Galaxies: Structure, Dynamics, and Evolution
Problem Set 1
Instructor: Dr. Bouwens

Here is problem set #1. The entire problem set will be due before noon on Monday, October 3, 2016 (email them to Daniel Lam or put them in his mailbox).

1. In class, we calculated the relaxation time for a star in some dynamical system (e.g., a galaxy) by considering the effect of its interactions with other stars in a galaxy. In calculating the relaxation time, we only considered impact parameters b from b_{min} to the size of the galaxy R (where b_{min} was the minimum impact parameter where our simple formula for the velocity kick was approximately valid). What about the effect of impact parameters $b = 0$ to b_{min}? How does this impact the relaxation time?

(a) Calculate the probability that a star will pass by another star with impact parameter b less than or equal to b_{min}? (Ignore the curvature of the orbit.) Adopt the standard variables N, v, m, and G used in the derivation during lecture.

(b) If any star passes by another star with impact parameter $b < b_{min}$, its velocity will be so perturbed that it "will lose all memory of its initial orbit." Let us say it relaxes with just one encounter. Calculate the relaxation time assuming that $b < b_{min}$ encounters are the only meaningful relaxation process. Calculate the relaxation time for the same choice of N, m, G, and v considered in class for a galaxy (i.e., $N = 10^{10}$, $v = 100$ km s^{-1}, $r = 10$ kpc, $t_{cross} = 10^8$ yr). How does this compare with the relaxation time derived in class considering only $b > b_{min}$ encounters?

2. Derive the potential $\Phi(r)$ from the isothermal density profile $\rho(r) = \frac{v_c^2}{4\pi G r^2}$.

3. A galaxy has a gas disk with a constant circular velocity v_c. Calculate the mass enclosed inside any radius r. Use the result to calculate the density at r. (Nota bene: this is not the average density inside r).

4. (a) Consider that there was some overdense region in the universe which had a density ρ which was $2\rho_{crit}$ (the critical density) which otherwise had spherical symmetry. What was the density of that sphere relative to ρ_{crit}

when that sphere were 10 times smaller?

(b) Imagine that the universe as a whole had an average density ρ equal to $2\rho_{crit}$ at the present time. How overdense was the universe when the universe was 10 times smaller?