Galaxies: Structure, Dynamics, and Evolution
Problem Set 4
Instructor: Dr. Bouwens

Here is problem set #4. The entire problem set will be due before noon on Wednesday, November 9, 2015 (email them to Gaby or put them in her mailbox).

1. One particularly interesting question to consider regarding the orbit structure is for the homogeneous ellipsoid potential where the motion of a star in the x, y, and z directions can be described as harmonic oscillators. Suppose that we represent the motion of each particle as $x = a_x cos(\omega_x t + \phi_x)$, $y = a_y cos(\omega_y t + \phi_y)$, $z = a_y cos(\omega_z t + \phi_z)$ where ω_x does not equal ω_y does not equal ω_z and ω_x/ω_y, ω_y/ω_z, and ω_x/ω_z are not rational numbers. Particles in such a potential follow box orbits.

(a) Argue that a particle travels arbitrarily close to every spatial position in the entire volume $(-a_x, a_x) \times (-a_y, a_y) \times (-a_z, a_z)$. If you cannot prove it explicitly, demonstrate the plausibility of this statement by showing 2-D projections of the orbital tracks for two separate choices of $(\omega_x, \omega_y, \omega_z)$. Plot out the orbital tracks for varying integration times (short time, intermediate time, long time intervals).

(b) Write down a formula for the angular momentum of a particle in this potential. Is the angular momentum conserved? Why or why not? What is the average value of the angular momentum averaged over time?

(c) If a were equal to b (and hence ω_x were equal to ω_y), would the angular momentum be conserved? Why or why not? Would particles still travel on box orbits, or would the orbits be loop orbits?

2. Derive the third Jeans equation by subtracting the second Jeans equation from the first Jeans equation multiplied by $\bar{v_j}$. See the lecture notes to the Bachelor course for hints on how to do this.

3. Show that the distribution function DF

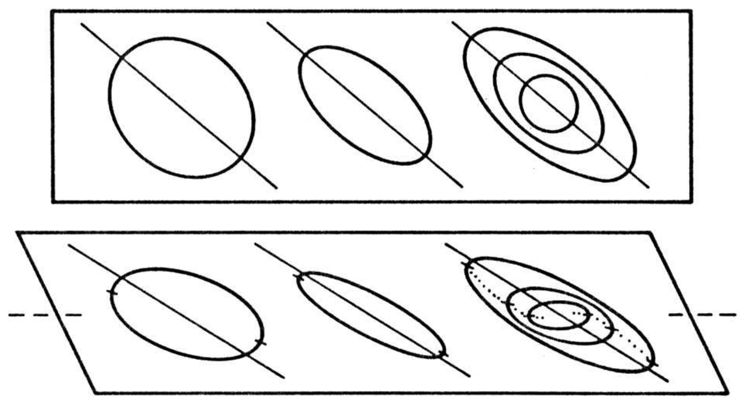
$$f(\epsilon, L) = \begin{cases} F\delta(L^2)(\epsilon - \epsilon_0)^{-1/2} & \text{for } \epsilon > \epsilon_0, \\ 0 & \text{otherwise.} \end{cases}$$

where F and ϵ_0 are constants and δ is the familiar delta function. Show that this distribution function self-consistently generates a model with density

$$\rho(r) = \begin{cases} Cr^{-2} & \text{for } r < r_0 \\ 0 & \text{otherwise.} \end{cases}$$

where C is a constant and the relative potential at r_0 satisfies $\Psi(r_0) = \epsilon_0$. This is the only analytic stellar system known to us in which all stars are on perfectly radial orbits.

4. Determine the impact of projection effects on the apparent isophotal twist (for elliptical galaxies). Consider two ellipses with their major axis oriented 45 degrees away from some line (that line would be horizontal on the following diagram):



Suppose that the axial ratio is 1.15 for the one ellipse (similar to the leftmost ellipse shown in the above figure) and 2.8 for the other ellipse (similar to the center ellipse shown in the above figure). Suppose that we are viewing the ellipses face on and then we rotate the ellipses by 60 degrees about an axis (parallel to the aforementioned line) so that the ellipses are viewed almost edge on. What ellipticity would we measure for each of our two ellipses? What would be the apparent position angle of the major axis of each ellipse relative to aforementioned line?