Relevant Material for Lecture 1

"Galaxies: Structure, Dynamics, and Evolution"

2 Gravitational force and potential (BT 2 - 2.1)

The matter in galaxies (whether stars, gas, dark matter, etc) is kept from escaping by gravity. Before we study the motions of individual particles, we show how we can calculate the gravitation force and potential from an extended density distribution.

The gravitational force caused by a point mass M at $\vec{x}_0$ on a unit mass at position $\vec{x}$ is

$$\vec{F}(\vec{x}) = GM\frac{\vec{x}_0 - \vec{x}}{|\vec{x}_0 - \vec{x}|^3}$$

In general, the gravitational force is related to the potential Φ by

$$\vec{F}(\vec{x}) = -\vec{\nabla}\Phi(\vec{x})$$

so that

$$\Phi(\vec{x}) = -\frac{GM}{|\vec{x}_0 - \vec{x}|}$$

The gravitational potential for **extended** density distribution $\rho(\vec{x})$ can be obtained by integrating over the density distribution

$$\Phi(\vec{x}) = -G\iiint\frac{\rho(\vec{x}_0)d^3\vec{x}_0}{|\vec{x}_0 - \vec{x}|}$$

Note:
The triple integration is often expensive
Easier for special geometries, mass stratifications

- Sphere $\rho = \rho(r)$
- Classical ellipsoid $\rho = \rho(m^2)$ where $m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$
- Thin disk

The density follows from the potential by Poisson's equation:

$$4\pi G\rho(\vec{x}) = \vec{\nabla}^2\Phi(\vec{x})$$

The mass in some volume can easily be derived from the force field: Integrate both sides of Poisson's equation over the volume enclosing a total mass M.
For the left hand side we obtain:

$$4\pi G\int_V \rho d\vec{x} = 4\pi GM$$

Using the divergence theorem, we obtain for the right hand side:

$$\int_V \nabla^2\Phi d\vec{x} = \int_S \vec{\nabla}\Phi \cdot d^2S$$

Combining left and right side gives Gauss's theorem:

$$4\pi GM = \int \vec{\nabla}\Phi \cdot d^2S$$

→the integral of the normal component of $\vec{\nabla}\Phi$ over any closed surface equals $4\pi G$ times the mass contained within that surface

The potential energy can be shown to be:

$$W = 1/2 \int \rho(\vec{x})\Phi(\vec{x})d\vec{x}$$

We derive this as follows. Assume that we "build" up the galaxy slowly. We have a galaxy with a density $f\rho$, with $0 < f < 1$. We add a tiny bit of density $\delta f\rho$, taking the mass from infinity to the galaxy. Ignoring the change in the potential, this costs an energy

$$\int \delta f \rho(\vec{x})\ f\Phi(\vec{x})d\vec{x}$$

where $f\Phi$ is simply the potential of density $f\rho$, and the integral is the integral over the full galaxy volume.
We now have to add all the contributions together to derive the full energy needed to "build" the full galaxy

$$W = \int_0^1 \int \rho(\vec{x})\ f\Phi(\vec{x})d\vec{x}\ df$$

$$= \int \rho(\vec{x})\Phi(\vec{x})d\vec{x} \int_0^1 f df$$

$$= 1/2 \int \rho(\vec{x})\Phi(\vec{x})d\vec{x}$$

3.1 Potential for spherical systems (BT 2.1, 2.2)

Newton's Theorems:

- First Theorem:
  A body inside an infinitesimally thin spherical shell of matter experiences no net gravitational force from that shell

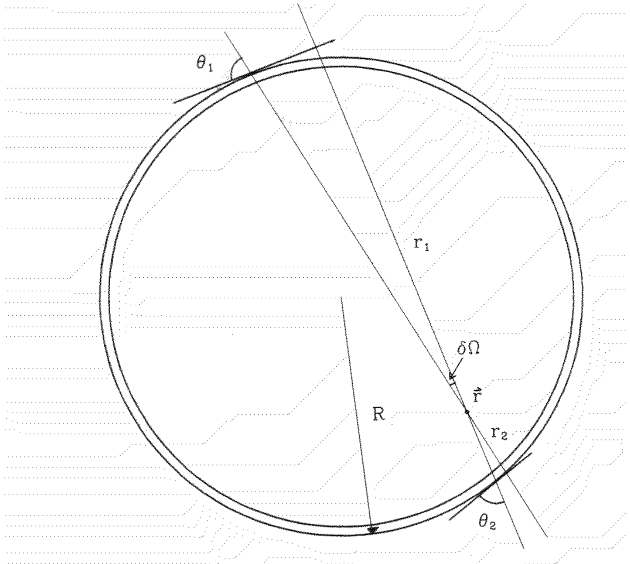


Figure 2-1. Proof of Newton's first theorem.

Consider contributions to the force at point $\vec{r}$, due to the matter in the shell in a very narrow cone $d\Omega$. The intersection angles at 1 and 2, Θ_1 and Θ_2, are equal for infinitely small $d\Omega$. The relative masses in the cone δm_1 and δm_2 satisfy $\delta m_1/\delta m_2 = (r_1/r_2)^2$. The gravitational forces are proportional to $\delta m_1/r_1^2$ and $\delta m_2/r_2^2$, and therefore equal, but of opposite sign. Hence the matter in the cone does not contribute any net force at the location $\vec{r}$. If we sum over all cones, we find no net force !

- Newton's Second Theorem:
  The gravitational force on a body outside a closed spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.

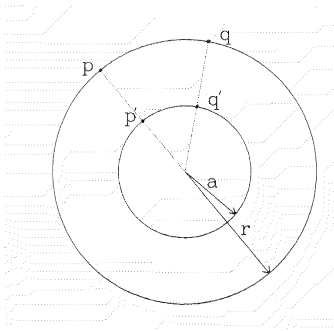


Figure 2-2. Proof of Newton's second theorem.

Calculate the potential at point $\vec{p}$ at radius r from the center of an infinitesimally thin shell with mass M and radius a. Consider the contribution from the portion of the sphere with solid angle $\delta\Omega$ at q':

$$\delta\Phi_1 = -\frac{GM}{|\vec{p} - \vec{q'}|}\frac{\delta\Omega}{4\pi}$$

Now take an infinitesimally thin shell with the same mass M, but radius r. Scale $\vec{p}$ down to $\vec{p'}$, so that it lies at a radius a inside the shell. Scale $\vec{q'}$ up, so that it lies on the shell. Calculate the potential at $\vec{p'}$. The contribution of the matter near $\vec{q}$

with the same solid angle $\delta\Omega$ is:

$$\delta\Phi_2 = -\frac{GM}{|\vec{p'} - \vec{q}|}\frac{\delta\Omega}{4\pi}$$

Since $|\vec{p} - \vec{q'}| = |\vec{p'} - \vec{q}|$, $\delta\Phi_1 = \delta\Phi_2$. Sum over all solid angles to obtain

$$\Phi_1 = \Phi_2$$

Since Φ_2 is the potential inside a sphere with mass M and radius r, it is equal to $\Phi_2 = -GM/r$, and this is equal to Φ_1. This is the same as the potential at r if all the mass is concentrated at the center.

We can now calculate potential of spherical system with density $\rho(r)$. Divide system up into shells, and add contribution from each shell. Distinguish between shells with radius r' , $r' < r$ and shells with $r' > r$:
$r' < r : \ \delta\Phi = -G\delta M/r$,
$r' > r : \ \delta\Phi = -G\delta M/r'$.

Hence total potential:

$$\Phi = -4\pi G\left[\frac{1}{r}\int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r' dr'\right].$$

Hence only single integration ! The force on the unit mass at radius r is determined by mass interior to r:

$$\vec{F}(r) = -\frac{d\Phi}{dr}\vec{e}_r = -\frac{GM(r)}{r^2}\vec{e}_r,$$

where

$$M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'.$$

The circular speed $v_c(r)$ is defined as the speed of a test particle with unit mass in a circular orbit around the center, with radius r. We derive

$$v_c^2(r) = r\frac{d\Phi}{dr} = rF = \frac{GM(r)}{r}.$$

THE CIRCULAR SPEED MEASURES THE MASS INSIDE r !
And is independent of the mass outside r.
The escape speed v_e is the speed needed to escape from the system, for a star at radius r. It is given by

$$v_e(r) = \sqrt{2|\Phi(r)|}$$

Only if a star has a speed greater than that, it can escape. It is dependent on the full mass distribution.

3.2 Simple potentials

- Pointmass

$$\Phi(r) = -\frac{GM}{r}, \quad v_c(r) = \sqrt{\frac{GM}{r}}, \quad v_e(r) = \sqrt{\frac{2GM}{r}}$$

If the circular speed declines like $\frac{1}{\sqrt{r}}$ we call it "Keplerian".

- Homogeneous Sphere

density is constant ρ within radius a, outside it is 0.
For $r < a$:

$$M(r) = \tfrac{4}{3}\pi r^3 \rho, \quad v_c = r\sqrt{\tfrac{4}{3}\pi G\rho}$$

The circular velocity is proportional to the radius of the orbit. Hence the orbital period is:

$$T = \frac{2\pi r}{v_c} = \sqrt{\frac{3\pi}{G\rho}}$$

independent of radius !

release a test mass from rest at position r. Equation of motion:

$$\frac{d^2r}{dt^2} = -\frac{GM(r)}{r^2} = -\tfrac{4}{3}\pi G\rho r$$

This is equation of motion of harmonic oscillator of angular frequency $2\pi/T$. The test mass will reach the center in a fixed time, independent of r. This time is given by

$$t_{dyn} = \frac{T}{4} = \sqrt{\frac{3\pi}{16G\rho}}$$

which we call the dynamical time. Even for systems with variable density we apply this formula (but then take the mean density).
Potential: $\Phi(r) =$

$$r < a: \quad -2\pi G\rho(a^2 - 1/3r^2)$$

$$r > a: \quad -\frac{4\pi G\rho a^3}{3r}$$

- Logarithmic Potential (for Singular Isothermal Sphere)

assume $\rho = \rho_o/r^2$. This density distribution is called the "Singular Isothermal Sphere". It is often used to approximate galaxies. Calculate the mass inside r:

$$M(r) = 4\pi \int_0^r \rho r'^2 dr' = 4\pi \int_0^r \rho_o dr' =$$

$$= [4\pi\rho_o r]_0^r = 4\pi\rho_o r$$

Hence the total mass is infinite. Now calculate the potential by comparing to potential at $r = 1$

$$\Phi(r) = \Phi(1) - \int_1^r F dr' = \Phi(1) - \int_1^r \frac{-GM(r')}{r'^2} dr' =$$

$$\Phi(1) + \int_1^r G4\pi\rho_o 1/r' dr' = \Phi(1) + 4\pi G\rho_o [\ln r']_1^r =$$

$$\Phi(1) + 4\pi G\rho_o \ln r$$

This model is therefore called the "logarithmic potential".

We have a special relation for the circular velocity:

$$v_c^2 = rF = r4\pi G\rho_o/r = 4\pi G\rho_o$$

$$v_c = \sqrt{4\pi G\rho_o}$$

The circular velocity is constant as a function of radius ! We can also express the potential and density in terms of v_c, instead of ρ_o:

$$\Phi(r) = v_c^2 \ln r$$

$$\rho(r) = \frac{v_c^2}{4\pi G}\frac{1}{r^2}$$

The logarithmic potential will return many times in this course, as many galaxies have such potentials.

Axisymmetric models

Generally much more complex, but always easy to get ρ from V:

Miyamoto & Nagai model

$$\Phi(R,z) = -\frac{GM}{\sqrt{R^2+(a+\sqrt{z^2+b^2})^2}}$$

Special cases: $a=0$ Plummer sphere
$b=0$ Kuzmin disk

Kuzmin disk: $\rho(R,z) = \Sigma(R)\delta(z)$

with $\Sigma(R) = \frac{Ma}{2\pi}\frac{1}{(R^2+b^2)^{3/2}}$

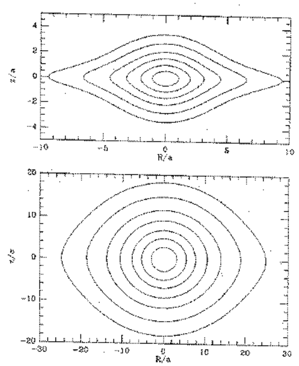


Figure 2-7. Contours of equal density in the (R,z) plane for Satoh's density distribution (2-53b) when: $b/a=1$ (top); $b/a=10$ (bottom). Contour levels are $f\times\{1, 0.3, 0.1, 0.03, \ldots\}$, where: $f=0.1M/a^3$ (top); $f=0.001M/a^3$ (bottom).

Satoh's model

$$\Phi(R,z) = -\frac{GM}{\sqrt{R^2+z^2+a(a+2\sqrt{z^2+b^2})}}$$

- Logarithmic potential

$$\Phi(R,z) = \frac{1}{2}v_0^2 \ln(R_c^2 + R^2 + \frac{z^2}{q^2})$$

$$\rho(R,z) = \frac{v_0^2}{4\pi G q^2} \frac{(1+2q^2)R_c^2 + R^2 + (2-1/q^2)z^2}{(R_c^2 + R^2 + z^2/q^2)^2}$$

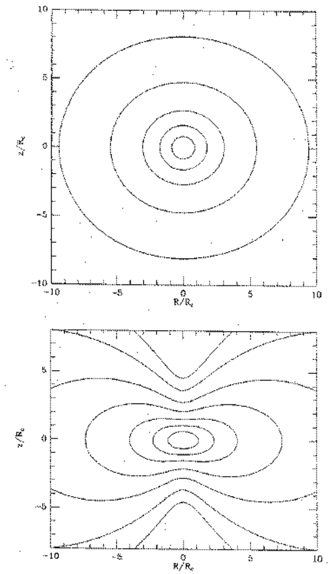


Figure 2-8. Contours of equal density in the (R, z) plane for

Homework assignments:
1) Derive the potential from the density for the point mass, homogeneous sphere, and logarithmic potential, using the equation on page 6.
2) The model given by $\rho = 1/(1+r^2)^{2.5}$ is a Plummer model. Derive the potential of this model. What is the total mass ?
3) Give the derivation of the density related to the axisymmetric logarithmic potential given here.

3.3 Virial Theorem: (not in BT)

relation for global properties: Kinetic Energy and Potential energy.

Again consider our system of point masses m_i with positions $\vec{x}_i$.
Construct $\sum_i \vec{p}_i \vec{x}_i$ and differentiate w.r.t. time:

$$\frac{d}{dt}\sum_i \vec{p}_i\vec{x}_i = \frac{d}{dt}\sum_i m_i \frac{d\vec{x}_i}{dt}\vec{x}_i = \frac{d}{dt}\sum_i \frac{1}{2}\frac{d}{dt}(m_i x_i^2)$$

$$= \frac{1}{2}\frac{d^2 I}{dt^2}$$

where $I = \sum_i m_i x_i^2$, which is the moment of inertia.

However, we can also write:

$$\frac{d}{dt}\sum_i \vec{p}_i\vec{x}_i = \sum_i \frac{d\vec{p}_i}{dt}\vec{x}_i + \sum_i \vec{p}_i \frac{d\vec{x}_i}{dt}$$

Then

$$\sum_i \vec{p}_i \frac{d\vec{x}_i}{dt} = \sum_i m_i \vec{v}_i^2 = 2K$$

with K the kinetic energy.

Since $\frac{d\vec{p}_i}{dt} = \vec{F}_i$ we have

$$\frac{1}{2}\frac{d^2 I}{dt^2} = \sum_i \vec{F}_i\vec{x}_i + 2K.$$

Now assume the galaxy is 'quasi-static', i.e. its properties change only slowly so that $\frac{d^2 I}{dt^2} = 0$. Then the equation above implies

$$K = -\frac{1}{2}\sum_i \vec{F}_i\vec{x}_i$$

.

Then assume that the force $\vec{F}_i$ can be written as a summation over 'pairwise forces' $\vec{F}_{ij}$

$$\vec{F}_i = \sum_{j, j\neq i} \vec{F}_{ij}$$

Now realize that

$$\sum_i \vec{F}_i\vec{x}_i = \sum_i \sum_{j\neq i} \vec{F}_{ij}\vec{x}_i$$

This summation can be rewritten. We sum the terms over the full area $0 < i \leq N$, $0 < j \leq N$, $i \neq j$. However, we can also limit the summation over just half this area: $0 < i \leq N$, $i < j \leq N$, and add the (j, i) term explicitly to the (i, j) term within the summation.
Hence instead of summing over $\vec{F}_{ij}\vec{x}_i$, we sum over $\vec{F}_{ij}\vec{x}_i + \vec{F}_{ji}\vec{x}_j$

Hence

$$\sum_i \sum_{j \neq i} \vec{F}_{ij}\vec{x}_i = \sum_i \sum_{j>i} (\vec{F}_{ij}\vec{x}_i + \vec{F}_{ji}\vec{x}_j)$$

This is simply a change in how the summation is done, it does not use any special property of the force field.

Because $\vec{F}_{ij} = -\vec{F}_{ji}$ (forces are equal and opposite for pairwise forces) the last term can be rewritten

$$\sum_i \vec{F}_i\vec{x}_i = \sum_i \sum_{j>i} \vec{F}_{ij}(\vec{x}_i - \vec{x}_j)$$

For gravitational force $\vec{F}_{ij} = -\frac{Gm_i m_j}{|\vec{x}_i - \vec{x}_j|^2} \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$

$$\sum_i \vec{F}_i\vec{x}_i = -\sum_i \sum_{j>i} \frac{Gm_i m_j}{|\vec{x}_i - \vec{x}_j|^3}(\vec{x}_i - \vec{x}_j)(\vec{x}_i - \vec{x}_j)$$

which equals

$$= -\sum_i \sum_{j>i} \frac{Gm_i m_j}{|\vec{x}_i - \vec{x}_j|} = -\frac{1}{2}\sum_i \sum_{j \neq i} \frac{Gm_i m_j}{|\vec{x}_i - \vec{x}_j|} = W$$

with W the total potential energy. Therefore for a galaxy in quasi-static equilibrium:

$$K = -\frac{1}{2}W,$$

which is the virial theorem for quasi-static systems. The more general expression for non-static systems is:

$$\frac{1}{2}\frac{d^2 I}{dt^2} = W + 2K.$$

3.4 Applications (BT pages 213, 214)

Consider a system with total mass M
Kinetic energy $K = \frac{1}{2}M\langle v^2\rangle$ with
$\langle v^2\rangle$ = mean square speed of stars (assumption: speed of star not correlated with mass of star)

Define gravitational radius r_g

$$W = -\frac{GM^2}{r_g}$$

Spitzer found for many systems that $r_g = 2.5 r_h$, where r_h is the radius which contains half the mass
Virial theorom implies:

$$M\langle v^2\rangle = \frac{GM^2}{r_g}$$

$$M = \langle v^2\rangle r_g G^{-1}$$

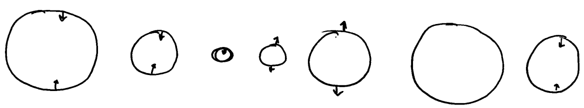
Hence, we can estimate the mass of galaxies if we know the typical velocities in the galaxy, and its size!

3.5 Binding Energy and Formation of Galaxies

The total energy E of a galaxy is

$$E = K + W = -K = 1/2W$$

- Bound galaxies have negative energy - cannot fall apart and dissolve into a very large homogeneous distribution
- A galaxy cannot just form from an unbound, extended smooth distribution $-> E_{total} = E_{start} \approx 0$, $E_{gal} = -K$, so energy must be lost or the structure keeps oscillating:



Possible energy losses through

- Ejection of stars
- Radiation (before stars would form)

3.6. Scaling Relations (not in BT)

Consider a steady state galaxy with particles m_i at location $x_i(t)$.
Can the galaxy be rescaled to other physical galaxies?

- scaled particle mass $\hat{m}_i = a_m m_i$
- scaled particle location $\hat{x}_i = a_x x_i(a_t t)$
- a_m, a_x, a_t are scaling parameters

In the 'rescaled' galaxy we have

$$\hat{\vec{F}}_i = a_m m_i \frac{d^2}{dt^2} a_x \vec{x}(a_t t) = a_m a_x a_t^2 \vec{F}_{i,orig}$$

The gravitational force is equal to

$$\hat{\vec{F}}_G = \sum_{j \neq i} \frac{a_x \vec{x}_j - a_x \vec{x}_i}{|a_x \vec{x}_j - a_x \vec{x}_i|^3} G a_m m_i a_m m_j = \frac{a_m^2}{a_x^2} \vec{F}_{G,orig}$$

Equilibrium is satisfied if the two terms above are equal

$$\hat{\vec{F}}_i = \hat{\vec{F}}_G$$

Since we have equilibrium when all scaling parameters are equal to 1, we obtain

$$a_m a_x a_t^2 = \frac{a_m^2}{a_x^2},$$

or

$$a_m = a_x^3 a_t^2.$$

Now how do velocities scale ?
$\hat{v} = \frac{d}{dt}\hat{x} = \frac{d}{dt} a_x x(a_t t) = a_x a_t v_{orig}$

Hence, the scaling of the velocities satisfies: $a_v = a_x\, a_t$. We can write the new scaling relation as

$$a_m = a_x a_v^2$$

Hence ANY galaxy can be scaled up like this !

Notice that it is trivial to derive this from the virial theorem - the expression for the mass has exactly the same form.

As a consequence, if we have a model for a galaxy with a certain mass and size, we can make many more models, with arbitrary mass, and arbitrary size.

Galaxy Formation

Summary

- We can't say from equilibrium physics what the structure is of a galaxy
- We don't know what a galaxy consists of
  Dark matter (90% ?)
  some stars, gas (10%?)

  we don't even know where a galaxy stops !

Why are galaxies like we see them ?
How is the structure of galaxies determined ?

the bad news

equilibrium physics does NOT fix galaxy structure
(No HR diagram for galaxies as for stars)

the good news

equilibrium physics does NOT fix galaxy structure
galaxy structure is determined by GALAXY FORMATION, i.e., the process by which galaxies formed

The big question in galaxy research is that of **GALAXY FORMATION**

HOW DO GALAXIES FORM ?

We know that

- Universe was very smooth at z=1000 from the Cosmic Background Radiation (fluctuations $\propto 10^{-5}$)
- Universe is not so smooth now: galaxies, clusters, large scale structure

Where does this come from ?

SIMPLEST HYPOTHESIS:

gravitational collapse of very small density enhancements
- We start with a homogeneous universe, with a very small section at slightly higher density
- we notice that the relative density contrast $\delta = \delta\rho/\rho$ grows with time.

this is easy to derive using simple equations, and we will show this below

The expanding universe

it is observed that the universe expands

- nearby: $v = H_0 D$
  - v=velocity , D = distance, H_0= 100 h km/s/Mpc

  h=0.73 $\pm$ 0.03

What are the equations of motion ?

- The complete answer follows from General Relativity
- the correct answer can also be derived from basic, Newtonian physics

Consider a homogeneous sphere, with density ρ, and uniformly expanding
Consider the force on a shell of the sphere, at radius r, and velocity $\dot{r}$:

$$\ddot{r} = -\frac{GM(< r)}{r^2}$$

As the sphere expands, the mass is conserved. Multiply both sides with $\dot{r}$

$$\dot{r}\ddot{r} = -\frac{GM\dot{r}}{r^2}$$

Integrate once:

$$\tfrac{1}{2}(\dot{r})^2 = \frac{GM}{r} + c$$

The terms here can be easily identified: on the left is the kinetic energy, on the right is the gravitational energy, and the integration constant. The total energy is given by

$$E = \tfrac{1}{2}(\dot{r})^2 - \frac{GM}{r} = c$$

Now write $M = \frac{4}{3}\pi r^3 \rho$

$$\tfrac{1}{2}(\dot{r})^2 = \frac{G\frac{4}{3}\pi r^3 \rho}{r} + c$$

$$= G\tfrac{4}{3}\pi \rho r^2 + c$$

$$\left(\frac{\dot{r}}{r}\right)^2 = \tfrac{8}{3}\pi G \rho + \frac{c}{r^2}$$

This is the final equation of motion. Remember, however, that ρ is not a constant, it varies like $\propto r^{-3}$

The left term is special, since it is equal to H, the Hubble "constant". The consequence is, that the hubble constant is not constant !

Universe models

Notice that E is the energy of the sphere, normalized to $r = \infty$. The value of E will determine the evolution of the sphere. The equations above imply that

$$E = c$$

We have different types of models:

- $E = c = 0$. The total energy is zero. The gravitational and kinetic energy compensate each other. The expansion halts at $t = \infty$. Require that $r = t^\alpha$, we find for α: $\alpha = 2/3$. Hence, the solution is $r = t^{2/3}$
- $E = c < 0$ Gravitational energy dominates. The universe will halt, and collapse again !
- $E = c > 0$ Kinetic energy dominates. The universe will keep expanding. Gravitational energy will become less and less important, and at some phase expansion will be at a constant rate.

We can rewrite the Energy criterium as a density criterium. If $c = E = 0$, we define a critical density ρ_c from the last equation:

$$H^2 = \tfrac{8}{3}\pi G \rho_c$$

which can be rewritten as

$$\rho_c = \frac{H^2}{\frac{8}{3}\pi G} = \frac{3H^2}{8\pi G}$$

This density has the property that it would halt the expansion at infinity.
The true density of the universe is usually expressed as Ω

$$\Omega = \rho/\rho_c = \frac{8\pi G\rho}{3H^2}$$

If $\Omega = 1$, then the expansion stops at infinity. If $\Omega < 1$, the expansion continues forever. If $\Omega > 1$, the expansion halts and reverses.

Looking back in time

The remarkable thing is, that as the universe expands, the photon "expands" as well.

first consider objects close to each other

$$v = H_0 D$$

where D is the distance between the 2 objects.
Hence the light will be shifted in wavelength by (simple Doppler)

$$\frac{\delta\lambda}{\lambda} = \frac{v}{c} = \frac{H_0 D}{c}$$

But now compare this to the expansion of the universe in the time it took for the photon to travel from the

moment of emission t_{emit} to the moment of detection t_{obs}

$$\delta D = v_{expand} \times (t_{emit} - t_{obs})$$
$$= H_0 \times D \times (t_{emit} - t_{obs})$$

It follows what

$$\frac{\delta D}{D} = H_0 \times (t_{emit} - t_{obs})$$

But since $(t_{emit} - t_{obs}) = \frac{D}{c}$

$$\frac{\delta D}{D} = \frac{H_0 \times D}{c}$$

and we obtain

$$\frac{\delta\lambda}{\lambda} = \frac{\delta D}{D}$$

Hence the wavelength of the photons expand like the distance between objects ! In short

$$\lambda \propto r$$

where r is the radius which we introduced earlier. This leads us to introduce the "redshift" z:

$$1 + z = \frac{\lambda(observed)}{\lambda(emitted)} = \frac{r(t(observed))}{r(t(emitted))}$$

The redshift is easily measured for galaxies from emission lines, absorption lines, etc. For nearby galaxies

$$v = cz$$

We know galaxies with redshifts of 5 and higher - i.e., the universe was 5 times smaller, and 125 times denser, when that light was emitted than it is now...

We generally use redshift to indicate the relative size of the universe, because it is so easy to use.

How can we derive time of emission from red-shift ?

for $\Omega = 1$ universe

$$r \propto t^{2/3}$$

Redshift is defined by

$$(1+z) = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{r_{obs}}{r_{emit}} = \left(\frac{t_{obs}}{t_{emit}}\right)^{2/3}$$

Hence

$$t_{emit} = \frac{t_{obs}}{(1+z)^{1.5}}$$

Take MicroWaveBackground: $z = 1000$, $t_{obs} = 13\ 10^9$ year, hence $t_{emit} = 4\ 10^5$ year.

How does Ω evolve with redshift ?

Take the equation

$$\left(\frac{\dot{r}}{r}\right)^2 = \tfrac{8}{3}\pi G\rho + \frac{c}{r^2}$$

- if $\Omega = 1$, the total energy is zero, and remains zero - hence Ω remains 1
- if $\Omega < 1$, the energy was defined as:

$$E = c = \tfrac{1}{2}(\dot{r})^2 - \frac{GM}{r}$$

and $\dot{r}$ increases at increasing redshift (i.e., decreases with increasing time). Since

$$\Omega = \frac{8\pi G\rho}{3H^2}$$

we can rewrite the energy equation

$$\frac{E}{r^2} = \tfrac{1}{2}\left(\frac{\dot{r}}{r}\right)^2 - \frac{G\frac{4}{3}\pi\rho r^3}{r^3}$$

$$= \tfrac{1}{2}(H^2 - \tfrac{8}{3}\pi G\rho)$$

Now devide both sides by $4/3\pi G\rho$:

$$\frac{3E}{4\pi G\rho r^2} = \frac{3H^2}{8\pi G\rho} - 1$$

or

$$\frac{3E}{4\pi G}\frac{1}{\rho r^2} = \frac{1}{\Omega} - 1$$

Now we know that $\rho \propto r^{-3}$, hence $\rho r^2 \propto r^{-1} \propto 1 + z$
In short, the left term is proportional to $1/(1 + z)$, and gets smaller and smaller with increasing redshift z. Hence the term on the right also gets smaller as $1/(1 + z)$. Now write $\Omega = 1 + \delta\Omega$. Hence

$$\frac{1}{\Omega} - 1 = -\delta\Omega$$

And since $\frac{1}{\Omega} - 1$ evolves like $(1/(1 + z))$

$$\delta\Omega \propto 1/(1 + z)$$

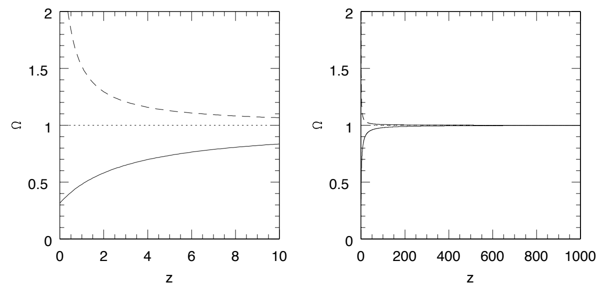
Hence, with increasing redshift, Ω gets closer and closer to 1
As a consequence, Ω was very, very close to 1 at high redshifts, independent of the current value !

How do we now form a galaxy ?

Answer: look back in the distant past (very high redshift). Ω was very close to 1 at that time. Assume that in some volume, the density ρ was enhanced by a minute fraction $\delta\rho$. Since Ω was almost 1 , the smallest $\delta\rho$ is high enough to push the *local* Ω above 1. This local volume has Ω higher than 1, and the total energy is lower than 0. In short, it will not keep expanding like the rest would (if, e.g., $\Omega \leq 1$). Hence it will collapse at some time, and will form a galaxy. Or a cluster...

Evolution of a overdense region

Ω will be close to 1 in the distant past of our universe, independent of the current value. This is put graphically below



Now, take the universe at a nominal redshift of say, $z = 10^6$. We know that Ω is very close to 1. Assume that for some reason or another, density fluctuations are present in this universe:

$$\delta = \rho/\bar{\rho} - 1$$

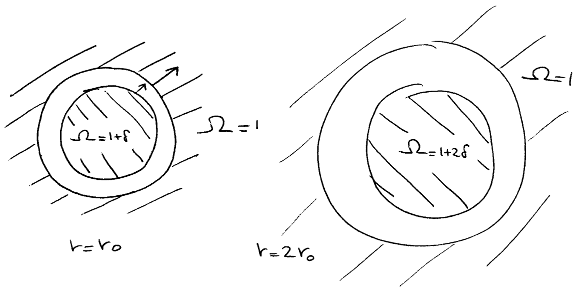
where $\bar{\rho}$ is the mean density of the universe at that epoch, which we take to be the critical density.

We now wish to understand how a overdensity $\delta > 0$ evolves with time.

It turns out that the overdensity grows with expansion like

$$\delta \propto r \propto t^{2/3}$$

We can prove this by considering a homogeneous sphere at $z = 10^6$ with overdensity δ. The sphere is embedded in a $\Omega = 1$ universe.



The particles in the sphere do not feel anything from the outside universe. Hence the sphere will evolve like it is a "separate universe", with $\Omega = (1 + \delta)$. Since a universe with $\Omega > 1$ will collapse at some time, the sphere will collapse at some time $t_{collapse}$. Before that, the density of the sphere will evolve like

$$\frac{1}{\Omega(t)} - 1 = \frac{\frac{1}{\Omega_0} - 1}{1 + z(t)} = \frac{\frac{1}{\Omega_0} - 1}{1/r(t)} = (\frac{1}{\Omega_0} - 1)r(t)$$

where r is the "radius" of the universe (expansion parameter). Take at $z = 10^6$: $\Omega_0 = 1 + \delta_0$, then we find

$$\frac{1}{\Omega_0} - 1 = \frac{1}{1 + \delta_0} - 1 = 1 - \delta_0 - 1 = -\delta_0$$

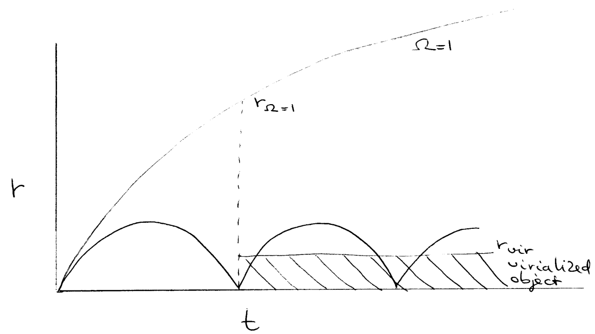
Hence, the equation results into

$$\delta(t) = \delta_0 \times \frac{r(t)}{r_0} = \delta_0 * \left(\frac{t}{t_0}\right)^{2/3}$$

Hence, the density contrast of the sphere increases linearly with expansion radius. As a result, if a fluctuation was present of 10^{-6} at $z = 10^6$, it would have grown to a fluctuation of $\delta = 1$ at $z = 0$ under its own gravity.

This mechanism is the basic mechanism to form galaxies

In detail, what happens is the following:



The sphere will collapse, and start oscillating (if we ignore the material just outside of the sphere). In reality, the sphere will have internal density fluctuations, and it will settle to an equilibrium structure, with a radius of about half the "maximum expansion " radius.

This radius is often called the "virialization radius" , r_{vir}. The sphere will obtain this radius at the first collapse time, which is also called the "virialization time" (or "formation time")
By comparing the maximum expansion radius of the sphere to the "normal expansion" radius of the universe (with $\Omega = 1$) at the virialization time, we can derive:

$$r_{max} = \frac{1}{\frac{1}{4}(12\pi)^{2/3}} r_{\Omega=1}(t_{collapse}) \quad (\approx 0.36 r_{\Omega=1}(t_{collapse}))$$

This is derived using analytical solutions for the expansion of the universe. Since $r_{vir} = 1/2 r_{max}$

$$r_{vir} = \frac{1}{\frac{1}{2}(12\pi)^{2/3}} r_{\Omega=1}$$

The relative density of the sphere, compared to the rest of the universe, is simply given by the ratio $(r_{\Omega=1}/r_{vir})^3$, since the mass of the sphere is conserved, but the density is increased compared to the $\Omega = 1$ universe since the mass is put in a smaller density structure.
Hence

$$\frac{\rho_{vir}}{\rho(universe)(z = z_{vir})} = (r_{\Omega=1}/r_{vir})^3$$

$$= (1/2(12\pi)^{2/3})^3 = 18\pi^2 = 178$$

This makes a very specific prediction for the density of objects (galaxies, clusters, etc : If a galaxy forms at a redshift z_{form}, it will have a density which is 178 times higher the density of the universe at z_{form}.

After the galaxy has formed, it will remain the same, whereas the universe will keep expanding. Hence, the density contrast will increase with time

$$\frac{\rho_{vir}}{\rho(universe)} = 178 * (r/r_{form})^3$$

$$= 178 * ((1 + z_{form})/(1 + z))^3 = 178 * (t/t_{form})^2$$

This can be used as a simple recipe: we now measure that galaxies have an overdensity of about 10^5 inside the optical radius. This part would be formed at a redshift of

$$(1 + z_{form}) = (10^5/178)^{1/3} = 8$$

The galaxy is much bigger, however, than the optical radius. The halo has a density profile which goes like $\rho \propto r^{-2}$. The average density goes down like r^{-2}, and the density contrast will be a lot smaller if we take the halo into account. If we assume that the halo extends to 100 kpc (10 times further), the density will be lower by a factor of 100, and the formation redshift will be

$$(1 + z_{form}) = (10^3/178)^{1/3} = 1.8$$

Hence, the fact galaxies have halos has a very important consequence for galaxy formation: it makes them bigger, have lower mean density, and thereby form much later !
At maximum expansion, the region has an overdensity of about 5, this increases very rapidly to 178 in the next half of the total collapse time.

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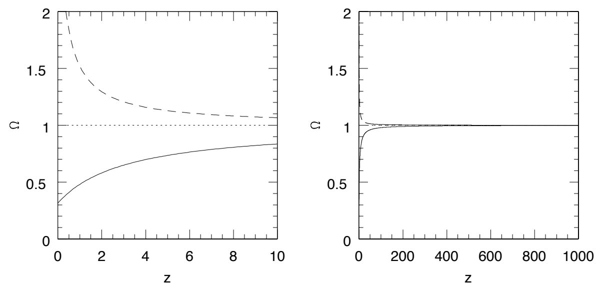

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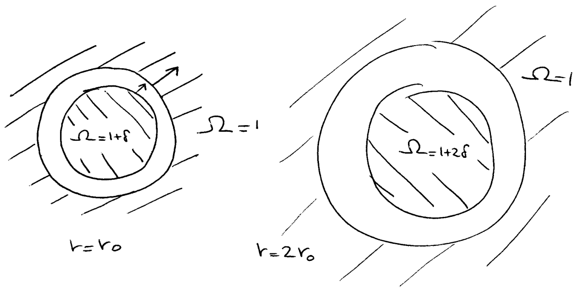

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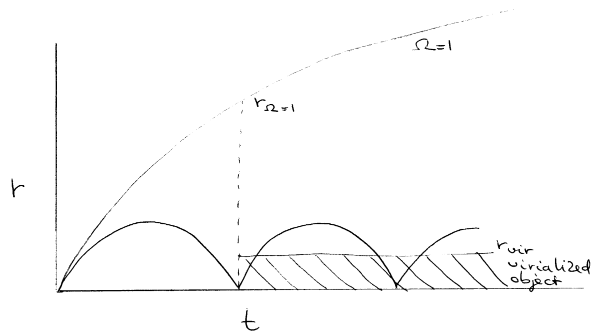

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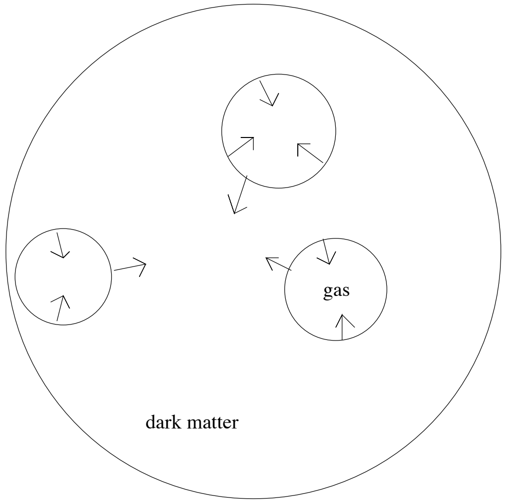
Galaxy Formation

Leading questions for today
- How do visible galaxies form inside halos ?
- Why do galaxies/halos merge so easily ?

How do visible galaxies form inside halos ?

density fluctuations and gravity produce:
dark matter halos
- halos much bigger than visible part of galaxy
- halos rotate slowly $< v > /\sigma \approx 0.3$

Halos entirely UNLIKE visible galaxies
So what happens ?



When new halo just formed: gas is distributed like dark matter
- gas supported by pressure
- dark matter supported by random motions

Gas can cool by radiation, and can collapse to the center if cooling efficient
Dark matter cannot cool ! Will not collapse to center

Gas cooling is expressed as:

cooling rate $= n^2\Lambda(T)$

cooling rate is cooling per unit volume element
n is number density of gas
$\Lambda(T)$ is cooling function

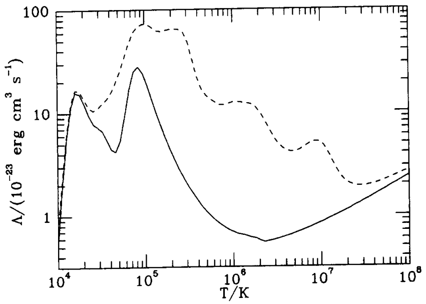


Figure 24.19 The cooling function $\Lambda(T)$. The solid line corresponds to a gas mixture of 90% hydrogen and 10% helium, by number. The dashed line is for solar abundances. (Figure from Binney and Tremaine, *Galactic Dynamics*, Princeton University Press, Princeton, NJ, 1987.)

Cooling mechanisms: bound-bound, bound-free, free-free, electron scattering

- 10^4 = ionization/recombination hydrogen
- 10^5 = ionization/recombination helium

$T > 10^6 K$: thermal bremsstrahlung and Compton scattering

Now take a halo with gas inside. Two options:
- $T_{cool} < T_{dyn}$, then cooling is efficient, and the gas will collect in the center
- $T_{cool} > T_{dyn}$, then cooling is inefficient, and the gas will NOT collect in the center

Draw figure of n_{lum}, the number density in luminous material (in units of particles/cm^3), versus temperature T. Draw the line of $t_{cool} = t_{dyn}$, and put in astronomical objects:
- galaxies
- groups and clusters of galaxies

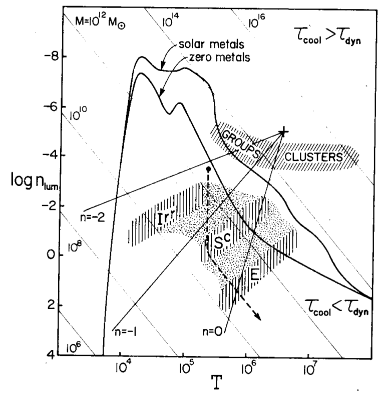


FIG. 1. Ostriker-Rees cooling diagram showing the actual locations of present-day galaxies and clusters. Data on galaxies came from the following sources - E's: σ's from Terlevich *et al.* (1981), M_{lum}/L from Faber and Gallagher (1979); Sc's: v_{rot}'s and radii from Burstein *et al.* (1981), M_{lum}/L_B assumed to be 1.6; Irr's: Thuan and Seitzer (1979). Data on groups and clusters from Rood and Dickel (1978). Cross is mean mass turning around today (White and Rees 1978). Heavy straight lines are clustering loci for various values of n. Light lines are the mass of a self-gravitating body composed purely of ordinary matter. Dashed line is a sample track for rapid dissipation within a heavy halo (see Figure 2).

We find
- gas in galaxies cools efficiently
- gas in clusters does not !

when galaxies cool, the density will increase, while T remains roughly constant.
When the gas becomes self gravitating, the T might increase (the circular velocity will increase, hence the temperature $\propto v_c^2$)

when the gas is self-gravitating, it can start to form stars !

As a result, the luminous galaxy forms in the center of the halo, and is much smaller than the dark halo

How to get disks in spiral galaxies?

Gas cools down and contracts.
Assume that specific angular momentum is conserved

$$j = \frac{J}{m} = r_{start} < v >= 0.3 r_{start} \sigma_{\rm dm}$$

Gas contracts by factor f:

$$f = \frac{r_{start}}{r_{end}}$$

specific angular momentum is conserved:

$$j_{end} = r_{end} < v_{end} >= j_{start} = 0.3 r_{start} \sigma_{\rm dm}$$

Hence

$$< v_{end} >= 0.3 \frac{r_{start}}{r_{end}} \sigma_{\rm dm}$$
$$= 0.3 f \sigma_{\rm dm}$$

The gas rotates faster and faster while the gas contracts
contraction is halted when the cooled gas is in circular orbits around the center. For such orbits

$$v = v_c = \sqrt{2} \sigma_{\rm dm}$$

Hence this occurs when

$$0.3 f = \sqrt{(2)}$$

or

$$f = 5$$

Hence, a cooling gas cloud will settle into a disk when it has contracted by about a factor of 5 or so.

Ellipticals and bulges can be made out of pre-existing disks when galaxies merge, or during mergers