Galaxies: Structure, Dynamics, and Evolution
Problem Set 3
Instructor: Dr. Bouwens

Here is problem set #3. The entire problem set will be due before noon on Tuesday, November 1, 2016 (email them to Daniel Lam or put them in his mailbox).

1. To help yourself visualize the Inner and Outer Lindblad Resonances work as well as the corotational radius, I will ask you to sketch out a number of snapshots of the movement of a spiral arm around a galaxy. Assume that a galaxy has two spiral arms and that it is an isothermal sphere.

(a) Suppose that circular velocity of the galaxy is 180 km/s and that the inner Lindblad resonance for a galaxy is at a radius of 2.5 kpc. What is the pattern speed (or frequency) Ω_p?

(b) What is the period of epicyclic motion at the Inner Lindblad resonance?

(c) What is the corotation radius and radius of the Outer Lindblad resonance for the pattern speed you computed in part (a)?

(d) In multiples of the epicyclic period of stars at the inner Lindblad resonance (consider multiples out to 10), please sketch out the motion of stars in a spiral galaxy (similar to what I do in lecture, but now moving indicative stars at the Inner and Outer Lindblad resonances and corotation radius self consistently). Please indicate the position of the spiral arms, a star at the radius of the Innerblad resonance, a star at the corotation radius, and a star at the radius of the outer Lindblad resonance. Assume that the position angle of the spiral arms and the stars at all three radii are all 0 at time t = 0.

(e) Redraw the first three time steps from part (d) but now adopting a frame that rotates at a frequency of $\Omega - n\kappa/m$. In what sense (if any) do the orbits appear invariant in this rotational frame?

2. How many integrals of motion are there for a particle with the following force law $F(r) = -\hat{r}/r^2$ where r is the radius?

3. Calculate the density related to the spherically symmetric potential $\Psi = 1/\sqrt{1+r^2}$. Show that this model is a polytrope (i.e., that we can write $\rho = cnst \times \Psi^g$.)

4. (a) For the parameterization of the distribution function as $f(\epsilon)$ where

$f(\epsilon) \geq 0$ for $\epsilon \geq 0$ and $f(\epsilon) = 0$ for $\epsilon < 0$, where $\epsilon = \Phi_0 - E$ and $\Psi = \Phi_0 - \Phi(r)$. Assume that the $\rho(r) = Cr^{-1.1}$. Based on $\rho(r)$, calculate $\Phi(r)$ and $\Psi(r)$. Suppose that $\Psi(r_0) = 0$ at some radius r_0.

(b) Assume a distribution function $f(\epsilon) = N\delta(\epsilon)$ where δ is the familiar delta function and use the $\Psi(r)$ calculate in part (a). Based on this distribution function, calculate $\rho(r)$. Does it match the $\rho(r)$ in part (a)?

(c) Assume that the distribution function $f(\epsilon) = \Sigma_{i=1}^{i=5} N_i \delta(\epsilon - \epsilon_i)$. Sketch the $\rho(r)$ you calculate in part (a). By sketching on the same diagram the $\rho(r)$ you would calculate from 5 different terms in the new expression for $f(\epsilon)$, illustrate how you might try to approximately match the $\rho(r)$ calculated in part (a) by appropriately choosing N_i and ϵ_i.

(d) What are the similarities between the approach in part (c) and Schwarzschild's method?

5. Finding a solution to the collisionless Boltzmann equation using Jeans theorem.

Derive ρ and Ψ for a spherically-symmetric system with some distribution function f of the form $f(\epsilon) = \begin{cases} \epsilon^{n-3/2}, & \text{if } \epsilon > 0 \\ 0, & \text{if } \epsilon < 0 \end{cases}$. where $n = 1$, $\epsilon = -E + \Phi_0$ and E is the energy of a particle orbitting around the system. Adopt the standard definition that $\Psi = \epsilon + (1/2)v^2$. Show that the total mass of the model is $(1/2)\Psi_0 G^{-3/2}\sqrt{\pi/c_1}$ where c_1 is defined by equation (4-85b) from BT. Hint this is problem 4-15 from Binney & Tremaine (BT) and is discussed in some depth on BT 300-302.

6. Orbits.

(a) Poincare Surfaces of Section. Go to the website http://burro.cwru.edu/JavaLab/SOSweb/SOSapplet.html. Run three different simulations with the java applet for galaxy shapes of 1.0. Take the galaxy type to be "singular" and the energy to be "-1.0." Also run three simulations for galaxy shapes of 0.5. Print out the Poincare' surfaces of section (shown in the right-hand panel for the applet) and the orbits in x/y space. Would you describe the orbits as box orbits or loop orbits? If not familiar with the concept of box or loop orbits, consult the lecture notes on the web.

(b) To obtain a feel for box orbits or other orbits in different potentials, please go to the website http://www.ifa.hawaii.edu/~barnes/transform.html and search for the text "box orbit," "X-Tube Orbit," and "Fish Orbit." Watch the movies. Attach a print out of the orbit structure with your solutions to the other problems.