What can we Learn from the Cosmic Microwave Background (Part II) +

What can we learn from the way galaxies distribute themselves in the universe?

Layout of the Course

Feb 3: Introduction / Overview / General Concepts
Feb 5: Age of Universe / Distance Ladder / Hubble Constant
Feb 10: Distance Ladder / Hubble Constant Distant Measures
Feb 12: SNe science / Baryonic Content / Dark Matter Content of Universe
Feb 17: Dark Matter + Cosmic Microwave Background
Feb 19: Cosmic Microwave Background + Large Scale Structure ← Today
Feb 26: Large Scale Structure / Baryon Acoustic Oscillations + Dark Energy
Mar 5: Dark Energy / Clusters / Cosmic Shear
Mar 12: Cosmic Shear / Dark Energy Missions
Mar 19: No Class
Mar 26: Review for Final Exam

Apr 11: Final Exam

This morning I sent around problem set 2

Will be due by March 4

Review Material from Last Week

Mass Density in Dark Matter

What is the evidence for dark matter?

Rotational Curves of Spiral Galaxies

Observations of Galaxy Cluster Collisions

Measurement of Masses for Galaxy Clusters

Peculiar Velocities of Galaxies in the Nearby Universe

One can determine Ω_{DM} by measuring the ratio of the masses in baryons + DM in galaxy clusters

A) Measure mass in baryons by exploiting SZ effect.

B) Measure total mass in cluster using 3 different techniques:

1. Use velocity dispersion (motion of galaxies in cluster)
2. Measurement of gas profile (hydrostatic equilibrium)
3. Model gravitational lensing of background sources

→ $\Omega_{\text{dark matter}} = 0.24$

Measured M/L ratios increase towards largest scales indicating the increasing importance of dark matter on large scales:

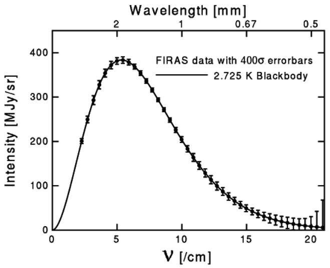
Solar Neighborhood: M/L = 1

$(M/L)_{galaxy} \sim$ 10-20 M_{solar}/L_{solar}

$(M/L)_{cluster} \sim$ 100-200 M_{solar}/L_{solar}

Universe: M/L = ~ 200

Photons from the CMB have a spectral energy distribution which is almost a perfect black body.



Cosmic Microwave Background is Isotropic

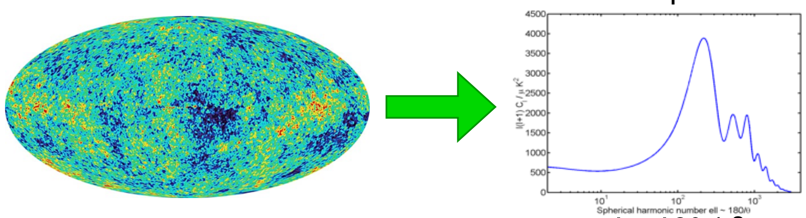
CMB radiation became decoupled from matter during recombination era (z~1100)

Ionized Plasma (z > 1100)	→ Neutral Gas (z < 1100)
< 380,000 years	> 380,000 years
Temperature > 3600 K	Temperature < 3600 K
Hydrogen ionized	Hydrogen neutral Almost no free electrons
Photons Thomson-scattering off of the ionized hydrogen	Photons unbound from plasma

How to represent or model anisotropies in the CMB?

-- Use the spherical harmonic expansion to construct a power spectrum to describe anisotropies of the CMB on the sky

Power Spectrum



l = 180 / θ

Expansion:

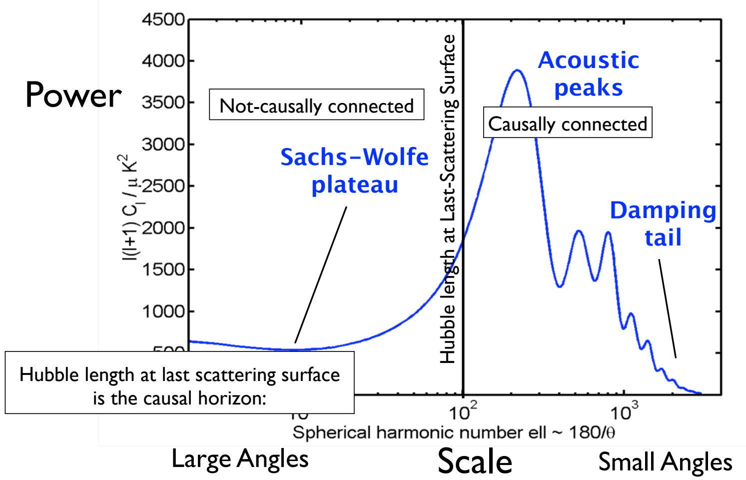
$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^\ell(\theta,\phi)$$

After deriving the a_{lm} coefficients from the data, determine the statistical average

$$c_\ell = \langle |a_{\ell m}| \rangle^2$$

What can we learn from the CMB power spectrum?

Different physics in causally connected vs. causally disconnected regions

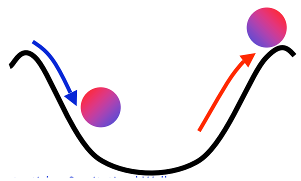


Sachs-Wolfe Effect (1967)

$\Delta\nu/\nu \sim \Delta T/T \sim \Phi/c^2$

Additional effect of time dilation while potential evolves (full GR):

$$\frac{\Delta T}{T} \sim \frac{1}{3}\frac{\Delta\Phi}{c^2}$$



Photons climbs out of potential minimum, loses energy ↔ lower temperature

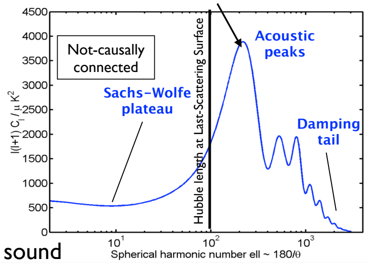
Photons falls out of potential maximum, gains energy ↔ higher temperature

But what distribution of potential minima and maxima do we expect?

This comes from inflation

For a Harrison-Zeldovich power spectrum $P(k) \propto k$ (expected from inflation), the CMB power spectrum is expected to be flat, i.e., $C_l \propto 1/l(l+1)$

Also discussed how the position and size of the first acoustic peak could constrain Ω_m and Ω_Λ:

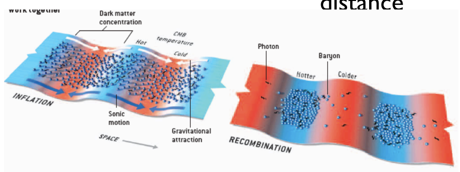


angular scale of 1st acoustic peak

$$\theta = \frac{L_S(z)}{D_A(z)}$$

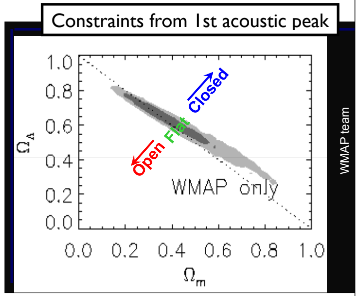
distance traversed by sound wave -- "standard rod"

angular diameter distance



[Image Credit: Hu & White 2004]

Constraints from 1st acoustic peak



End of Review from last week ==>

New Material

What can we learn from the other peaks?

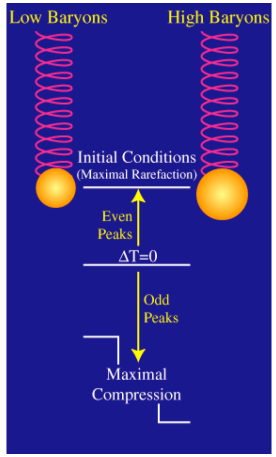
What can we learn from the other peaks?

Learn about baryon content

-- The presence of more baryons increases the amplitude of the oscillations

-- As a result, the fluid is compressed more before photon pressure can resist the compression

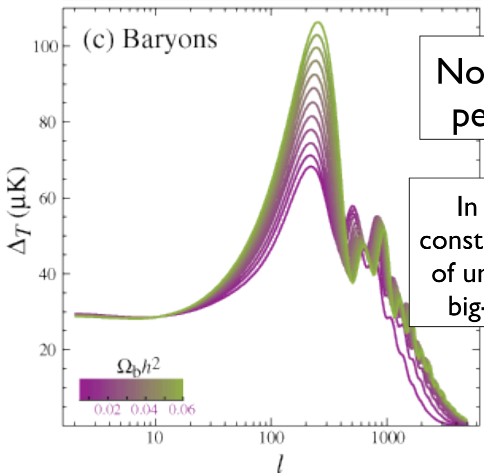
-- This results in an asymmetry between the even and odd peaks



Credit: Wayne Hu

What can we learn from the other peaks?

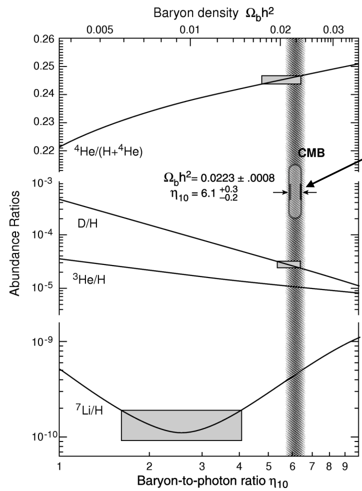
Learn about baryon content



Note how 1st and 3rd peaks are enhanced!

In fact, this provides best constraint on baryonic content of universe (even better than big-bang nucleosysnthesis)

How do constraints on Ω_{baryon} from the CMB compare with Big Bang Nucleosynthesis?



Constraints on Ω_{baryon} from CMB in perfect agreement with Big Bang Nucleosynthesis

What can we learn from the other peaks?

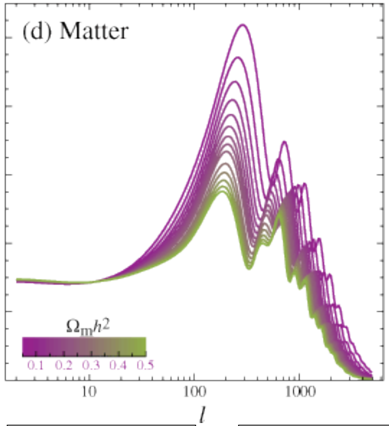
Learn about dark matter content

A higher matter density will result in the universe being matter dominated at an earlier time, allowing for more growth of structure at small scales.

Ω_m increase => More Small-Scale Structure

More Small-Scale Structure => Larger Acoustic Peaks Small Scales

Note how 3rd peak is enhanced when dark matter density higher!

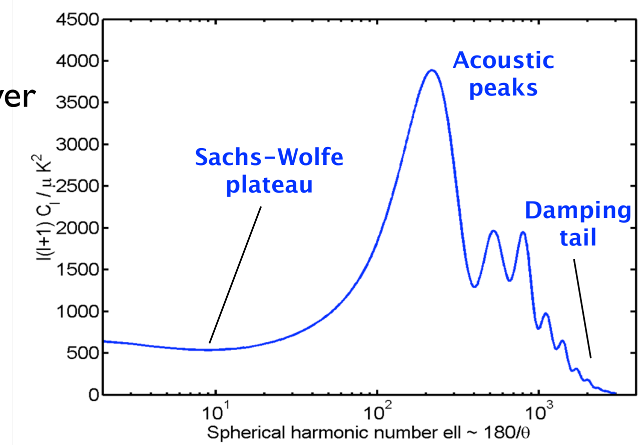


Larger Scales

Small Scales

What about the damping tail?

Power



Large Angles

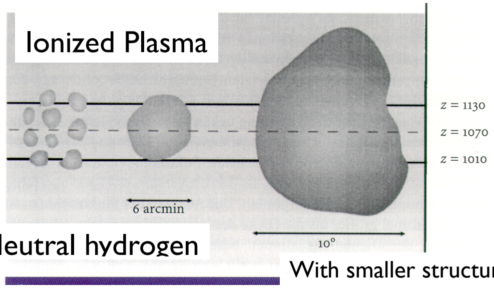
Scale

Small Angles

What about the damping tail?

-- (1) **Radial Smearing:** Decoupling does not happen instantaneously. This is not so important in viewing the last scattering surface for larger fluctuations. But for smaller fluctuations, the stuctures will overlap.

Note finite width of last scattering surface z = 1130 to 1010



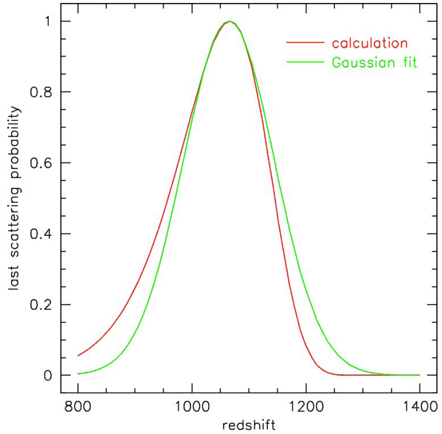
Redshift

With smaller structures, projection effects will play a significant role in diluting signal

How extended is the surface of last scattering?

-- Distribution describes the probability that a photon from the cosmic microwave background was last scattered at a given redshift.

-- Can roughly be described by a normal distribution with mean z = 1080 and standard deviation dz = 80



What about the damping tail?

-- **(2) Photon Diffusion / Silk Damping:** 2nd cause of the Damping tail results from photons in overdensities diffusing out of the overdensities via a random walk. This will wash out the over densities in the baryonic material since the baryons are coupled to the photons before recombination.

Mean Free Path of Photons

$$l_{\rm mfp} = \frac{1}{n_{\rm e}\sigma_{\rm T}},$$

Equation for Random Walk

$$\langle {\rm d}l^2 \rangle = N l_{\rm mfp}^2$$

Number of Scatterings

$$N = \frac{c\,{\rm d}t}{l_{\rm mfp}}.$$

→

Length Scale of Photon Diffusion

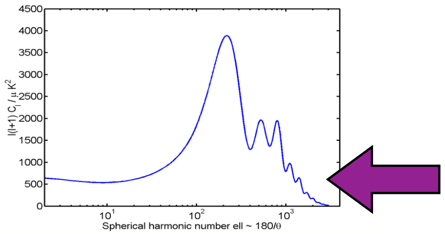
$$\lambda_{\rm S}^2 = \int_0^{t_{\rm dec}} c\, l_{\rm mfp} \frac{{\rm d}t}{a^2}.$$

scale factor for universe

What about the damping tail?

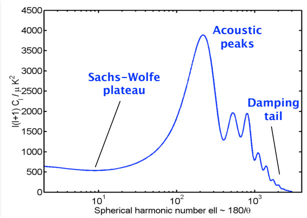
-- (1) **Radial Smearing:** Decoupling does not happen instantaneously, and so light from many smaller structures will overlap -- diluting the overall signal

-- **(2) Photon Diffusion / Silk Damping:** 2nd cause of the Damping tail results from photons in overdensities diffusing out of the overdensities via a random walk. This will wash out the over densities in the baryonic material since the baryons are coupled to the photons before recombination.



Power falls off

There are several other features in the CMB power spectrum to discuss

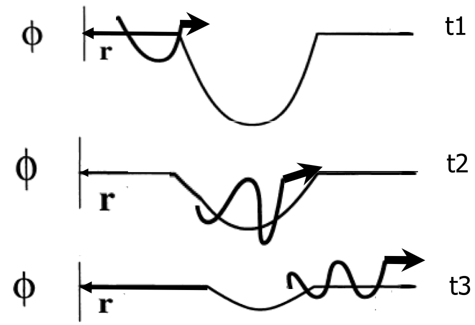


In addition to the temperature perturbations created by density fluctuations in the early universe (early Sachs Wolfe Effect), there is also the Integrated Sachs-Wolfe Effect

Late Integrated Sachs Wolfe Effect

When CMB photons travel from the last scattering surface to us, they occasionally cross deep collapsed regions

When they traverse these regions, the photons gain energy when the fall into the potential and lose energy climbing out.



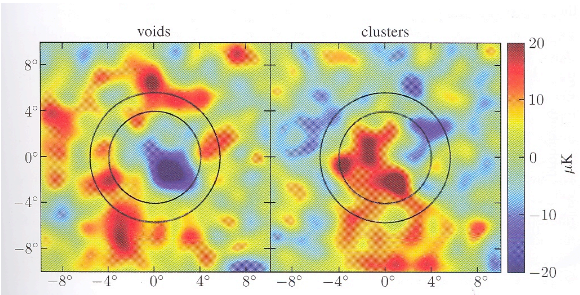
If the depth of the potential did not change as a function of cosmic time, you would expect the energy lost to be the same as the energy gained.

However, this is not the case -- if there is the universe has a substantial amount of dark energy

Late Integrated Sachs Wolfe Effect

One can measure this effect by looking at the CMB through overdense / underdense regions of the universe.

By looking at spots in the CMB which traverse through voids in the universe (underdensities), one expects the photons to be colder in general.



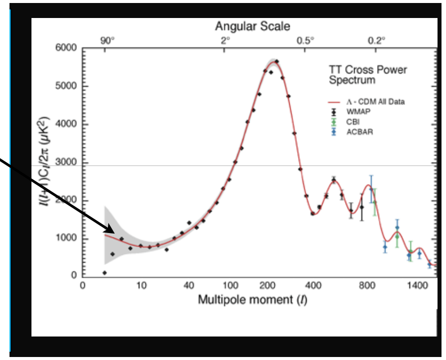
By looking at spots in the CMB which traverse through overdensities (galaxy clusters), one expects to find hotter photons in general.

One can use this approach to constrain the amount of dark energy in the universe!

Late Integrated Sachs Wolfe Effect

What is the effect of this on the power spectrum?

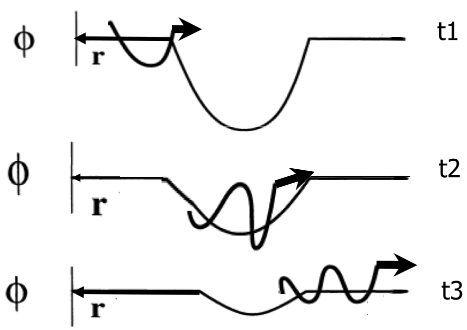
Results in slight tilt to C_l vs. l relationship



This is a difficult effect to observe in the general CMB power spectrum, since it results in a slight tilt at large scale (small l numbers) where spectrum is noisy due to cosmic variance

There is also an early integrated Sachs Wolfe effect

This occurs due to the fact that the depth of potential wells is affected by radiation escapes from the potentials due to recombination



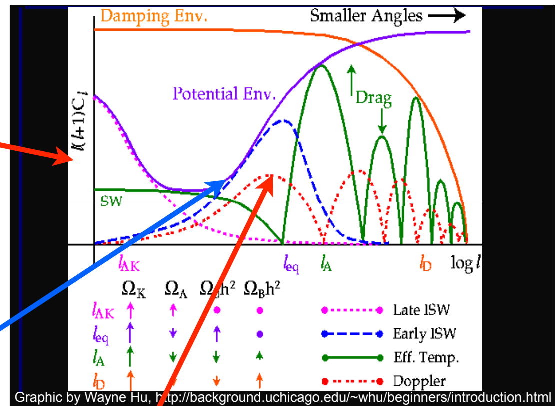
Before recombination, potential wells are will include a contribution from radiation and be deeper

After recombination, potentials will lose this contribution from radiation and be shallower

Illustration of the many effects impacting the CMB power spectrum

It's possible to break-down the power spectrum in a detailed manner

Also an early ISW effect due to the effect of photons climbing out of overdensities immediately after recombination



Doppler effect due to the expected motions of material in between modes at recombination epoch

Won't be tested on this figure

One important limitation comes from cosmic variance

One thing that is important to remember is that actual density fluctuations in the real universe we see in the CMB are just one realization of a Gaussian-random process and may be different than the average perturbation size given an infinite number of universes

How well we can measure the fluctuation strength on a given physical (angular) scale, therefore, depends on how many fluctuation modes on this physical (angular) scale are available on the sky...

We can only see so many in the visible universe.

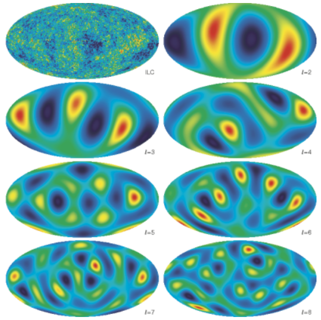
How many fluctuation modes are available on a given scale?

It depends on the l number...

2l+1

from the Expansion:

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^{\ell}(\theta,\phi)$$



Each of the components a_{lm} are gaussian random variables.

and are used to determine $c_\ell = \langle |a_{\ell m}| \rangle^2$

How many fluctuation modes are available on a given scale?

It depends on the l number...

2l+1

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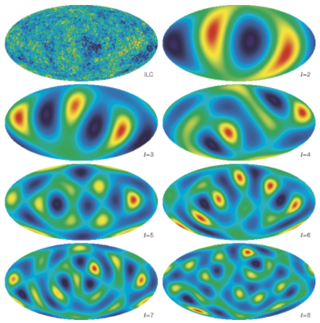
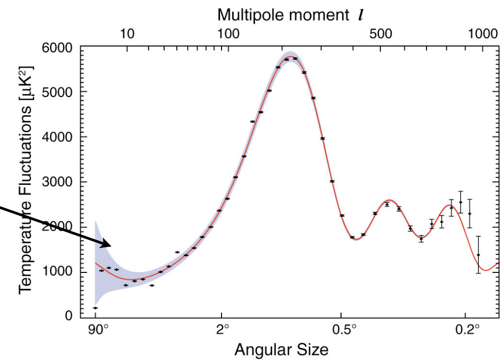


Clearly, we would expect the C_l to show less variance at smaller scales where more realizations (and l numbers) exist

Concept of Cosmic Variance

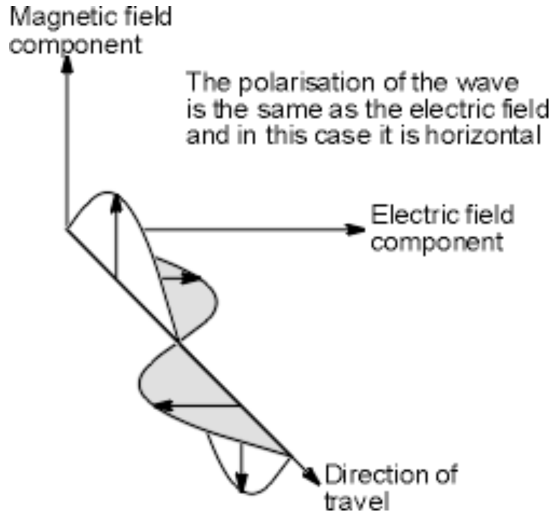
As a result, the intrinsic uncertainty on the CMB TT power spectrum became very large at low multipoles l

Note how uncertainty blows up at small multipoles l



Polarisation Information in the Cosmic Microwave Background

Polarization of Light



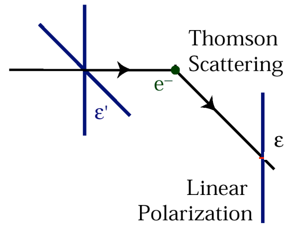
How polarized is the cosmic microwave background overall?

most of the CMB light shows no net polarization

however there is a ~10% net polarization

Why are photons from the CMB polarized?

They are polarized from Thomson scattering
(valid in the limit that photon is much less than mass energy in the particle)

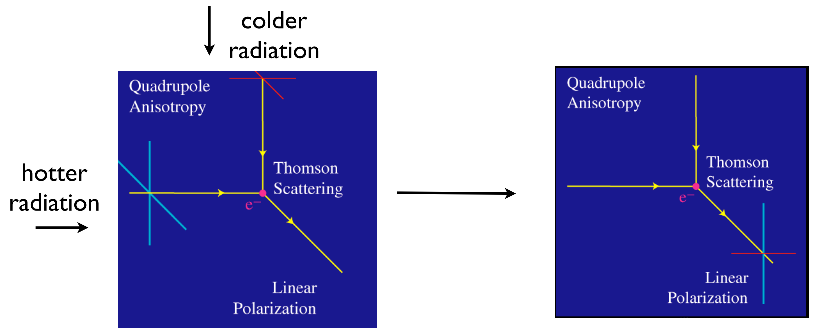


photons polarized perpendicular to incidence direction

How can this result in a polarized signal from the microwave background?

because of the relationship between the temperature structure of the CMB and polarization one gets from Thomson scattering

How can this result in a polarized signal from the microwave background?

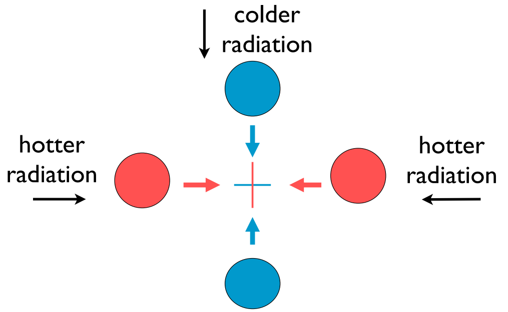


photons from hotter region will be observed with one polarization and those from colder region will be observed with another

No net polarization for an isotropic (or dipole) radiation field from the CMB. Only if the temperature structure has a quadrapole.

How can this result in a polarized signal from the microwave background?

Same diagram but in plane of last scattering surface

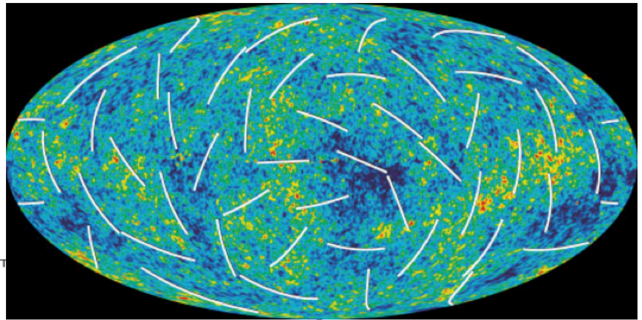


No net polarization for an isotropic (or dipole) radiation field from the CMB. Only if the radiation field has a quadrapole.

So as a result of this process, one finds a net polarization to the CMB radiation as a whole.

One can map out a polarization field for the entire CMB sky

e.g. with WMAP



One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

$$v = E + B$$

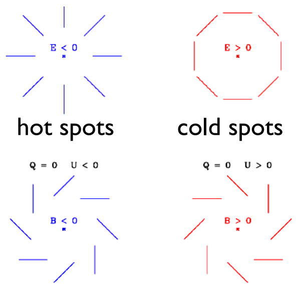
$$E = \nabla\phi \qquad \nabla \cdot B = 0$$

$$\nabla \times E = 0$$

One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

E-modes

B-modes



E-modes are curl free and can be written as the gradient of a potential

$$\nabla \times E = 0$$

B-modes have no divergence.

$$\nabla \cdot B = 0$$

The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

What is physical origin of E and B modes?

i.e., why look at them separately?

E-modes have their origin in normal density perturbations such as make up the early universe

B-modes are only expected to arise from gravity waves in early universe (inflation) and from gravitational lensing (beween us and the last scattering surface)

Also have a temperature component to the CMB light which is entirely unpolarized, this is called the T mode (distinct from E and B modes)

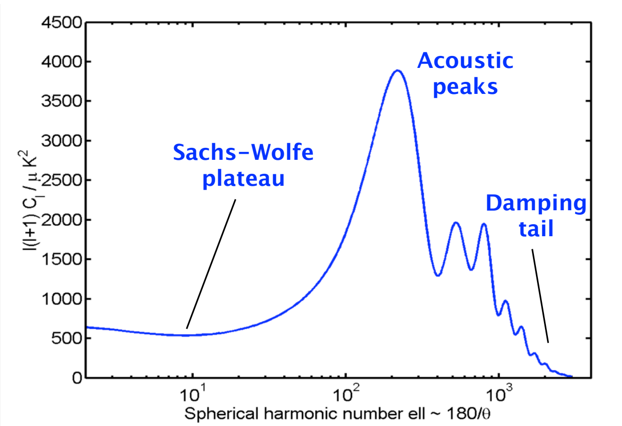
Why look at the polarized light separately?

teaches us new things..
tests our assumptions...

What about these three components to temperature structures T, E, and B?

How does this relate to what we did before?

So, far what I have shown you the TT angular power spectrum...



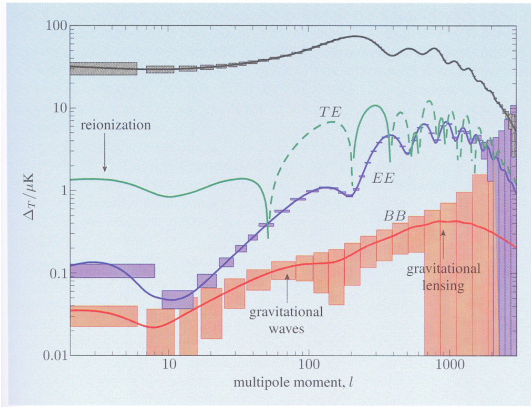
So, in addition to maps of the temperature T of the cosmic microwave background (all polarizations), we can look at maps of the temperature with an E-mode type polarization and B-mode type polarization

By cross-correlating the difference in temperature of the light from these different components T, E, and B, you can look at four different power spectra TT, TE, EE, BB...

Might imagine there could also TB and EB type power spectra...

However, since T and E modes have one type of symmetry and B modes have another, TB and EB always equal zero.

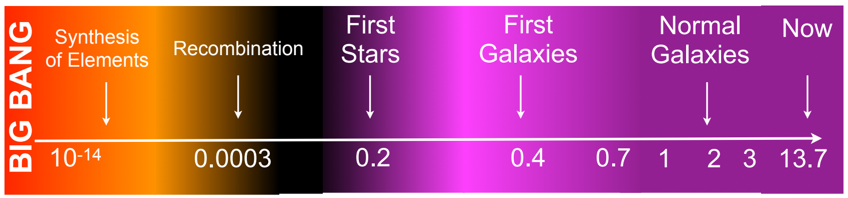
But we can also look the TE, EE, and BB angular power spectra



Note that the EE, TE, and BB power spectra are not nearly as prominent as the TT power spectrum. This is because only 10% of the light from the CMB is polarized!

What new information do the TE, EE, and BB spectra provide?

Allows us to answer question how long did hydrogen in the universe in a neutral state, i.e., from 400,000 yrs after Big Bang to 1 Gyr

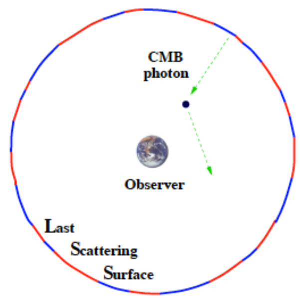


state of hydrogen

ionized ⟶ ⟵ neutral ⟶ ⟵ ionized

What new information do the TE, EE, and BB spectra provide?

The microwave background helps us answer this question -- since photons from the microwave background scatter off of ionized elecrons in the universe

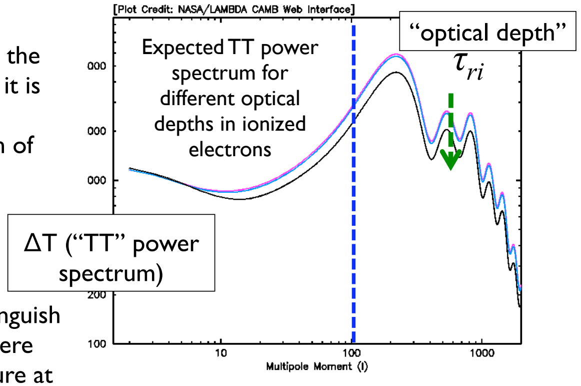


Obviously, the longer the hydrogen remains in an ionized state, the more photons from the CMB we would expect to be scattered.

Credit: Porciano

What new information do the TE, EE, and BB spectra provide?

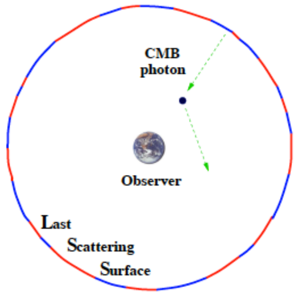
Information about reionization is present in the TT power spectrum, but it is degenerate with the underlying normalisation of the power spectrum.



Difficult to know to distinguish between scenarios where universe had less structure at early times and where the apparent structure washed out by Thomson scattering.

What new information do the TE, EE, and BB spectra provide?

Photons from the CMB are expected to have a certain polarization symmetry relative to the temperature structure of the CMB.

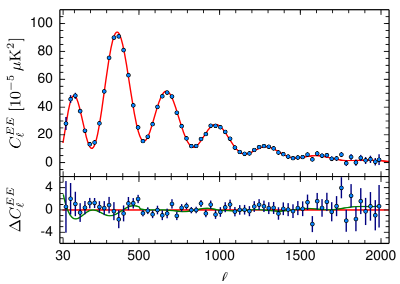


Scattering by ionized matter would mix up this polarization information => teaching us about the intervening ionized hydrogen

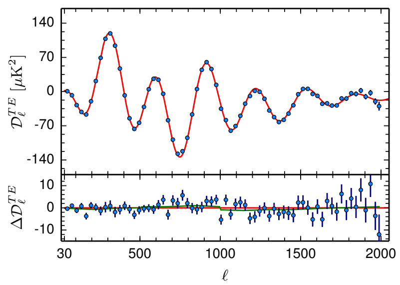
Credit: Porciano

What new information do the TE, EE, and BB spectra provide?

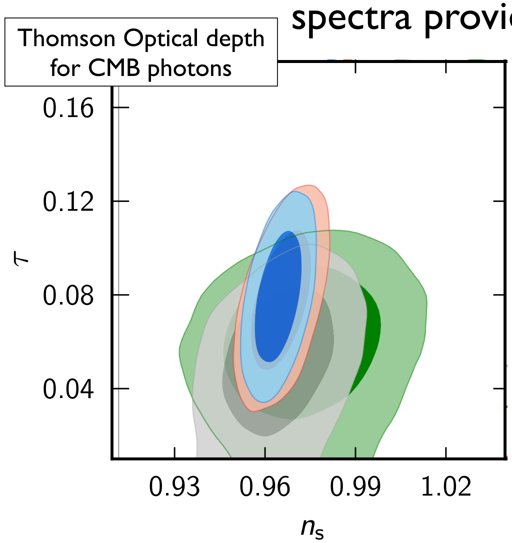
TE Cross Power Spectrum (Planck)

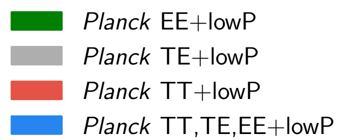


EE Power Spectrum (Planck)



What new information do the TE, EE, and BB spectra provide?





~8-10% of the photons from CMB are scattered

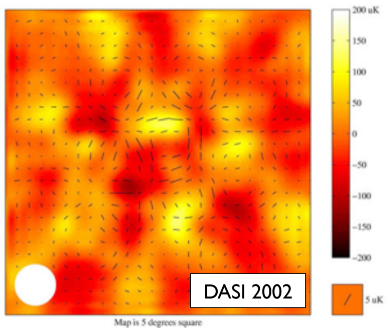
Slope of primordial Power Spectrum

First detection of polarization in CMB

-- DASI South Pole experiment (interferometer) first to detect E mode polarization (2002)

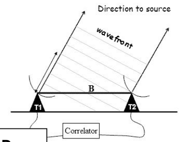
-- This was followed by WMAP reporting a measure of the C_{TE} power spectrum at low angular scales

-- Measurements of the E-mode polarization also made with CARMAP, MAXIPOL, and QUAD



interferometer: collect coherent signals over certain angular scale on sky

DASI in South Pole



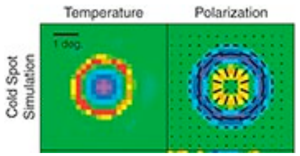
Credit: Basu

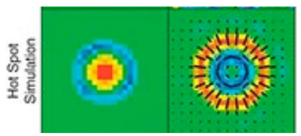
It is interesting that we can actually test whether our understanding of the polarization of CMB is correct

Around cold or hot spots, we expect a certain structure to the polarization signal

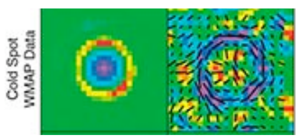
Can test this by looking at the polarization signal around hot or cold spots in the observations.

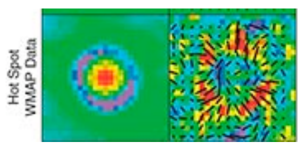
From theory





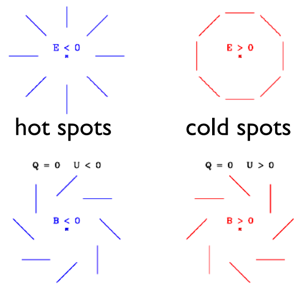
As observed by WMAP





One tends to break down the polarization map into two modes

E-modes



E-modes are curl free and can be written as the gradient of a potential

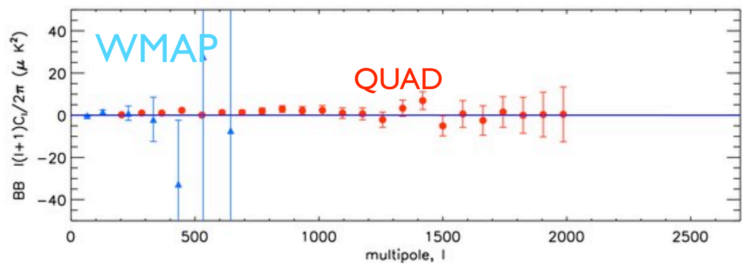
B-modes

B-modes are curl free and can be written as the gradient of a potential

The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

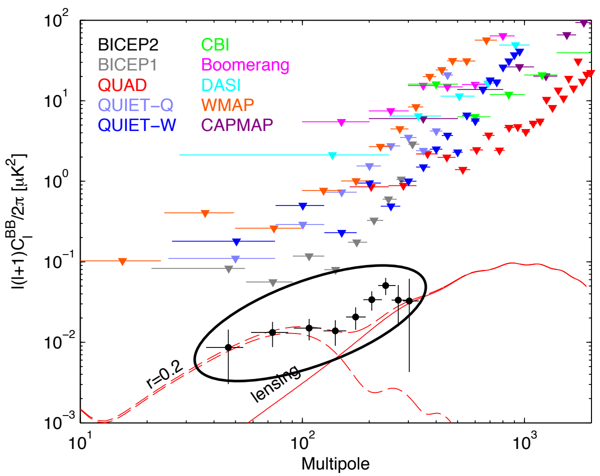
What about probes of the CMB BB power spectrum?

No power in BB power spectrum detected as of 2013 -- goal of Planck!



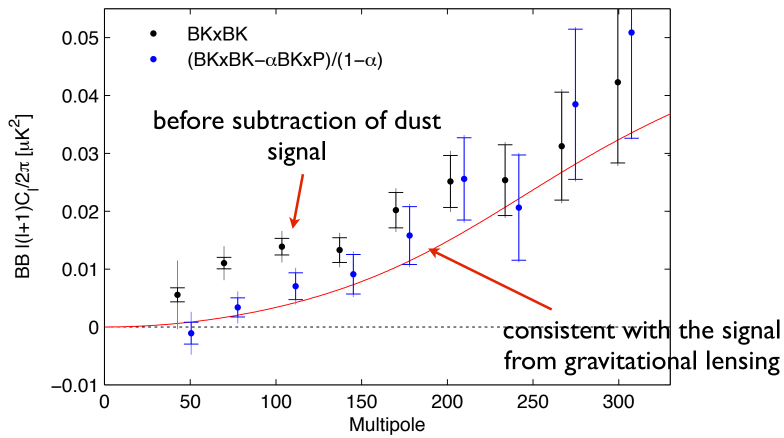
Was expected to the smoking gun test of inflation -- since the signal is expected to originate from gravity waves (from inflation) -- signal on smaller scales comes from gravitational lensing

Significant BB signal detected by BICEP II!



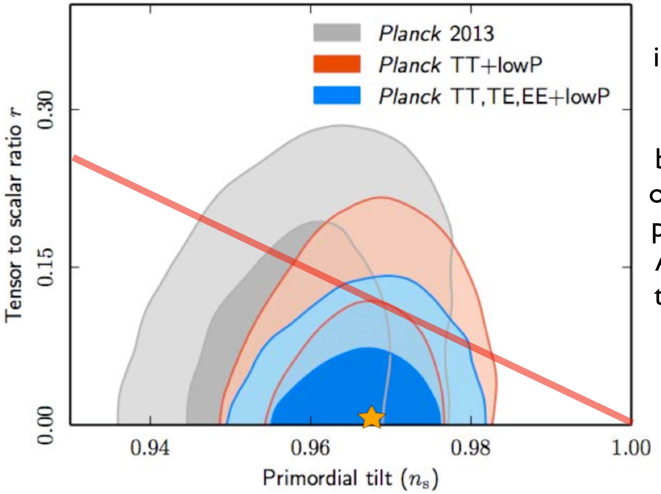
BICEP2 results show a positive detection of BB modes. Attempted fit to gravity waves from inflation... Lensing contributes at small scales

But current BB signal from BICEP II appears consistent with arising from dust in our own galaxy..



Credit: BICEP/Keck Collaboration

Current constraints on BB allow us to set constraints on r, the ratio of power tensor-to-scalar modes.



For simplest inflation models, there is a relationship between the tilt of the primordial power spectrum And the tensor-to-scalar ratio r.

$r = 8(1-n_s)$

=> r = 0.1-0.3

Planck - BICEP-II/Keck Joint Results

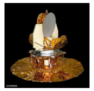
Ruled out at 95% confidence

Both WMAP + Planck have provided us with an immense amount of information on the cosmological parameters

Constraints on the cosmological parameters from WMAP observations (7-year)

WMAP launched June 2001

Credit: NASA



Note the same dual receivers as COBE. This design, added with the very stable conditions at the L2, minimizes the "1/f noise" in amplifiers and receivers.

Thus after 7 years, the data can still be added and noise lowered (of course, the improvement will be marginal).

WMAP Cosmological Parameters
Model: lcdm+sz+lens
Data: wmap7

$10^2\Omega_b h^2$	$2.258^{+0.057}_{-0.056}$	$1-n_s$	0.037 ± 0.014
$1-n_s$	$0.0079 < 1-n_s < 0.0642$ (95% CL)	$A_{BAO}(z=0.35)$	$0.463^{+0.021}_{-0.020}$
C_{220}	5763^{+38}_{-40}	$d_A(z_{eq})$	14281^{+158}_{-161} Mpc
$d_A(z_*)$	14116^{+160}_{-163} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.43 \pm 0.11) \times 10^{-9}$
h	0.710 ± 0.025	H_0	71.0 ± 2.5 km/s/Mpc
k_{eq}	$0.00974^{+0.00041}_{-0.00040}$	ℓ_{eq}	137.5 ± 4.3
ℓ_*	302.44 ± 0.80	n_s	0.963 ± 0.014
Ω_b	0.0449 ± 0.0028	$\Omega_b h^2$	$0.02258^{+0.00057}_{-0.00056}$
Ω_c	0.222 ± 0.026	$\Omega_c h^2$	0.1109 ± 0.0056
Ω_Λ	0.734 ± 0.029	Ω_m	0.266 ± 0.029
$\Omega_m h^2$	$0.1334^{+0.0056}_{-0.0055}$	$r_{hor}(z_{dec})$	285.5 ± 3.0 Mpc
$r_s(z_d)$	153.2 ± 1.7 Mpc	$r_s(z_d)/D_v(z=0.2)$	$0.1922^{+0.0072}_{-0.0073}$
$r_s(z_d)/D_v(z=0.35)$	$0.1153^{+0.0038}_{-0.0039}$	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc
R	1.719 ± 0.019	σ_8	0.801 ± 0.030
A_{SZ}	$0.97^{+0.68}_{-0.97}$	t_0	13.75 ± 0.13 Gyr
τ	0.088 ± 0.015	θ_*	0.010388 ± 0.000027
θ_*	0.5952 ± 0.0016 °	t_*	379164^{+5187}_{-5243} yr
z_{dec}	1088.2 ± 1.2	z_d	1020.3 ± 1.4
z_{eq}	3196^{+134}_{-133}	z_{reion}	10.5 ± 1.2
z_*	$1090.79^{+0.94}_{-0.92}$		

Constraints on the cosmological parameters from Planck observations (final results)

2010-2014: The Planck satellite

Credit: ESA

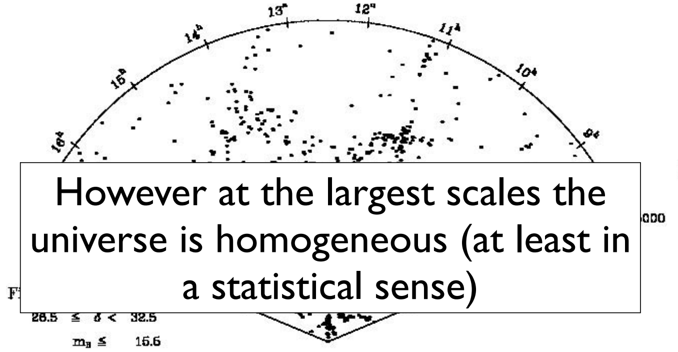
Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10}A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$.	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8\Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029
z_{drag}	1059.39 ± 0.46	1060.03 ± 0.54	1063.2 ± 2.4	1059.93 ± 0.30	1059.94 ± 0.30	1060.01 ± 0.29
r_{drag} [Mpc]	147.21 ± 0.48	147.59 ± 0.49	146.46 ± 0.70	147.05 ± 0.30	147.09 ± 0.26	147.21 ± 0.23
k_D [Mpc^{-1}]	0.14054 ± 0.00052	0.14043 ± 0.00057	0.1426 ± 0.0012	0.14090 ± 0.00032	0.14087 ± 0.00030	0.14078 ± 0.00028
z_{eq}	3411 ± 48	3349 ± 46	3340^{+81}_{-92}	3407 ± 31	3402 ± 26	3387 ± 21
k_{eq} [Mpc^{-1}]	0.01041 ± 0.00014	0.01022 ± 0.00014	$0.01019^{+0.00025}_{-0.00028}$	0.010398 ± 0.000094	0.010384 ± 0.000081	0.010339 ± 0.000063
$100\theta_{s,eq}$	0.4483 ± 0.0046	0.4547 ± 0.0045	0.4562 ± 0.0092	0.4490 ± 0.0030	0.4494 ± 0.0026	0.4509 ± 0.0020
f^{143}_{2000}	31.2 ± 3.0			29.5 ± 2.7	29.6 ± 2.8	29.4 ± 2.7
$f^{143\times217}_{2000}$	33.6 ± 2.0			32.2 ± 1.9	32.3 ± 1.9	32.1 ± 1.9
f^{217}_{2000}	108.2 ± 1.9			107.0 ± 1.8	107.1 ± 1.8	106.9 ± 1.8

So what can we learn from the spatial distribution of galaxies on the sky?

In forming the Big Bang model of the universe and the Friedmann equations, one thing we assumed is that the universe is isotropic and homogeneous

This is true in a statistical sense

But as you all know it isn't

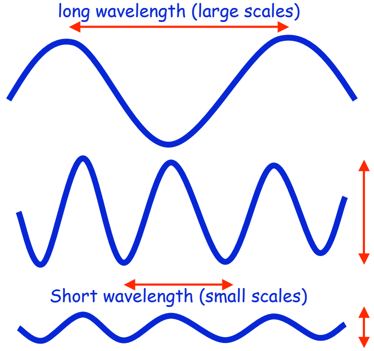


However at the largest scales the universe is homogeneous (at least in a statistical sense)

Spatial Distribution of Galaxies on some part of sky

How do we express this spatial structure (density perturbations in the universe)?

Convenient to express it in terms of Fourier modes:



Subtract off mean density: $\delta(\vec{r}) = \frac{\rho - \bar{\rho}}{\bar{\rho}} = \frac{\Delta\rho}{\bar{\rho}}$

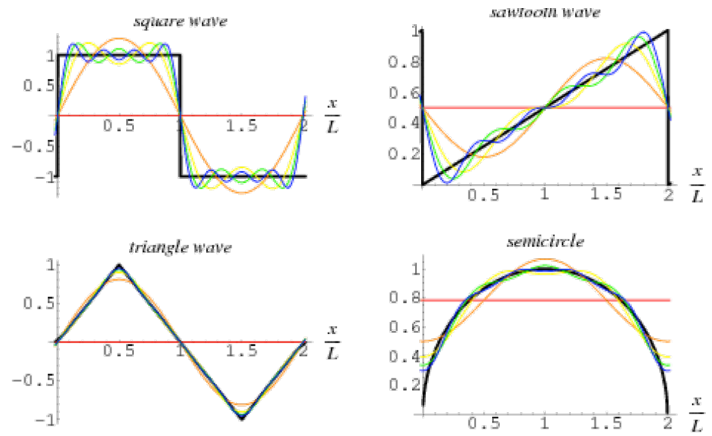
Fourier Transform: $\delta_k = \sum \delta(\vec{r}) e^{-i\mathbf{k}\cdot\mathbf{r}}$

Power Spectrum: $P(k) = \left\langle |\delta_k|^2 \right\rangle$

Analogous to the way we looked at fluctuations in the CMB

Similar concept to Fourier Series

-- Most of you are probably familiar with the fact that one can use a fourier series to represent an arbitrary one-dimensional function



Can express structure in universe in terms of power spectrum

What is the primordial power spectrum?

What is the primordial power spectrum?

The initial power spectrum of fluctuations is the following:

$$P_0(k) = A\,k^{n_s}$$

A is the normalization and n_s is the power-law slope. From inflation, n_s is thought to be almost exactly equal to one. This is the Harrison-Zeldovich power spectrum.

A power law makes sense for the primordial power spectrum since it has no characteristic scale.

What is the primordial power spectrum?

The initial power spectrum of fluctuations is the following:

$$P_0(k) = A\,k^{n_s}$$

A is the normalization and n_s is the power-law slope. From inflation, n_s is thought to be almost exactly equal to one. This is the Harrison-Zeldovich power spectrum.

Density Fluctuations in Universe expected to be Gaussian, homogeneous, isotropic (modes are uncorrelated)

$$\wp(\delta) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{\delta}{2\sigma^2}\right)}$$

Gaussian-random field

all information in power spectrum

What is the primordial power spectrum?

The initial power spectrum of fluctuations is the following:

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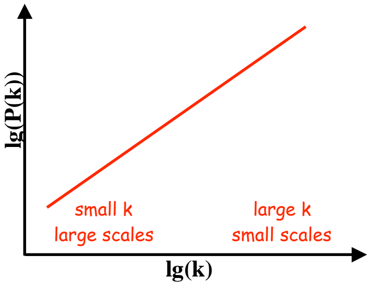


Credit: Pearson

To simplest approximation, it can be expressed in terms of growing modes...

$$P(k,t) = D_+^2(t)\; P(k,t_0) =: D_+^2(t)\; P_0(k)$$

where P(k,t) is the power spectrum at some later time and D(t) is the growth factor.

In the linear growth regime (before modes start turning around and collapsing and virializing), the time t and mode k are totally separable in the above equation.

How fast does the power spectrum grow?

During the epoch where radiation dominates the energy density ($z > 3500$), no significant growth in structure occurs -- except at scales larger than the horizon, where the growth goes as R^2 (R = scale of universe)

During the epoch where matter dominates ($z < 3500$), the growth goes as R (R = scale of universe)

How fast does the power spectrum grow?

During the epoch where radiation dominates the energy density ($z > 3500$), no significant growth in structure occurs -- except at scales larger than the horizon, where the growth goes as R^2 (R = scale of universe)

Implication is that growth in causally connected regions (i.e., within the horizon) will not grow at early times

But structure at large scales (super horizon scale) will grow

Recall from earlier in semester

Epoch of Matter-Radiation Equality (z=3500)

Energy Density in Dark Energy

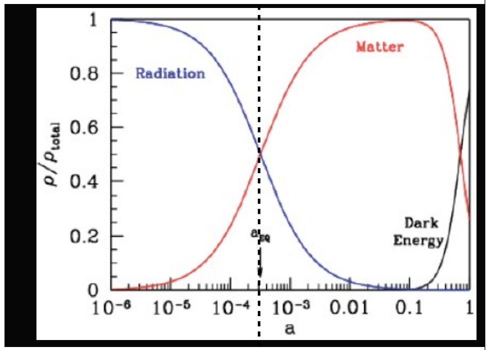
Λ = const (dominant at late times)

Energy Density in Matter

$\rho_m \propto R^{-3}$

Energy Density in Radiation

$\rho_r \propto R^{-4}$ (dominant at earliest times)



P(k) below horizon does not grow

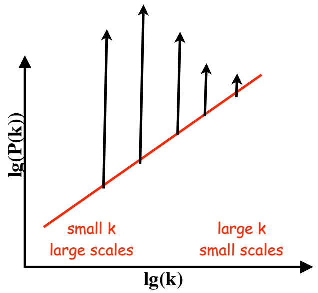
P(k) can grow

What is the primordial power spectrum?

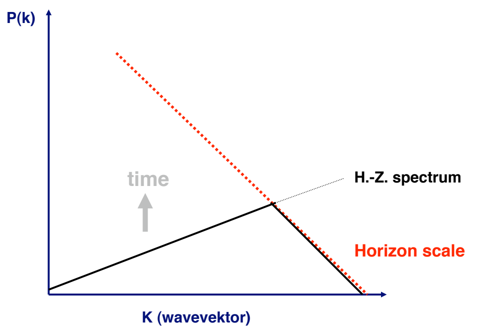
The initial power spectrum of fluctuations is the following:

$$P_0(k) = A\,k^{n_s}$$

Therefore we could expect P(k) at large scales to grow much more than at small scales



Evolution of the Matter Power Spectrum



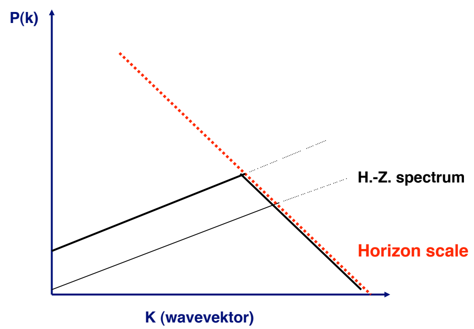
H. Böhringer 48

large scales
small k

small scales
large k

Credit: Bohringer

Evolution of the Matter Power Spectrum



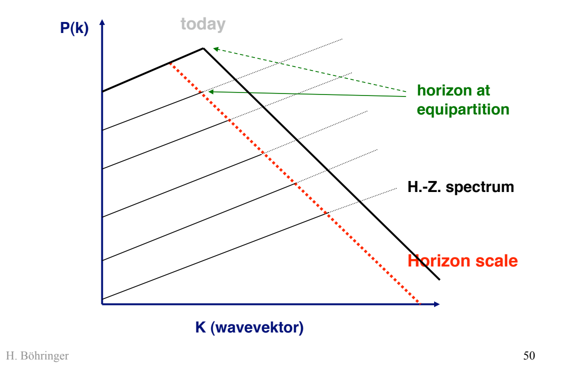
H. Böhringer 49

large scales
small k

small scales
large k

Credit: Bohringer

Evolution of the Matter Power Spectrum



large scales
small k

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Credit: Bohringer

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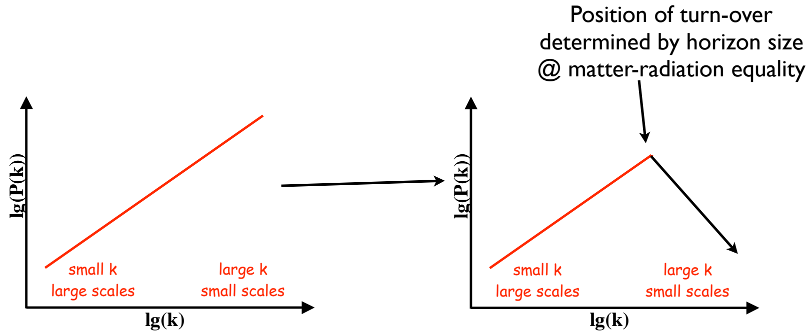


How does the power spectrum grow after the point of matter-radiation equality?

The power spectrum grows in proportion to R the size of the universe (won't derive this for you -- it is done in Joop's origin and evolution of universe)

How does one treat this formally?

Formally, one utilizes a transfer function to include these physics:

$$P_0(k) = A\, k^{n_s}\, T^2(k)$$

where T(k) is the transfer function.

The transfer function T(k) depends on the cosmological model and in particular on the quantity $\Gamma = \Omega_m h$. Γ is called the shape parameter.

The transfer function T(k) includes all physics involved in the growth of the primordial power spectrum to after recombination (and so some additional physics beyond what I mentioned)

How does one treat this formally?

Formally, one utilizes a transfer function to include these physics:

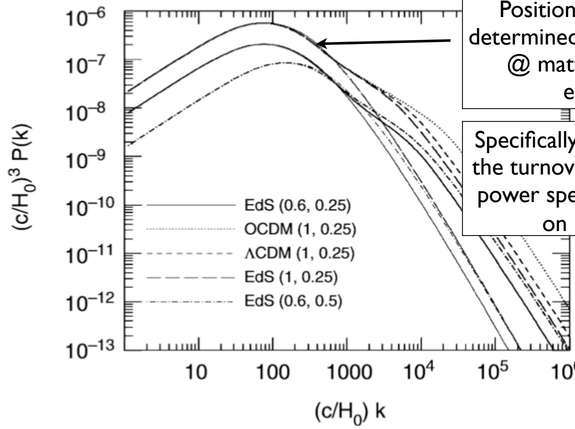
$$P_0(k) = A\, k^{n_s}\, T^2(k)$$

where T(k) is the transfer function.

Other important effects to include are the free streaming of relativistic matter/radiation (energy density in relativistic components of universe at recombination) which washes out power at small scales

Other important effects to include are baryons falling into the dark matter potential after recombination.

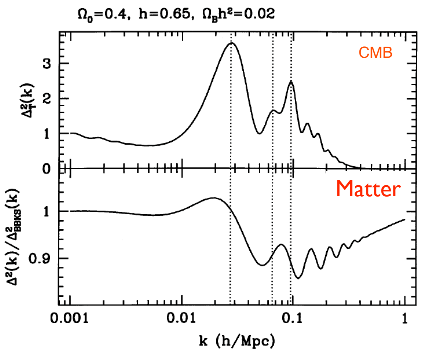
What does the matter power spectrum look like when all of these effects are included?



Position of turn-over determined by horizon size @ matter-radiation equality

Specifically the position of the turnover in the matter power spectrum depends on $\Gamma = \Omega_m h$

How does the CMB and large scale structure fit into this?



Matter Power Spectrum after Dividing Out Rough Shape

CMB and LSS out of phase

LSS amplitude smaller than CMB

The matter power spectrum is one of the most important parameters to derive in observational cosmology.

Different techniques/sources probe different regimes in matter power spectrum