Galaxies: Structure, Dynamics, and Evolution
Problem Set 5
Instructor: Dr. Bouwens

Here is problem set #5. The entire problem set will be due before noon on Wednesday, November 28, 2016 (email them to Gabriela Calistro or put them in her mailbox).

1. Derive the Fundamental Plane that one would find if the mass-to-light ratio is a function of mass only $M/L = M^{0.25}$ and more generally $M/L = M^{\gamma}$. (The Fundamental Plane is the relation of the form $R_e \propto \sigma^{\alpha}\mu_e^{\beta}$.) Assume that the galaxies are homologous, i.e., they have similar density profiles, but scaled up or down with respect to each other.

2. Consider the collapse of a uniform cloud of stars initially at rest. Assume the cloud has a total mass of 5×10^{10} $M_{\odot}$, is entirely composed of stars with 1 $M_{\odot}$, and has approximate dimensions of 2 kpc $\times$ 2 kpc $\times$ 2 kpc. Assume that the collapse finishes in one free fall time, $1/\sqrt{G\rho}$. What is the time scale for violent relaxation? [Approximate order-of-magnitude estimates are fine for this first step.] If the system were instead in equilibrium (i.e., not undergoing collapse), what relaxation time scale would we estimate for stars in this system using the equations we derived in Lecture #2? How do these time scales compare?

3. Determine what the b_n normalization factor in the Sersic law must be such that the integral of the surface brightness profile $10^{b_n[(R/R_e)^{1/n}-1]}$ over all radii is equal to one. What is this normalization factor in the case $n = 1$ and $n = 4$?

4. Look at the angular correlation functions for luminous galaxies $-22 < M_{UV,AB} < -21$ and lower luminosity galaxies $-19 < M_{UV,AB} < -18$ (shown in the last lecture). What is the ratio of bias factors for these galaxies at a scale of 1.5 h^{-1} Mpc? What about the ratio of bias factors for red and blue galaxies at a scale of 1.5 h^{-1} Mpc for the other figure shown in lecture? [Make your best guess for the bias factors based on the figures.]

5. One result which has been found in the astronomical literature is that the observed clustering of quasars does not depend on the luminosity of the quasar. What does this suggest about the relationship between the quasar luminosity and the mass of the underlying halo in which it lives. Can you

think of any physical reason why this might be the case?

6. (a) Derive the enclosed mass M($< r$) for the NFW profile $\rho(r) = \rho_s/[(r/r_s)(1+r/r_s)^2]$. Use $r/(1+r)^2 = 1/(r+1) - 1/(1+r)^2$
(b) Use this to show $\rho_s = \frac{200}{3}\rho_{cr}(z)\frac{c^3}{\ln(1+c)-c/(1+c)}$ given our parameterization $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$
(c) Derive the circular velocity as a function of radius for an NFW profile.

7. We can see from the figure from Springel et al. that about 30-40% of the mass of a halo is in subhalos. This appears quite different from the situation in clusters, where the light is dominated by the ensemble of regular cluster galaxies, and NOT by the brightest cluster galaxy. Can you think of an explanation ?