Relevant Material for Lecture 7

"Galaxies: Structure, Dynamics, and Evolution"

2. **Structure of Ellipticals**

2.1 **Photometry (BM4.3)**

Photometry can be done accurately with modern CCDs. For many galaxies, you don't have to take your own data - go to Sloan !

Images of ellipticals generally show great regularity

- light is (nearly) constant on elliptical "isophotes"

- deviations from ellipses generally quite small - level is few percent
- intensity profiles are regular, no peaks or dips, unless dust is present
- intensity profiles fairly well described by sersic law:

$$I(R) = I_e 10^{b_n[(R/R_e)^{1/n}-1]}$$

R_e is half light radius, n between 2 and 5 (or so) for most ellipticals

n=4 is special case, de Vaucouleurs profile

- ellipticities generally below 0.5, median about 0.3
- position angles vary with radius, ellipticities vary with radius
- deviations from elliptical: "boxy" or "disky" or other distortions
- many have weak dust extinction
- weak color gradients: central parts are redder

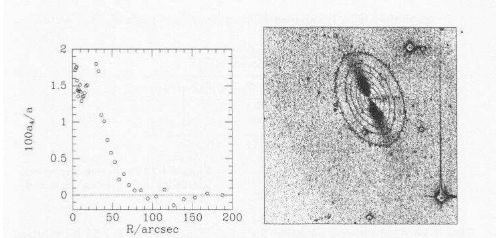
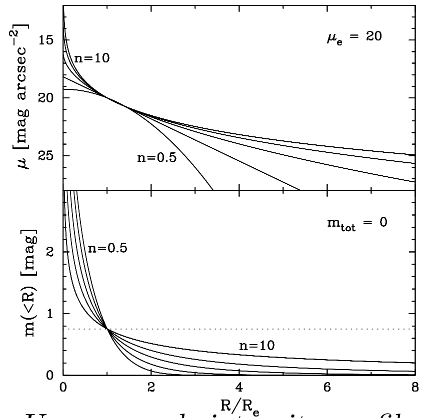


Figure 4.21 The diskiness [equation (4.10)] of NGC 4697 is large from the center out to $R \simeq 40$ arcsec, and from there out is effectively zero (left panel). The right panel demonstrates that this result reflects the existence of a disk at the center of NGC 4697 by showing an image of the galaxy from which a best-fitting elliptical model has been subtracted. [After Peletier *et al.* (1990) from data kindly supplied by R. Peletier]



Upper panel: intensity profiles of Sersic laws (from astroph-0503176v1). The n parameter varies from n=0.5 to 10. Lower panel: the total magnitude in an aperture, as a function of aperture size

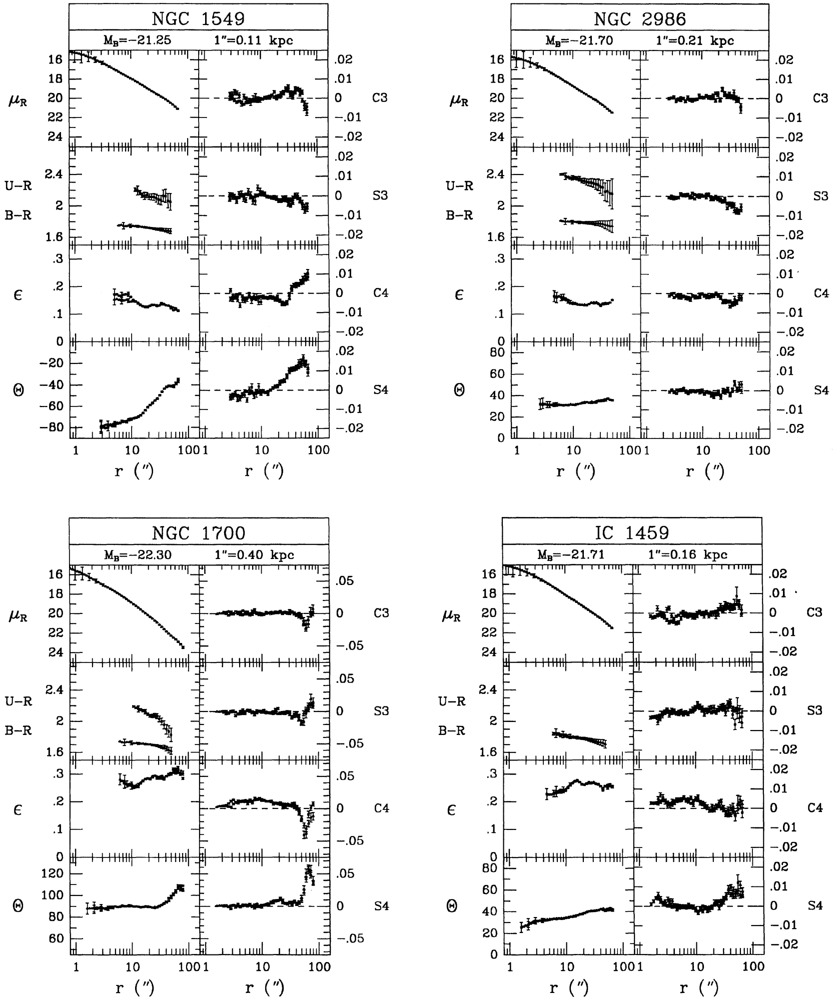


FIG. 9. (continued)

2.2 Kinematics

We generally cannot measure velocities of individual stars
way too many stars in a patch of 1x1 arcsec
Hence measure average motion of the stars.

- Assume that the velocity distribution of the stars along the line-of-sight is (BM 11.1.1)

$$F(v_{los})$$

usually it is assumed that F is a Gaussian, or close to a Gaussian

$$F(v_{los}) = exp\left[\frac{-(v_{los} - \overline{v_{los}})^2}{2\sigma_{los}^2}\right]$$

where $\overline{v_{los}}$ is the average line-of-sight velocity of the stars.

Measure $\overline{v_{los}}$ and σ_{los} by taking a spectrum of the galaxy $G(\lambda)$, and fitting a stellar spectrum $S(\lambda)$ broadened by the velocity distribution $F(v_{los})$

$$\text{fit} \quad G \quad \text{with} \quad \text{convolution of } S \text{ with } F$$

This is most easily done by writing G and S as a function of $L = \ln \lambda$.
A velocity shift δv corresponds to a wavelength shift $\delta\lambda = \lambda v_{los}/c$. Hence $\delta L = v_{los}/c$.

$$\text{Model(L)} = \text{convolved star spectrum}(L) =$$

$$= \int S(L - v_{los}/c) F(v_{los}) dv_{los}$$

This is a simple convolution which can be carried out fast on the computer. The best fitting value for the dispersion and average velocity is derived by minimizing

$$\chi^2 = \Sigma \left(\frac{(G(L) - Model(L))^2}{error(L)^2} \right)$$

The fit can be done either in real space, or in Fourier space (In Fourier space the convolution is simply a multiplication).
By minimizing the residuals from the fit, we can select the best-fitting stellar spectrum, and obtain the average velocity, and velocity dispersion of the stars.
In practice, the stellar continuum is never fitting well, and is subtracted from both the galaxy spectrum and the stellar spectrum. A scaling factor γ is allowed in the fit (i.e., using γS instead of S) to compensate for the fact that the stellar absorption lines often have different strengths, due to different temperatures and metallicities of the stars and the galaxies.

Using correlation function

Another method of deriving velocities is by using the correlation function. Assume that the galaxy has spectrum $G(L)$, the star spectrum $S(L)$. The correlation function is

$$Corr(s) = \int G(L) S(L+s) dL$$

By finding the peak of the correlation function, one finds the value of the average line-of-sight velocity $s_{peak} = v_{los}/c$

why ?

This can be easily derived from the minimization we did earlier. We minimize

$$\chi^2 = \Sigma \left(\frac{(G(L) - Model(L))^2}{error(L)^2} \right)$$

Write out:

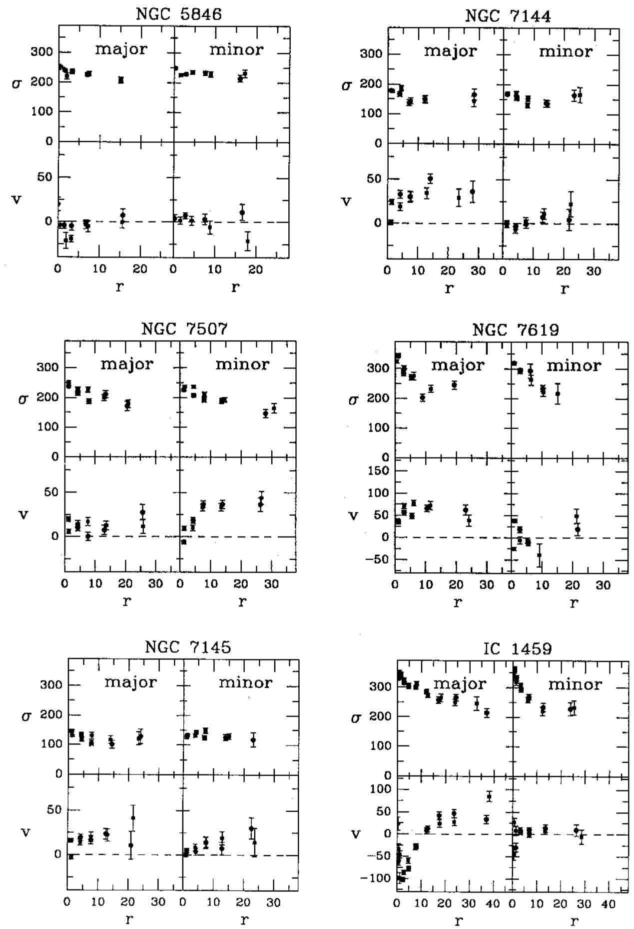
$$\Sigma \left(\frac{(G(L) - Model(L))^2}{error(L)^2} \right) =$$

$$\Sigma[G(L)^2/error(L)^2 + Model(L)^2/error(L)^2$$

$$-2(G(L)Model(L))/error(L)^2]$$

Since the first two terms in the last expression are constant as a function of the line-of-sight velocity, it is clear that χ^2 is minimized if the last term is maximized. At the best fitting velocity, this is exactly the peak of the correlation function. This assumes that the error(L) is a constant.
Some examples of results

1) major and minor axis rotation and dispersion curves



2) full 2D results

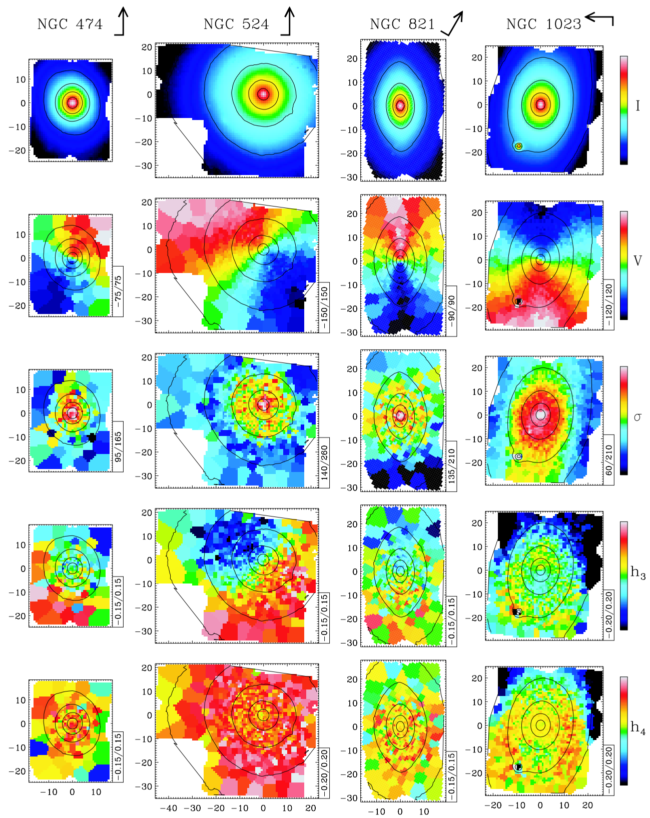


Figure 4. (a)–(l) Maps of the stellar kinematics of the 48 E and S0 galaxies in the SAURON representative sample. The SAURON spectra have been spatially binned to a minimum S/N of 60 by means of the Voronoi 2D-binning algorithm of Cappellari & Copin (2003). All maps are plotted to the same spatial scale. The arrow and its associated dash at the top of each column mark the north and east directions, respectively; the corresponding position angle of the vertical (upward) axis is provided in Table 3. From top to bottom: (i) reconstructed total intensity, (ii) stellar mean velocity V, (iii) stellar velocity dispersion σ, (iv) and (v) Gauss–Hermite moments h_3 and h_4. The cuts levels are indicated in a box on the right-hand side of each map.

general results

- Ellipticals have dispersions around 100-300 km/s (with exceptions)
- Ellipticals rotate gently, with $v/sigma$ from 0 to 1, 1.5
- Most of the rotation on the major axis, but also rotation along the minor axis
- Most of the stellar motions are random, and NOT systematic
  - Elliptical Galaxies are supported by random motions, not rotation

Why are elliptical galaxies flattened ?

Is it just because of rotation ?
Assume that the galaxies are axisymmetric and rotate around the z-axis.

We can extend virial theorem to 3D virial theorem (BTold 4-78/BTnew 4.8.3)

$$1/2\frac{d^2 I_{j,k}}{dt^2} = 2T_{jk} + \Pi_{jk} + Wjk$$

where

$$T_{jk} = 1/2\int \rho\,\overline{v_j}\,\overline{v_k}\,d^3x$$

$$\Pi_{jk} = 1/2\int \rho\sigma^2_{jk} d^3x$$

$$\overline{v_j} = \int v_j f(x,v) d^3v/\rho$$

$$\sigma_{jk} = \int (v_j - \overline{v_j})(v_k - \overline{v_k}) f(x,v) d^3v/\rho$$

and W is the potential energy tensor

$$W_{jk} = -\int \rho(x) x_j \frac{\delta\Psi}{\delta x_k} d^3x$$

For a stationary system, I_{jk} is zero, and the tensors on the right hand side are zero when added.
We have assumed that the rotation is along the z-axis, and symmetrical. Hence

$$W_{xx} = W_{yy}; W_{ij} = 0 \text{ for} i \neq j$$

and similar for Π and T. Hence we are left with the equations

$$2T_{xx} + \Pi_{xx} + W_{xx} = 0; 2T_{zz} + \Pi_{zz} + W_{zz} = 0$$

Dividing the first by the second equation:

$$\frac{2T_{xx} + \Pi_{xx}}{2T_{zz} + \Pi_{zz}} = \frac{W_{xx}}{W_{zz}}$$

Notice that the $T_x x$ and $T_z z$ are due to streaming motions (they are an integral over the average velocities $\overline{v_x}, \overline{v_y}\ \overline{v_z}$). We are assuming that the galaxy is axisummetric around the z-axis. Streaming motion in the direction of the z-axis is not allowed, hence $T_{zz} = 0$, and

$$2T_{xx} = 1/2\int \rho\,\overline{v_\phi}^{\,2} d^3x = 1/2 M v_0^2$$

where M is the mass of the system and v_0^2 is the mass-weighted mean square rotation speed. In the same way

$$\Pi_{xx} = M\sigma_0^2,$$

where σ_0^2 is the mass weighted mean square velocity along the line of sight to the galaxy. We parametrize the dispersion in the z direction by δ

$$\Pi_{zz} = (1-\delta)\Pi_{xx} = (1-\delta)M\sigma_0^2.$$

$\delta < 1$ measures the anisotropy of the velocity-dispersion tensor. At $\delta = 0$, the velocity dispersion in the z and x direction is the same, and the system is "isotropic". For other δ, the system is "anisotropic". We can write

$$\frac{v_0^2}{\sigma_0^2} = 2(1-\delta)\frac{W_{xx}}{W_{zz}} - 2$$

As it turns out, for galaxies with isodensity surfaces which are similar ellipsoids, the term $\frac{W_{xx}}{W_{zz}}$ depends only on the ellipticity of the ellipsoids. Hence $\frac{v_0}{\sigma_0}$ is a function of ϵ and δ only. If the model is isotropic, v/σ should depend in a very simple way on the observed ellipticity. The deviation of the observations from this relation tells us how anisotropic the galaxies are.

In practice, we also have to take into account projection effects: the observer measures not the intrinsic motions, but only the motions along the line-of-sight.

The figure below shows the model predictions.

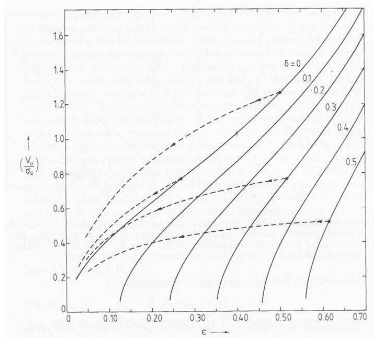


Figure 4-5. The relationship between the rotation parameter v/σ and ellipticity ϵ predicted by (4-95) for elliptical galaxies whose isodensity surfaces are similar coaxial oblate spheroids. The dashed curves show the movement of the point corresponding to the observable quantities $\tilde{v}/\tilde{\sigma}$ and $\tilde{\epsilon}_a$ when the galaxy's inclination angle i is decreased from $i = 90°$.

The observations indicate that many galaxies are not isotropic - i.e., not flattened by rotation:

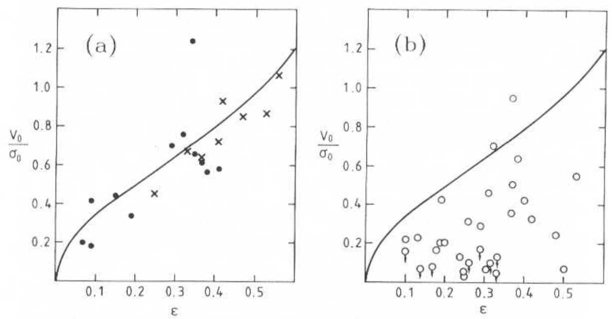


Figure 4-6. (a) The positions in the $(v/\sigma, \epsilon)$ plane of elliptical galaxies (dots), and of spheroids (crosses), that have luminosities smaller than $L = 2.5 \times 10^{10}\,L_\odot$. (b) The same as (a) but for elliptical galaxies brighter than $L = 2.5 \times 10^{10}\,L_\odot$. (After Davies et al. 1983.)

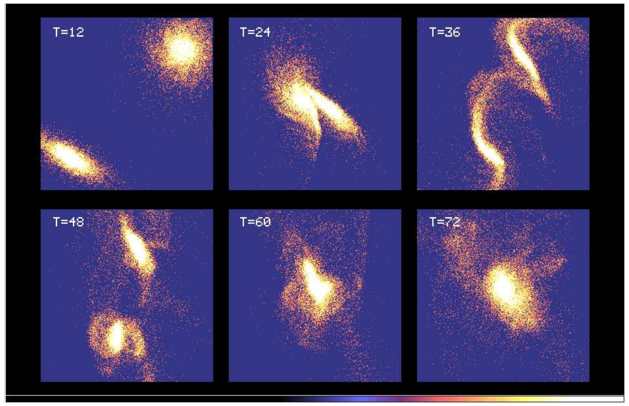
Bright elliptical galaxies rotate very slowly, and hence must be anisotropic to generate their shapes. Fainter ellipticals generally rotate faster - and are more or less consistent with being isotropic.

The conclusion is that the bright elliptical galaxies must be anisotropic. It is very well possibly that these galaxies are triaxial - i.e., have no axis of symmetry. This follows from the simple fact that if their shape is not caused by rotation, the velocity dispersions do not have to be the same in the 3 dimensions. Triaxial models are fairly straightforward to build (due to the box orbits allowed in such potentials). Hence the modeling of galaxies is quite complex: spherical symmetry, or even axisymmetric symmetry does not apply !

This is a major complication in the study of galaxies.

Mergers of galaxies (BT 7.4 1(a,b))

Theories of galaxy formation predict that halos grow by the mergers of lower mass halos. What happens to the galaxies inside these halos ? Simulations can give us the answer. Below is an example.



Merging galaxies quickly form 1 large galaxy.

Why do halos/ galaxies merge so easily ?

BT 7.1 abbreviated

- dynamical friction slows down two galaxies meeting each other
- even galaxies with hyperbolic orbits can merge !

Dynamical friction

consider massive point mass M, moving in sea of small point masses m

- the motion of the massive object will produce a wake behind it
- the wake is an overdensity, and will exert a force which slows down the moving massive object
- the wake is proportional to GM, hence the gravitational force on the object is proportional to G^2M^2

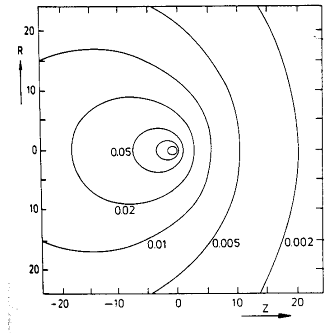


Figure 7-3. A mass travels from left to right at speed v through a homogeneous Maxwellian distribution of stars with one-dimensional dispersion $\sigma = v$. Deflection of the stars by the mass enhances the stellar density downstream more than upstream. Contours of equal stellar density are labeled with the corresponding fractional density enhancement. (From Mulder 1983.)

the friction can be estimated effectively by considering individual interactions between the massive object, and the small point masses

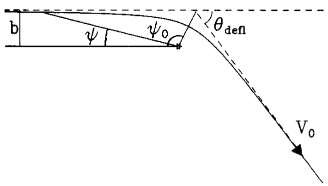


Figure 7-2. The motion of the reduced particle during a hyperbolic encounter.

The motion of the pointmass and the massive object is simply given by the solution for the two-body problem.

m is the mass of small pointmass.
M is the mass of big pointmass.
V_0 is velocity difference at the start. $\vec{V}_0 = \vec{V}_m - \vec{V}_M$
b is impact parameter.
The deviations in velocity are

$$|\Delta v_{M,\perp}| = \frac{2mbV_0^3}{G(M+m)^2}\left[1+\frac{b^2V_0^4}{G^2(M+m)^2}\right]^{-1}$$

$$|\Delta v_{M,||}| = \frac{2mV_0}{(M+m)}\left[1+\frac{b^2V_0^4}{G^2(M+m)^2}\right]^{-1}$$

the parallel deviation $\Delta v_{M,||}$ will always be in the same direction, whereas the perpendicular component will be in a random direction.

Hence, as a result, when the contributions from all particles are taken (by integration), the perpendicular component can be ignored to first order, and the parallel component will dominate.

This will slow down the moving mass M.

Assume that phase-space density of stars is $f(\vec{v})$. M encounters stars with velocities in volume $d\vec{v}_m$ at impact parameters between b and $b+db$

$$rate = 2\pi b\ db \times V_0 \times f(v_m) d\vec{v}_m$$

Integrate over impact parameters b, then the change in $\vec{v}_M$ is

$$\frac{d\vec{v}_M}{dt} = \vec{V}_0 f(\vec{v}_m) d\vec{v}_m \int_0^{b_{max}} \frac{2mV_0}{M+m} \times \left[1+\frac{b^2V_0^4}{G^2(M+m)^2}\right]^{-1} 2\pi b db$$

where b_{max} is the largest impact parameter that needs to be considered.
Now calculate the integral, using $\vec{V}_0 = \vec{v}_m - \vec{v}_M$. We get:

$$\frac{d\vec{v}_M}{dt} = 2\pi \ln(1+\Lambda^2) G^2 m(M+m) f(\vec{v}_m) d\vec{v}_m \frac{(\vec{v}_m - \vec{v}_M)}{|\vec{v}_m - \vec{v}_M|^3}$$

where

$$\Lambda \equiv \frac{b_{max}V_0^2}{G(M+m)}$$

Usually Λ very large. Globular cluster in Milky Way: $M = 10^6$, $V_0 \approx 100$ km/s. $\Lambda = 4.6\ 10^3$, $\ln(1+\Lambda^2) \approx 17$. In the following we use $\ln(1+\Lambda^2) \approx 2\ln\Lambda$. We ignore the variations of Λ with the velocities v_m of the background stars. We assume that the galaxy is isotropic.

The resulting integral is valid for encounters with stars in volume element $d\vec{v}_m$. We have to integrate over all $\vec{v}_m$ to get the total force on M. Notice that the force depends on $\vec{v}_m$ only through the terms. Hence we have to integrate

$$\int f(\vec{v}_m)\frac{(\vec{v}_m - \vec{v}_M)}{|\vec{v}_m - \vec{v}_M|^3}d\vec{v}_m$$

The equation simplifies if the distribution function is isotropic. Notice that the equation is equivalent to the equation for the gravitational force at a location $r = v_M$ for a density denstribution $\rho = f$. Newtons theorem states that this is equivalent to the force caused by a pointmass M with mass equivalent to the mass inside r.
Hence

$$\int f(\vec{v}_m)\frac{(\vec{v}_m - \vec{v}_M)}{|\vec{v}_m - \vec{v}_M|^3}d\vec{v}_m = \frac{\int_0^{v_M} f(v_m)4\pi v_m^2 dv_m}{v_M^3}\vec{v}_M$$

Use $\ln(1+\Lambda^2) \approx 2\ln\Lambda$ to obtain

$$\frac{d\vec{v}_M}{dt} = -16\pi^2 \ln(\Lambda)G^2 m(M+m)\frac{\int_0^{v_M} f(v_m)v_m^2 dv_m}{v_M^3}\vec{v}_M$$

Only stars moving slower than v_M contribute to the force. The drag always opposes the motion.

This is the CHANDRASEKHAR DYNAMICAL FRICTION FORMULA (BT 7.1)
for very large v_M, one derives approximately

$$\frac{dv_M}{dt} = -\frac{4\pi\ln(\Lambda)G^2(M+m)\rho_m}{v_M^2}$$

Two interesting points:

- the drag depends on ρ_m, but not m itself
- the larger M, the stonger the deceleration big objects fall in faster than small objects

Dynamical friction for object with mass M in isothermal halo

For mass M in orbit in isothermal halo one derives the following. The density in the halo is given by $\rho = \sigma^2/(2\pi G r^2) = v_c^2/(4\pi G r^2)$.
Fill this in the equation above to derive:

$$\frac{dv_M}{dt} = -0.428\ln\Lambda\frac{GM}{r^2}$$

Application to Magellanic Clouds (BT 7.1 1(b)

The LMC and SMC have distances of 50 and 63 kpc, and masses of 2e10 and 2e9 solar masses.
If our galaxy has a massive halo extending to the clouds, then the friction time can be estimated in the following way:

The force causes the mass M to loose angular momentum per unit mass L

$$\frac{dL}{dt} = \frac{Fr}{M} = -0.428\frac{GM}{r}\ln\Lambda$$

At all times $L = rv_c$, and v_c is constant. Hence

$$r\frac{dr}{dt} = -0.428\frac{GM}{v_c}\ln\Lambda$$

Solution

$$r(t)^2 = -2 * 0.428\frac{GM}{v_c}\ln\Lambda t + constant$$

Hence, if $r(0) = r_i$, the mass reaches $r = 0$ at $t_{fric}) = 1.17r_i^2 v_c/(\ln\Lambda GM)$
For the clouds this gives:

$$t_{fric} = \frac{1\times10^{10}}{\ln\Lambda}\left(\frac{r}{60kpc}\right)^2\left(\frac{v_c}{220km/s}\right)\left(\frac{2\times10^{10}}{M_{tot}}\right)yr.$$

approximately: $\ln\Lambda = 3$

Hence they will spiral inwards quickly, in a couple of Gyr !

This mechanism makes galaxies merge well !!

Simulations indicate that the orbits may be elliptical. Since the Clouds are close to peri-galacticon, the average friction time may be higher than the previous estimate.

The Magellanic Stream (BM 8.4.1 page 530) is gas that escaped from the clouds during their orbit around the galaxy.
The stream can be found over 60 degrees in the Southern Sky

2. **Structure of Ellipticals**

2.3 **Formation**

Why do ellipticals have radial profiles which are so alike?

Since we can make a large variety of models, the answer is not because we can make only deVaucouleurs profiles.
Hence the formation of the galaxies much have played an important role.
Now notice: our Vlasov equation stated that $df/dt = 0$: from the values of the distribution function at the beginning, the values of the distribution function at the end can be calculated. This suggests that the initial conditions fully determine the outcome...

This is not quite right either, however. One argument is phase mixing. (BTnew 4.10-page 379)

The distribution function remains constant, but while the system progresses, areas with zero and non-zero distribution function will come closer and closer together. At some stage, the invidual particles will start to play a role: we have a very thin area of non-zero distribution function, and it will be filled by only 1 particle. At that moment, our distribution function will not be representative anymore - we have to average over larger volumes. Since these larger volumes will incorporate “empty” distribution function space, the value of the distribution function will go down in those areas. In short, phase mixing will make df/dt go down where relevant.

The second process of relevance is violent relaxation. Fluctuations in the potential will accelerate and decelerate particles. Their energy will change (but the value of the distribution function is conserved). As a result, the energy distribution of the particles will be broadened. This causes the density profiles of galaxies to become more similar: the more violent relaxation, the more the galaxy forgets about the initial conditions. An example is given in figure below (4.28 from BTnew).

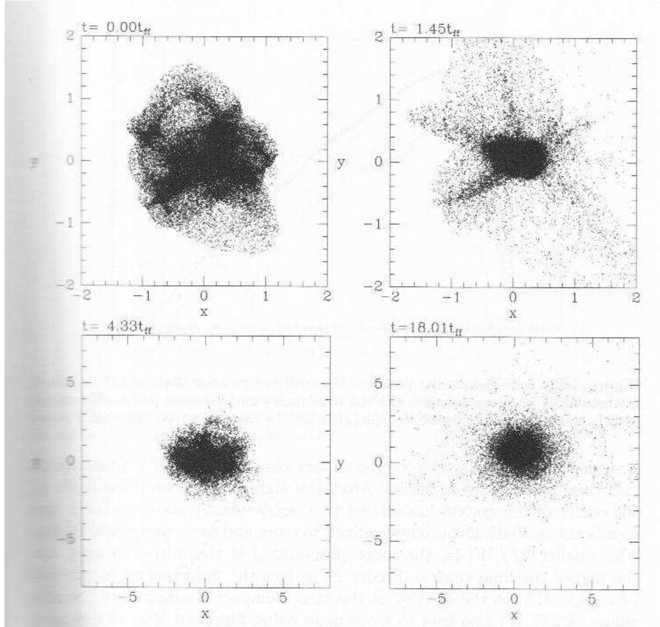


Figure 4.28 Four stages of a collapse simulation. Time is shown at the top of each panel in units such that the initial free-fall time is $\pi/2^{3/2} \simeq 1.1$ (Problem 3.4 with $GM = r = 1$). Top left: initially the 50 000 particles have positions and velocities obtained by sampling them from a homogeneous distribution within the unit sphere. Top right: gravity causes the system to fragment into lumps that fall together to form a tight minimum configuration. Bottom left: after several pulsations of ever-decreasing intensity, the core has settled to a quasi-steady state. Bottom right: after a much longer time a low-density halo of violently ejected stars is in place. Notice that the linear scale of the lower panels is four times that of the upper ones.

It shows the stages in a simulation of a collapse of a cloud of particles. The collapse is fairly strong (i.e., a lot of violent relaxation). These simulations were initially done by van Albada in Groningen.
The intensity profiles at the end were fairly much like the $r^{1/4}$ profiles !
We have to realize that violent relaxation does something important: it only works through the potential fluctuations, and hence the mass of the particle is irrelevant. Hence it does NOT cause equipartition : the massive stars and the low mass stars will have similar velocity distributions (as the normalized forces are the same).

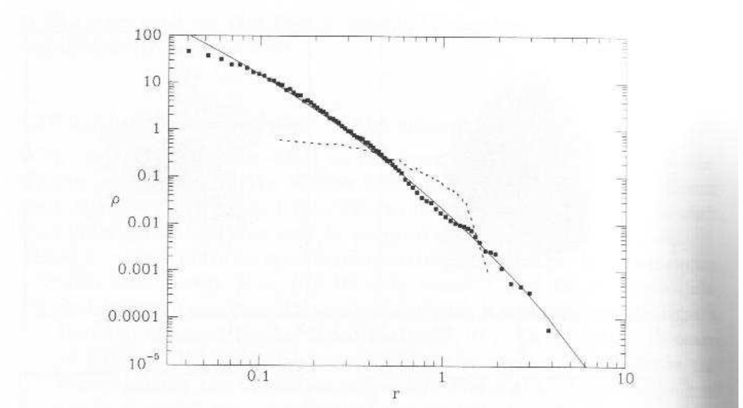


Figure 4.29 The radial density profile of the final configuration of the simulation shown in Figure 4.28 (squares), together with the simulation's initial density profile (dashed line) and the density profile of the $R^{1/4}$ model that has the same half-mass radius (full curve).

An example of the energy distribution is shown below, at the beginning, intermediate, and the end. As can be seen, the distribution function is positive over only a small range of energy in the beginning. The values of the distribution function are going down, due to phase mixing, and the range in energy is increasing, due to violent relaxation. At the end , the distribution is very smooth. $r^{1/4}$ profiles are naturally produced by processes which produce smooth distributions in energy !

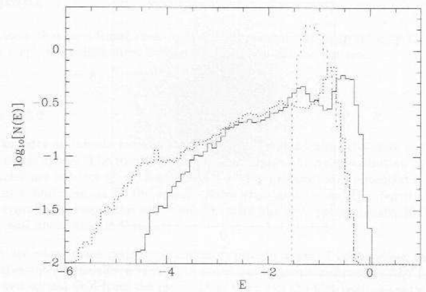


Figure 4.30 The evolution of the differential energy distribution of the model shown in Figure 4.28 at $t = 0$ (dashed curve), at $t = 1.45t_{\rm ff}$ (dotted curve) and $t = 18t_{\rm ff}$ (full curve). Energy is measured in units of GM/R_0, where M is the system's mass and R_0 is the radius of the initial particle distribution.

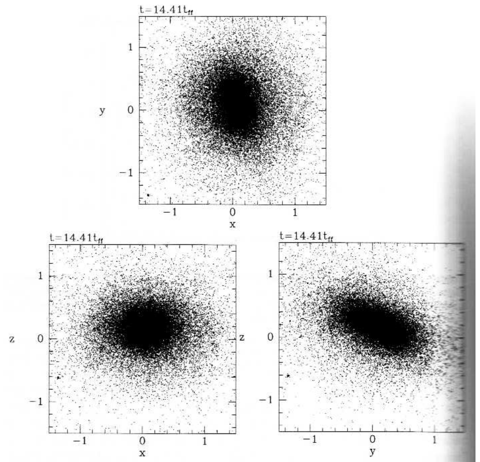


Figure 4.32 Three orthogonal projections of the final configuration to which 50 000 particles settled after they had been released from a cold, elliptical initial configuration. The initial conditions were generated by changing the initial conditions used to make Figure 4.28 from $(\mathbf{x}, \mathbf{v})$ to $(\mathbf{x} + \nabla\chi, \mathbf{v} + \nabla\chi/t_{\rm ff})$, where $\chi \equiv 0.075(3x^2 - y^2 - 2z^2)$.

Furthermore, such distributions are also naturally triaxial.

Interestingly, mergers also produce profiles which are rather similar. This is maybe not too surprising (the profiles are already a bit similar at the start) - and apparently there is enough violent relaxation during the merger to smooth things out. Here are the profiles as derived by Hopkins et al (2008) from a large set of merger simulations

6 Hopkins et al.

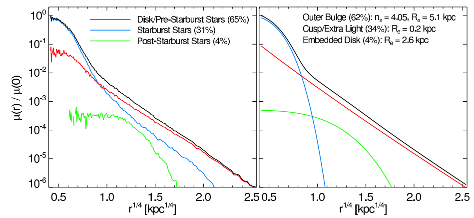


FIG. 1.— Projected surface mass density of stars in the remnant of a highly gas-rich merger. *Left:* Total profile (black) is shown with contribution from various physical components: stars in pre-simulation disks and those formed in the simulated disks before the final merger (disk/pre-starburst stars), stars formed in the compact, merger-induced starburst at final coalescence (starburst stars), and stars formed from gas which survives the merger (post-starburst stars). *Right:* Sersic profile fits to each of these components, labeled as "outer bulge," "cusp/extra light," and "embedded disk," respectively (sum shown in black). Mass fractions, Sersic indices (n_s) and effective radii of each component are plotted ($n_s = 1$ for the embedded disk and cusp/extra light component).

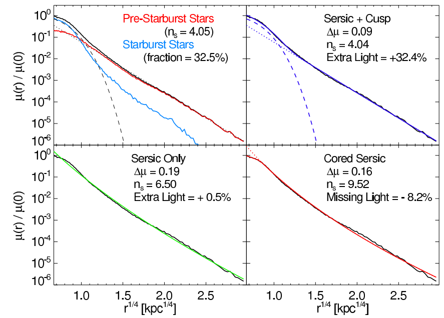


FIG. 2.— *Upper Left:* Surface mass density profile of remnant of a highly gas-rich merger (black), decomposed into stars formed prior to the final merger (which are then violently relaxed; red) and stars formed in the merger-driven dissipational starburst (blue). The Sersic fit to each component is shown, with the Sersic index of the pre-starburst component and mass fraction of the starburst component labeled. *Upper Right:* Two-component (Sersic plus cusp or extra light; where the cusp is fit with an exponential profile) fit to the total light profile, with the Sersic index of the outer component and mass fraction of the inner ($n_s = 1$) component, and rms scatter ($\Delta\mu$) about the fit. *Lower Left:* Single Sersic function fit to the profile. *Lower Right:* Cored Sersic function fit. The two-component fit as we parameterize it accurately recovers the Sersic profile of the violently relaxed component and mass fraction of the starburst component. The other fits give physically misleading results in this case.

Intrinsic correlations

We have seen that ellipticals can be scaled up and down as far as the equilibrium laws are concerned. In reality, they span a relatively narrow range in properties. The figure below shows the distribution.

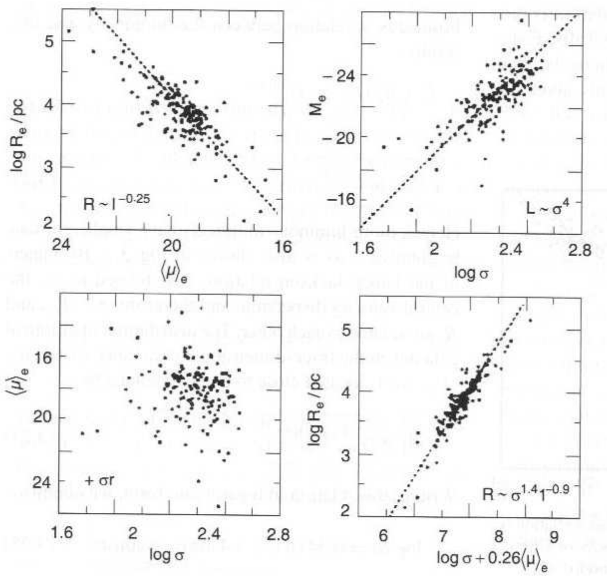


Fig. 3.23. Projections of the fundamental plane onto different two-parameter planes. Upper left: the relation between radius and mean surface brightness within the effective radius. Upper right: Faber–Jackson relation. Lower left: the relation between mean surface brightness and velocity dispersion shows the fundamental plane viewed from above. Lower right: the fundamental plane viewed from the side – the linear relation between radius and a combination of surface brightness and velocity dispersion

The figure on the top right shows the relation between absolute magnitude and velocity dispersion. This is the Faber Jackson relation, the equivalent of the Tully-Fisher relation for spirals.

The other figures show the additional relations between size (R_e), surface brightness ($< \mu_e >$), and velocity dispersion (σ). If we combine these 3 properties optimally, we find a very tight relation (at the bottom right).

$$R \propto \sigma^{1.4} \mu_e^{-0.9}$$

This relation is called the Fundamental Plane (there is not too much fundamental about it). It is much tighter than the Faber Jackson relation. What causes such a relation ?

Origin of the Fundamental Plane

The fundamental plane can best be understood as a relation between the mass-to-light ratio and the mass of the galaxy. Use the fundamental plane to remove the "light" term in the mass-to-light ratio:

$$\frac{M}{L} \propto \frac{M}{R_e^2 \mu_e} \propto \frac{M}{R_e^{-1.1} \sigma^{1.5} R_e^2} \propto \frac{M}{M^{0.75} R_e^{0.15}} \propto$$

$$\propto M^{0.25} R_e^{-0.15}$$

Here we used $M \propto R_e \sigma^2$. We see that the mass-to-light ratio is a function of mass (and a tiny bit of effective radius). This is probably due to the stellar populations in early-type galaxies: the metallicity, and possibly the age of the galaxies are a function of its mass. If so, one would automatically expect that the mass-to-light ratio is a function of the mass.

Some people also think that more aspects play a role than just the stellar populations. It could be, for example, that the profiles of the galaxies vary systematically with mass - so that the simple equations which we used above are not quite valid. We assumed that the relation between velocity dispersion, mass, and size is of a simple form $M = constant R_e \sigma^2$. This may not be the case if the profiles depend on M. Another possibility is that dark matter plays a stronger role in the larger galaxies. In short, whereas stellar population variations must play a role, the other parameters are not yet fully understood.