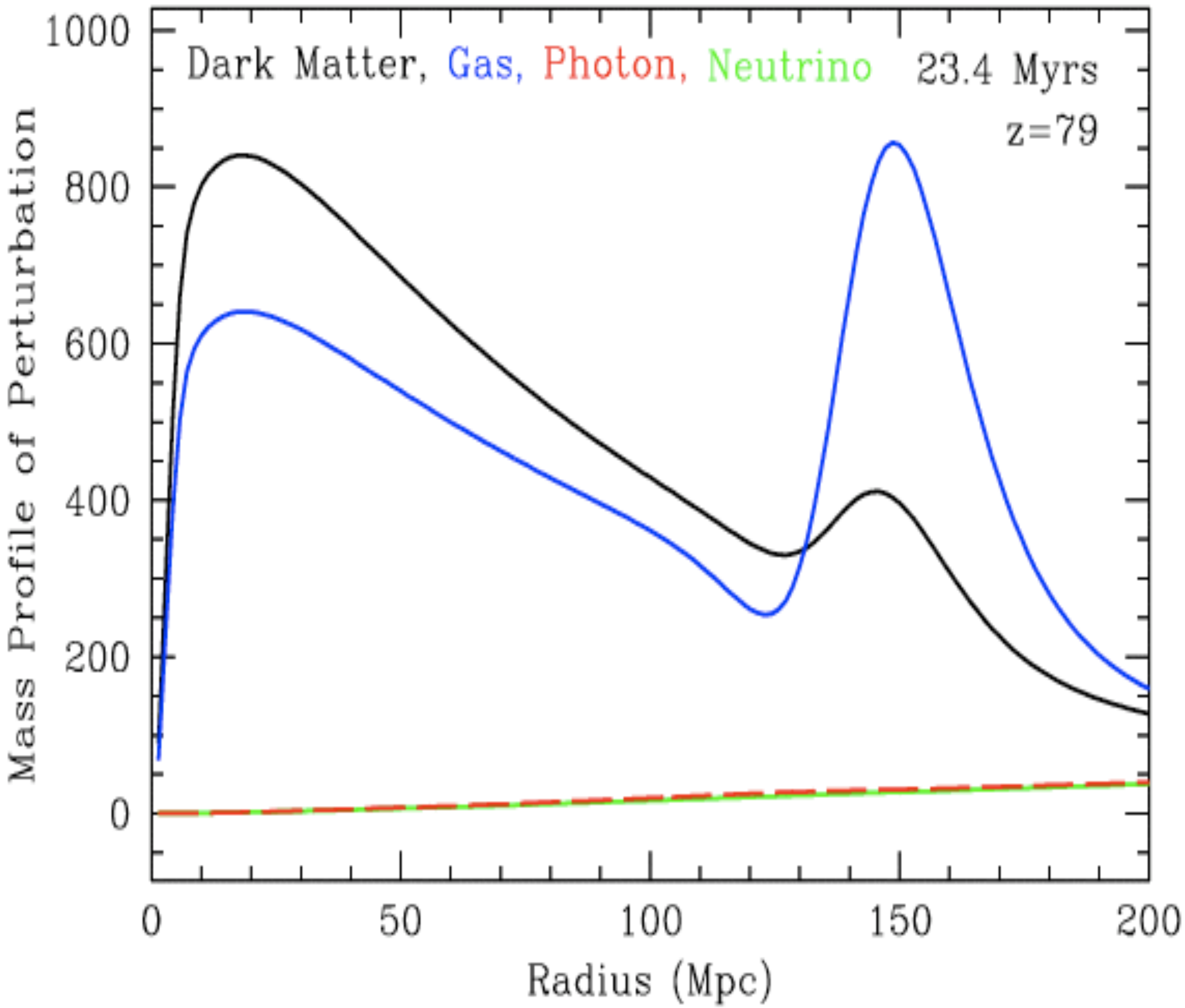
dark matter will fall towards it

but baryons and radiation will bounce

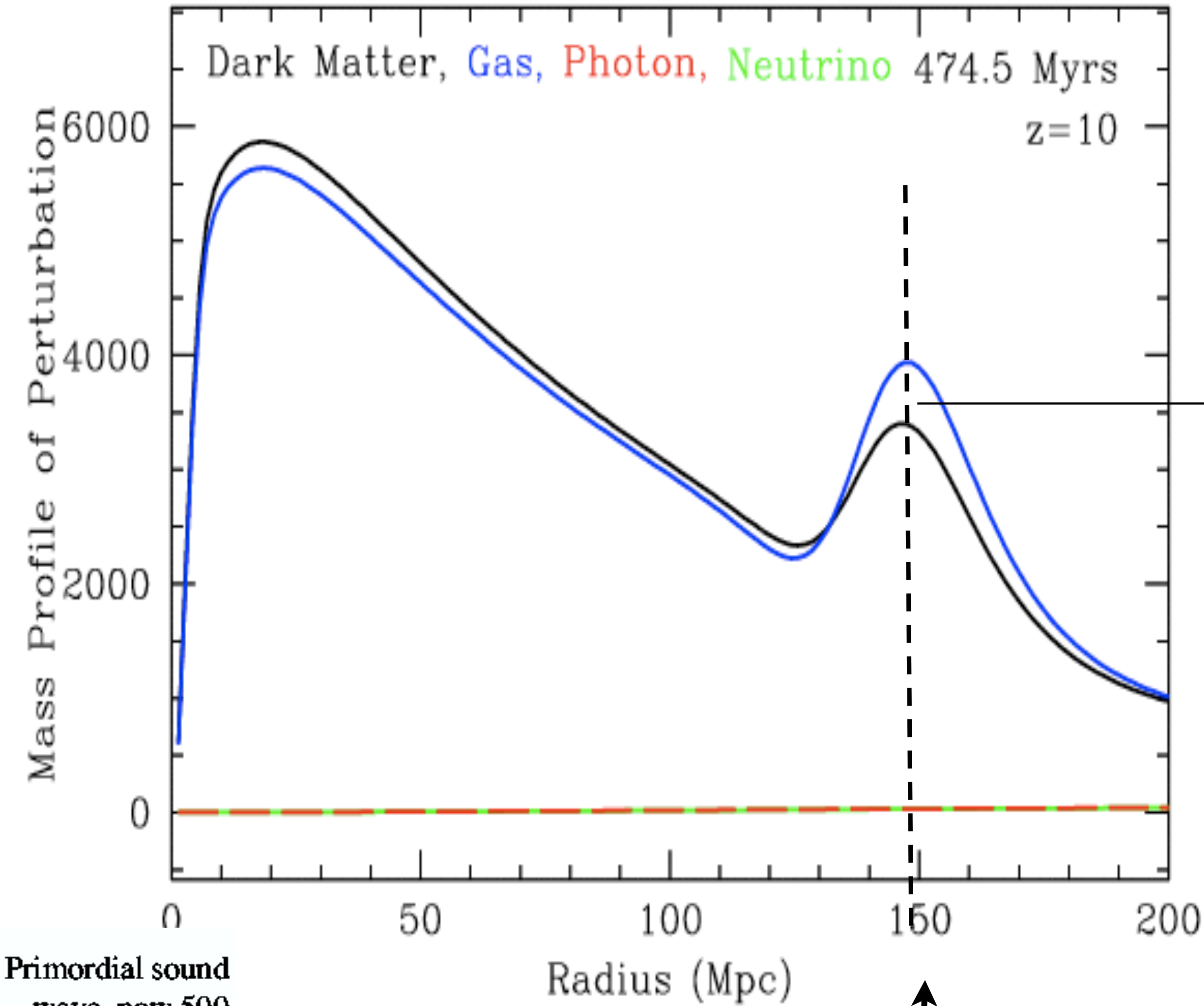


Dark Matter, Gas, Photon, Neutrino
0.38 Myrs
z=1081
Mass Profile of Perturbation
Radius (Mpc)
0
20
40
60
80
100
0
50
100
150
200

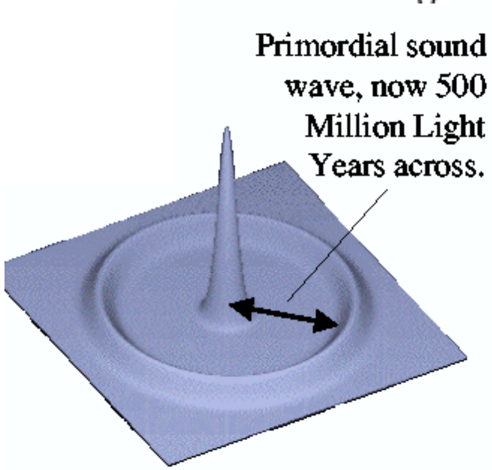

Dark Matter, Gas, Photon, Neutrino
1.45 Myrs
z=478
Mass Profile of Perturbation
Radius (Mpc)
0
100
200
300
0
50
100
150
200



Dark Matter, Gas, Photon, Neutrino
23.4 Myrs
z=79
Mass Profile of Perturbation
Radius (Mpc)
0
200
400
600
800
1000
0
50
100
150
200



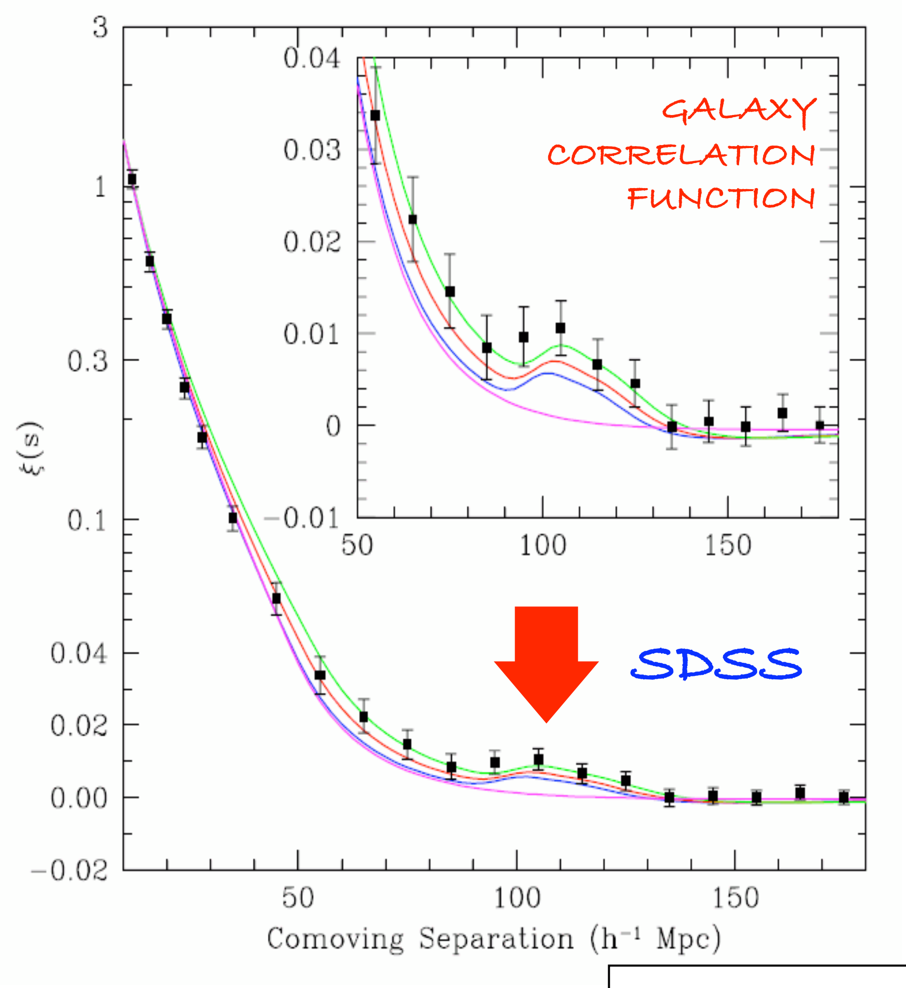
Sound horizon at matter-radiation decoupling



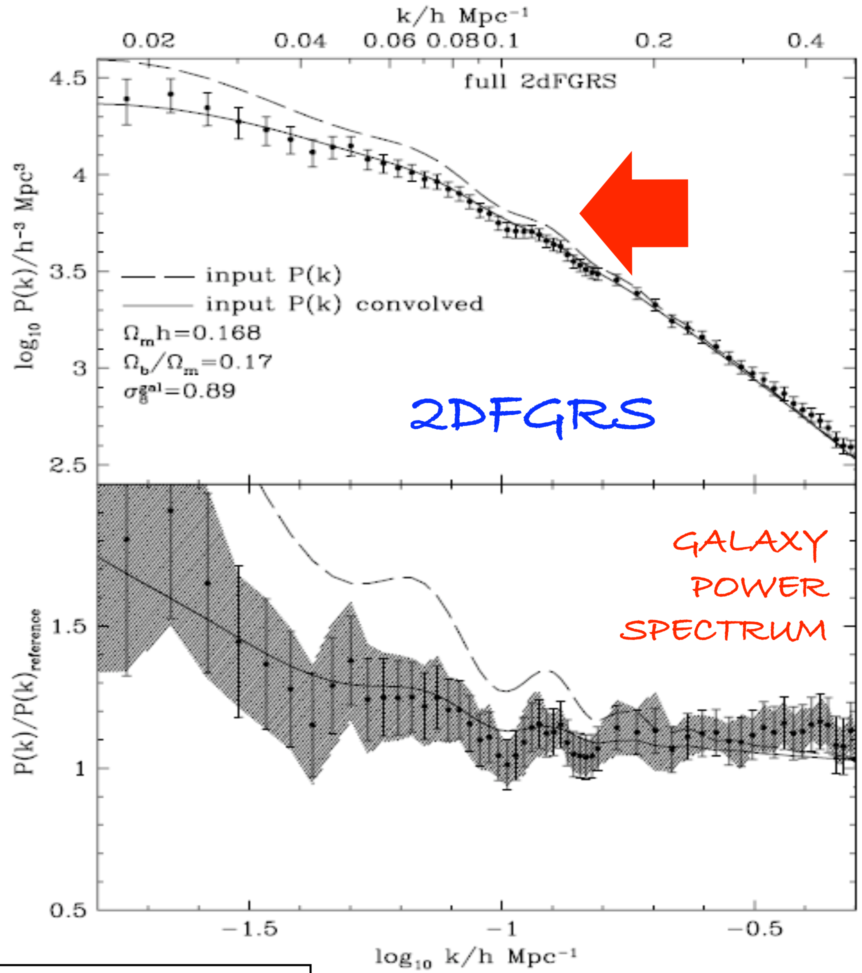
this bump is at 150 Mpc!

By measuring the correlation function for a galaxy survey we can look for this bump (from baryon acoustic oscillations)

SDSS: EISENSTEIN ET AL. (2005)



2DFGRS: COLE ET AL. (2005)



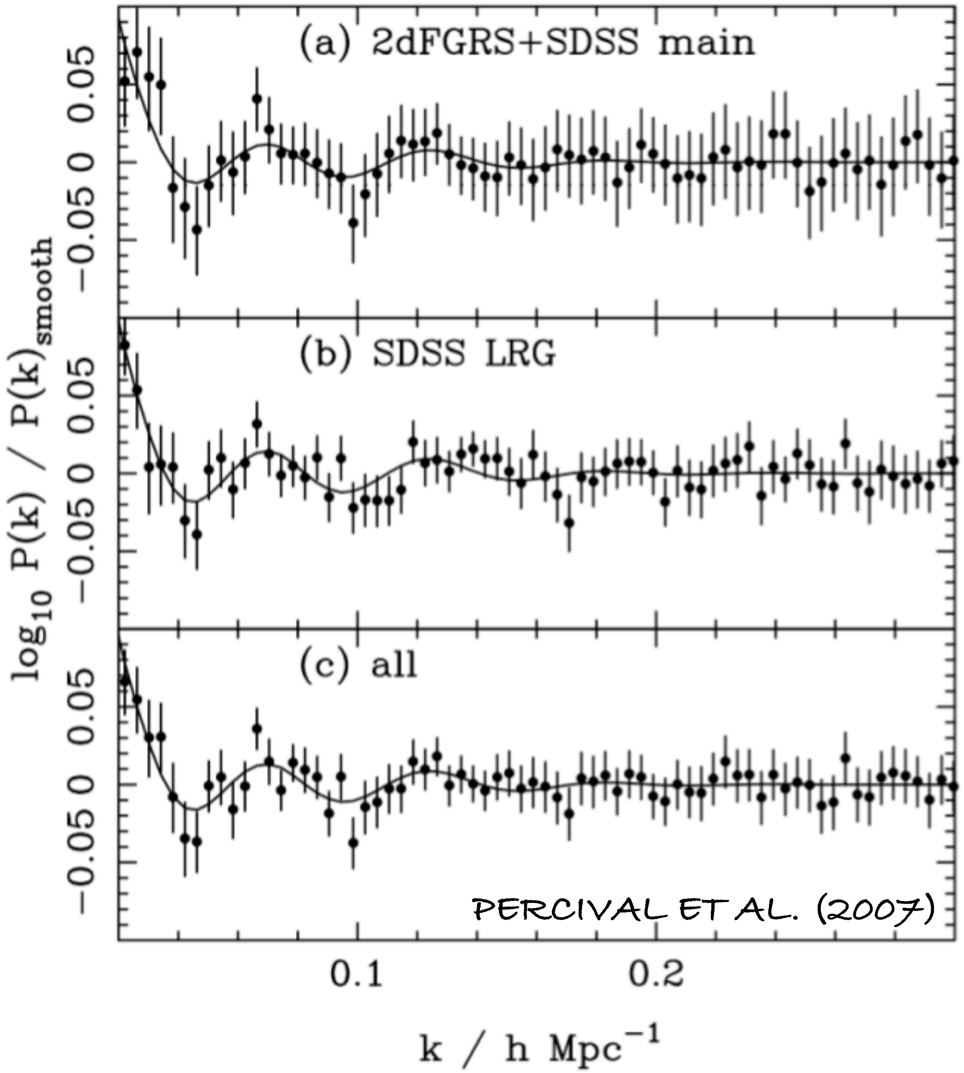
First Measurements

More recent State-of-the-art measurements of the baryonic acoustic oscillations

Detected at 99.74% confidence!

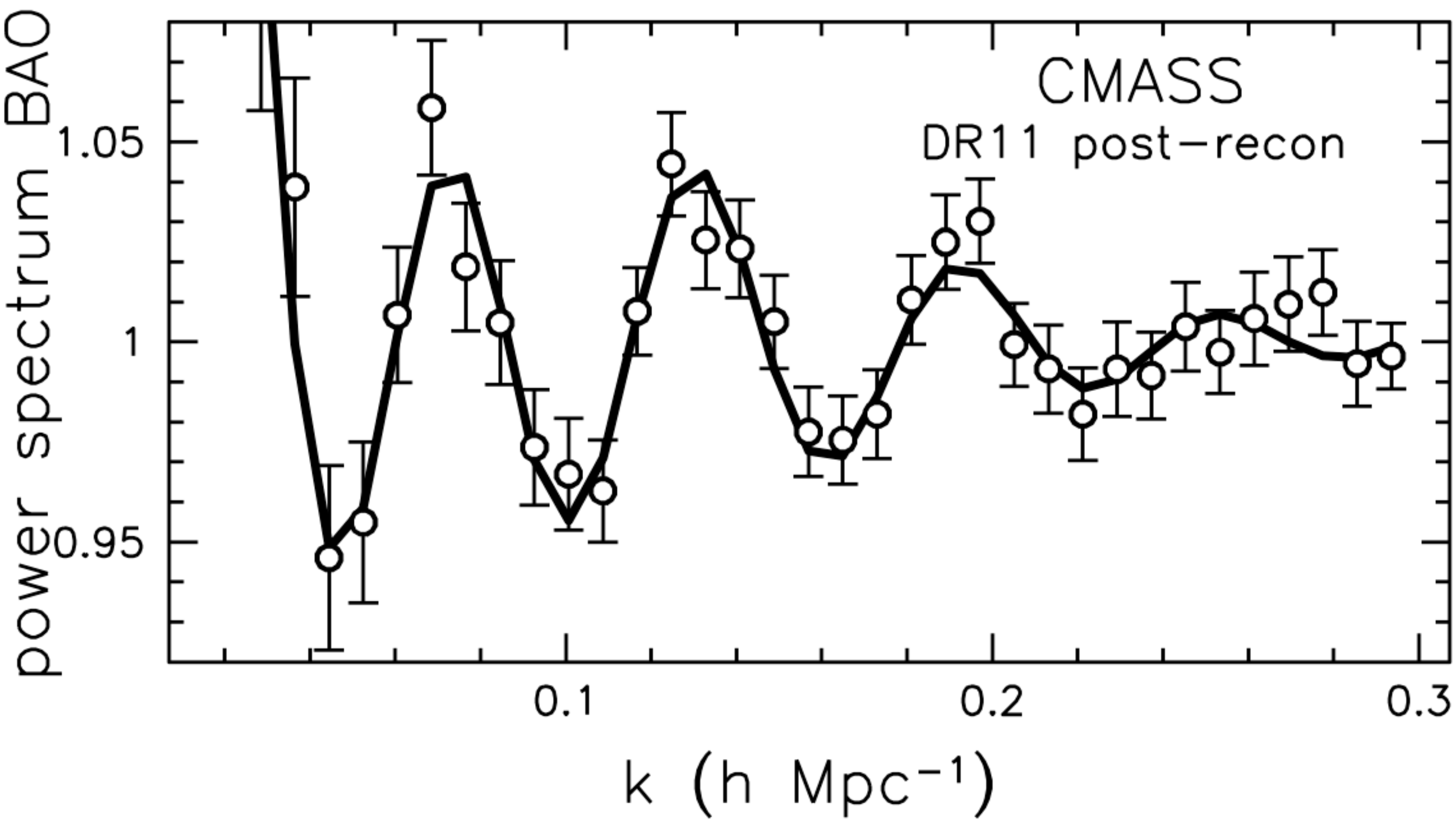
$\Omega_m = 0.256 \pm 0.027$

Allows us to examine same basic standard rod at both z = 0.35 and z = 1100 (CMB)



Now the BAO technique has been used out to z>~0.6...

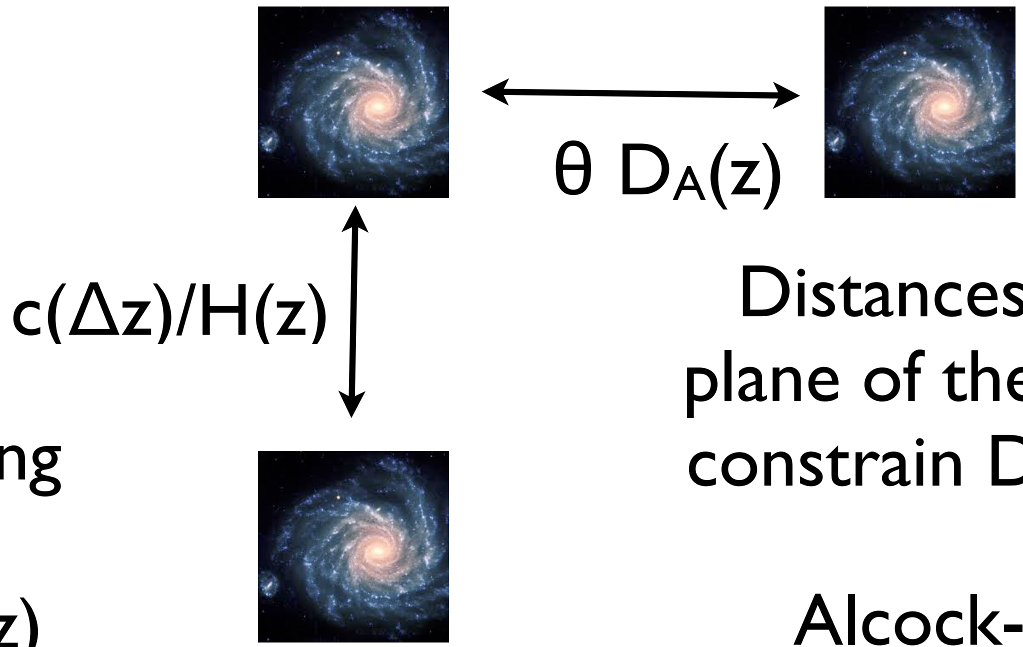
Results of BOSS survey at z~0.55



Anderson+2013

The Baryon Acoustic Oscillation Method can be used to look for structure in the plane of the sky, but also along the line of sight

Observables of interest for constraining the cosmology: $D_A(z)$, $H(z)$



Distances in plane of the sky constrain $D_A(z)$

Distances along line of sight constrain $H(z)$

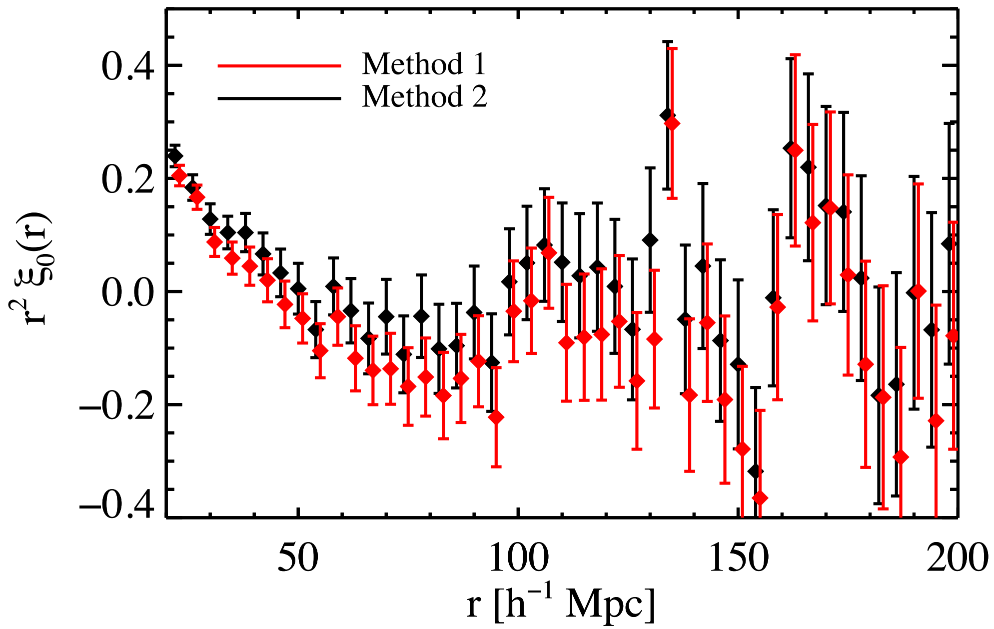
telescope

Alcock-Paczynski constraints

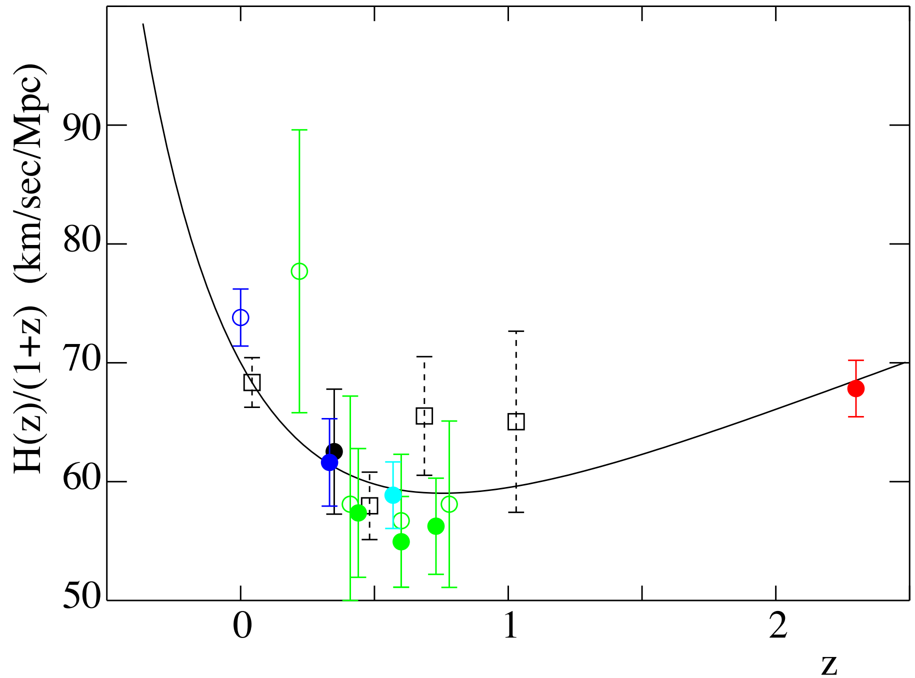
both length scales must be the same

BAO have also been used to constrain H(z)... amazing out to z~2.3...

Power spectrum measured for absorption lines from gas at z~2.3 in z~2.5 quasars



Constraints on the evolution of the Hubble parameter to z~2.3



Busca+2013

Baryonic Acoustic Oscillations are one of the four main techniques being used for dark energy experiments at present.

The other three are the following:

1. Galaxy Clusters
2. Cosmic Shear
3. Supernovae Ia Search Experiments

Enigma of Dark Energy

Already up to this point in the course, you have already seen many different pieces of evidence for some form of dark energy, which we have expressed as $\Omega_\Lambda > 0$

There is an overwhelming amount of evidence for its existence

→ SNe Search Experiments

Observed SNe in distant galaxies are observed to be fainter than they would otherwise be without dark energy

→ Late Integrated Sachs-Wolfe Effect

Dark Energy Affects the Differential Redshifting of CMB photons as they move in and out of gravitational potential. By cross correlating known galaxy clusters with CMB, we can observe this effect.

→ First Acoustic Peak of CMB Implies Universe is Flat, while other evidence indicates $\Omega_M \sim 0.3$ (Large Scale Flows, Kaiser Effect, Ratio of Baryons and Total Matter in Galaxy Clusters, Large Scale Structure, Baryon Acoustic Oscillations)

Enigma of Dark Energy

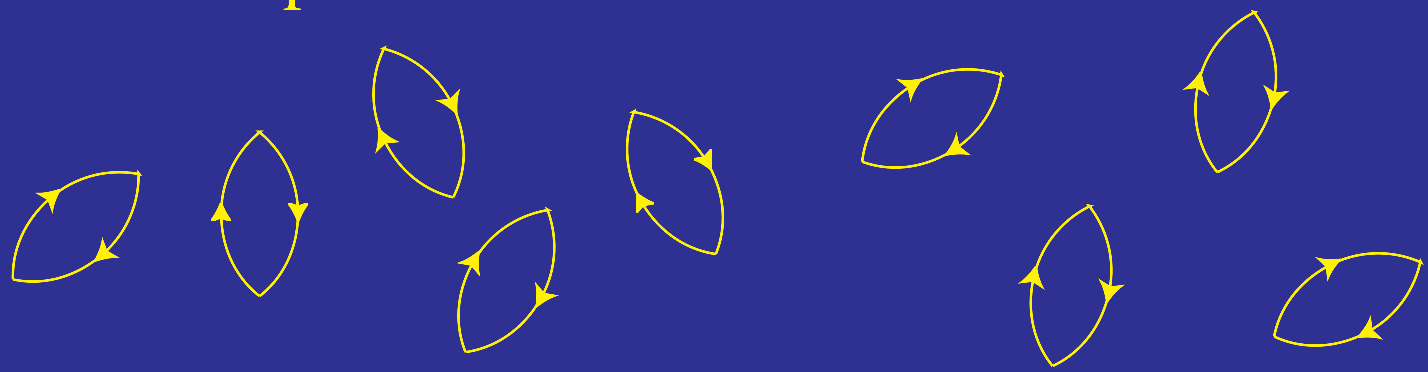
As you all know, a key component of universe is dark energy. Very roughly, it exerts a repulsive force on the space fabric -- increasing its acceleration.

There is an overwhelming amount of evidence for its existence

However, its nature remains an enigma

Enigma of Dark Energy

- Constant energy density, hence increasing net energy as universe expands consistent with data
- Quantum mechanics allows/predicts such phenomena in the form vacuum energy: empty space is alive with virtual particles



- Naive prediction is 10^{120} times too big and more sophisticated models still 10^{60} off

Credit Hu

→ Possibly more natural to explain dark energy as a scalar field that evolves with cosmic time...

Enigma of Dark Energy

As a result of there is a lot of interest in exploring forms of dark energy that are not constant, but evolve with cosmic time

Quote from Dark Energy Task Force

VI. A Dark Energy Primer

In General Relativity (GR), the growth of the Universe is described by a scale factor $a(t)$, defined so that at the present time t_0, $a(t_0) = 1$. The time evolution of the expansion in GR obeys

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}, \quad > 0$$

This implies that

1. The Universe is dominated by some particle or field (*dark energy*) that has negative pressure, in particular $w = P/\rho < -1/3$; ***or***
2. There is in fact a non-zero cosmological constant; ***or***
3. The theoretical basis for this equation, GR or the standard cosmological model, is incorrect.

Enigma of Dark Energy

In order to ascertain the form of dark energy, we parameterize its effects in terms as the w parameter:

$$P = w\rho c^2$$

Typically take c = 1

There are a few important cases:

Type dark energy	w	redshift scaling of DE density	dynamical significance
Cosmological Constant λ	-1	Constant	z<1
Quintessence	$-1 < w < -1/3$	$(1+z)^{+1}$ for w=-2/3	earlier
Phantom Energy	w<-1	$(1+z)^{-1}$ for w=-4/3	later

Enigma of Dark Energy

In the case of quintessence or phantom energy (w < -1 or w > -1), the dark energy density evolves with cosmic time.

How does it evolve?

Friedmann's equations:

w = P/ρ

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 + k \qquad \ddot{a} = -\frac{4\pi G}{3}\rho(1+3w)a \qquad (\text{for } w = \text{const.})$$

differentiate with respect to time →

multiply by da/dt

$$2\dot{a}\ddot{a} = \frac{8\pi G}{3}\frac{d}{dt}\left(\rho a^2\right) \qquad \dot{a}\ddot{a} = -\frac{4\pi G}{3}\rho(1+3w)a\dot{a}$$

$$\frac{d}{dt}\left(\rho a^2\right) = -\rho(1+3w)a\dot{a}$$

$$\frac{d}{dt}\left(\rho a^2\right)a + \rho a^2\dot{a} = -\rho(3w)a^2\dot{a}$$

$$\frac{d}{dt}\left(\rho a^3\right) = -\rho(3w)a^2\dot{a}$$

$$\dot{\rho}a^3 + 3\rho\dot{a}a^2 = -3w\rho a^2\dot{a}$$

$$\frac{d\log\rho}{dt} = -3(1+w)\frac{d\log a}{dt} \quad \Rightarrow \quad \rho \propto a^{-3(1+w)}$$

For w = -1 the density is constant.

Enigma of Dark Energy

Given this evolution in the energy density of dark energy, the second Friedmann equation can be rewritten as follows:

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_X a^{-3(1+w)}\right],$$

The term Ω_X represents the cosmological constant if $w = -1$. Otherwise, it represents dark energy with constant w.

Based on the above equation, we can derive all the standard formulas for the distances, evolution of the Hubble constant, growth factors, etc., but let us before doing this, let us consider another case first.

Time Varying Dark Energy

the most generic model for Dark Energy allows for a time variation in the equation of state parameter: $w = w(z)$

common parameterizations: $w(z) = w_0 + w_1 z$

$w(z) = w_0 + w_a(1-a) = w_0 + w_a z/(1+z)$

For this parameterization, we can rewrite the $a^{-3(1+w)}$ factor in the second term of the Friedmann equation in the following manner:

$$a^{-3(1+w)} \rightarrow \exp\left(3\int_a^1 \frac{da'}{a'}\left[1+w(a')\right]\right) .$$

for a time-independent w(a), this just reduces to $a^{-3(1+w)}$

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_X a^{-3(1+w)}\right],$$

How does the energy density in dark energy evolve relative to other components of universe for these more generic models?

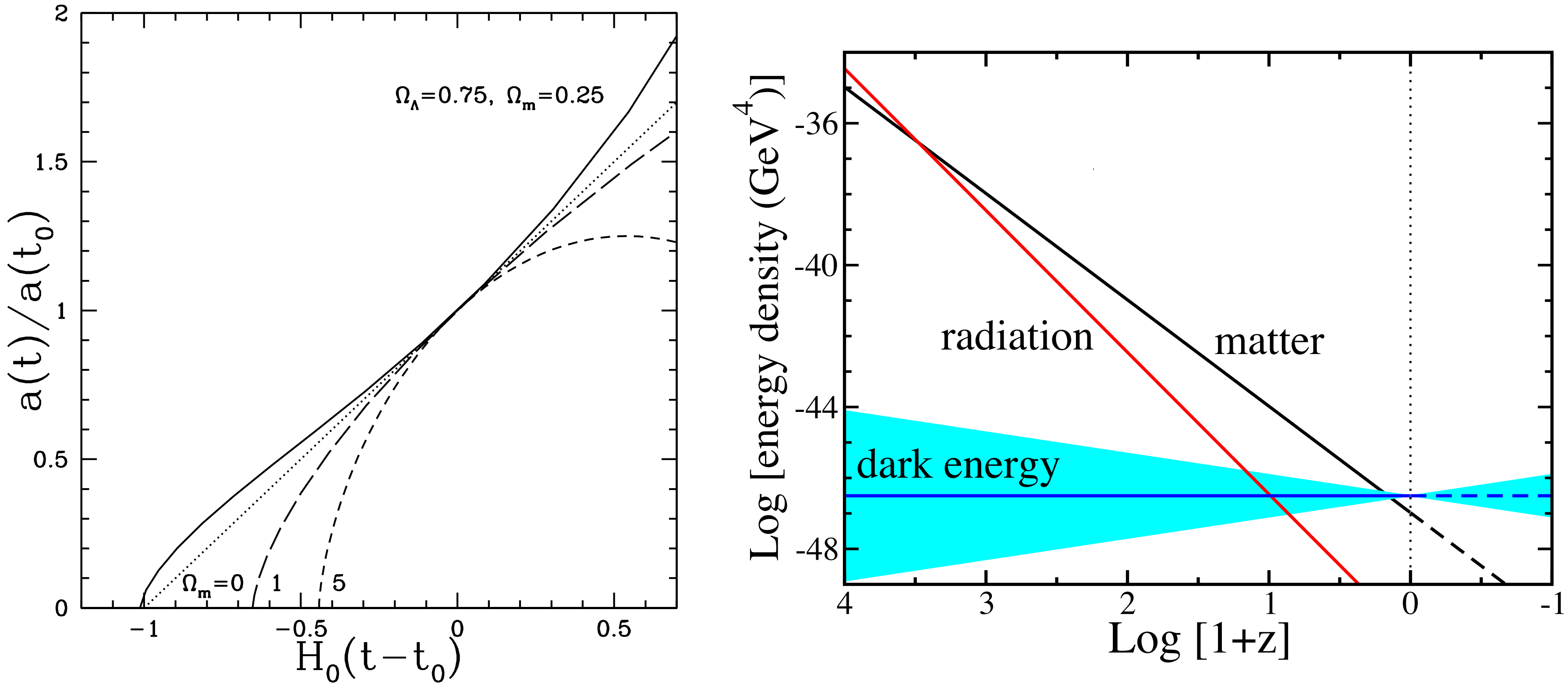
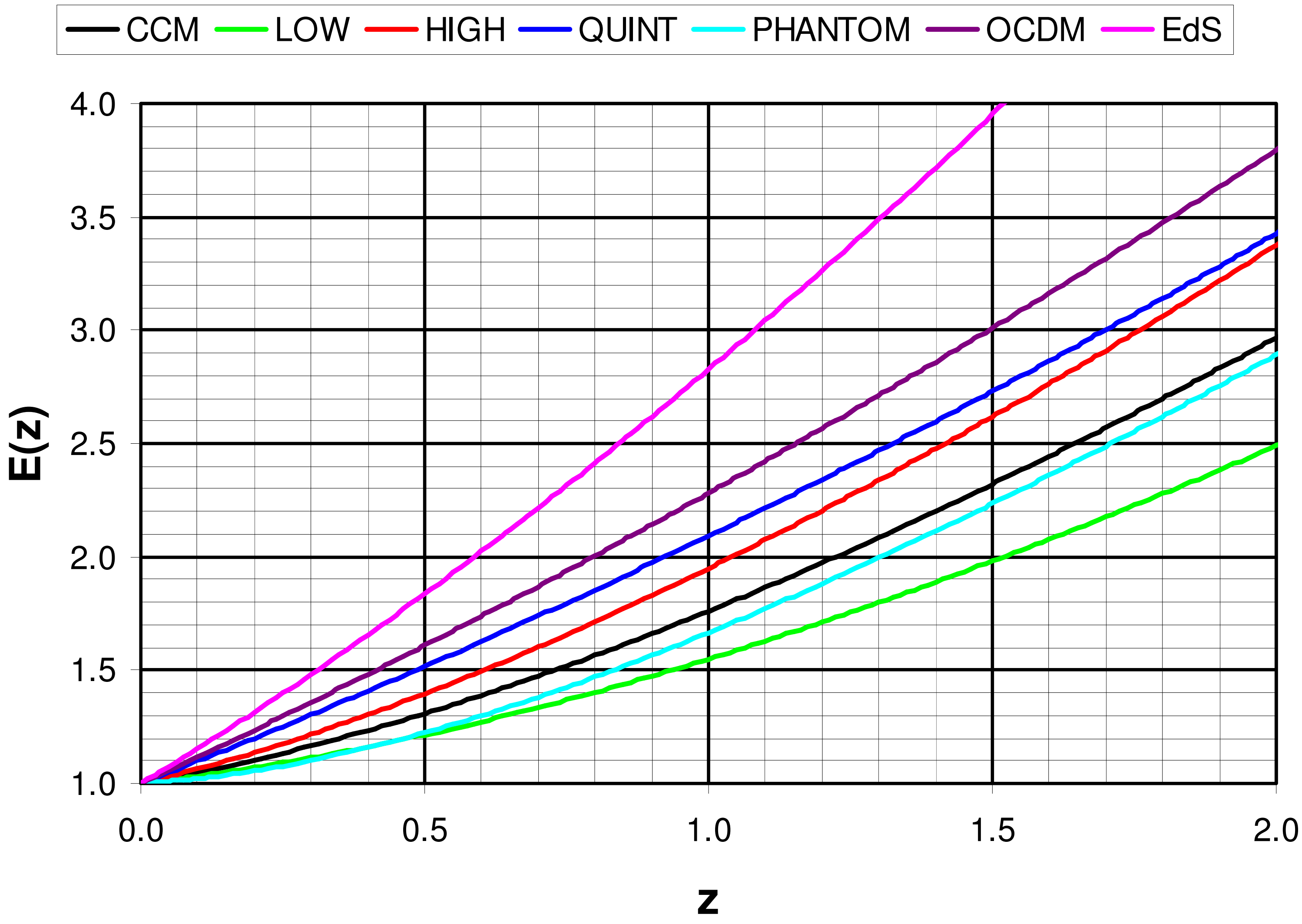


FIGURE 1. Left panel (a): Evolution of the scale factor vs. time for four cosmological models: three matter-dominated models with $\Omega_0 = \Omega_m = 0, 1, 5$, and one with $\Omega_\Lambda = 0.75, \Omega_m = 0.25$. Right panel (b): Evolution of radiation, matter, and dark energy densities with redshift. For dark energy, the band represents $w = -1 \pm 0.2$. From Frieman et al. [13].

How does this change the behavior of quantities we calculated before?

Evolution Function E(z)

$H(z) = H_0 E(z)$



$[\Omega_m, \Omega_{DE}, w]$

EdS [1.0,0,0]

OCDM [0.3,0,0]

QUINT [0.3,0.7,-0.5]

HIGH [0.4,0.6,-1]

CCM [0.3,0.7,-1]

PHANTOM [0.3,0.7,-1.3]

LOW [0.2,0.8,-1]

for flat and open geometries, E(z) is a monotonic function of z

Credit: Fassbender

We can also apply this modified E(z) factor to our calculation of distances.....

Comoving Distance :

$$D(z) = R_0 \cdot r = \begin{cases} \dfrac{d_H}{\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|} \displaystyle\int_0^z \dfrac{dz}{E(z)} \right) & \text{closed Universe } k=1,\ \Omega_k<0 \\ d_H \displaystyle\int_0^z \dfrac{dz}{E(z)} & \text{flat geometry } k=0,\ \Omega_k=0 \\ \dfrac{d_H}{\sqrt{|\Omega_k|}} \sinh\left(\sqrt{|\Omega_k|} \displaystyle\int_0^z \dfrac{dz}{E(z)} \right) & \text{open Universe } k=-1,\ \Omega_k>0 \end{cases}$$

closed Universe
k=1, Ω_k<0

flat geometry
k=0, Ω_k=0

open Universe
k=-1, Ω_k>0

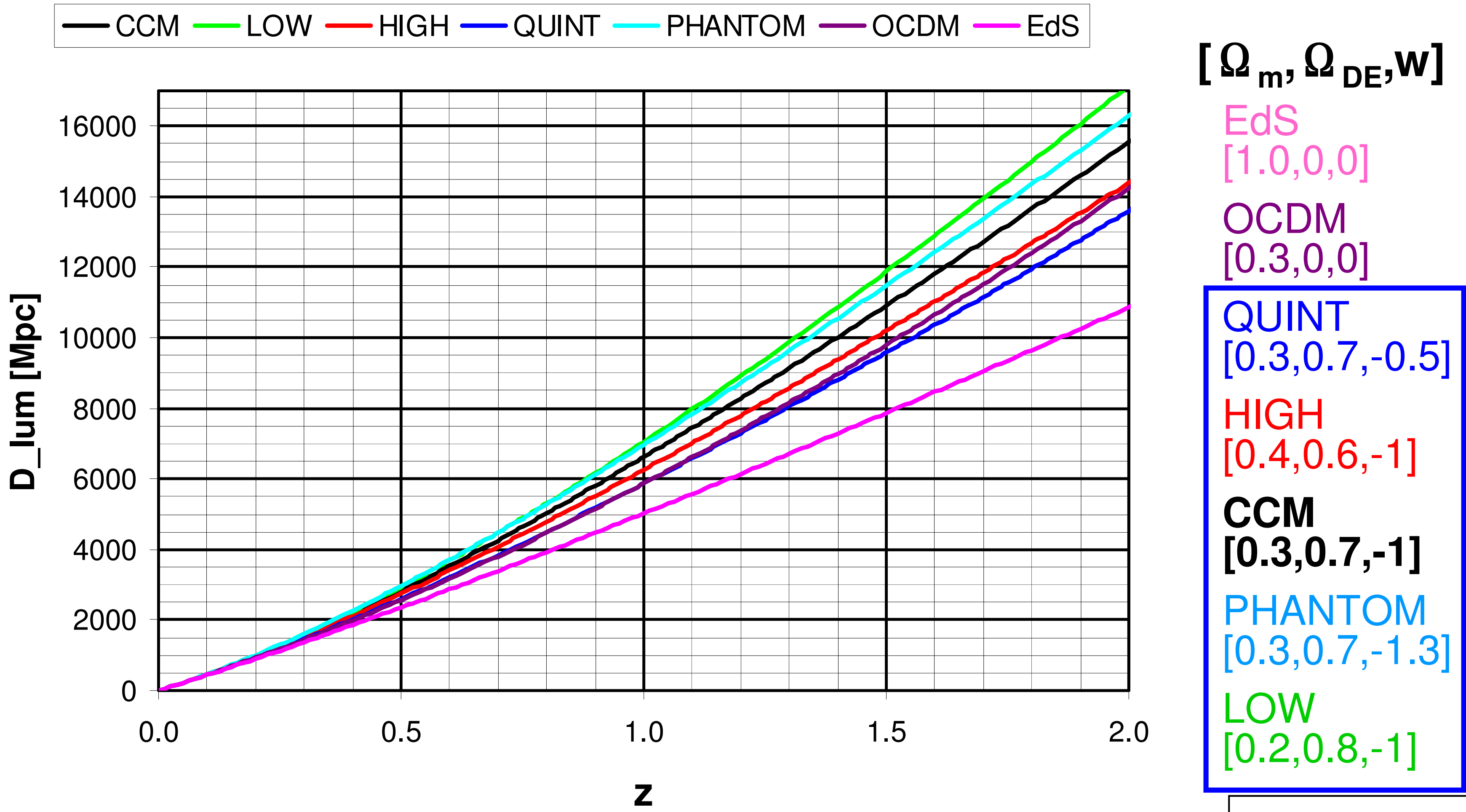
$$\text{mit}: \quad \Omega_k \equiv 1 - \Omega_m - \Omega_\Lambda \quad \& \quad d_H = 4280 h_{70}^{-1}\,\text{Mpc}$$

Luminosity Distance : $$D_L(z) = (1+z) \cdot D(z)$$

Angular Diameter Distance : $$D_A(z) = \xi(z) = \frac{D(z)}{1+z}$$

What is the effect on the Luminosity Distance D_L?

- cosmic distances are proportional to the integral over 1/E(z), i.e. the area under this function out to redshift z
- higher expansion rates in the past, i.e. larger values for the evolution function E(z), translate into shorter cosmic distances D(z) [for flat geometries]
- the larger the influence of Dark Energy, the larger the cosmic distances



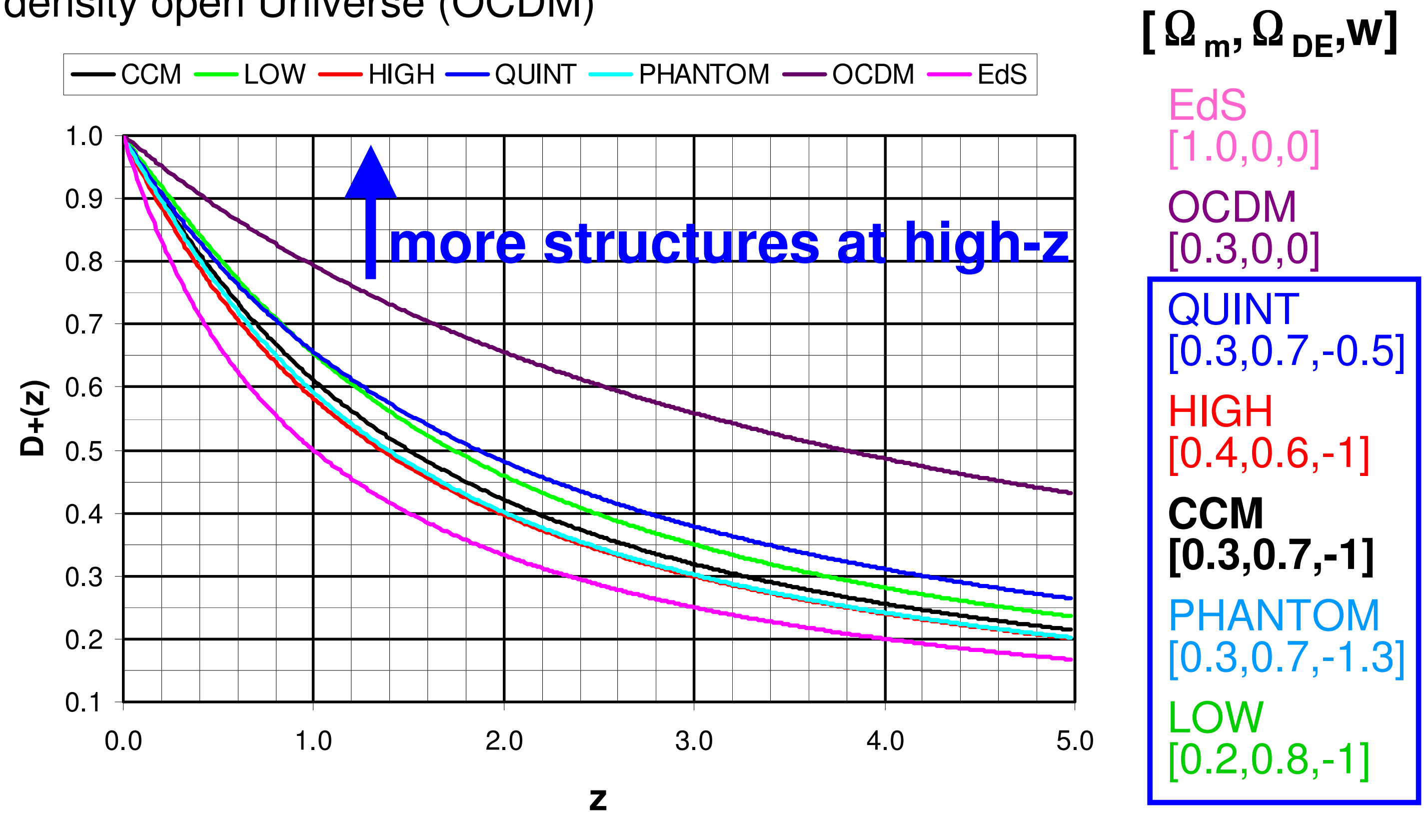
Credit: Fassbender

What is the effect on the Growth Factor?

- the linear structure growth function D+(z) is a solution to the density perturbation growth equation for the linear regime (L3) $\delta(\mathbf{x},t) = D_+(t) \cdot \delta_{i+}(\mathbf{x},t_i)$

$$D_+(z) = \frac{5}{2}\Omega_\mathrm{m} E(z) \cdot \int_z^\infty \frac{1+z'}{E(z')^3} dz'$$

- flat cosmologies with a dark energy component exhibit structure growth in between the Einstein-de Sitter (EdS) case of $D_+=(1+z)^{-1}$ and the slow structure growth of a low density open Universe (OCDM)



Credit: Fassbender

How can we constrain the w parameter?

Generally, we constrain the w parameter in the same way we constrain many other cosmological parameters.

We constrain it by looking at the following quantities versus redshift (cosmic time, see earlier lecture):

Useful in computing distances along the line of sight

$H(z) = H_0 E(z)$ → $E(z)$

Distances

Volume Element

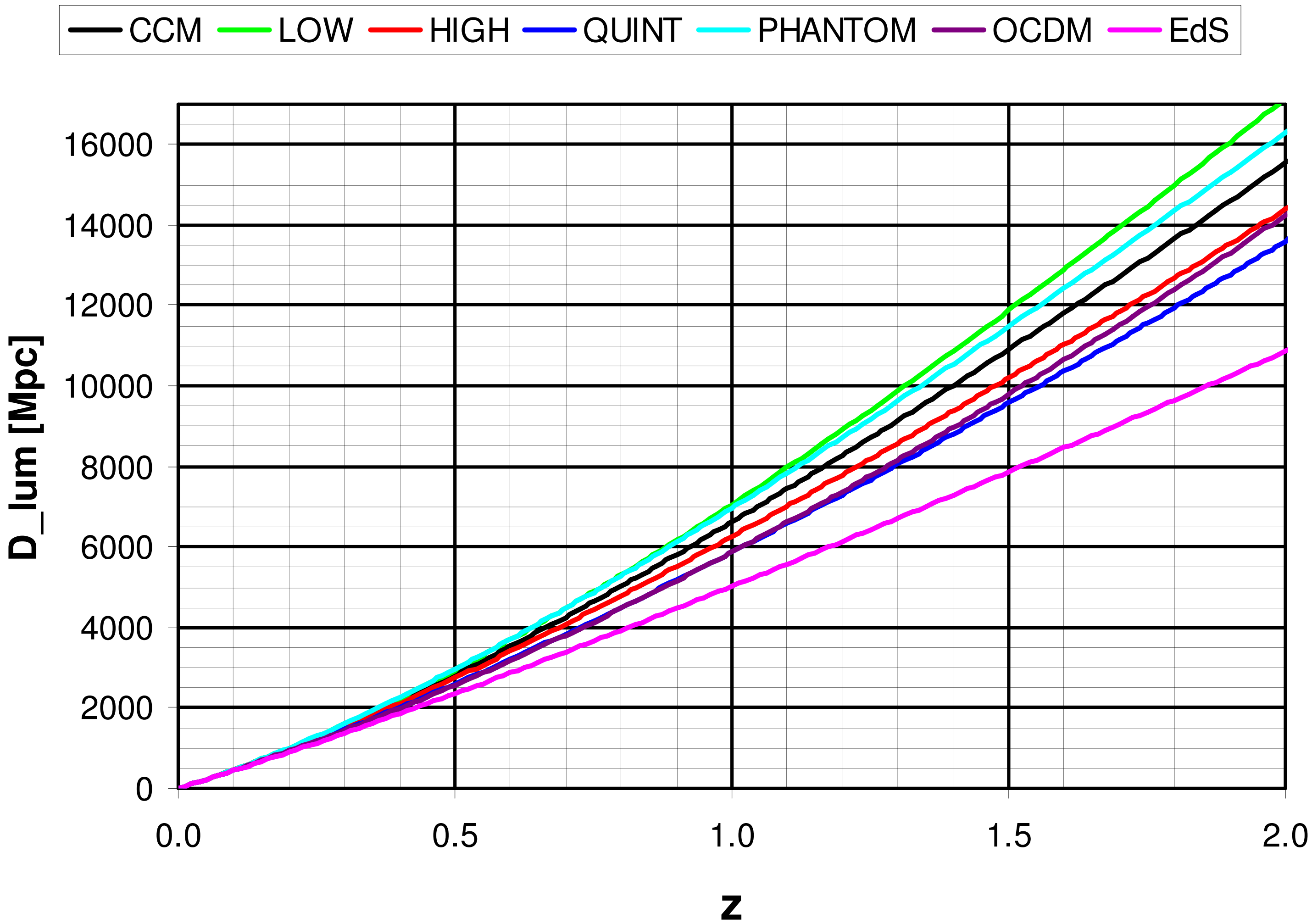
$E(z)$

Geometrical Tests

Growth Factor (Rate at which structures in Universe Grow)

How can we probe this?

Luminosity Distance



[Ω_m, Ω_{DE}, w]

EdS
[1.0,0,0]

OCDM
[0.3,0,0]

QUINT
[0.3,0.7,-0.5]

HIGH
[0.4,0.6,-1]

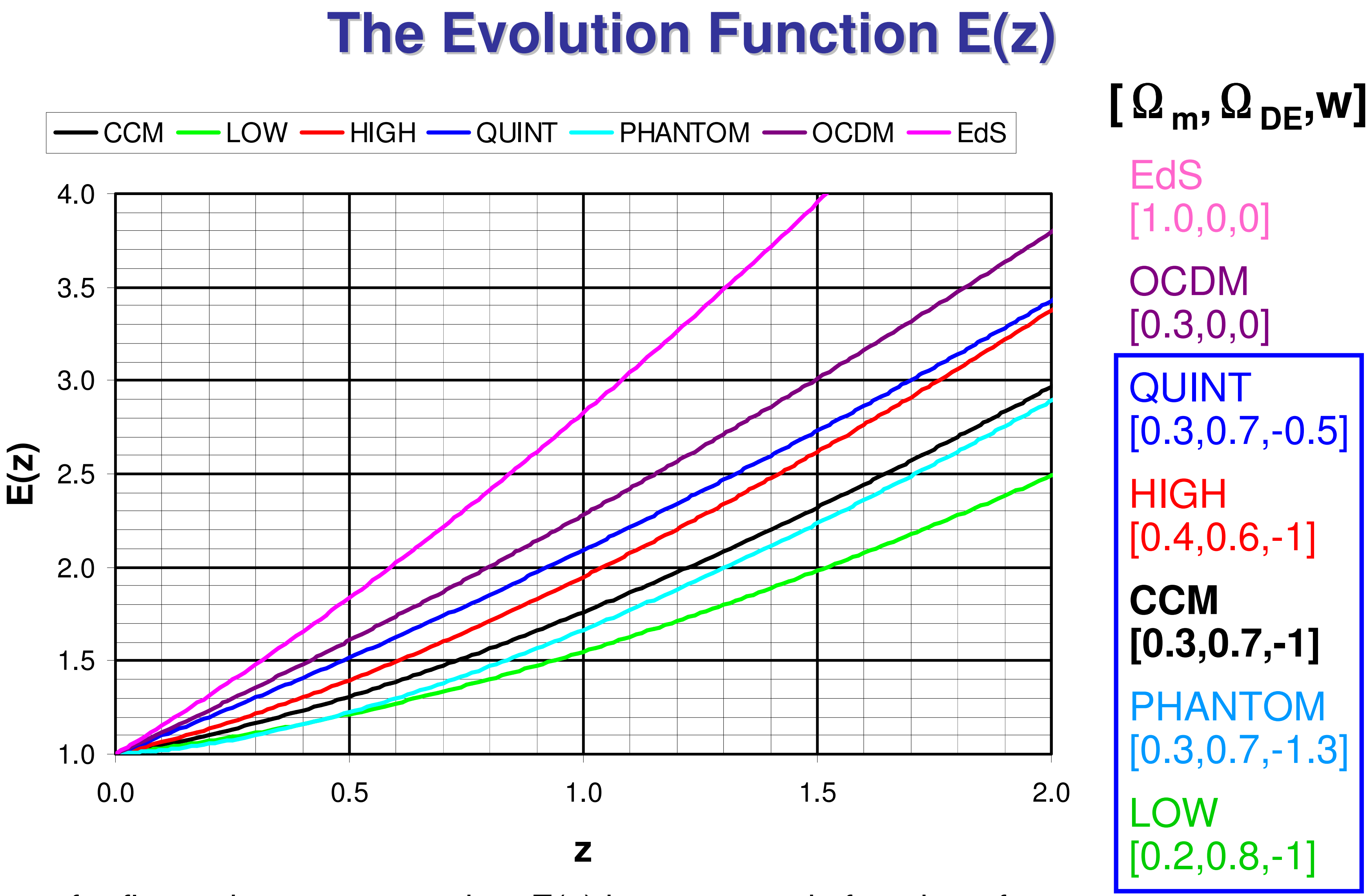
CCM
[0.3,0.7,-1]

PHANTOM
[0.3,0.7,-1.3]

LOW
[0.2,0.8,-1]

Credit: Fassbender

How can we probe this?

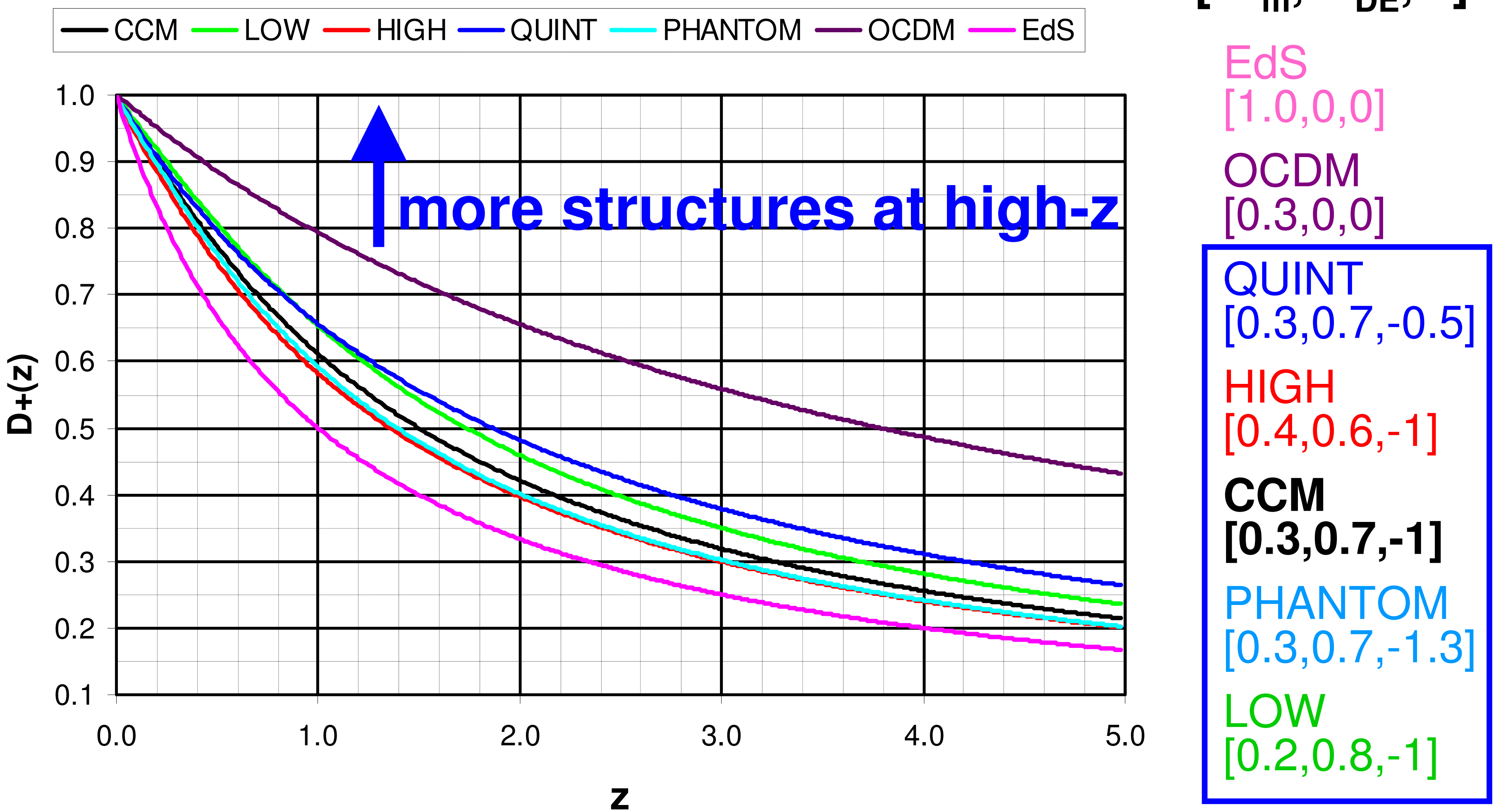


for flat and open geometries, E(z) is a monotonic function of z

Credit: Fassbender

How can we probe this?

Growth Factor



structure grow efficiently when Ω = 1 (since density is clos[e to ...]
where slight overdensities cause collapse)

Credit: Fassbender

Here's an alternate set of plots:

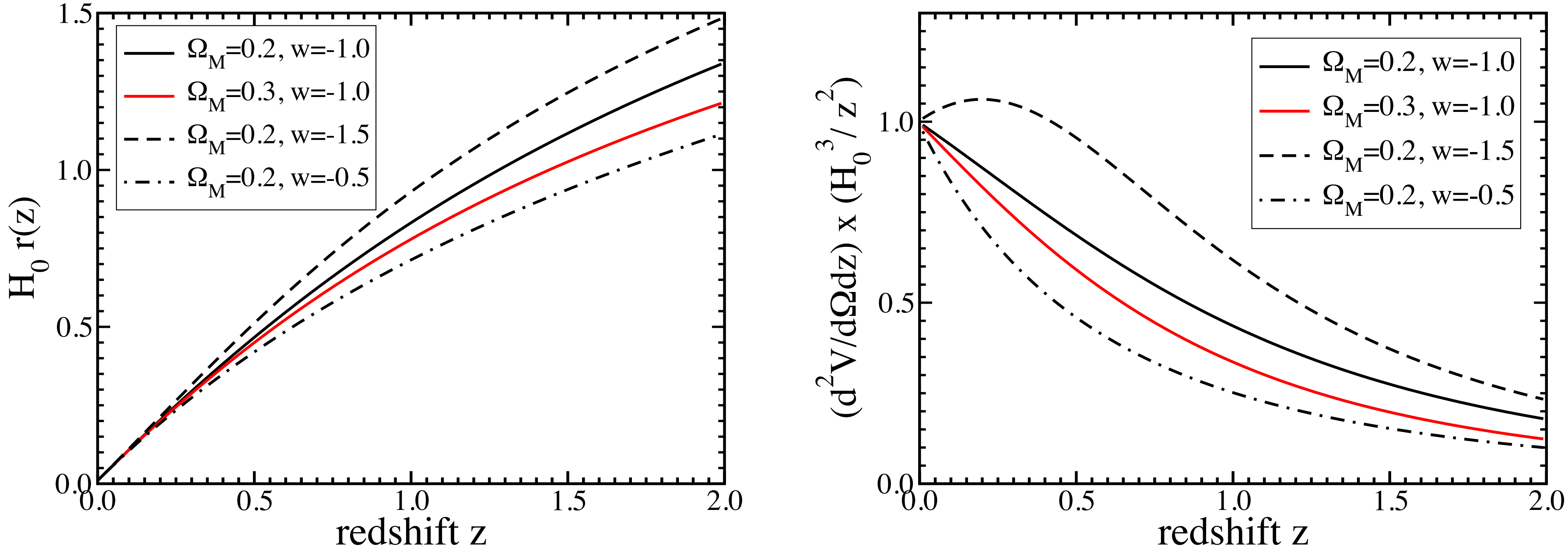
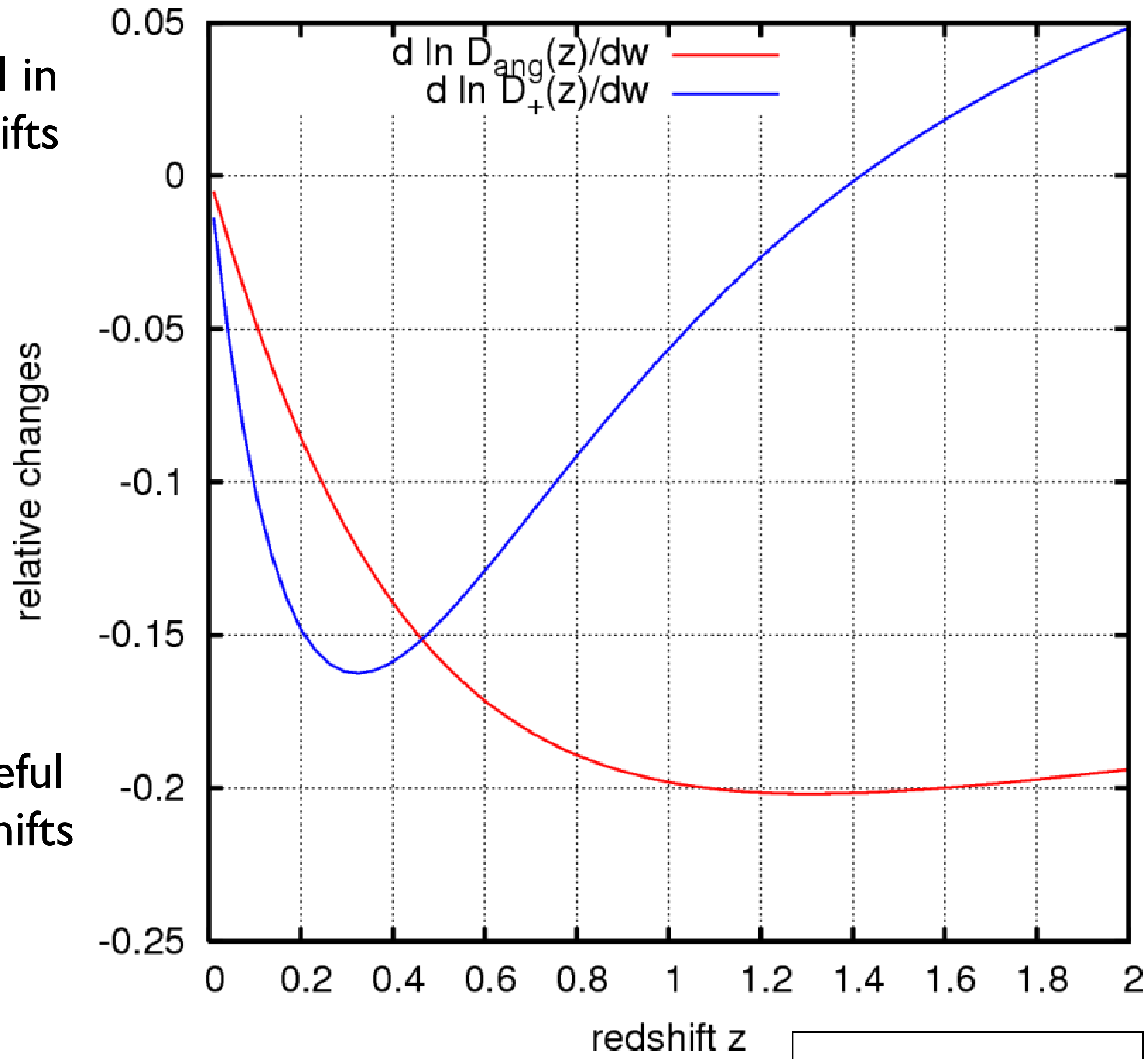


FIGURE 2. Left: Distance vs. redshift in a flat Universe with different values of the cosmological parameters Ω_m and w. Right: volume element vs. redshift for same models. From Frieman et al. [13].

Where do they provide the strongest constraints?

growth factor most useful in examining w at low redshifts

distance measures most useful in examining w at high redshifts



Credit: Bartelmann

So the game is to determine the w parameter and how it depends on redshift

There are four standard methods:

1. Supernovae Ia (lecture 4)
   -- use of standard candles to establish distance-redshift relation
   -- first established existence of dark energy >20 years ago
2. Baryonic Acoustic Oscillations (this lecture)
   -- gives us a standard rod to establish distance-redshift relation with low systematics
3. Galaxy Clusters (next lecture)
   -- provide us with sensitive probe of growth of structure
   -- early evidence for low Ω_m
4. Weak Gravitational Lensing (next lecture)
   -- provide us with sensitive probe of growth of structure
   -- powerful technique still in process of realizing full potential