Connecting Galaxies to their Halos

(through large scale structure / clustering and abundance matching)

+

Probing the Star Formation Histories of Galaxies

Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics
Feb 9: Disk Galaxies (I)
Feb 12: Disk Galaxies (II)
Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics
Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations
Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)
Mar 9: Elliptical Galaxies (II)
Mar 23: Dark Matter Halos
Mar 30: Connecting Galaxies to Dark Matter Halos ←
Apr 13: Galaxy Stellar Populations
Apr 20: Lessons from Large Galaxy Samples at $z<0.2$
Apr 23: Evolution of Galaxies with Redshift
May 4: Galaxy Evolution at $z>1.5$ + Gas Flows
May 11: Galaxy Evolution at $z>6$. / Review for Final Exam

Problem Set 3 - Due on April 6

Include "GSD" in subject line in message to Wout

Galaxies: Structure, Dynamics, and Evolution
Problem Set 3
Instructor: Dr. Bouwens

Here is Problem Set 3. The entire problem set will be due before class on Monday, April 6 (email them to Wout and include GSD in the subject line). Be sure to pay extra attention to problem 1, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. Consider the case of the homogeneous ellipsoid potential discussed on page 3 of this handout. For this potential, the motion of a star in the x, y, and z directions can be described as harmonic oscillators. Suppose that we represent the motion of each particle as $x = a_x cos(\omega_x t + \phi_x)$, $y = a_y cos(\omega_y t + \phi_y)$, $z = a_y cos(\omega_z t + \phi_z)$ where ω_x does not equal ω_y does not equal ω_z and ω_x/ω_y, ω_y/ω_z, and ω_x/ω_z are not rational numbers. Particles in such a potential follow box orbits.
(a) Argue that a particle travels arbitrarily close to every spatial position in the entire volume $(-a_x, a_x) \times (-a_y, a_y) \times (-a_z, a_z)$. If you cannot prove it explicitly, demonstrate the plausibility of this statement by showing 2-D projections of the orbital tracks for two separate choices of $(\omega_x, \omega_y, \omega_z)$. Plot out the orbital tracks for varying integration times (short time, intermediate time, long time intervals).
(b) Write down a formula for the angular momentum of a particle in this potential. Is the angular momentum conserved? Why or why not? What is the average value of the angular momentum averaged over time?
(c) If a were equal to b (and hence ω_x were equal to ω_y), would the angular momentum be conserved? Why or why not? Would particles still travel on box orbits, or would the orbits be loop orbits?

2. Finding a solution to the collisionless Boltzmann equation using the Jeans theorem.
(a) Derive ρ and Ψ for a spherically-symmetric system with some distribution function f of the form $f(\epsilon) = \begin{cases} \epsilon^{n-3/2}, & \text{if } \epsilon > 0 \\ 0, & \text{if } \epsilon < 0 \end{cases}$. where $n = 1$, $\epsilon = -E + \Phi_0$ and E is the energy of a particle orbitting around the system. Adopt the standard definition that $\Psi = \epsilon + (1/2)v^2$. Show that the total mass of the model is $(1/2)\Psi_0 G^{-3/2}\sqrt{\pi/c_1}$ where c_1 is defined by equation (4-107b) from BT. Hint this is problem 4-16 from Binney & Tremaine (BT) and is discussed in some depth on BT 223-225.

(b) Derive ρ and Ψ for some spherically symmetric system with the distribution function f with the form $f(\epsilon) = \begin{cases} \epsilon^{n-3/2}, & \text{if } \epsilon > 0 \\ 0, & \text{if } \epsilon < 0 \end{cases}$. where $n = 5$. This is the distribution function for a Plummer model. Find the expression for ρ and Ψ. Derive also the formula for the total mass of the system.

3. Derive the third Jeans equation by subtracting the second Jeans equation from the first Jeans equation multiplied by $\bar{v_j}$. See the supplementary reading posted on the course website for hints on how to do this.

4. How many integrals of motion does a particle have in a Kepler potential? What are they?

5. Dynamical Friction. Dynamical friction is an important mechanism which causes colliding galaxies to rapidly merge. How important would this mechanism be, if we consider the collision of a galaxy with an isolated star (wandering alone through the universe)? Make use of the following Chandrasekhar dynamical friction formula presented in class:

$$\frac{dv_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 (M+m)\rho_m}{v_M^2}$$

Assume that a galaxy is a 3 kpc x 3 kpc x 3 kpc cube with mass $3 \times 10^{10}\ M_\odot$ and is entirely composed of stars with 1 $M_\odot$. Assume that a star with one solar mass $M = 1\ M_\odot$ approaches the galaxy at velocity $v_M = 200$ km/s and at an angle perpendicular to the surface of the cube. How much will dynamical friction change the velocity of the star if it falls in from infinity and continues to infinity? Feel free to assume that the dynamical friction is constant throughout the entire passage of the star through the galaxy (i.e., that the slowing velocity of the star has no effect on the amplitude of the dynamical friction). How would the impact of dynamical friction change if the star (unrealistically) had a mass of $10^9\ M_\odot$? We ignored the effect of dynamical friction in calculating the relaxation time for a star in lecture 5. Is this assumption justified?

Practical Sessions

Feb 19: Board Work + Problem Set 1
Mar 12: Board Work + Problem Set 2
Mar 26: Problem Set 3 / Paper Presentations (4 slots)
Apr 2: Problem Set 3 (cont'd) / Paper Presentations (6 slots) ←
Apr 16: Problem Set 4 / Paper Presentations (4 slots)
Apr 30: Problem Set 5 / Paper Presentations (4 slots)
May 7: Problem Set 6 / Paper Presentations (4 slots)

April 2 Practical Session

(In 3 days)

Paper Presentations (12 + 3 minutes)

Lindblad Resonances, Spiral Density Waves: Dobbs & Baba 2014; Sellwood 2011

Yara Beelhuizen & Myrdhin van der Zwet

Elliptical Galaxy Scaling Relations: Emsellem+2007, Emsellem+2011

Ids Nieuwstraten & Philip Stoot

Gas, Dust in Spiral Galaxies: PHANGS ALMA+JWST

Ines Heitor & Margarida Fonseca

Disk Galaxy Scaling Relations: Fall & Romanowsky 2013, 2018

Garrett Coey & Nikolaos Ladopoulus

Jeans Equations & Milky Way: Loebman+2014, Bird+2022

Lotte Langerak & Leonor Ferro

Elliptical Galaxy Scaling Relations: Cappellari+2011, 2013

Dylan Gavron & Jacqueline Beran

April 2 Practical Session

(Today or In 3 days)

Problem Set 3 - Problems 5 (to be discussed)

Andreea Suta
Naomi Schutte

Galaxies: Structure, Dynamics, and Evolution
Problem Set 3
Instructor: Dr. Bouwens

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Note also that I’ve begun adding some of your grades to bright space

First 2 Problem Sets + Solution Presentations + Attendance

Still need to grade the paper presentations!

First, let's review the important material from last week

Fundamental Plane for Elliptical Galaxies

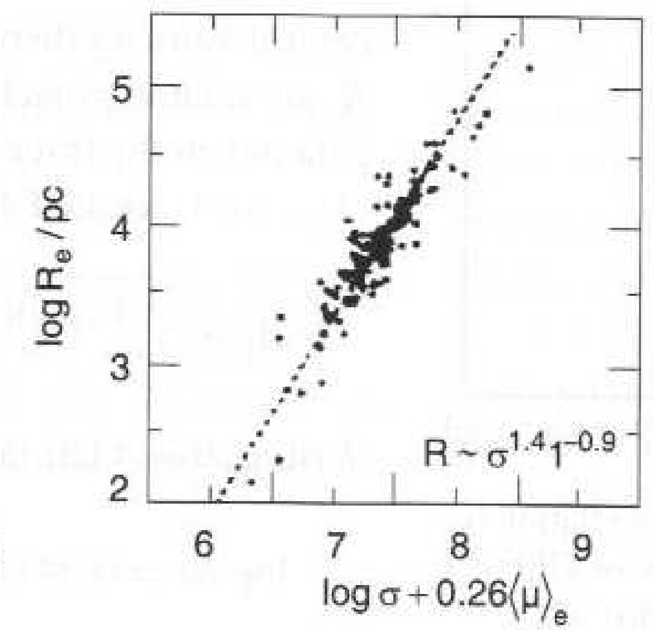
Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

These correlations are such that galaxies populate a fundamental plane in these parameters, so that if you know two of the following three variables for a galaxy, you can determine the third.

$$R \propto \sigma^{1.4} \mu_e^{-0.9}$$

where R is the size (radius), σ is the velocity dispersion, and μ is the galaxy surface brightness.

half-light radius

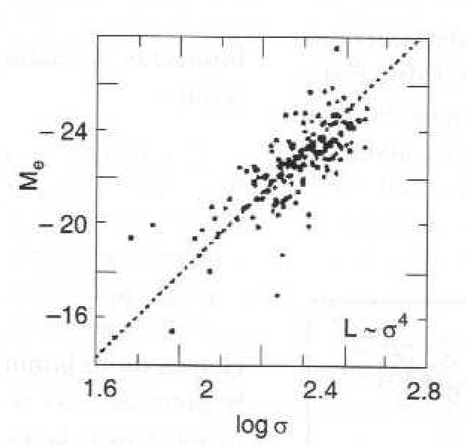


velocity dispersion

Intrinsic Correlations between Galaxy Properties

Note similarities between the Faber-Jackson relation (also used as a distance indicator);

$$L \propto \sigma^4$$



And the Tully-Fisher relationship: $L \propto (v_c)^4$

From lecture 2 (for disk galaxies):

What type of scaling relations might we expect to hold?

If we assume that there is a fixed circular velocity at large radii (as is the case for many disk galaxies):

the mass enclosed in some radius is

$$M = Rv_c^2/G$$

Assuming that galaxies form at a fixed redshift, we would expect

$$M \propto <\rho> R^3$$

Manipulating the second expression to derive an equation for R and substituting it in the first equation, we find

$$M \propto v_c^3$$

If the galaxy luminosity is proportional to mass, then

$$L \propto v_c^3$$

which isn't quite true, but is close to the observed scaling.

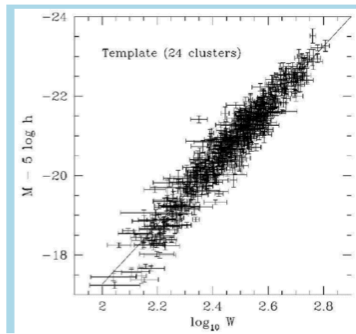
How do their structural parameters correlate?

The global properties of spiral galaxies are observed to correlate with each other:

Tully Fisher Relationship

Luminosity $\propto v_c^4$

luminosity



~2 x circular velocity

Slope of Tully-Fisher Relationship depends on which wavelength one measures the luminosity

Second: Dark Matter Halos

Navarro-Frenk-White Density Profiles

One of the most important steps in galaxy formation is the collapse of overdensities early in the universe. The density profile of the collapsed objects can have important impact on the formation of galaxies.

Simulations show that collapsed halos approximately have the following density profile:

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

where r_s and ρ_s are some scaling parameters.

At small radii ($r < r_s$), the density profile ρ scales approximately as r^{-1}

and at large radii ($r > r_s$), the density profile ρ scales approximately as r^{-3}

At $r \sim r_s$, the density profile ρ changes slope

Since ρ_s is completely determined by the total mass and concentration parameter $c = r_{200} / r_s$, all of the properties of dark matter halos can be expressed in terms of two variables (1) the mass of the dark matter halo and (2) the concentration parameter.

Navarro-Frenk-White Density Profiles

How does the median concentration parameters depend on the mass of the halo and redshift of the galaxy we are examining?

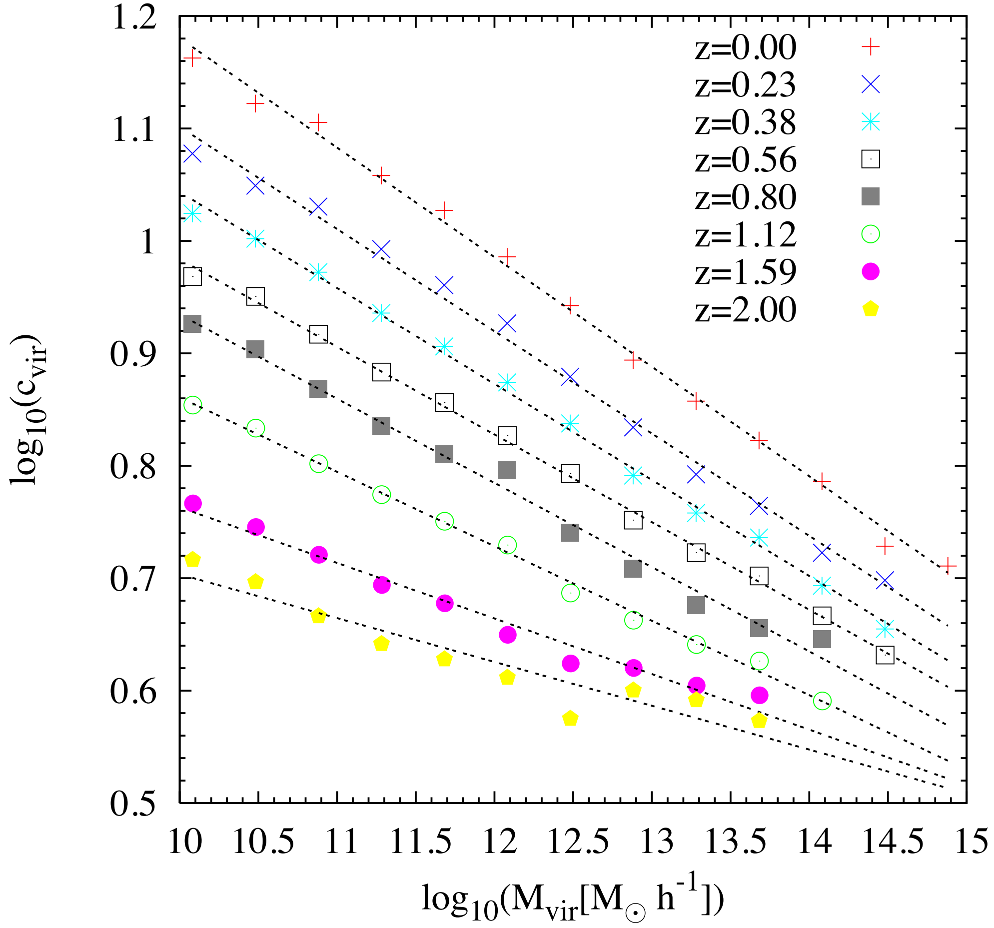


Figure 1. Mass and redshift dependence of the concentration parameter. The points show the median of the concentration as computed from the simulations, averaged for each mass bin. Lines show their respective linear fitting to eq. 5.

Munoz-Cuartas et al. 2010

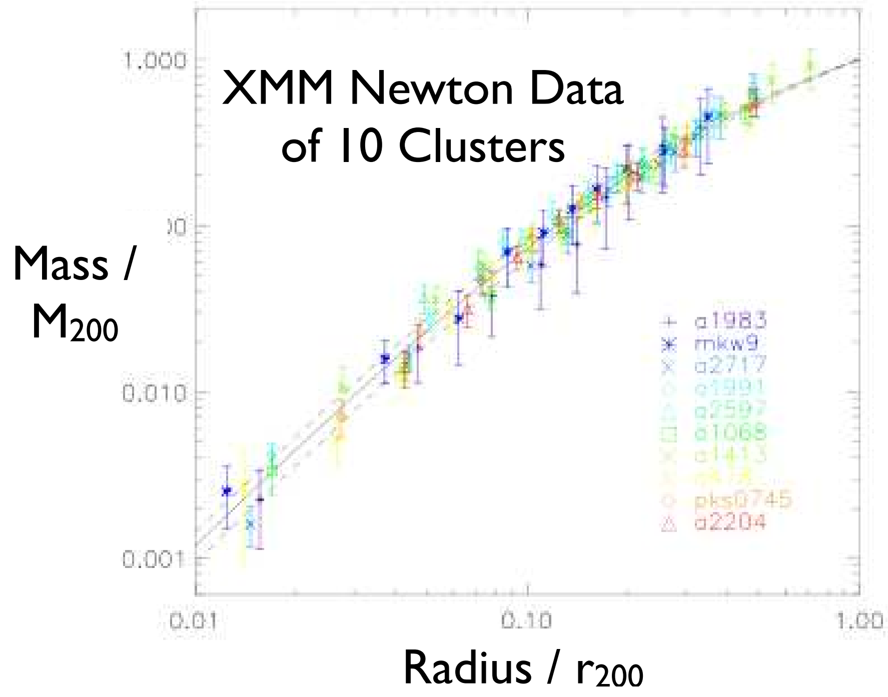
Observational Support for NFW profiles:

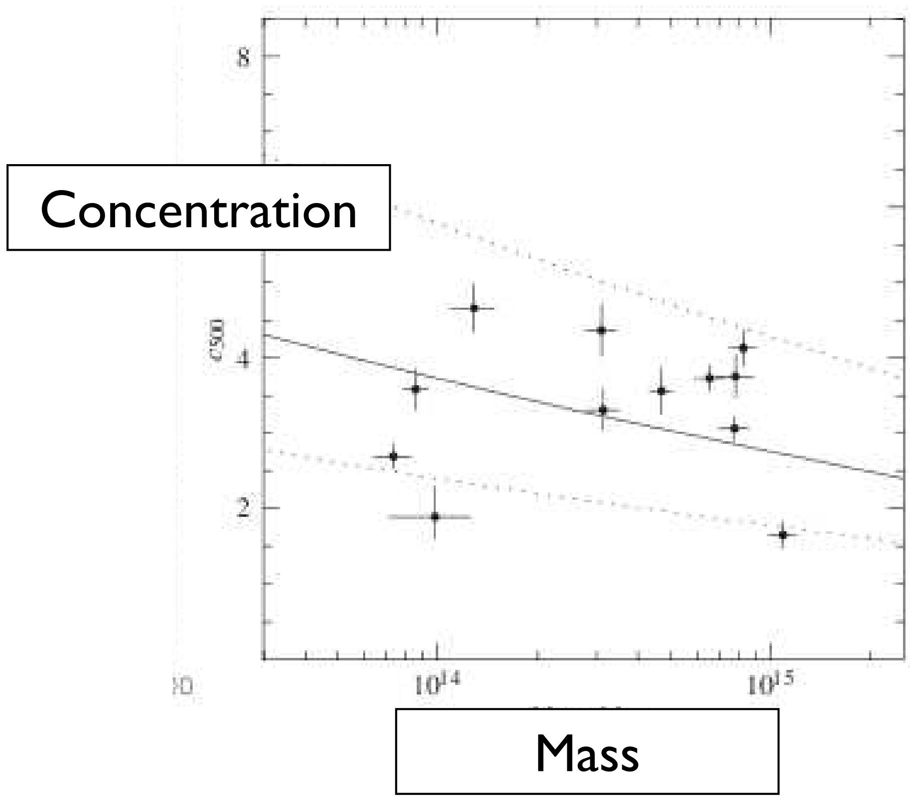
Galaxy Clusters:

1. x-ray profiles of galaxy clusters

Excellent Agreement →

Radial Profile





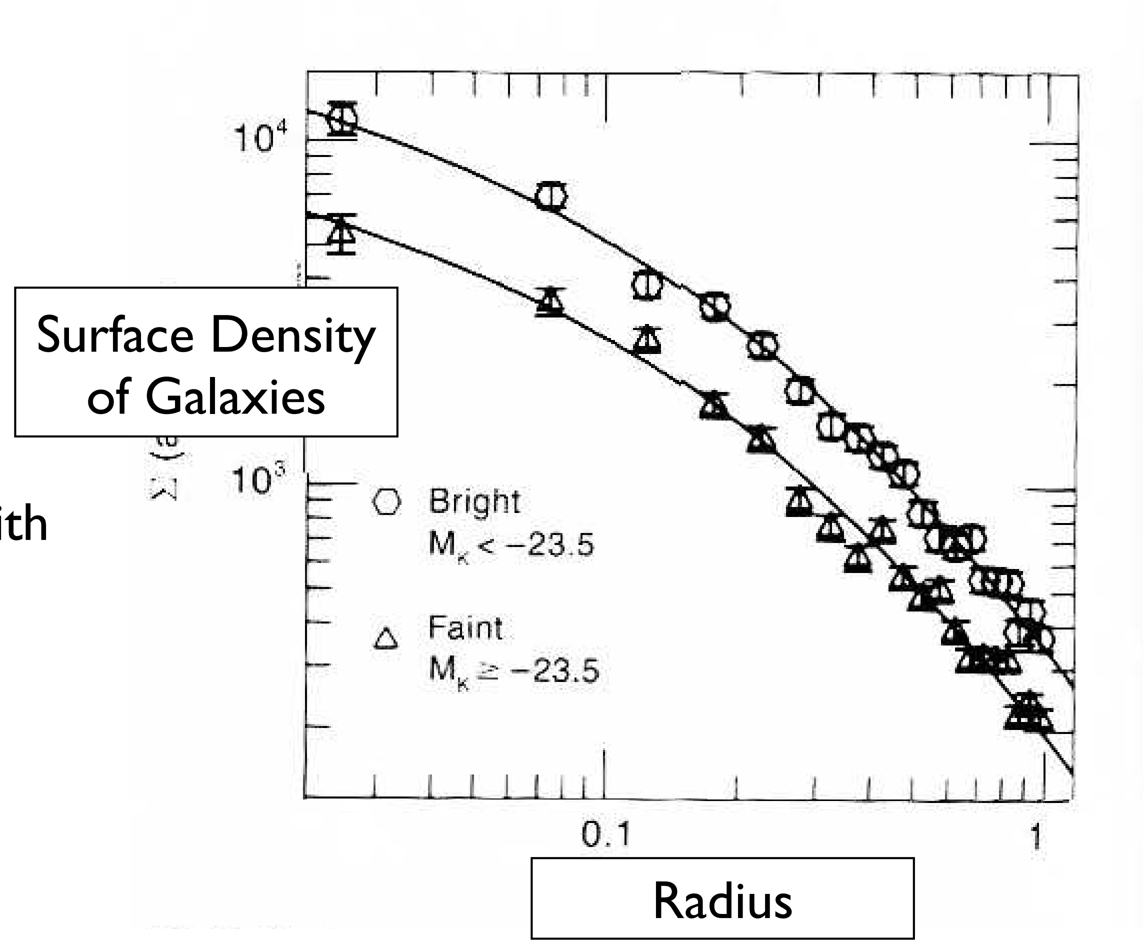
Distribution of Observed Concentration Parameters in Excellent Agreement with Theoretical Predictions!

Observational Support for NFW profiles:

Galaxy Clusters:

2. radial distribution of galaxies in clusters

Also in Excellent Agreement with NFW Profile →

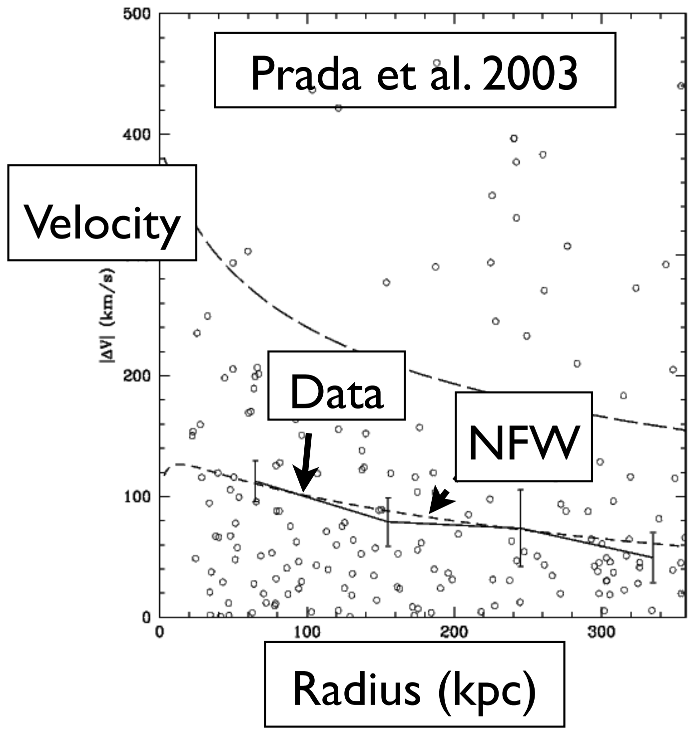


Observational Support for NFW profiles:

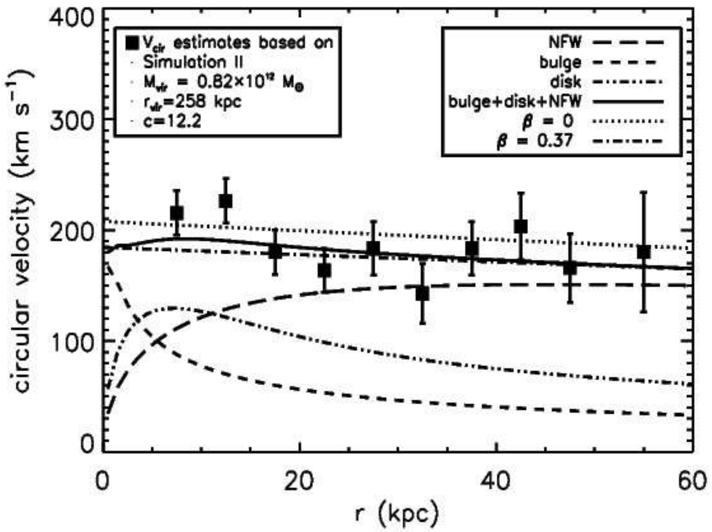
Galaxies:

1. Kinematics of satellite galaxies

Need to take the kinematic information of ~1-2 satellite galaxies around nearby galaxies and combine to create a composite "galaxy"



2. Kinematics of distant stars



Observational Support for NFW profiles:

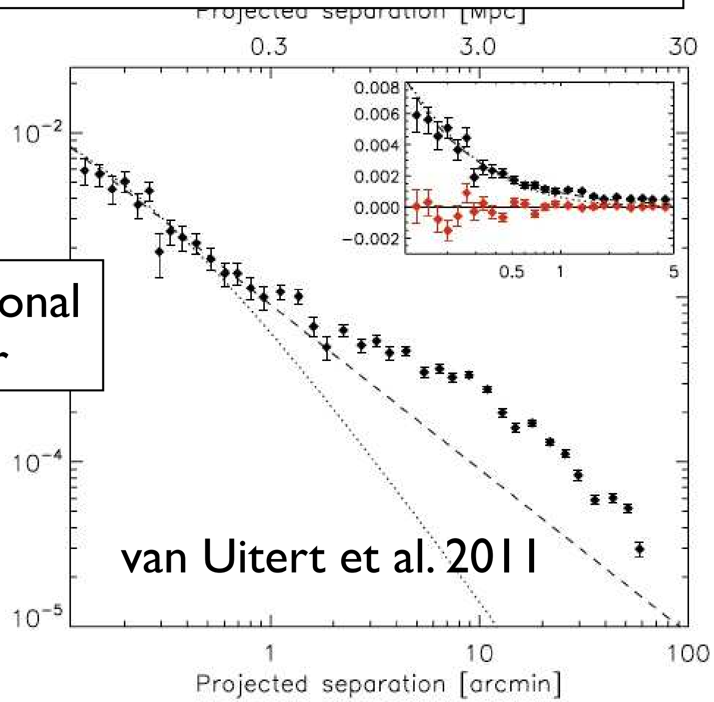
Galaxies:

3. From Gravitational Lensing

Using the impact of gravitational lensing from foreground galaxies on background galaxies can infer the mass internal to a radius in foreground galaxies

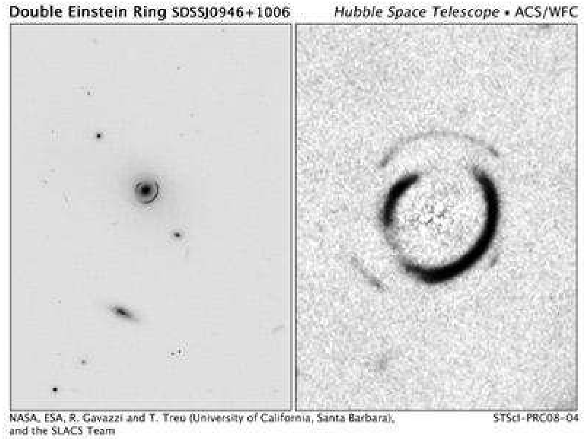
From weak lensing effects on galaxy shapes

Gravitational Shear



van Uitert et al. 2011

Using the impact of strong lensing on galaxies

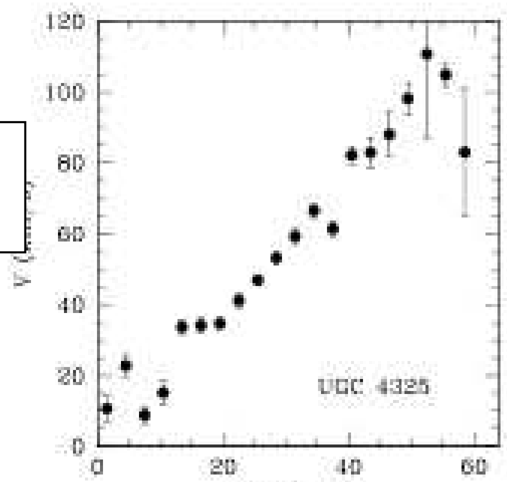


Large samples of foreground galaxies with einstein rings around them.

Problems / Challenges for NFW Halo Model

1. Rotational Curve at center of low surface brightness galaxies (which give us the best view of the mass profile in dark matter halos themselves)

Line of Sight Velocity



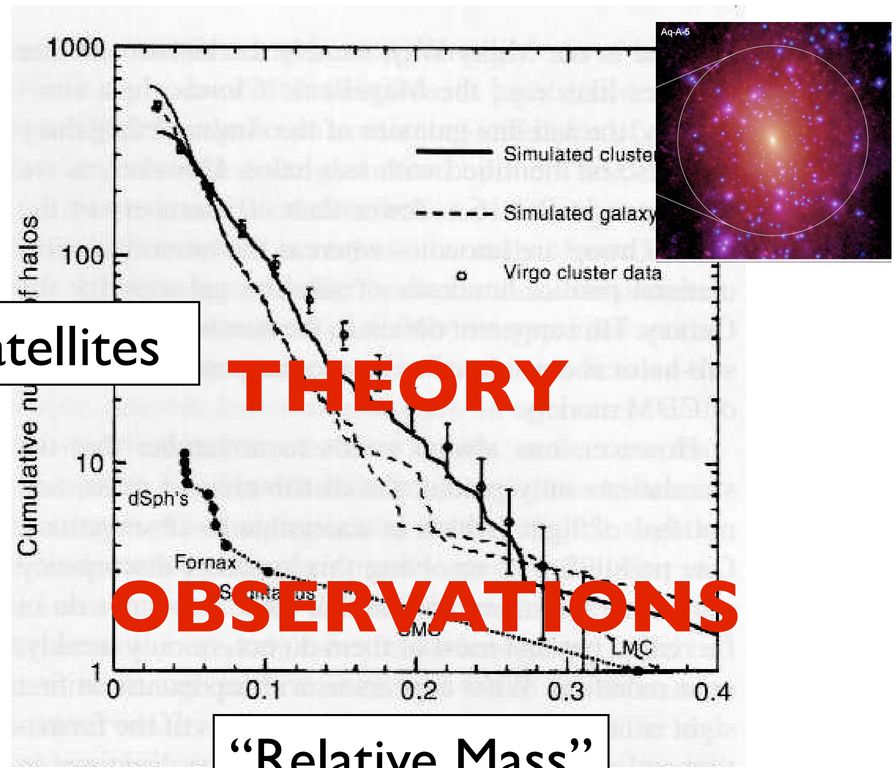
Radius

Velocity ∝ Radius

→ Mass Density ∝ constant,
not 1/radius

2. Predicted number of satellite galaxies for galaxies like the Milky Way are much greater than expected from the substructure in model dark matter halos

# of Satellites

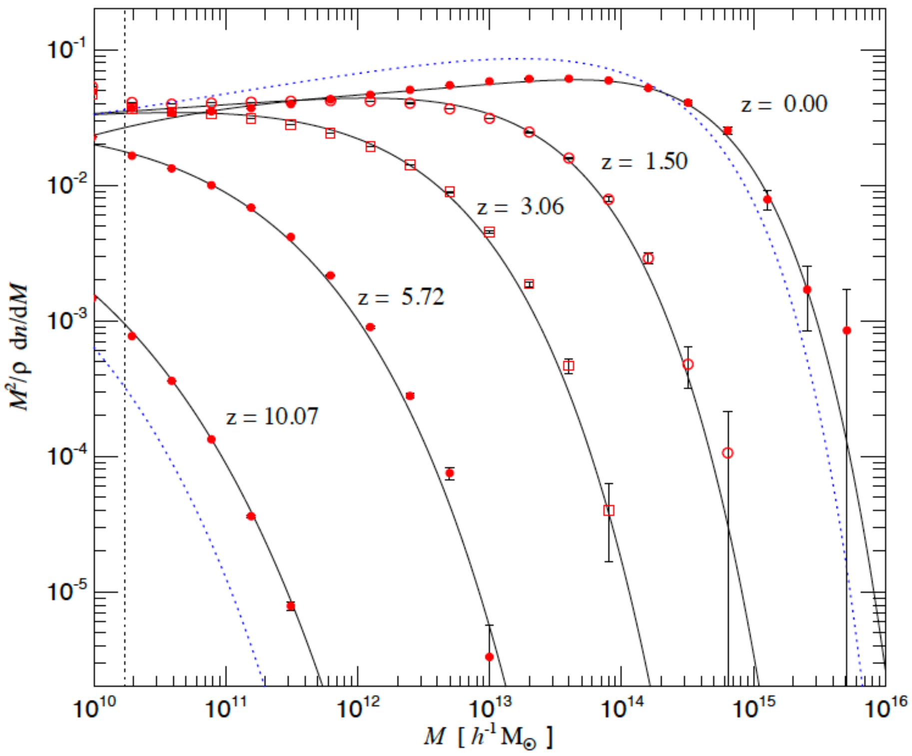


"Relative Mass"

"Missing Satellite Problem"

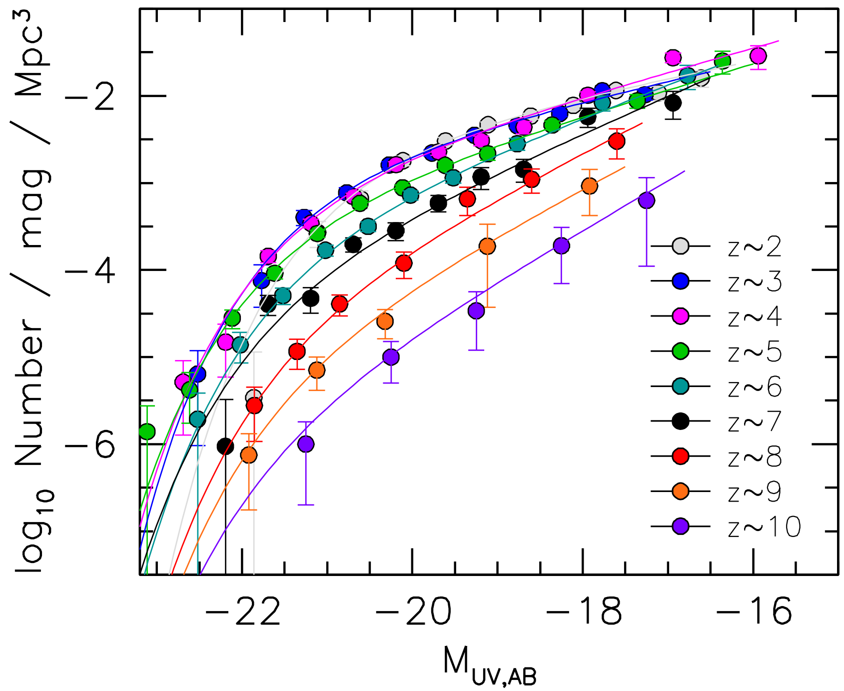
Impact Dark Matter Halo Have on the Density of Galaxies with Various Properties

Evolution of Dark Matter Halo Mass Function



Springel+2005

Evolution of Luminosity Function at UV Wavelengths

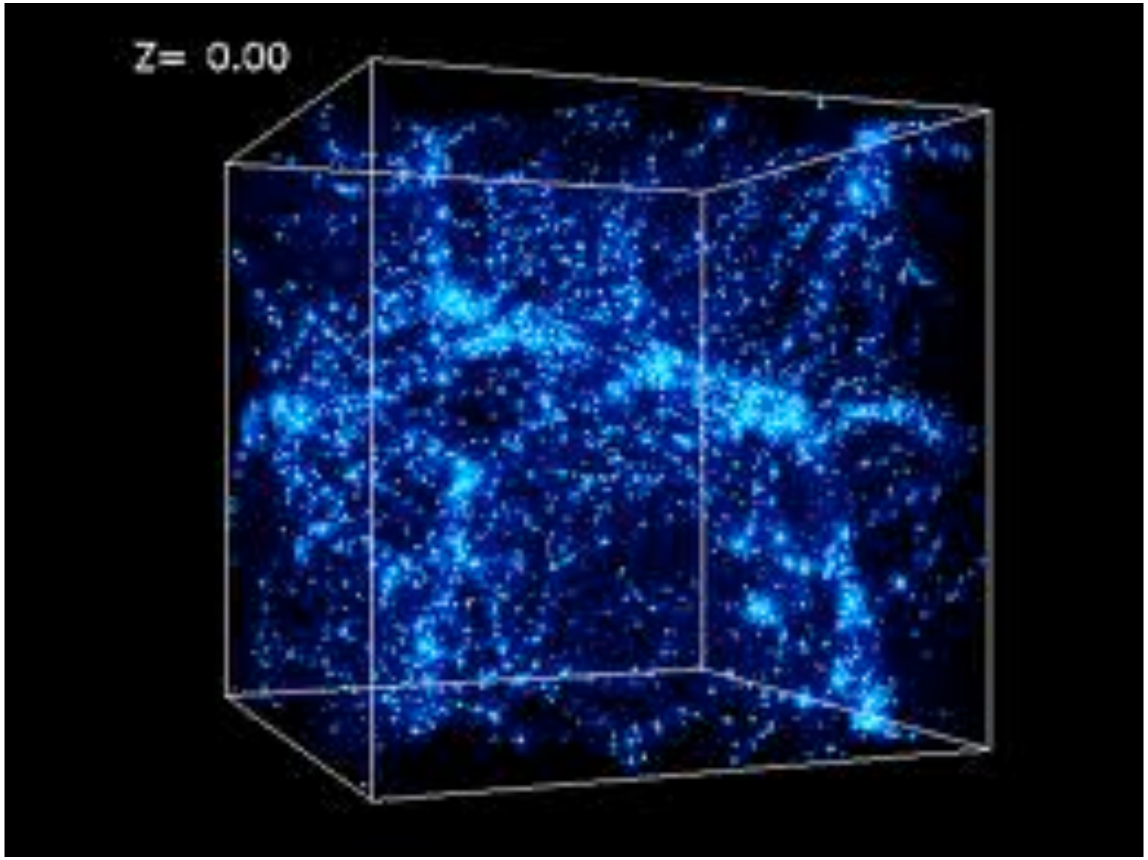


Bouwens+2021

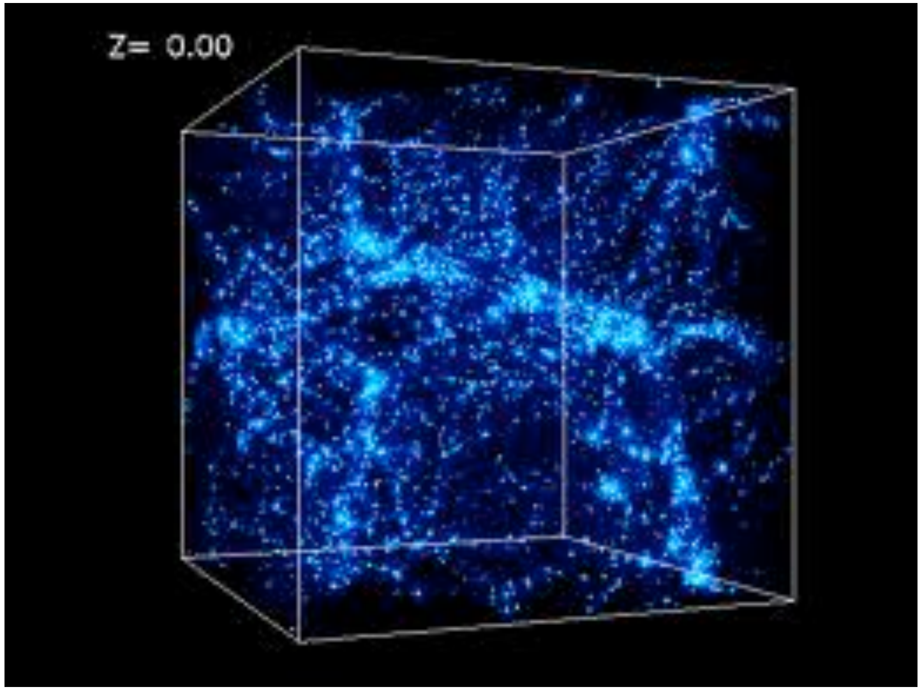
NOW new material for this week

How Do Galaxies Distribute Themselves in Space?

What does this teach us?



"How Do Galaxies Distribute Themselves in Space?

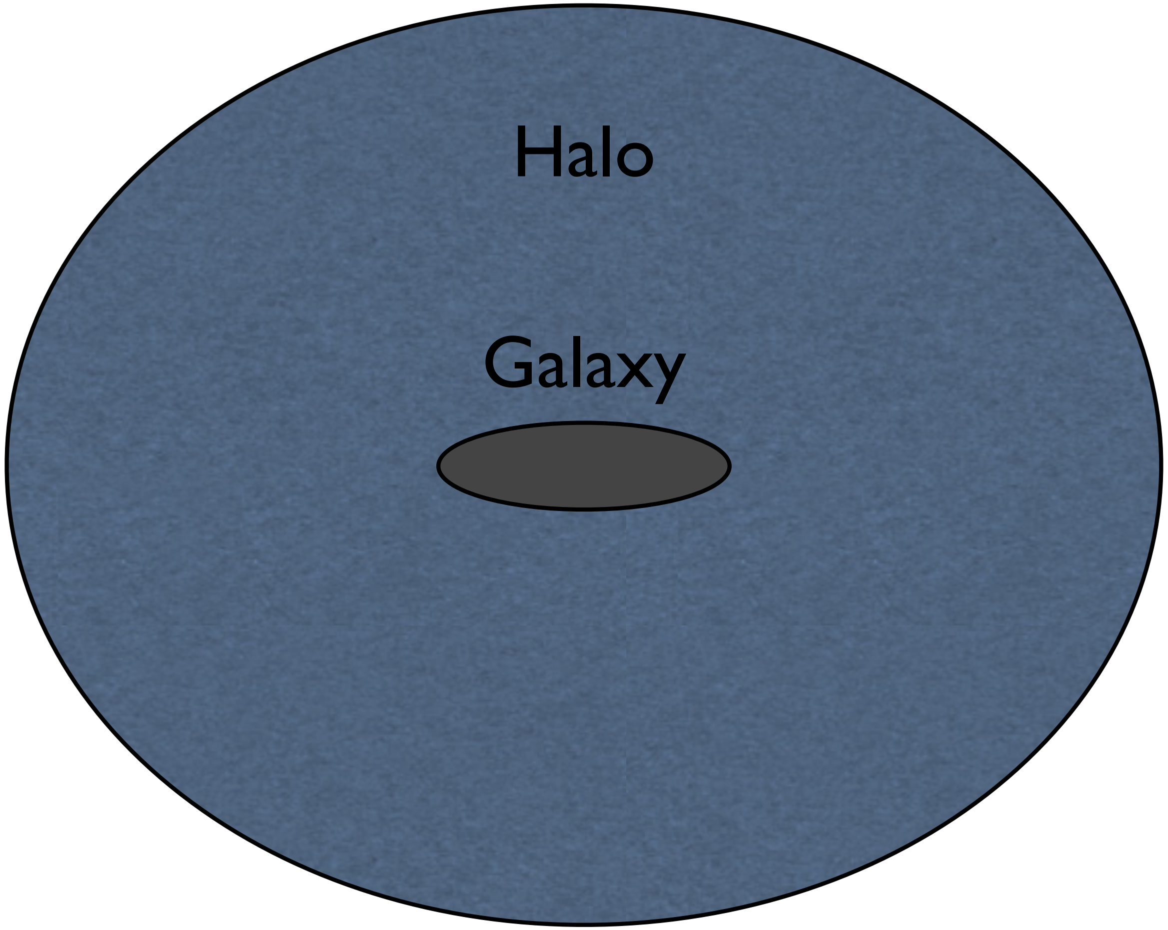


What does this teach us?"

It provides insight into the masses or properties of the collapsed dark matter halos in galaxies form and evolve.

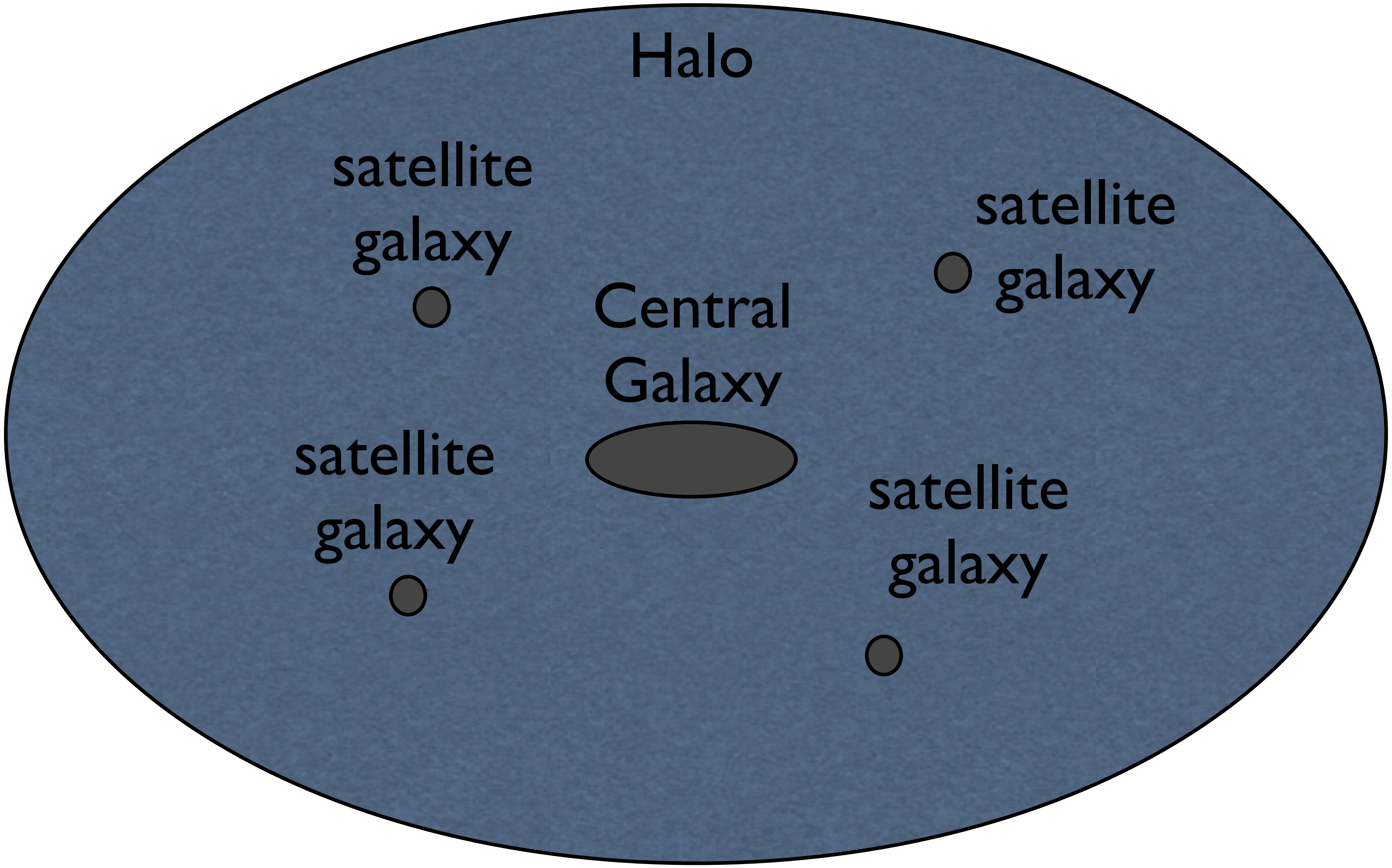
(first a prologue)

By now, you should be familiar with the idea that collapsed halos contain galaxies at their center.

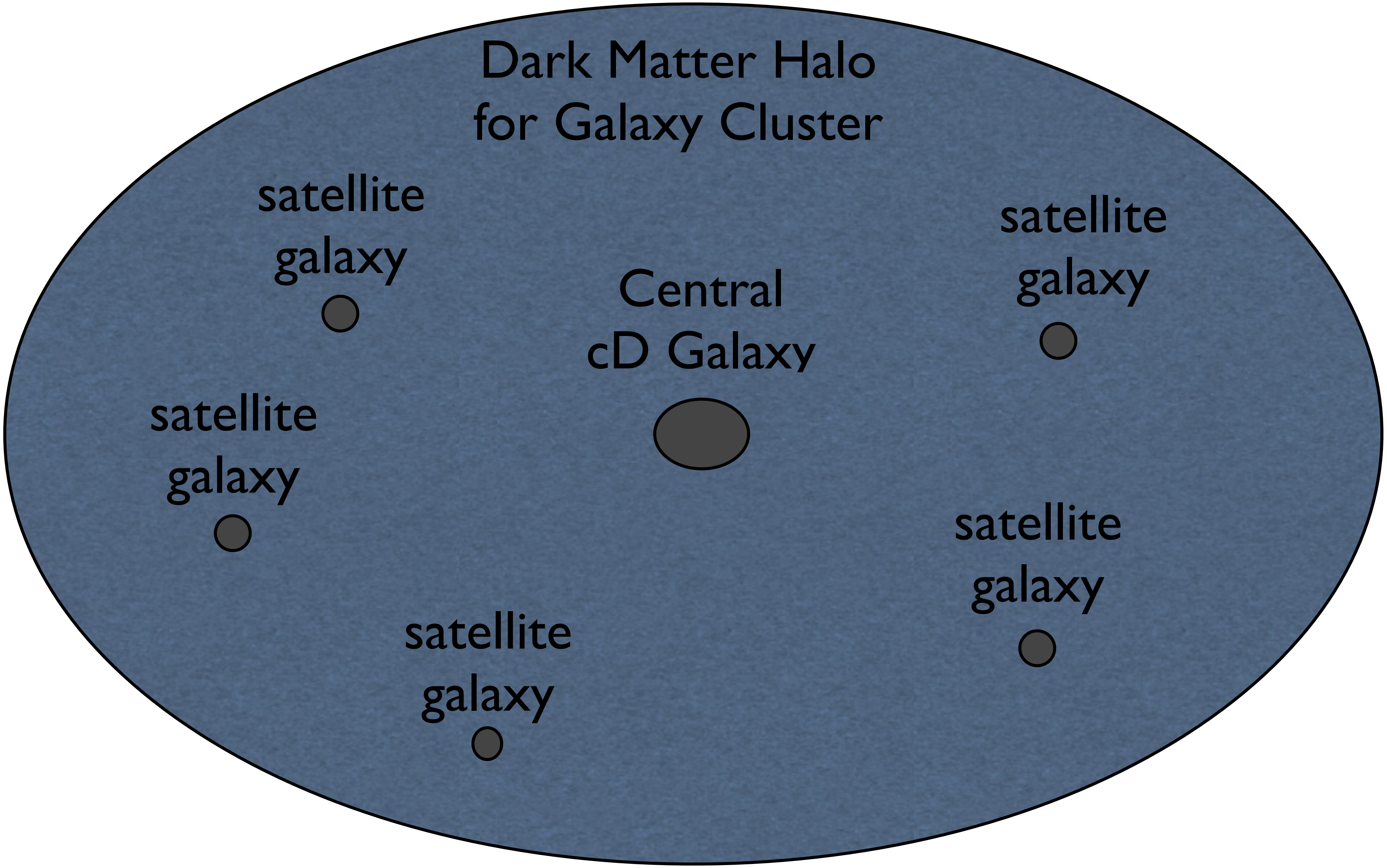


these galaxies form from the cooling of gas onto the center of the halo and forming a gas disk however, many dark matter halos contain more than one galaxy...

each halo almost always one most massive galaxy at the center (called the "central galaxy") and any number of satellite galaxies (orbitting around the center)



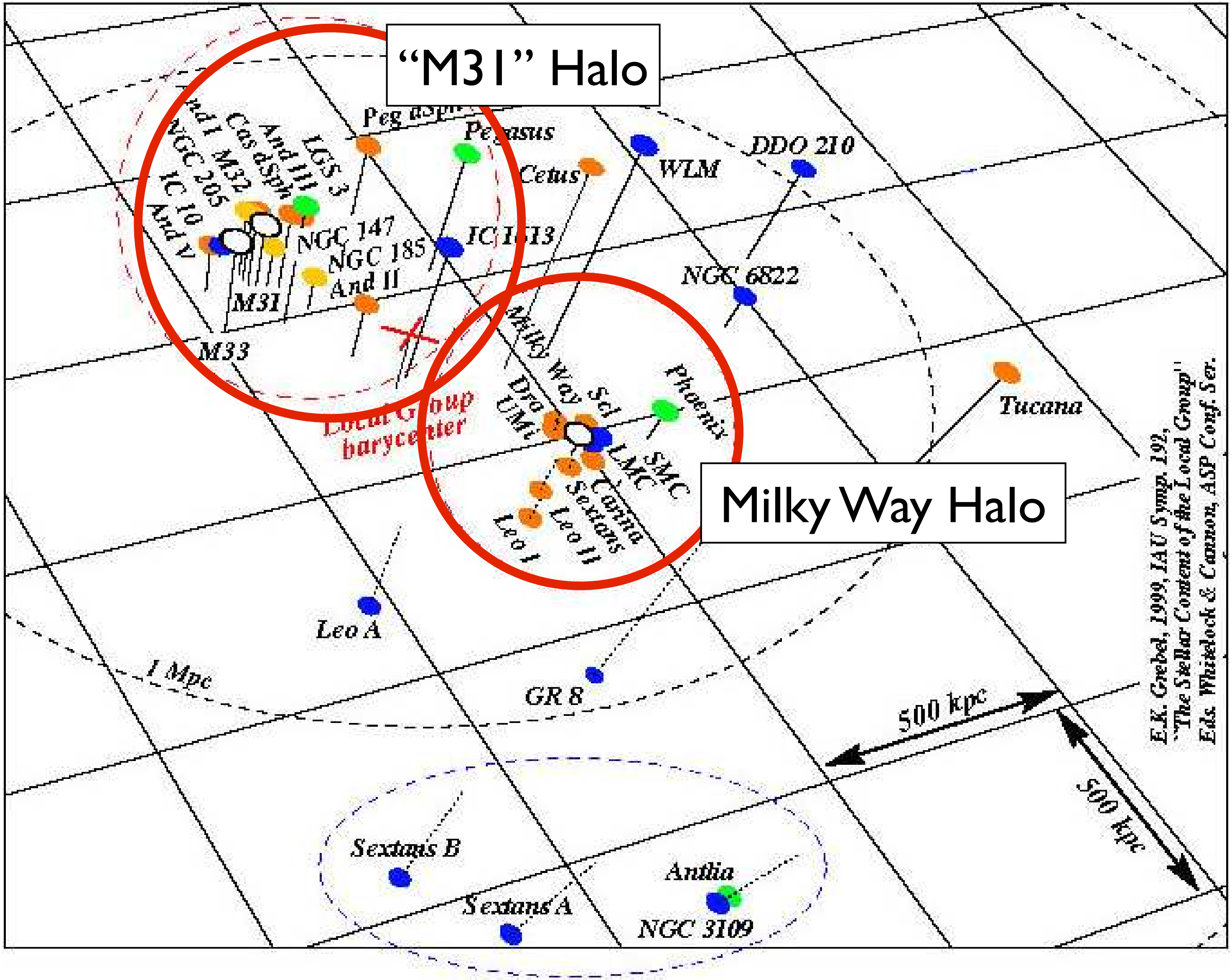
You should all be quite familiar with halos that contain many satellite galaxies. An excellent example is a galaxy cluster:



A less extreme version of a galaxy cluster is a galaxy group.

Our own local group is a good example:

The 3D distribution is



Lots of satellite galaxies in the M31 and Milky Way Halos!

Table 4.3 Local Group members

Name	Alternate Name	Coordinates RA (1950) Dec	Type	Distance (kpc)	M_V
M31	NGC 224	00 40.0 +40 59	Sb	725	−21.1
Milky Way	Galaxy	17 42.4 −28 55	Sbc	8	−20.6
M33	NGC 598	01 31.1 +30 24	Sc	795	−18.9
LMC		05 24.0 −69 48	Irr	49	−18.1
IC 10		00 17.7 +59 01	Irr	1250	−17.6
NGC 6822	DDO 209	19 42.1 −14 56	Irr	540	−16.4
M32	NGC 221	00 40.0 +40 36	dE2	725	−16.4
NGC 205		00 37.6 +41 25	dE5	725	−16.3
SMC		00 51.0 −73 06	Irr	58	−16.2
NGC 3109	DDO 236	10 00.8 −25 55	Irr	1260	−15.8
NGC 185		00 36.2 +48 04	dE3	620	−15.3
IC 1613	DDO 8	01 02.2 +01 51	Irr	765	−14.9
NGC 147	DDO 3	00 30.5 +48 14	dE4	589	−14.8
Sextans A	DDO 75	10 08.6 −04 28	Irr	1450	−14.4
Sextans B	DDO 70	09 57.4 +05 34	Irr	1300	−14.3
WLM	DDO 221	23 59.4 −15 45	Irr	940	−14.0
Sagittarius		18 51.9 −30 30	dSph/E7	24	−14.0
Fornax		02 37.8 −34 44	dSph/E3	131	−13.0
Pegasus	DDO 216	23 26.1 +14 28	Irr	759	−12.7
Leo I	DDO 74	10 05.8 +12 33	dSph/E3	270	−12.0
Leo A	DDO 69	09 56.5 +30 59	Irr	692	−11.7
And II		01 13.5 +33 09	dSph/E3	587	−11.7
And I		00 43.0 +37 44	dSph/E0	790	−11.7
SagDIG		19 27.9 −17 47	Irr	1150	−11.0
Antlia		10 01.8 −27 05	dSph/E3	1150	−10.7
Sculptor		00 57.6 −33 58	dSph/E3	78	−10.7
And III		00 32.6 +36 12	dSph/E6	790	−10.2
Leo II	DDO 93	11 10.8 +22 26	dSph/E0	230	−10.2
Sextans		10 10.6 −01 24	dSph/E4	90	−10.0
Phoenix		01 49.0 −44 42	Irr	390	−9.9
LGS 3		01 01.2 +21 37	Irr	760	−9.7
Tucana		22 38.5 −64 41	dSph/E5	900	−9.6
Carina		06 40.4 −50 55	dSph/E4	87	−9.2
Ursa Minor	DDO 199	15 08.2 +67 23	dSph/E5	69	−8.9
Draco	DDO 208	17 19.2 +57 58	dSph/E3	76	−8.6

SOURCE: From data kindly provided by M. Irwin.

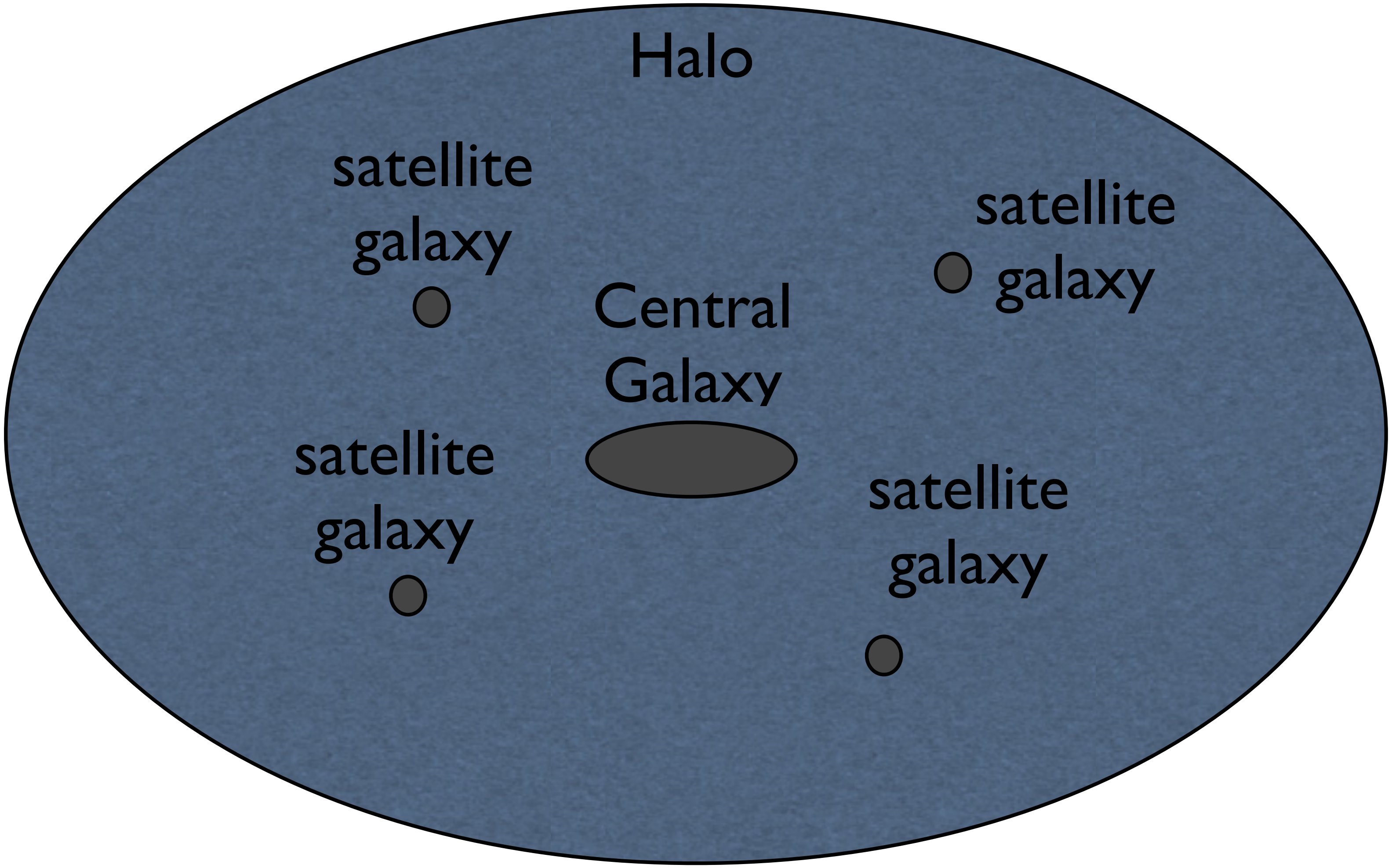
Of course, this list may be incomplete as the Milky Way itself blocks our field of view!

Note that this is prior to Gaia mission!

how can we explain the origin of the satellite galaxies?

can we explain them as a result of gas cooling?

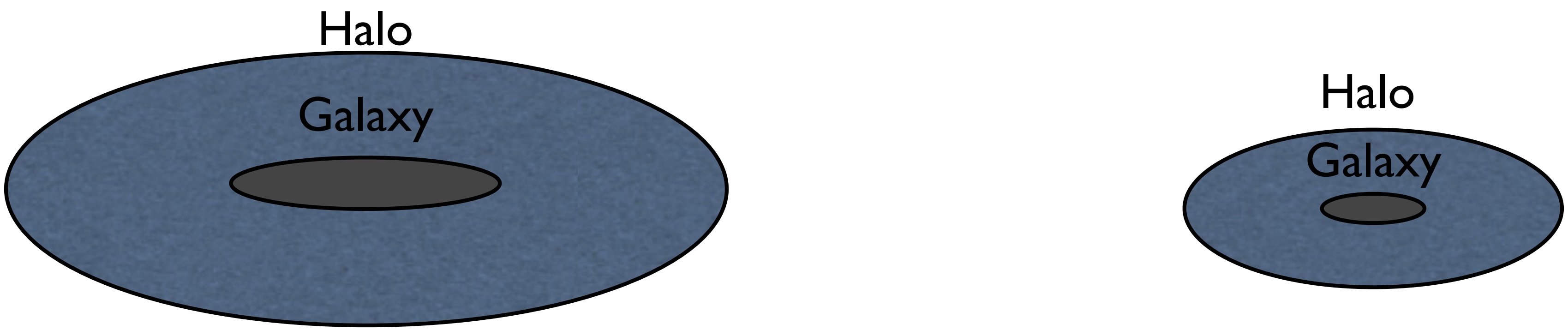
no -- since gas only efficiently cools onto the central position in the dark matter halo (gravitational potential)



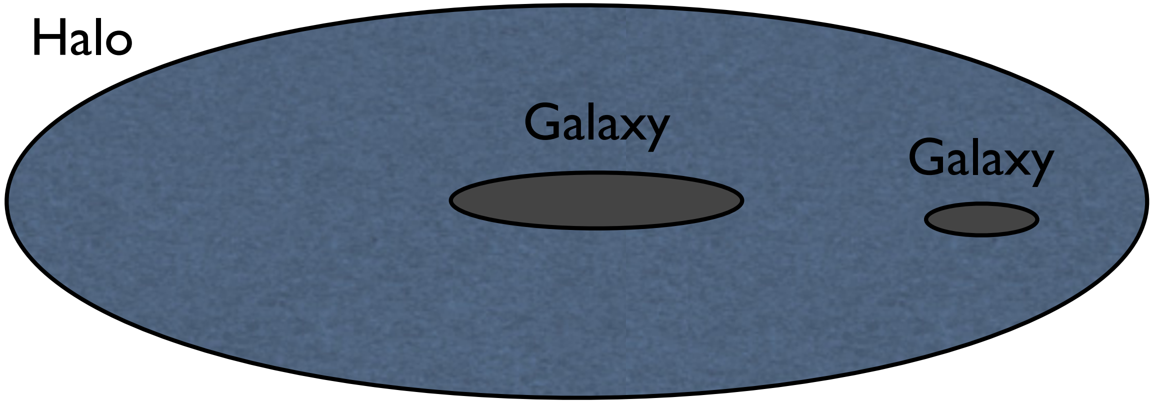
How can we explain the origin of the satellite galaxies?

A better explanation is through merging:

Step #1: We have two dark matter halos. Gas cools to the center of each halo to form disk galaxies in each:

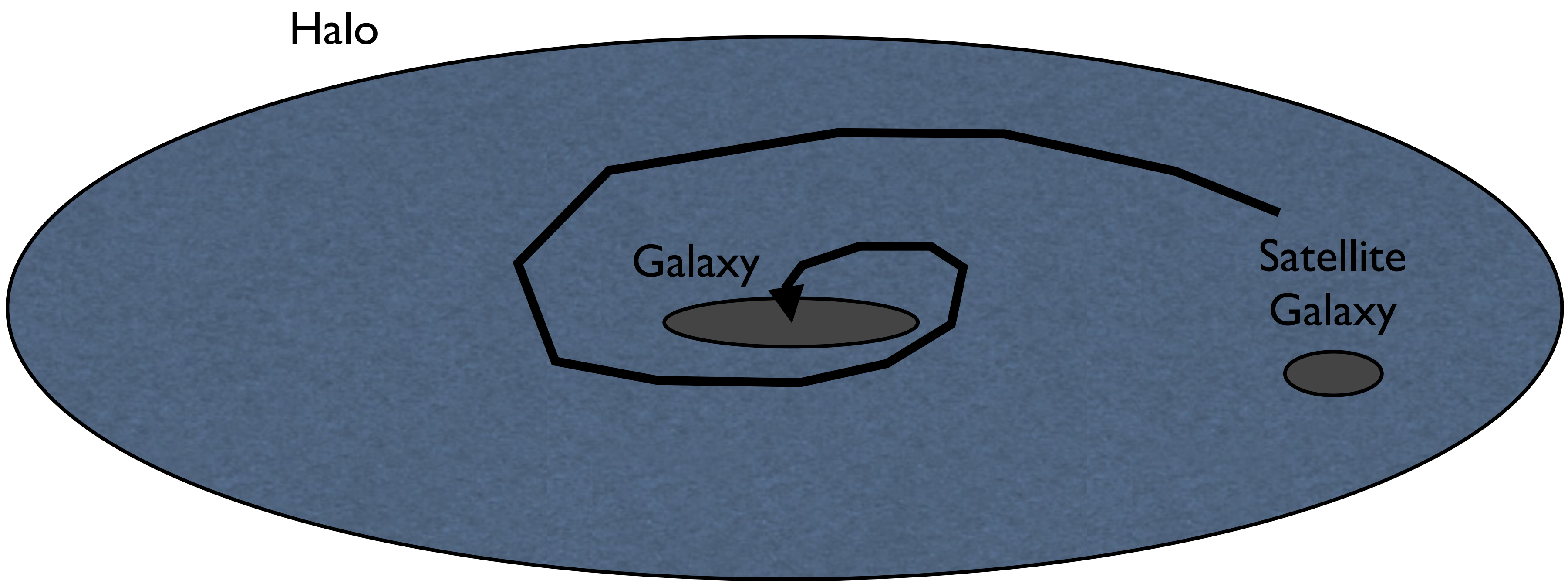


Step #2: These two dark matter halos merge to form a single more massive halo



The most massive halos contain large numbers of satellite galaxies, while less massive galaxies contain less. The merger origin of satellite galaxies explains why this is the case.

NOTA BENE: In many cases, satellite galaxies merge with the central galaxy as a result of dynamical friction.



The Local Group is not in equilibrium!

M31 and the Milky Way galaxy are traveling towards each other at 120 km/s!

Since M31 and the Milky Way are 700 kpc away from each other, these two galaxies will collide in perhaps ~4 billion years!

Can we use this information to estimate the mass of the Milky Way galaxy? Yes!

The Local Group is not in equilibrium!

First, consider the fact that the Milky Way and M31 will be initially flying away from each other with the Hubble flow.

Due to the self gravity of the mass within the local group, the Milky Way and M31 will stop expanding with the Hubble flow and start to fall towards each other.

Assuming that it takes the two main galaxies in the local group 14 Gyr to start falling towards each other at 120 km/s and have a distance of 700 kpc, we can calculate the mass of the local group to be $3 \times 10^{12}\ M_{solar}$

Comparing this mass to the total luminosity of the local group, one derives 70 M_{sol} / L_{sol}.

To put these numbers in context, I remind you of the mass-to-light ratios presented in lecture #1

Table 10-2. Estimates of the density parameter

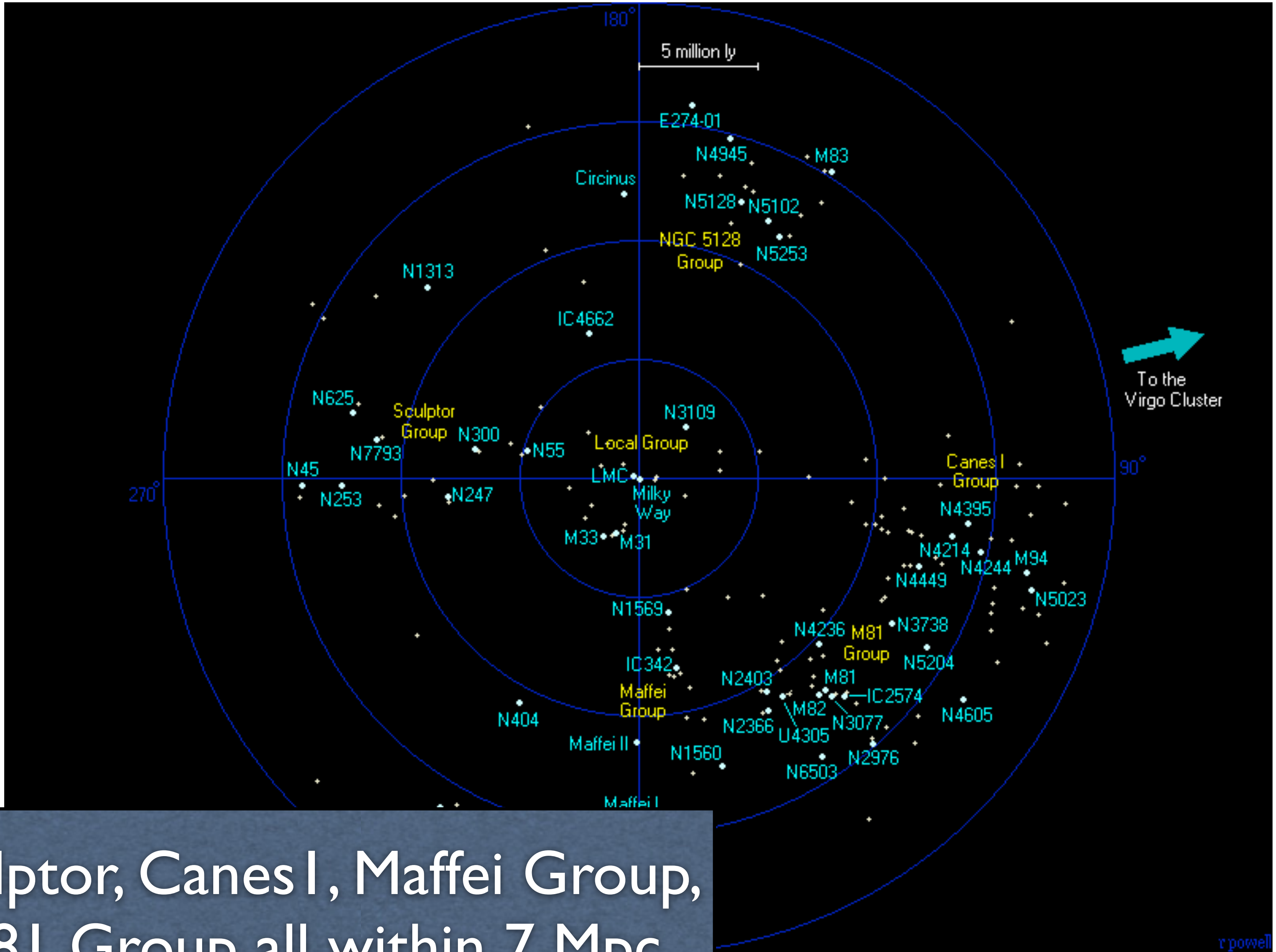
Method	$\Upsilon_V/\Upsilon_\odot$	Ω_0
Solar neighborhood	5	$0.003h^{-1}$
Elliptical galaxy cores	$12h$	0.007
Local escape speed	30	$0.018h^{-1}$
Satellite galaxies	30	$0.018h^{-1}$
Magellanic Stream	> 80	$> 0.05h^{-1}$
Rotation curve of NGC 3198	$> 28h$	> 0.017
X-ray halo of M87	> 750	$> 0.46h^{-1}$
Local Group timing	100	$0.06h^{-1}$
Groups of galaxies	$260h$	0.16
Clusters of galaxies	$400h$	0.25
Virgocentric flow	–	0.25
Nucleosynthesis	–	$(0.01 - 0.05)h^{-2}$
Inflation	–	1

NOTES: All lines except the last three are based on the luminosity density (10-24). Nucleosynthesis estimate omits density in non-baryonic matter. Several methods, such as Local Group timing and X-ray halo of M87, depend on h in complicated ways, and this dependence has been suppressed. See text for further detail.

Mass-to-Light Ratio

As we probe larger spatial scales, the mass-to-light ratio increases!

What collapsed clusters or groups are nearby the local group?

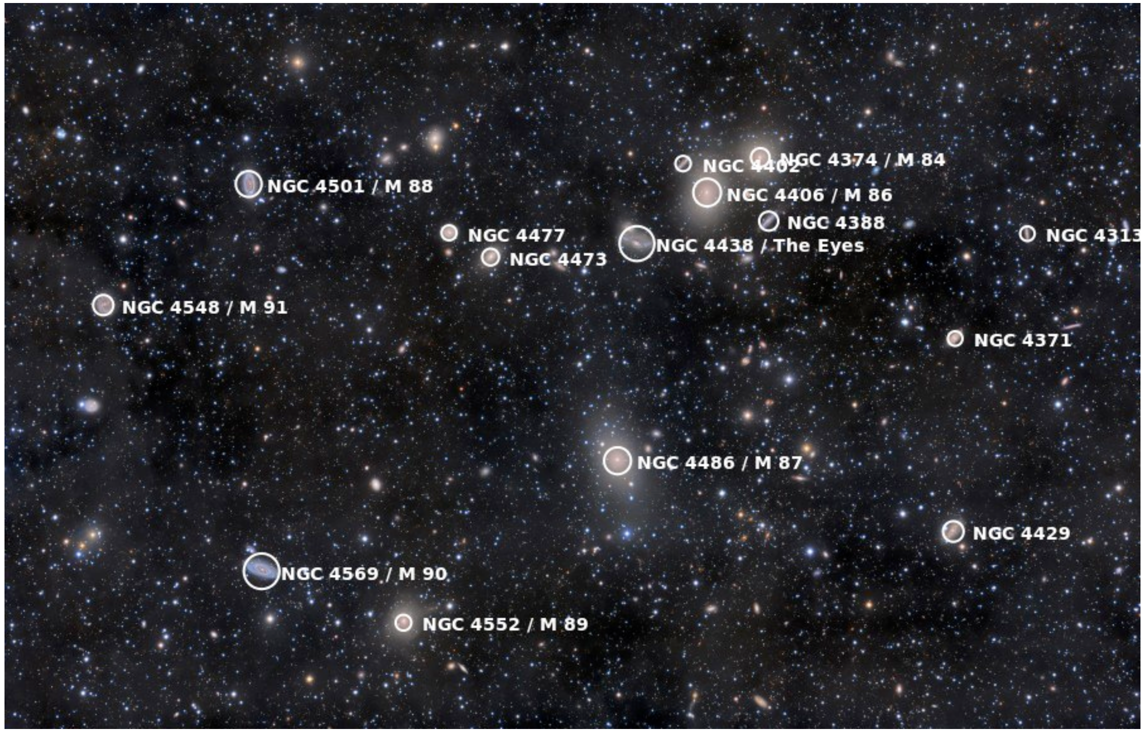


Sculptor, Canes I, Maffei Group, M81 Group all within 7 Mpc

What collapsed clusters or groups are nearby the local group?

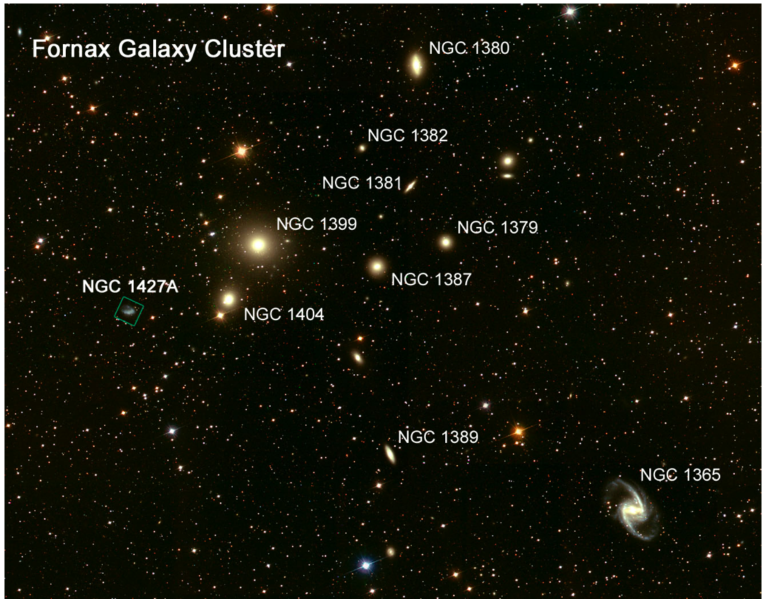
Virgo Cluster

- contains >250 large galaxies
- contains 2000 smaller galaxies
- covers 10 x 10 degrees on sky
- 18 Mpc away
- 3 Mpc diameter



Fornax Cluster

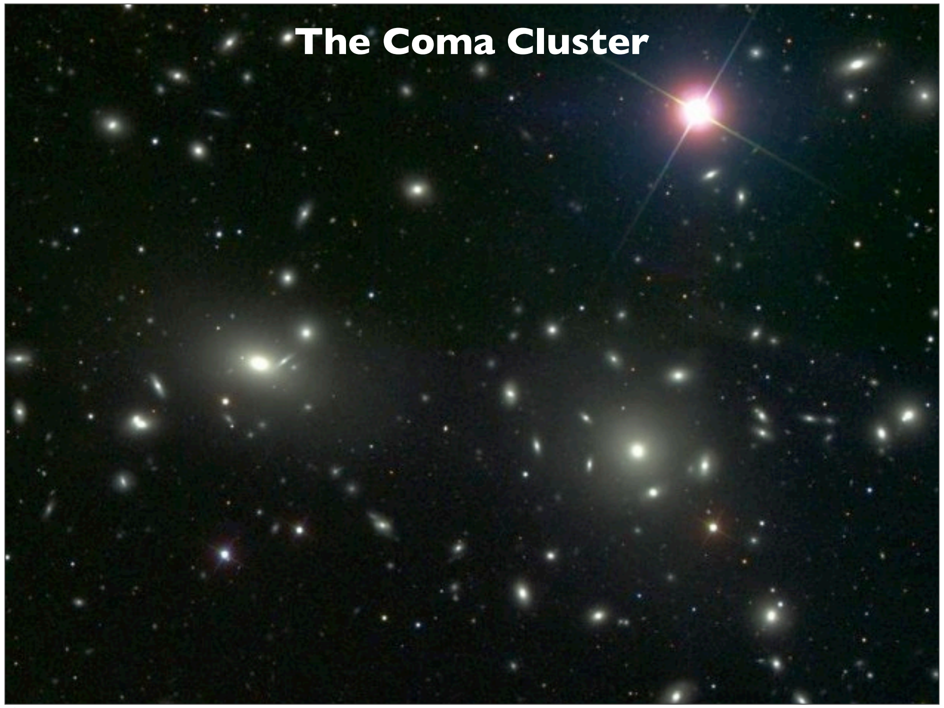
- contains >50 large galaxies
- 19 Mpc away
- less massive than Virgo



What collapsed clusters or groups are nearby the local group?

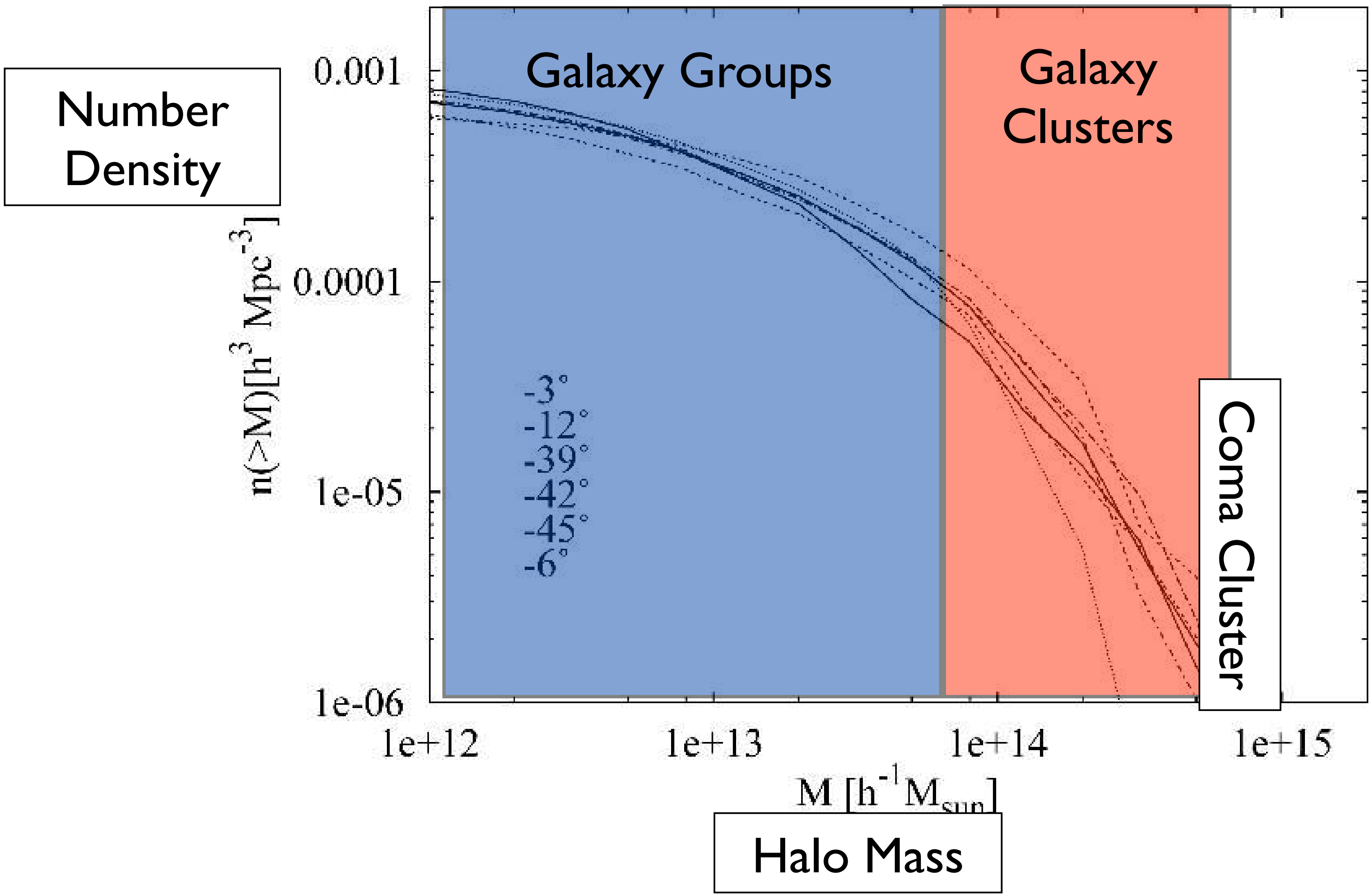
Coma cluster

contains >1000 large galaxies
contains 10000 smaller galaxies
~100 Mpc away
6 Mpc diameter
largest galaxies are giant ellipticals



How common are galaxy groups or clusters of various masses?

from Heinamaki et al (A&A 397, 63)



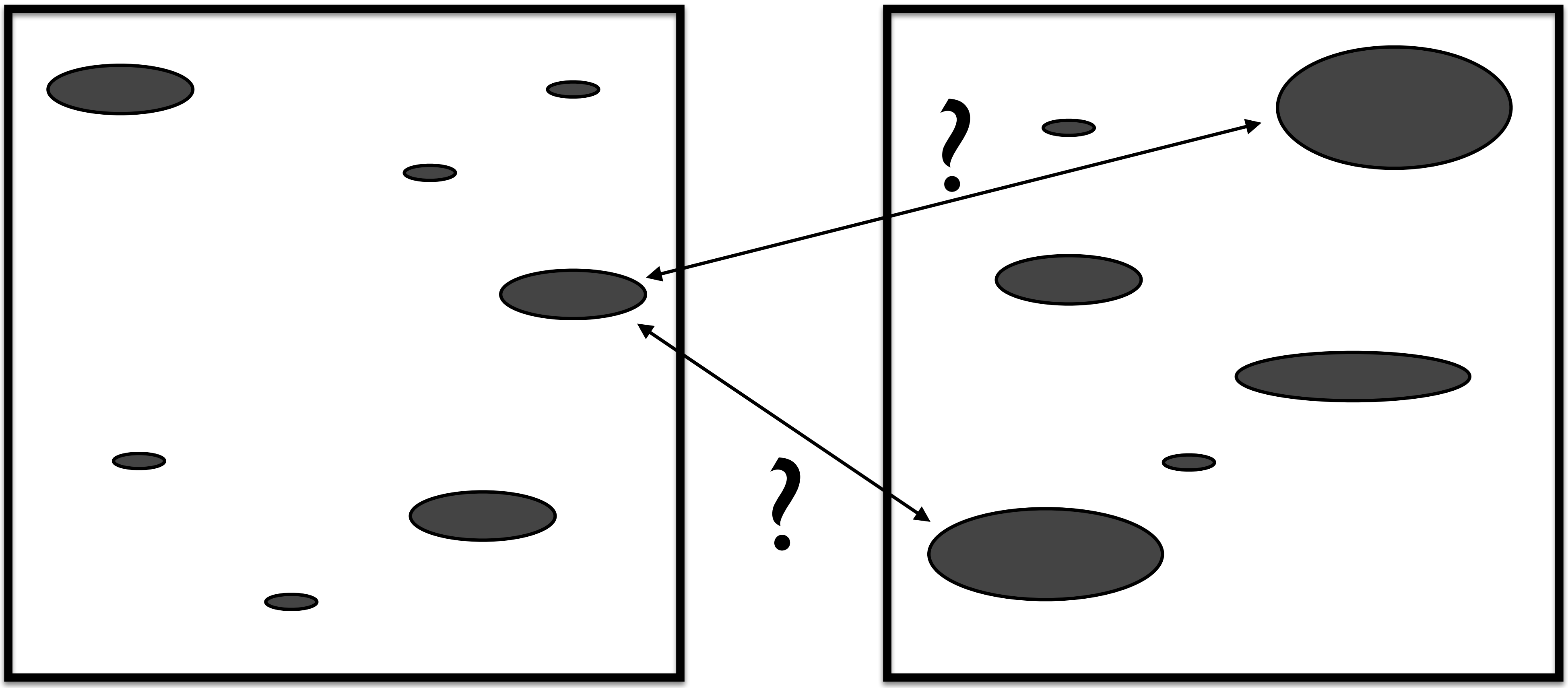
Important Endeavor:

Determining the Mass of the Dark Matter Halo in Which A Galaxy or Galaxies Live

Key Question: How do we determine what a given galaxy at z~1 looked like at both earlier and later times?

Survey of a volume of the universe at z~1

Survey of a volume of the universe at z~0



By constraining the halo mass of galaxies here

By constraining the halo mass of galaxies here

⇒ Since we can accurately model how the mass in halos grow with time due to gravity, we can associate galaxies in both time slices.

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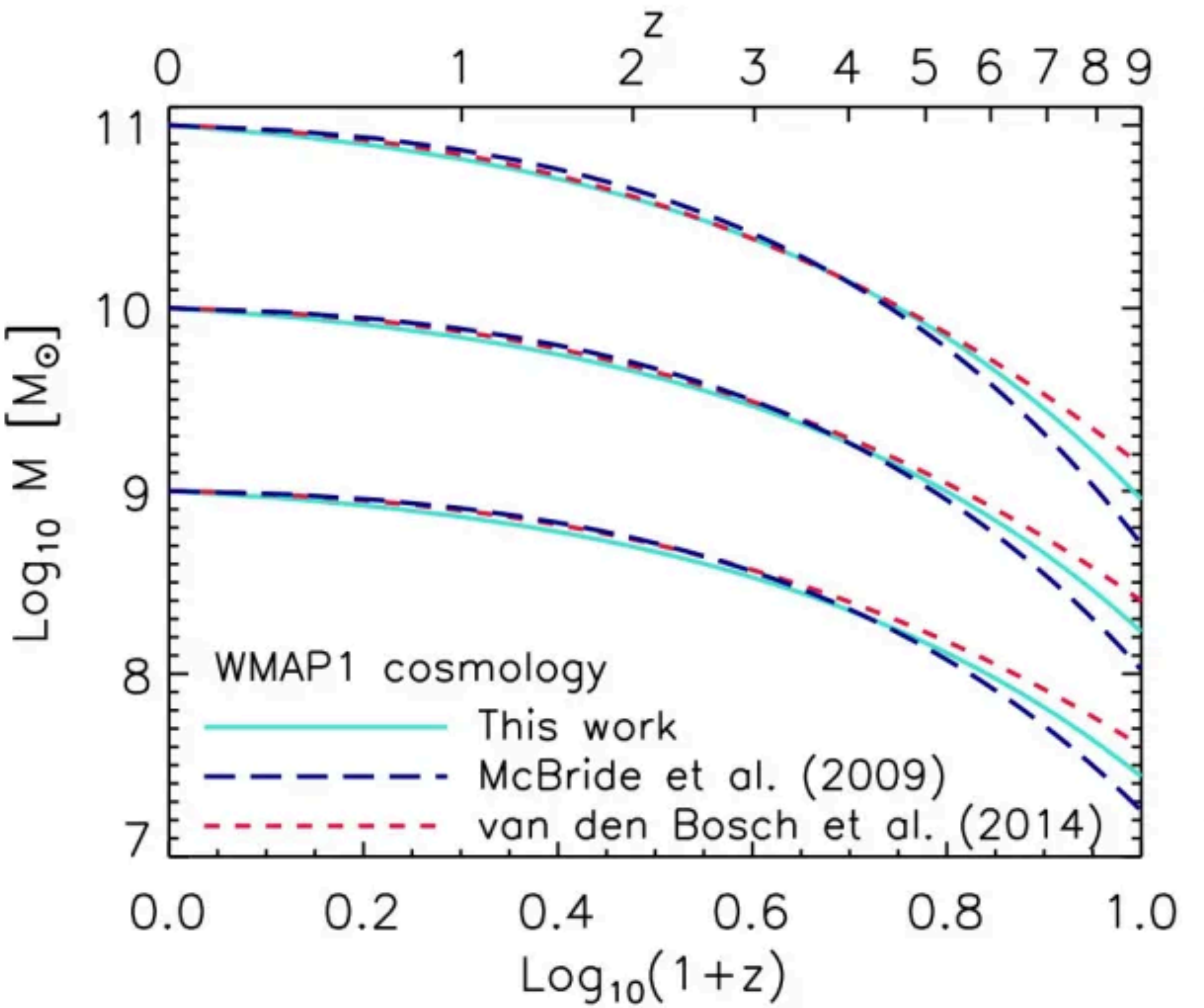
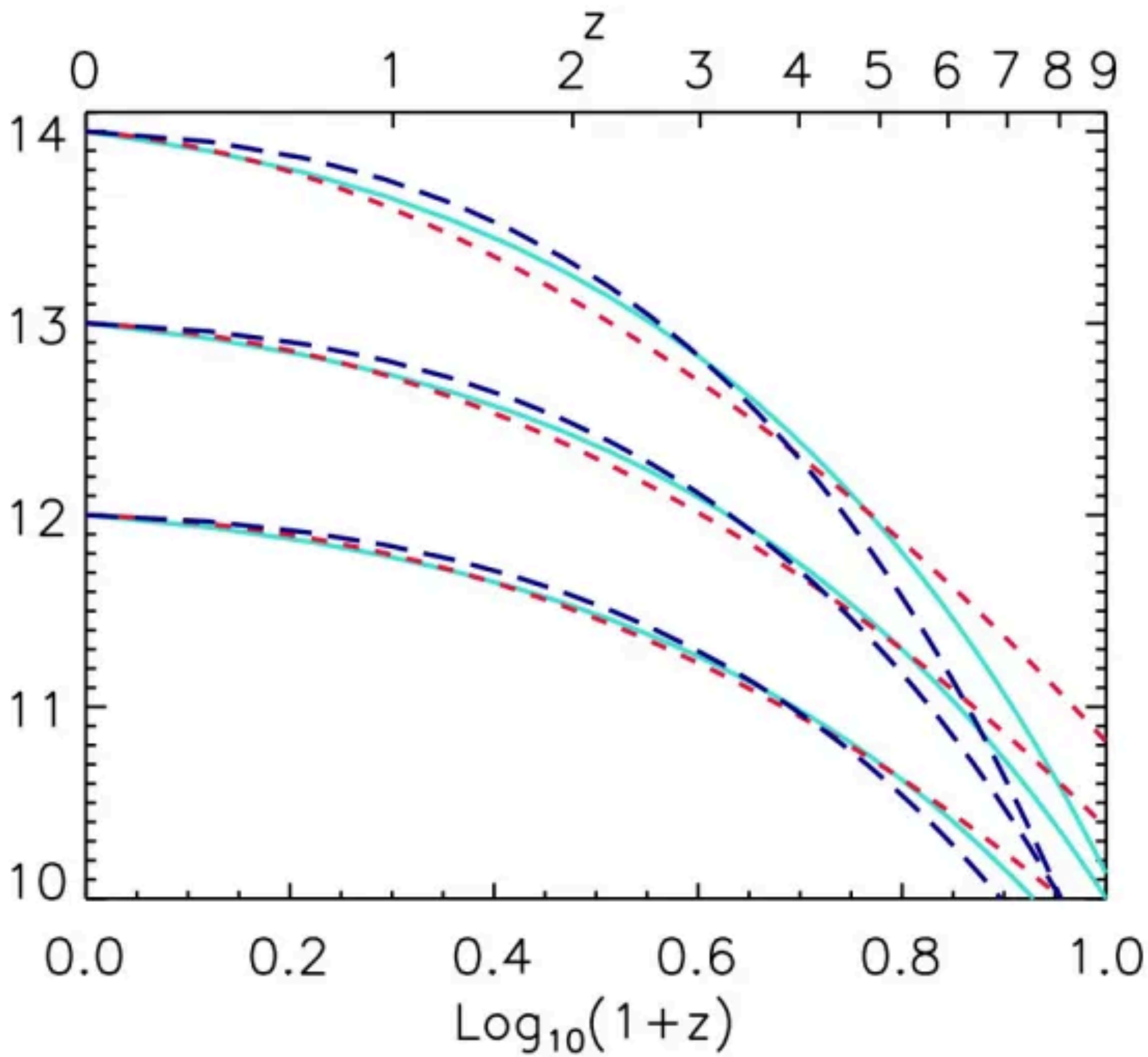


Correa+2014

The figure shows the analytic model presented in this work (turquoise solid lines), the median mass history obtained from the Bolshoi simulation and merger trees from van den Bosch et al. (2014) (purple dashed lines) and the best-fit relations from the Millennium simulation from McBride et al. (2009) (dark blue dot dashed lines). We provide numerical routines for the analytic model online (available at https://bitbucket.org/astroduff/commah).

⇒ Since we can accurately model how the mass in halos grow with time due to gravity, we can associate galaxies in both time slices.



Correa+2014

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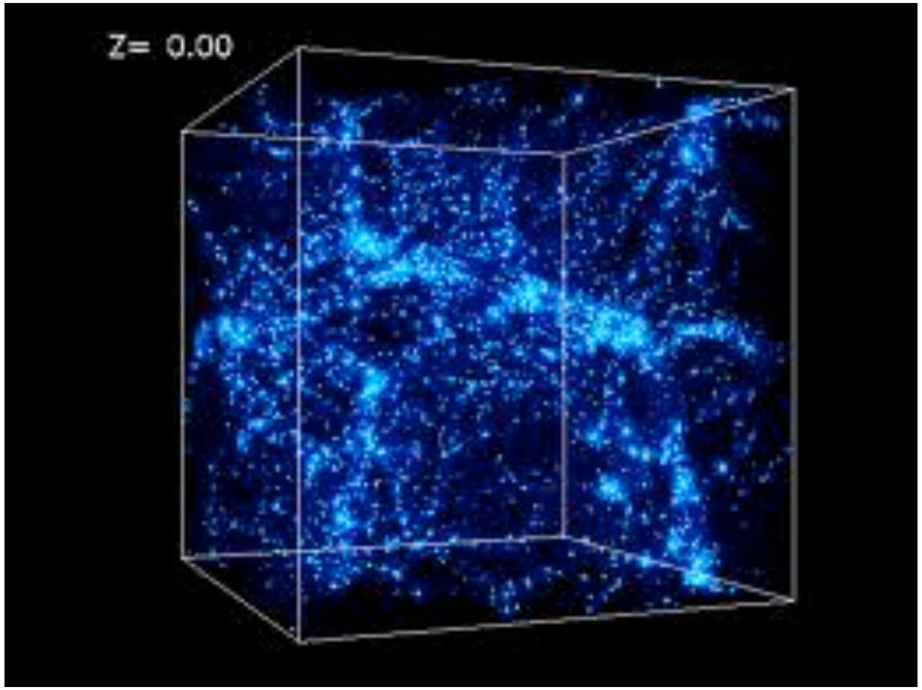
As collapsed systems are forming due to gravity, they are not forming in isolation, but in extended structures of halos with significant clustering

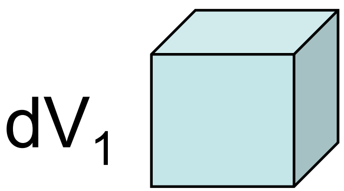


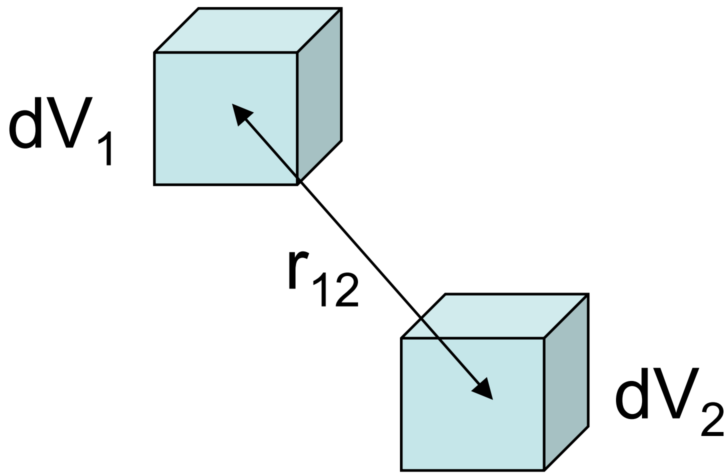
We can quantify the non-uniform distribution of galaxies on the sky through a measurement of the clustering signal.

This clustering signal ultimately helps us learn about the dark matter halos in which galaxies live.

We quantify clustering in terms of correlation functions

The Correlation function ξ is not equal to zero -- since the presence of a galaxy at some place in space makes it more likely another one will be close by....





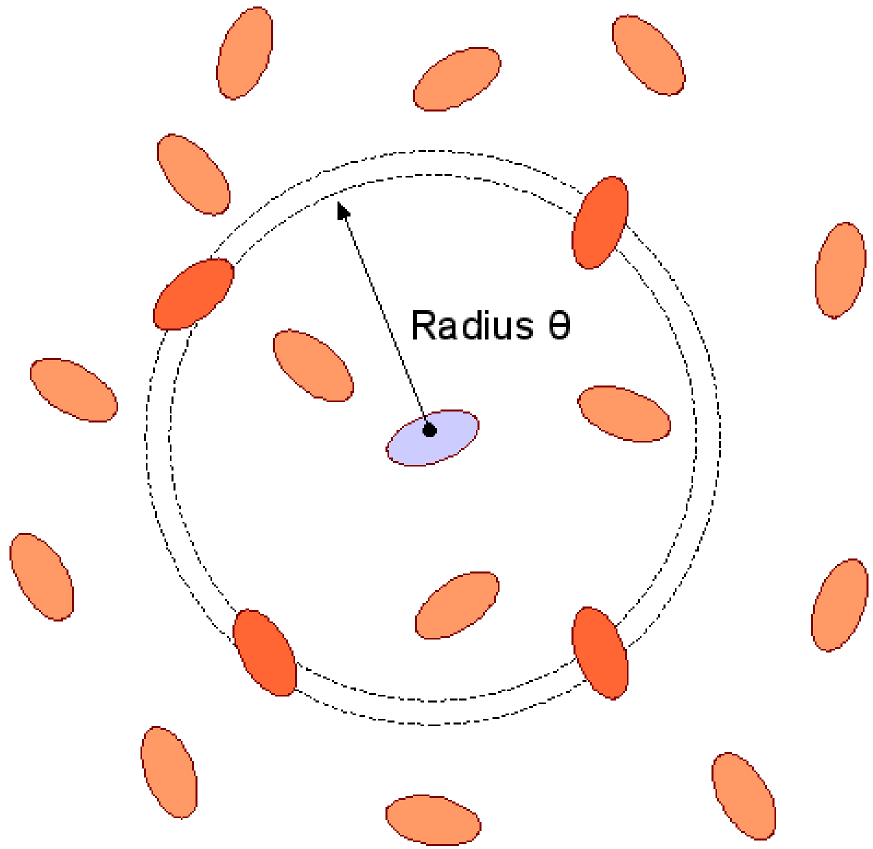
$$dP_1 = n\, dV_1$$

$$dP_{12} = n^2\,(1 + \xi(r_{12}))\, dV_1\, dV_2$$

n = average density of galaxies

How do we quantify the correlation function?

The Correlation function ξ is calculated by examining the distances between every pair of galaxies in a survey and comparing it to a random distribution



Correlations between points can be determined by counting pairs.

How do we quantify the correlation function?

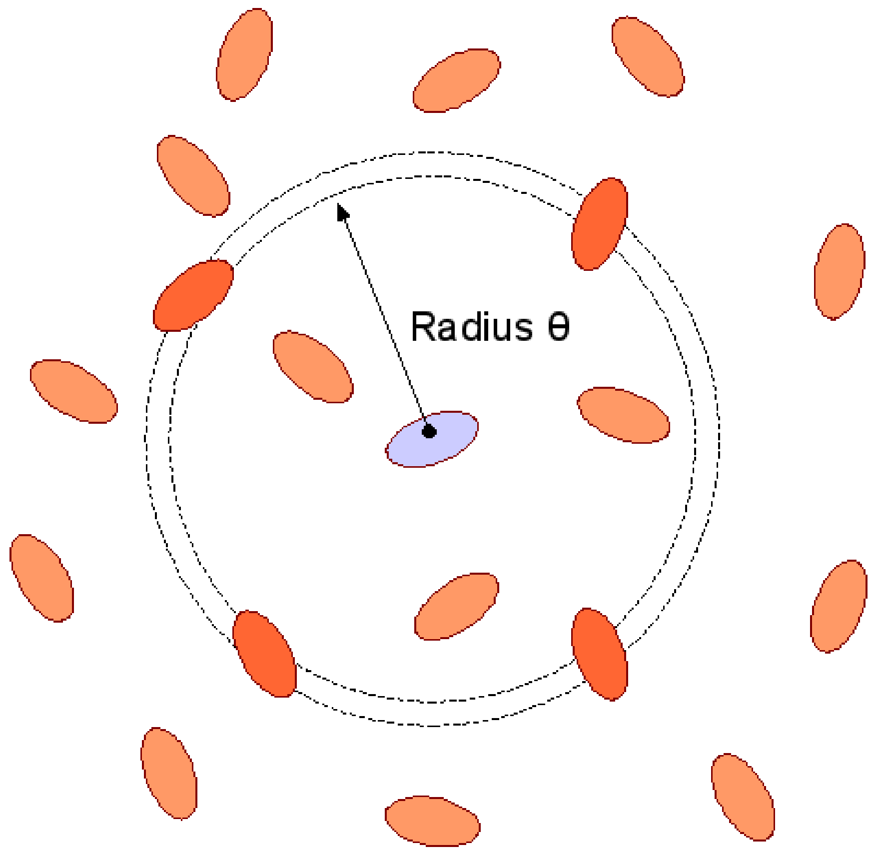
Fundamentally, this involves counting the number of galaxies at a certain distance from each other on the sky and then comparing that with a random distribution

$$\xi(r) = DD / RR - 1$$

DD = number of pairs in the data at a distance r

RR = number of pairs in some mock data set at a distance r

(in mock data sets pairs laid down randomly with uniform distribution)



Correlations between points can be determined by counting pairs.

How do we typically express the correlation function?

The Correlation function ξ is typically parametrized as a power-law in radius:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Typical values for γ are 1.8. r_0 is known as the correlation length and it tells us the typical distance from a source we can expect a large enhancement in neighboring sources

The power spectrum is the Fourier transform of the correlation function ξ

$$P(k) = \int \xi(r) e^{ik \cdot r} d^3 r \equiv \int \xi(r) \frac{\sin(kr)}{kr} r^2 \cdot dr$$

What can we learn about galaxies from the observed clustering / power spectrum?

It provides us with fundamental information about the dark matter halos in which galaxies live...

How does it provide this information?

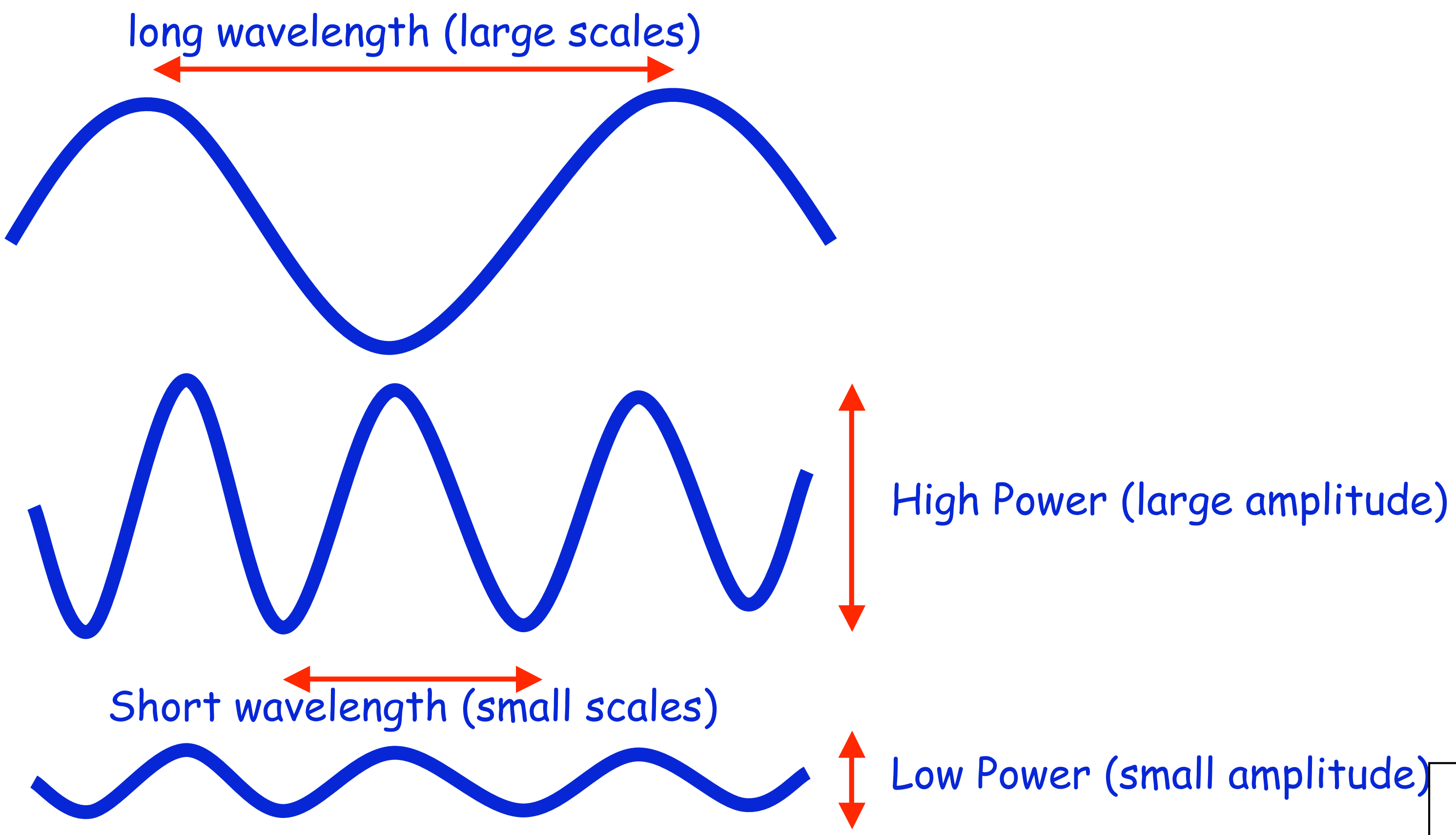
To understand this, we should review the basics of the growth of density perturbations in the universe.

(which we will discuss in a few slides...)

How are the dark matter power spectrum and power spectrum for dark matter halos calculated?

How can we describe dark matter fluctuations?

Convenient to express it in terms of Fourier modes and power spectrum:



Credit: Pearson

How can then we can calculate the power spectrum?

Subtract off mean density:

$$\delta(\vec{r}) = \frac{\rho - \bar{\rho}}{\bar{\rho}} = \frac{\Delta\rho}{\bar{\rho}}$$

Fourier Transform:

$$\delta_k = \sum \delta(\vec{r}) e^{-i\mathbf{k}\bullet\mathbf{r}}$$

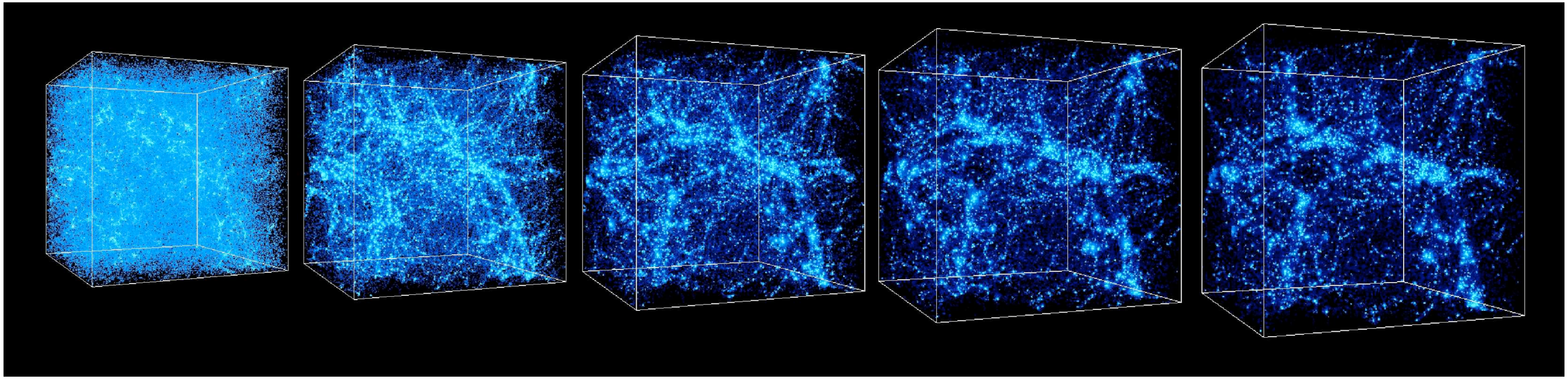
Power Spectrum:

$$P(k) = \left\langle \left|\delta_k\right|^2 \right\rangle$$

What happens to density perturbations over time?

They grow through gravitational forces.

cosmic time →



more uniform density

less uniform density

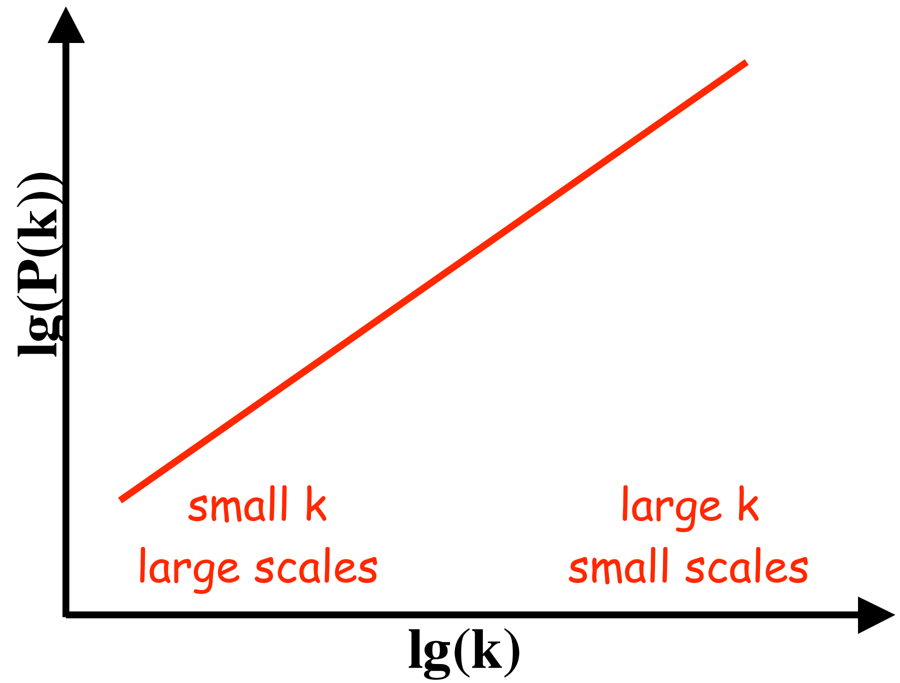
How does this operate in the current universe?

What is the primordial power spectrum?

The initial power spectrum of fluctuations is the following:

$$P_0(k) = A\,k^{n_s}$$

$n_s \sim 1$: "Harrison-Zeldovich Spectrum"



Credit: Pearson

How fast does the power spectrum grow?

z>3500: Radiation-dominated Epoch:

No significant growth in structure occurs -- except at scales larger than the horizon, where the growth goes as R^2 (R = scale of universe)

For causally connected regions (i.e., within the horizon), there is no growth

For regions not casually connected (i.e., super horizon scale), growth occurs as R^2

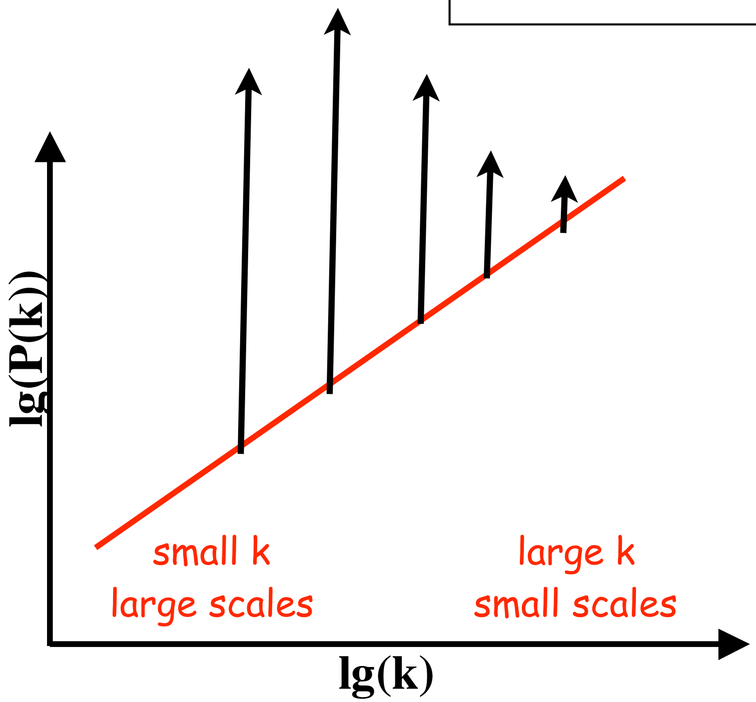
z<3500: Matter-Dominated Epoch:

growth goes as R (R = scale of universe)

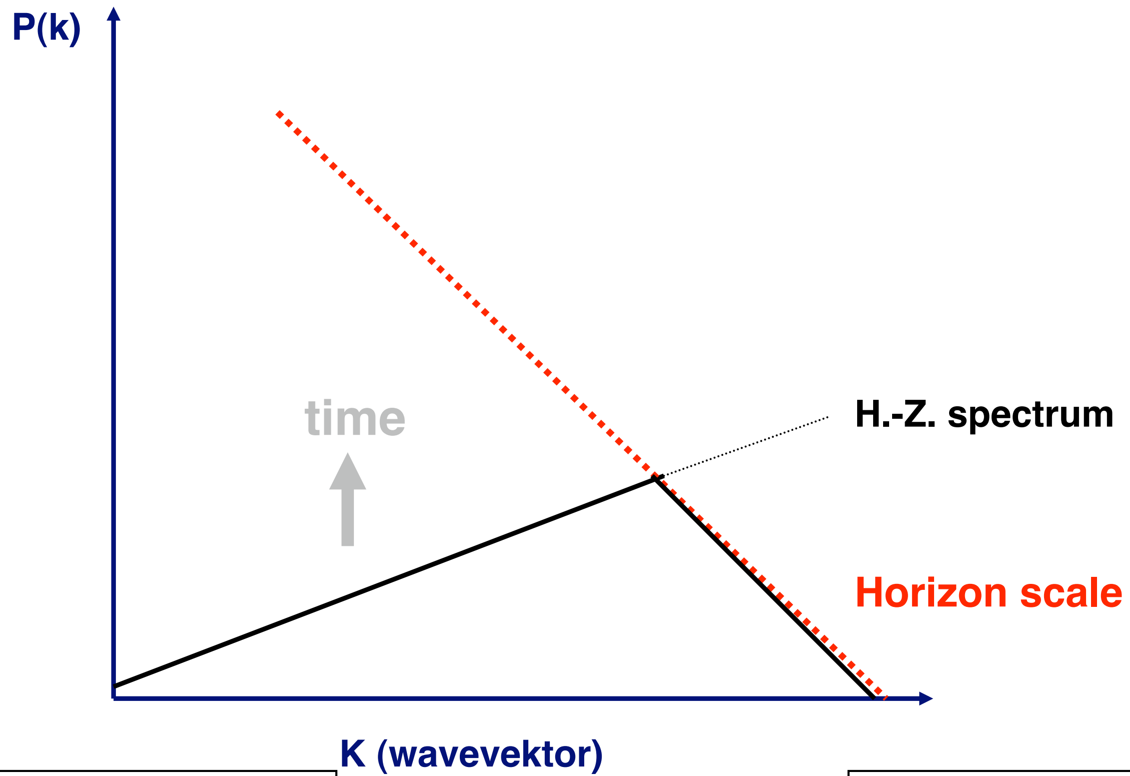
Most of the growth in the power spectrum will occur on large scales!

identical growth of structure at large scales

less growth of structure at small scales as universe becomes casually connected



Evolution of the Matter Power Spectrum

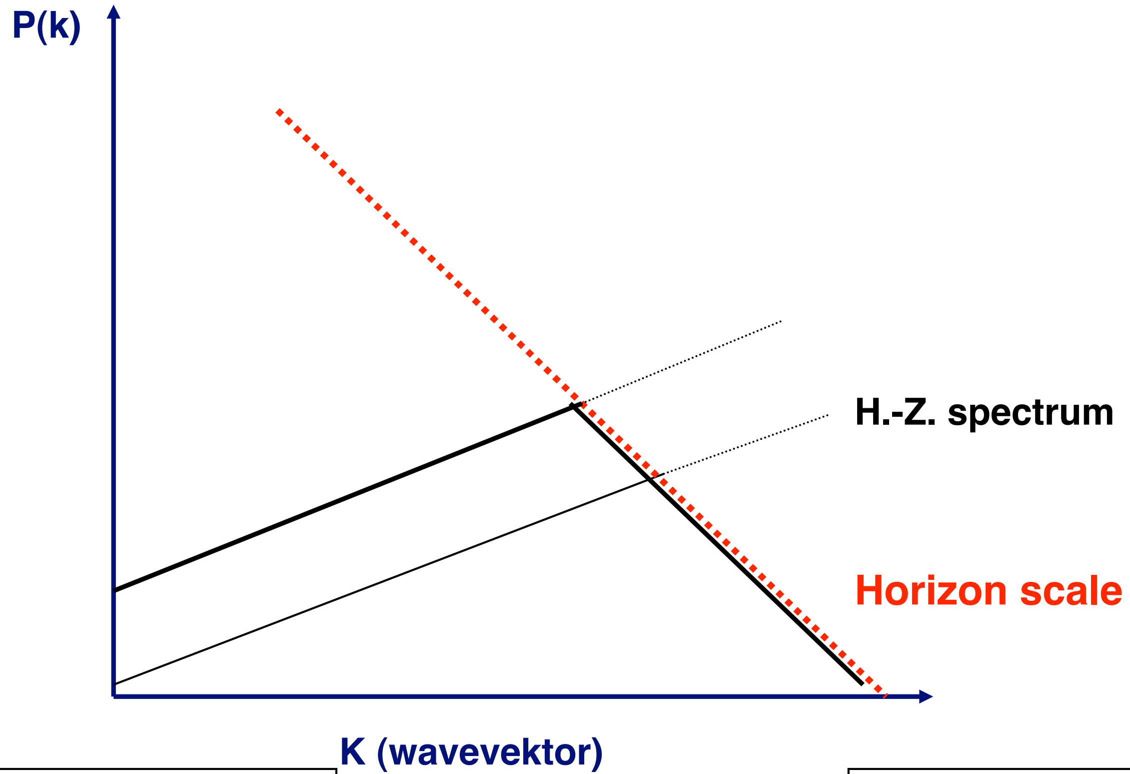


Large Scales

Small Scales

Credit: Bohringer

Evolution of the Matter Power Spectrum

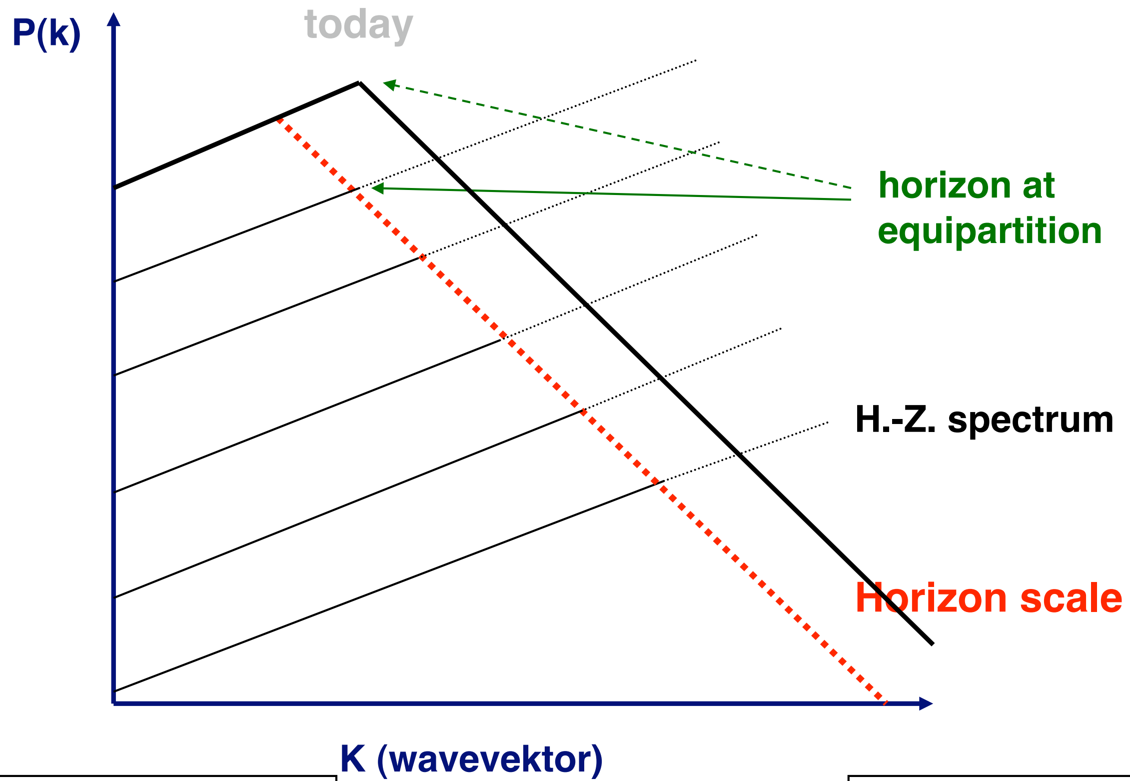


Large Scales

Small Scales

Credit: Bohringer

Evolution of the Matter Power Spectrum



H. Böhringer Large Scales

Small Scales 50

Credit: Bohringer

How does one calculate the power spectrum for matter fluctuations in early universe?

Formally, one utilizes a transfer function to include these physics:

$$P_0(k) = A\, k^{n_s}\, T^2(k)$$

where T(k) is the transfer function.

The transfer function T(k) depends on the cosmological model and in particular on the quantity $\Gamma = \Omega_m h$. Γ is called the shape parameter.

How does the power spectrum grow after epoch of radiation-matter equality and recombination?

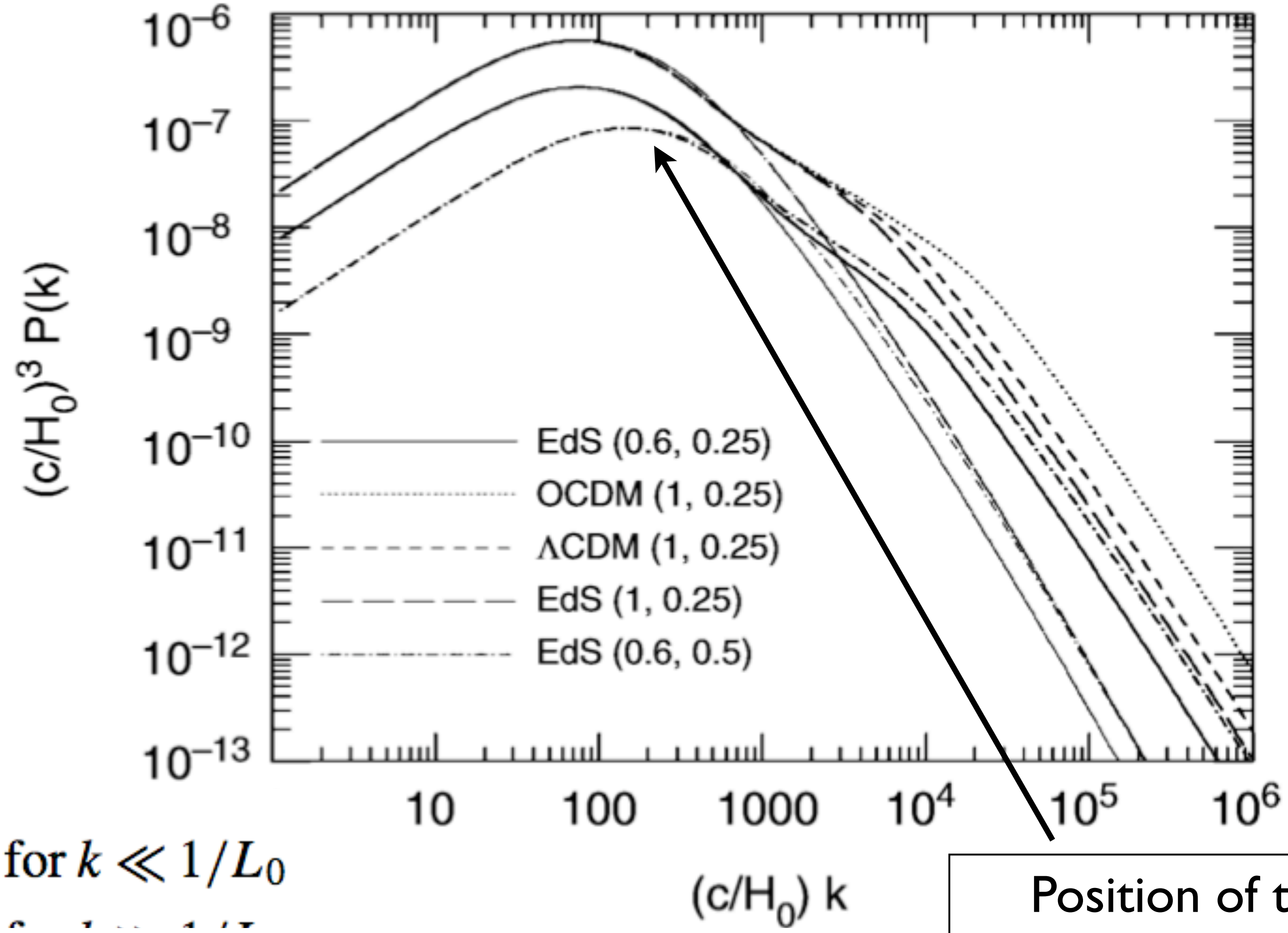
To simplest approximation, it can be expressed in terms of growing modes...

$$P(k, t) = D_+^2(t)\, P(k, t_0) =: D_+^2(t)\, P_0(k)$$

where P(k,t) is the power spectrum at some later time and $D_+(t)$ is the growth factor.

In the linear growth regime (before modes start turning around and collapsing and virializing), the time t and mode k are totally separable in the above equation.

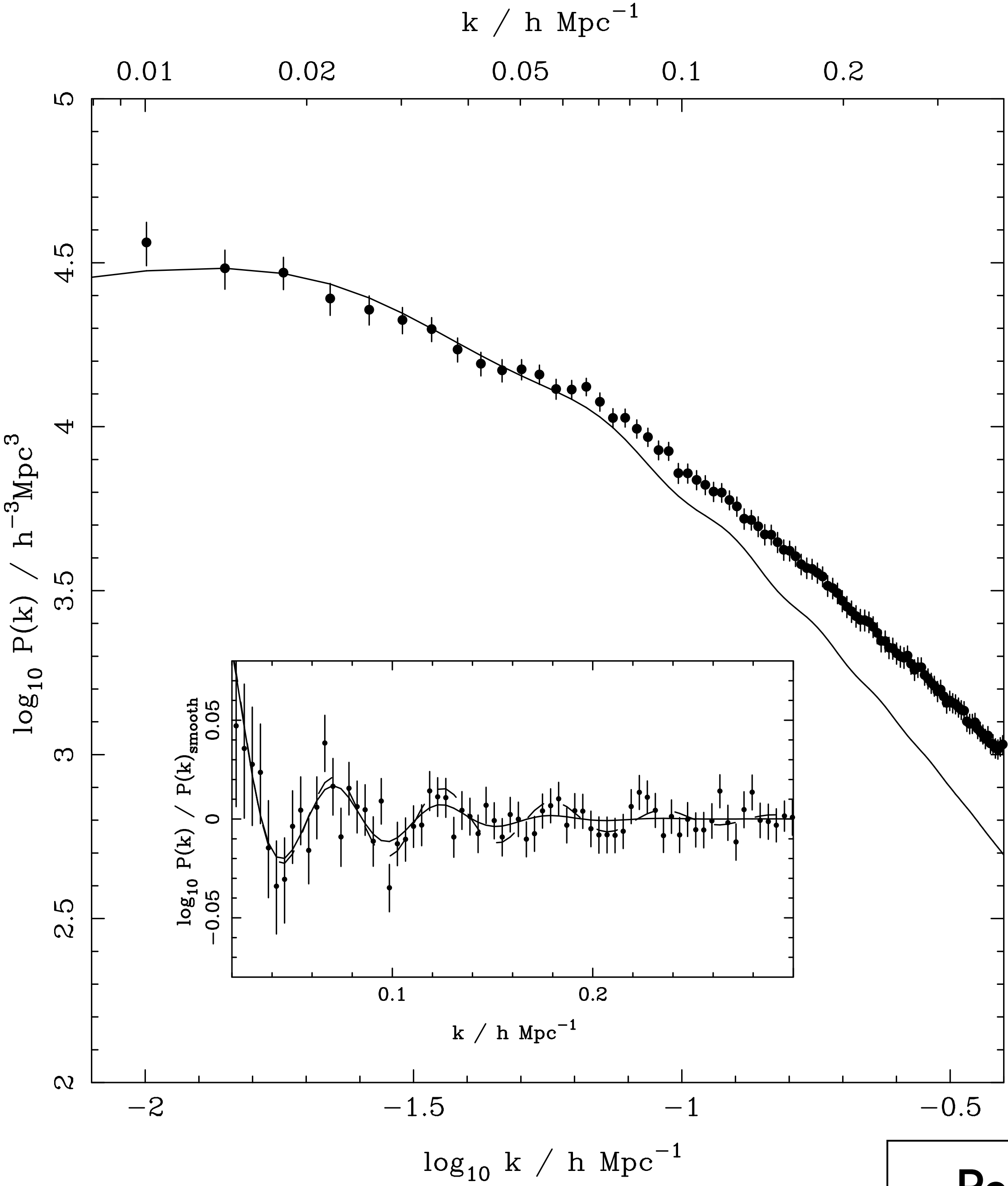
What does the matter power spectrum look like when all of these effects are included?



l for $k \ll 1/L_0$

Position of turn-over determined by horizon size @ matter-radiation equality

This is very similar to the observed power spectrum!



Percival et al. 2007

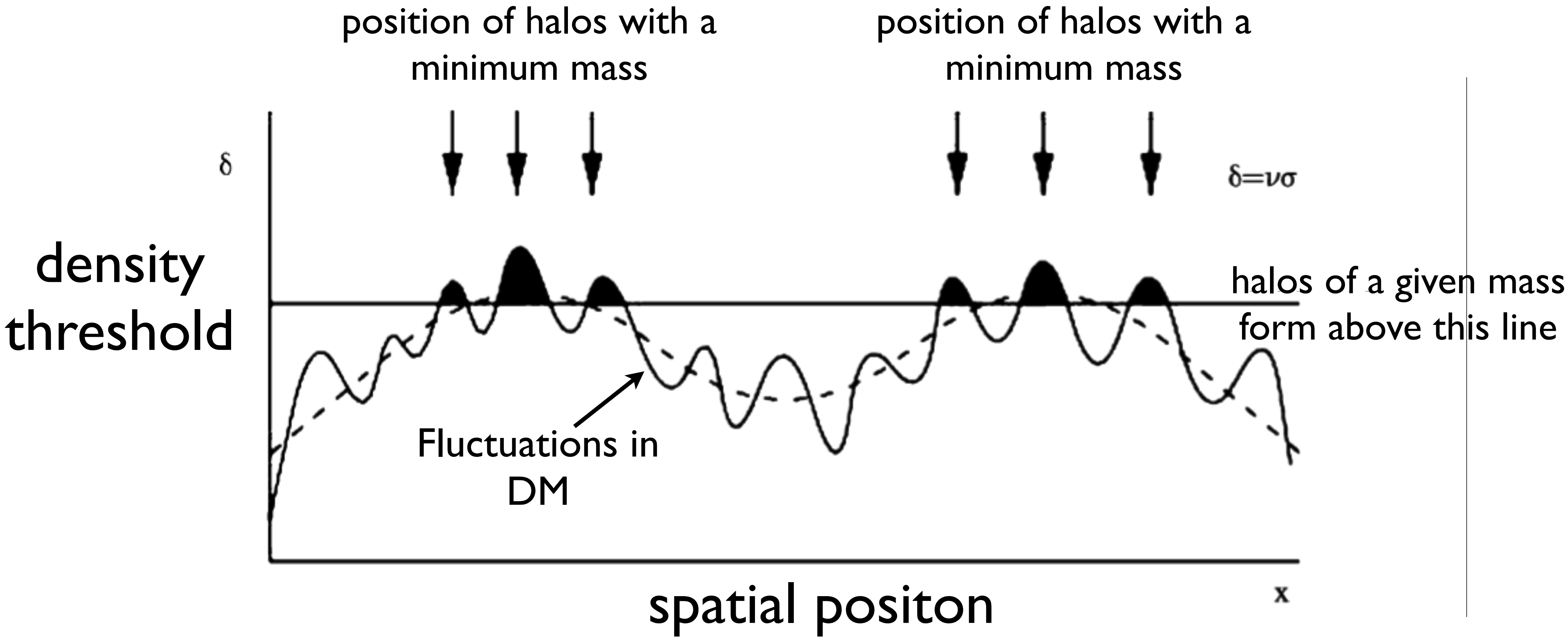
What is the power spectrum of collapsed halos of a given mass? How does it compare with the power spectrum of underlying matter?

Halos only form in overdense regions of the universe which have had time to collapse.

Collapsed halos with particularly high masses would only form in regions with the most significant overdensities.

This would lead to a spatial distribution of halos that shows considerably more structure than the underlying dark matter.

Why dark matter halos show a less uniform distribution than the underlying matter distribution?



In the regions around collapsed objects (particularly massive ones), there are substantial large-scale (long range) overdensities present, which make it easy for small fluctuations to push them over the requisite density threshold

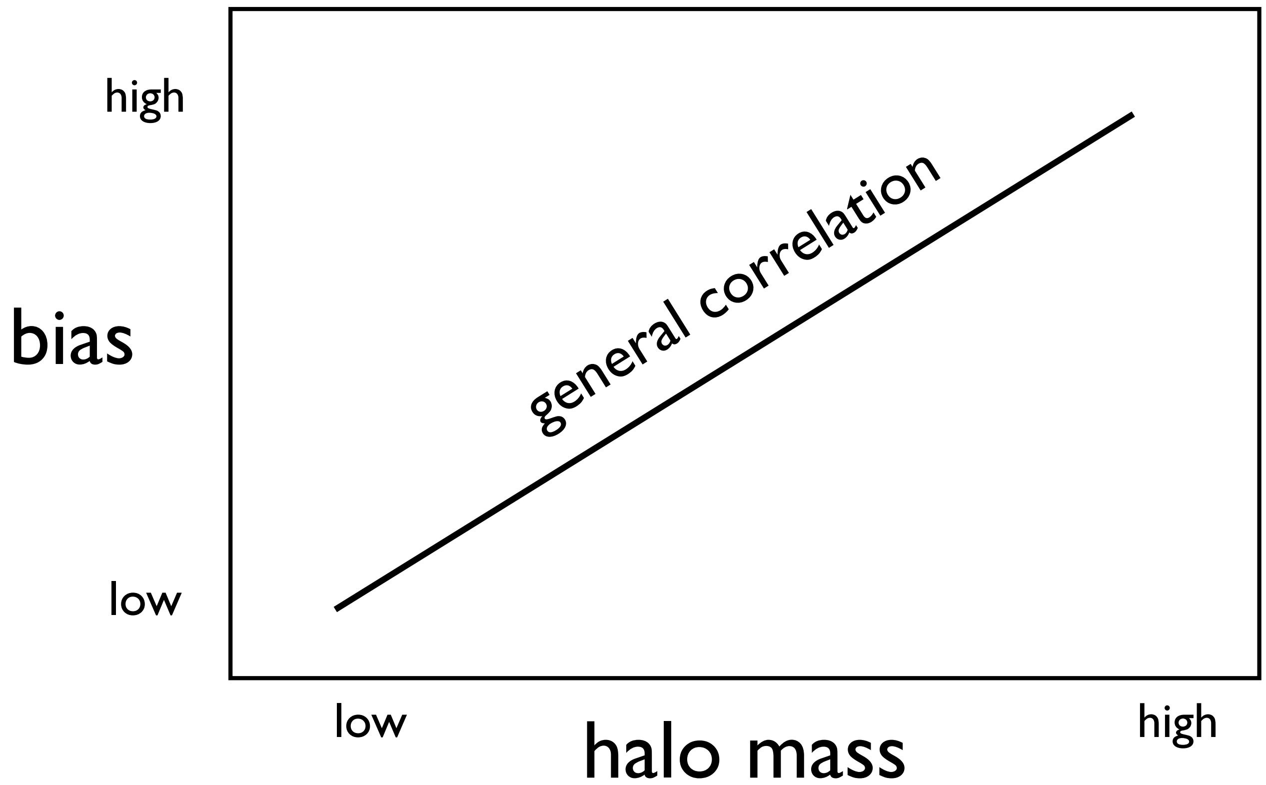
Because dark matter halos overestimate fluctuations in the underlying density distribution, they are said to be **biased**.

The most massive galaxy halos are expected to be distributed in the least uniform way relative to the underlying dark matter density distribution and be the most biased.

In summary, in terms of the bias factor b,

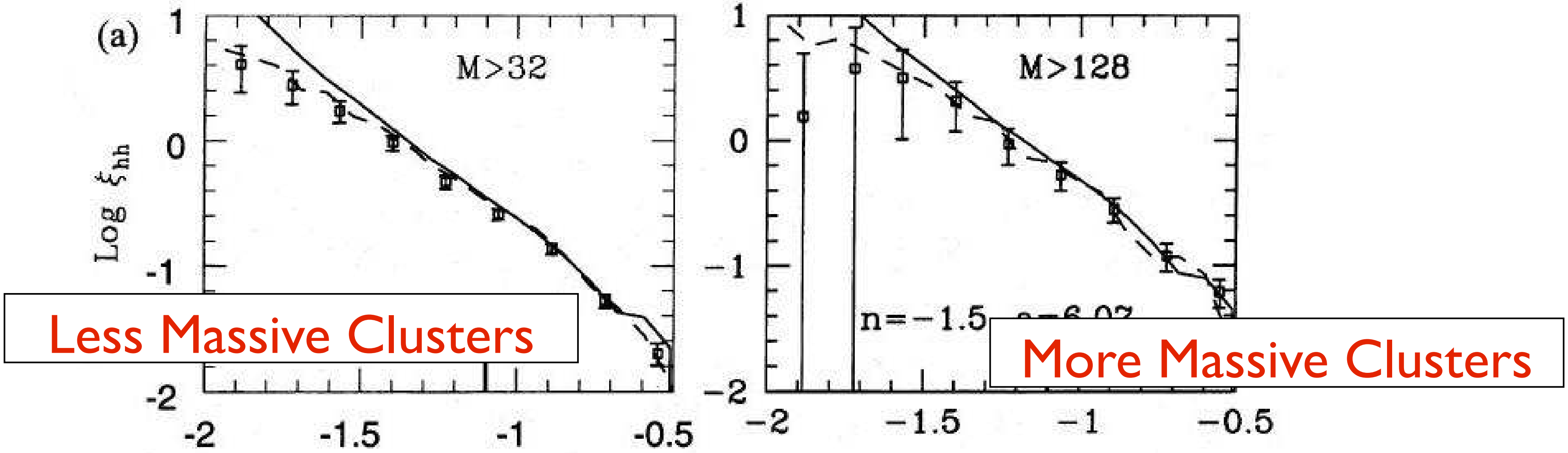
$$b^2 \; P(k)_{DM} \; = \; P(k)_{DMHalos}$$

Less massive galaxies, on the other hand, are less biased tracers of the underlying dark matter distribution.



The classic demonstration of this result (i.e., relationship between the bias/clustering properties and halo mass) is found in Mo & White (1996).

Here are some results from that paper:



Correlation functions of clusters, in a simulation

As can be seen, the low mass clusters (left panel) are less strongly clustered than the high mass clusters.

How can we define bias for galaxy halos?

-- Express fluctuations in the number of collapsed halos vs. spatial position in terms of fluctuations in the mass density times biasing factor:

$$\delta_h := \frac{\Delta n}{\bar{n}} = b\frac{\Delta\rho}{\bar{\rho}} = b\,\delta$$

In general, bias b >= 1

How can we use this to learn about the dark matter halos in which galaxies live?

Compute $P(k)_{galaxies}$ from the observations

Take $P(k)_{DM}$ from theory

Take $P(k)_{DMHalo}$ from theory

$$P(k)_{galaxies} = b^2 \, P(k)_{DM} = P(k)_{DMHalo}$$

↓

solve for the bias b

Use the bias b to determine the approximate halo mass in which galaxies with a given clustering properties live.

Could do the same exercise based on the correlation function!

Compute $\xi(r)_{galaxies}$ from observations

Take $\xi(r)_{DM}$ from theory

Take $\xi(r)_{DMHalos}$ from theory

$$\xi(r)_{galaxies} = b^2 \ \xi(r)_{DM} = \xi(r)_{DMHalos}$$

↓

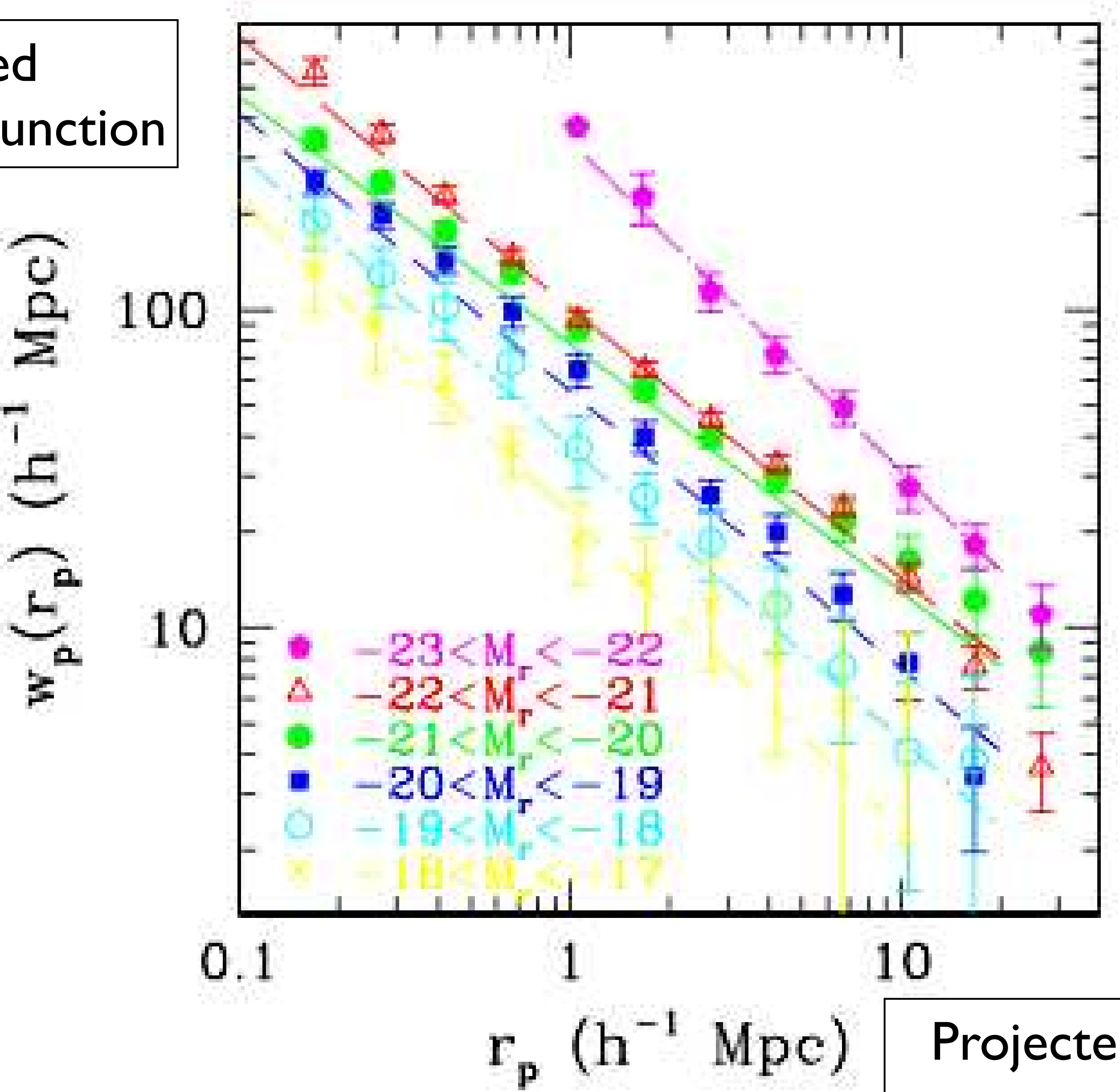
solve for the bias b

Use the bias b to determine the approximate halo mass in which galaxies with a given clustering properties live.

This implies that the correlation function of galaxies will be higher for galaxies living in more massive halos

More Luminous Galaxies Show A Stronger Spatial Correlation

Projected Correlation Function



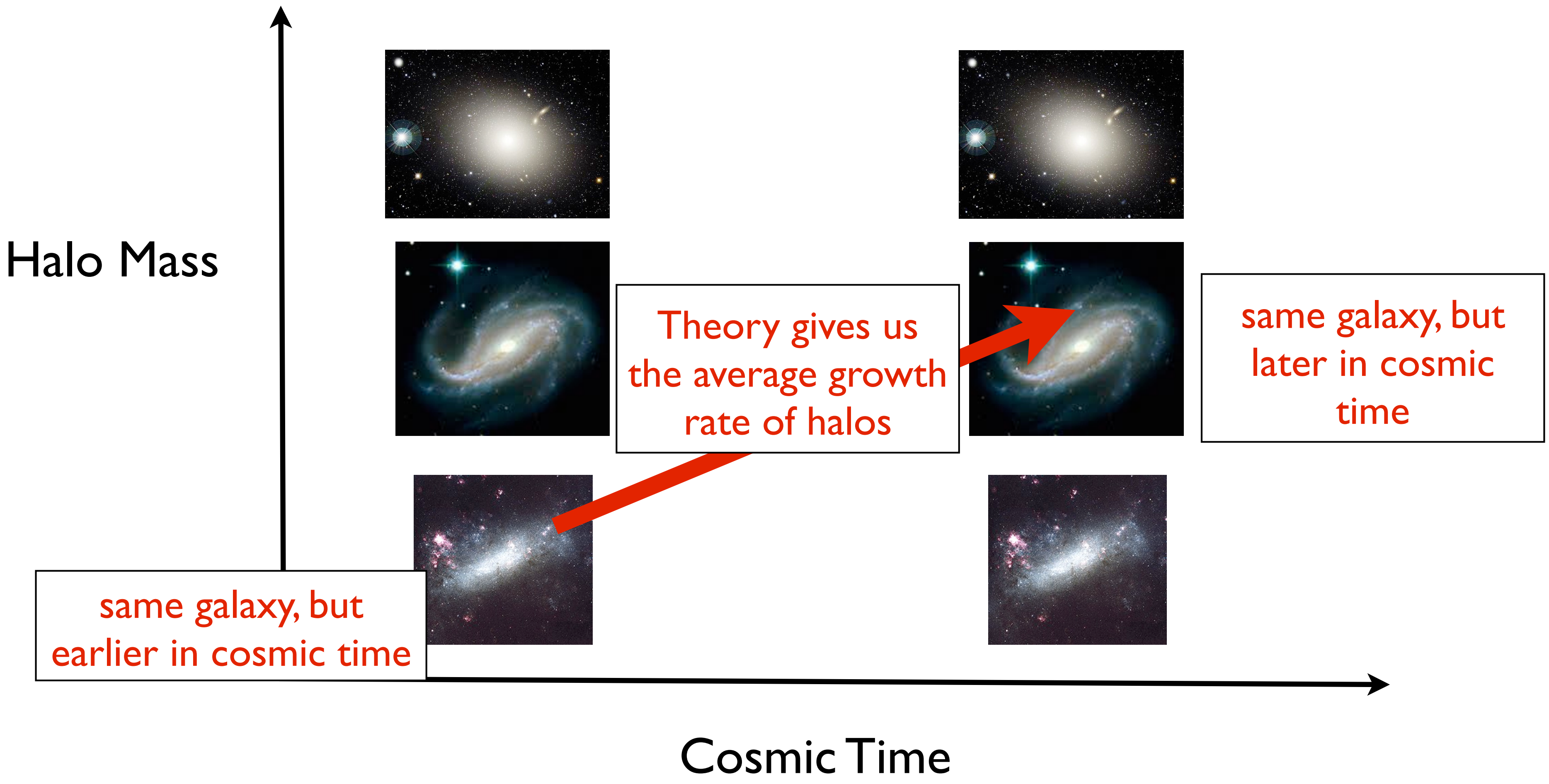
Projected physical separation

Implies more luminous galaxies live in more massive halos!

This demonstrates that the clustering properties of galaxies can be a powerful way of making inferences about the dark matter halos in which galaxies live!

The stronger the clustering properties (corresponding to systems with longer correlation lengths), the higher the mass of the galaxies under study.

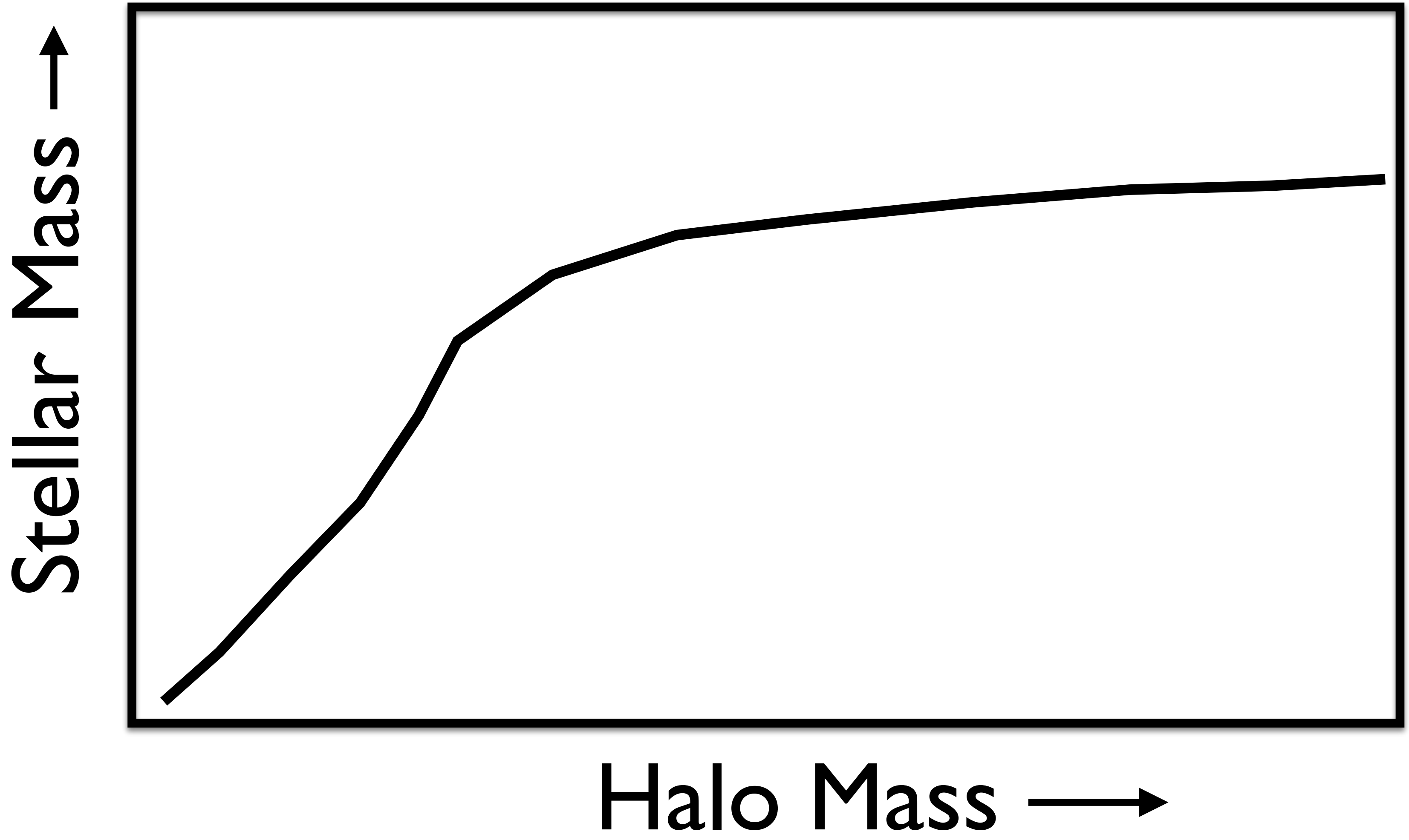
Halo mass provides us a powerful tool for tracing the same population of galaxies through cosmic time.



Besides clustering, is there another commonly used method to estimate halo mass?

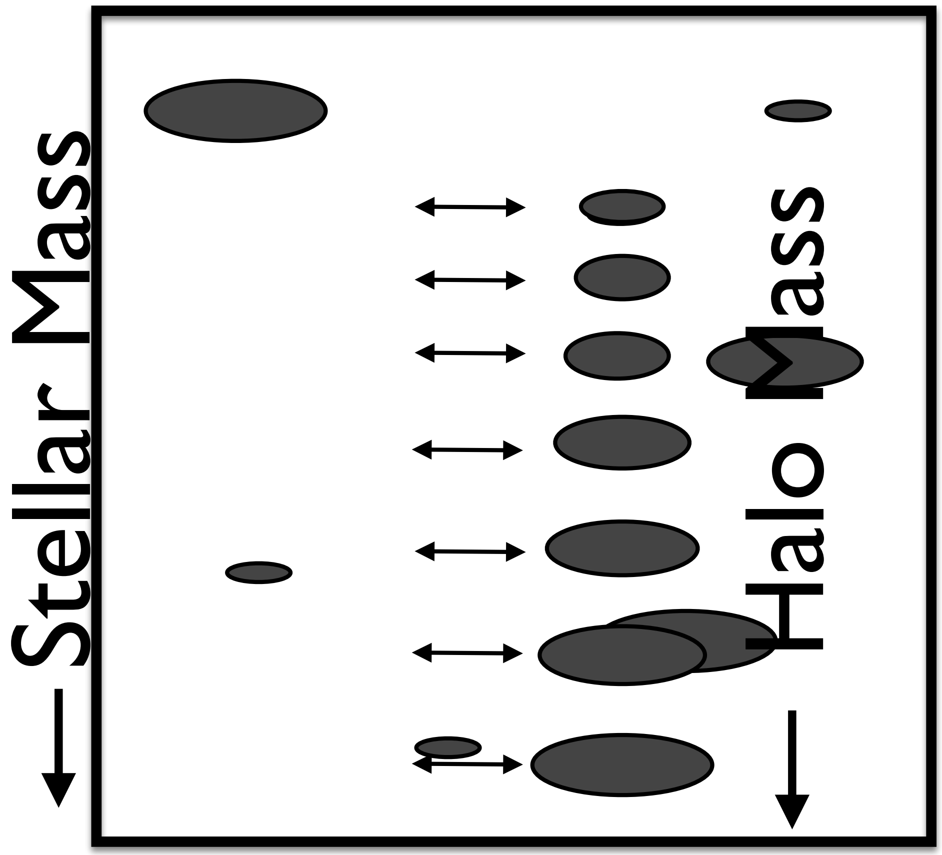
Yes — and it is called abundance matching

Basic idea leverages stellar mass in galaxies likely being a strictly increasing function of halo mass



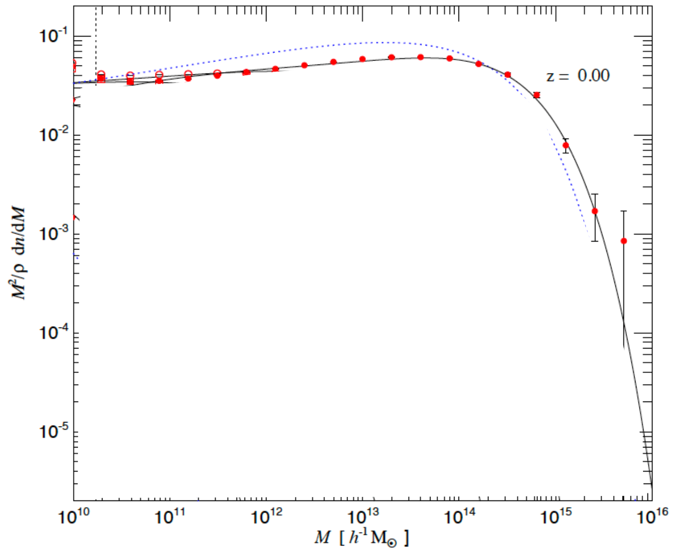
What are the halo masses of galaxies in this volume?

Survey of a volume of the universe at z~1



Halo Mass Function

Volume Density of Dark Matter Halos of a Given Mass



Dark Matter Halo Mass

Assuming a monotonic 1-1 correspondence between stellar and halo mass, abundance matching allows for an estimate of the halo masses for individual galaxies

What can learn about the formation and evolution of galaxies from the stars we find in these galaxies?

What can learn about the formation and evolution of galaxies from the stars we find in these galaxies?

Ideally, we would use the observed stars to reconstruct the history of star formation in a galaxy.

We would like to determine the function:

SFR(t)

where t is time.

Generally, this area of study is divided into two subfields:

1) Resolved Stellar Population Analyses

Can measure the luminosity and color of individual stars in the nearby object

Useful for studies of Nearby Galaxies and Star Clusters

2) Integrated (Unresolved) Stellar Population Analyses

One cannot resolve the light from individual stars and they all blend together.

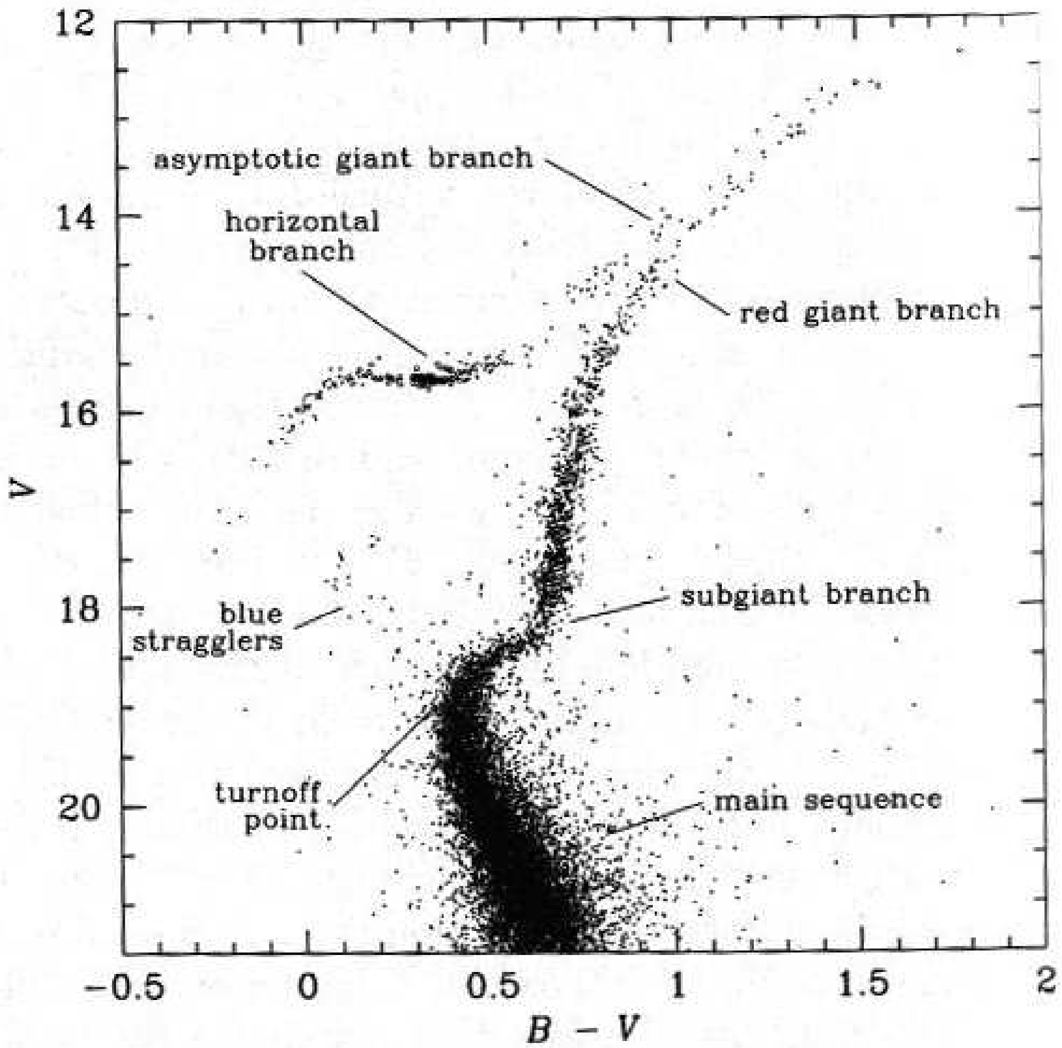
Used in Studying Distant Galaxies (galaxies greater than a few Mpc away)

1) Resolved Stellar Population Analyses

Nearby Star Clusters:

Advantage of star clusters is that all the stars in these clusters are at essentially the same distance from us. They also have the same age and metallicity.

Typical color magnitude diagram can be found to the right:

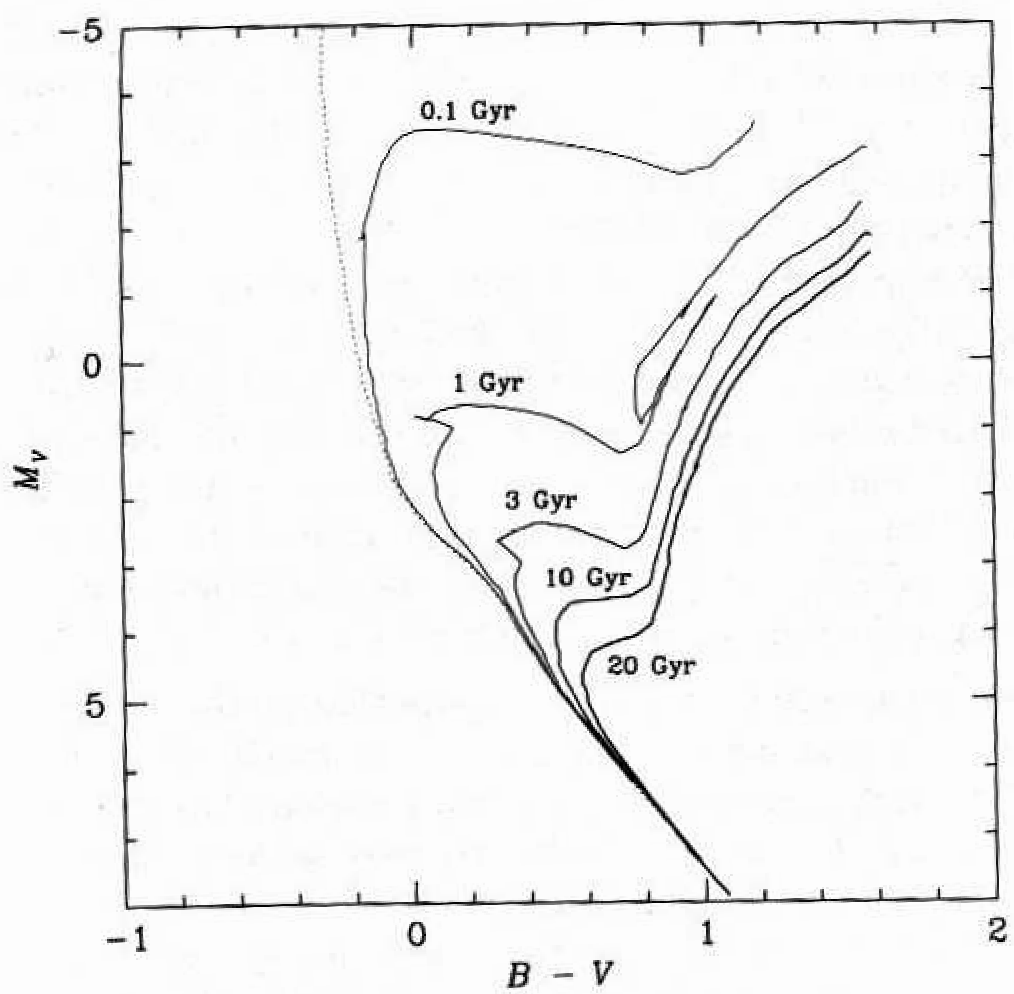


Nearby Star Clusters:

One can compare the color-magnitude diagram one observes with the expected color-magnitude tracks for stars of different fixed ages:

It is clear that one can measure the age of a star cluster quite accurately in this way.

These loci on the color-magnitude diagram (or hertzsprung russell) are called "isochrones"



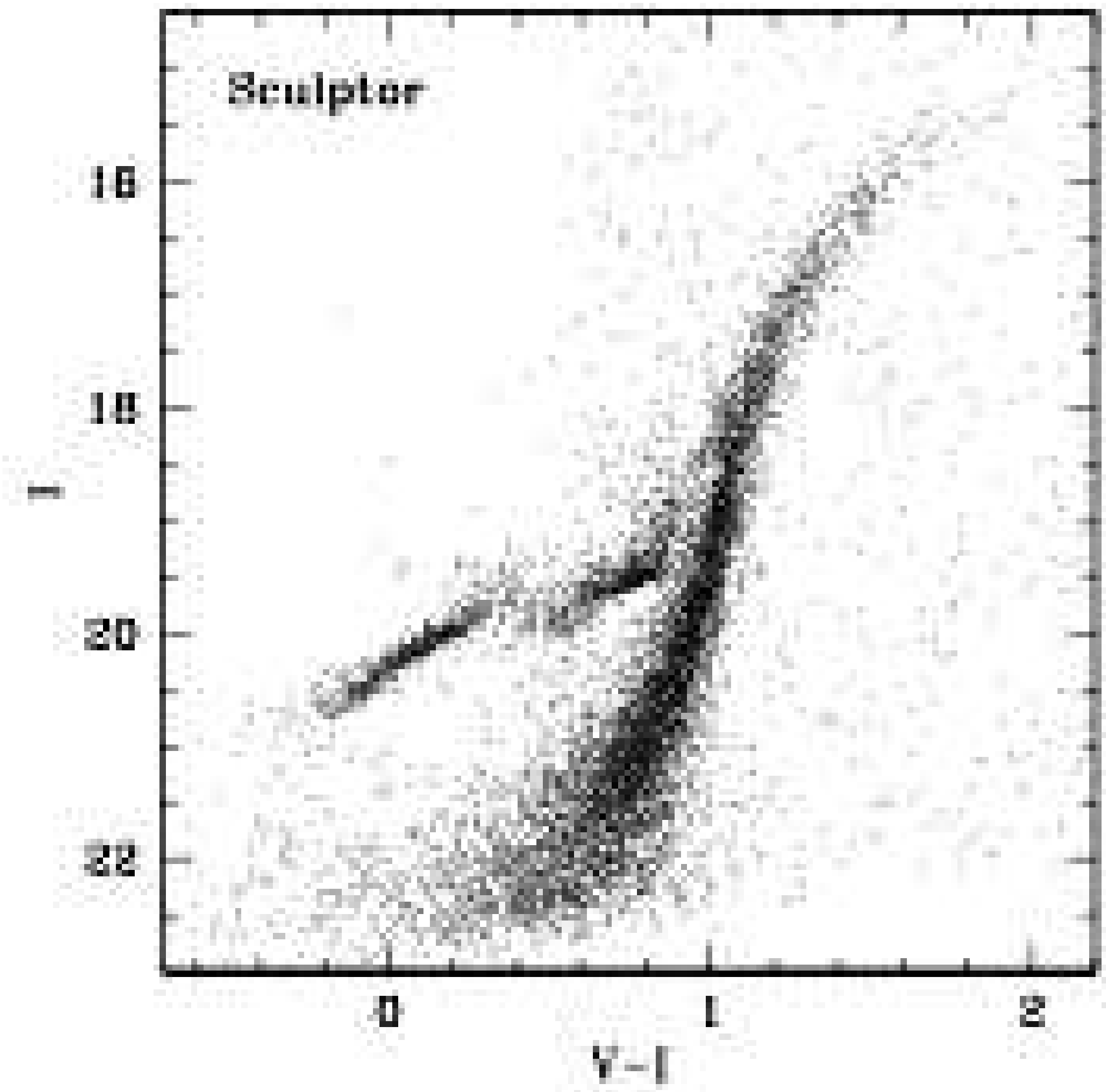
1) Resolved Stellar Population Analyses

Nearby Dwarf Galaxies

Here is the color magnitude diagram of a nearby dwarf galaxy Sculptor.

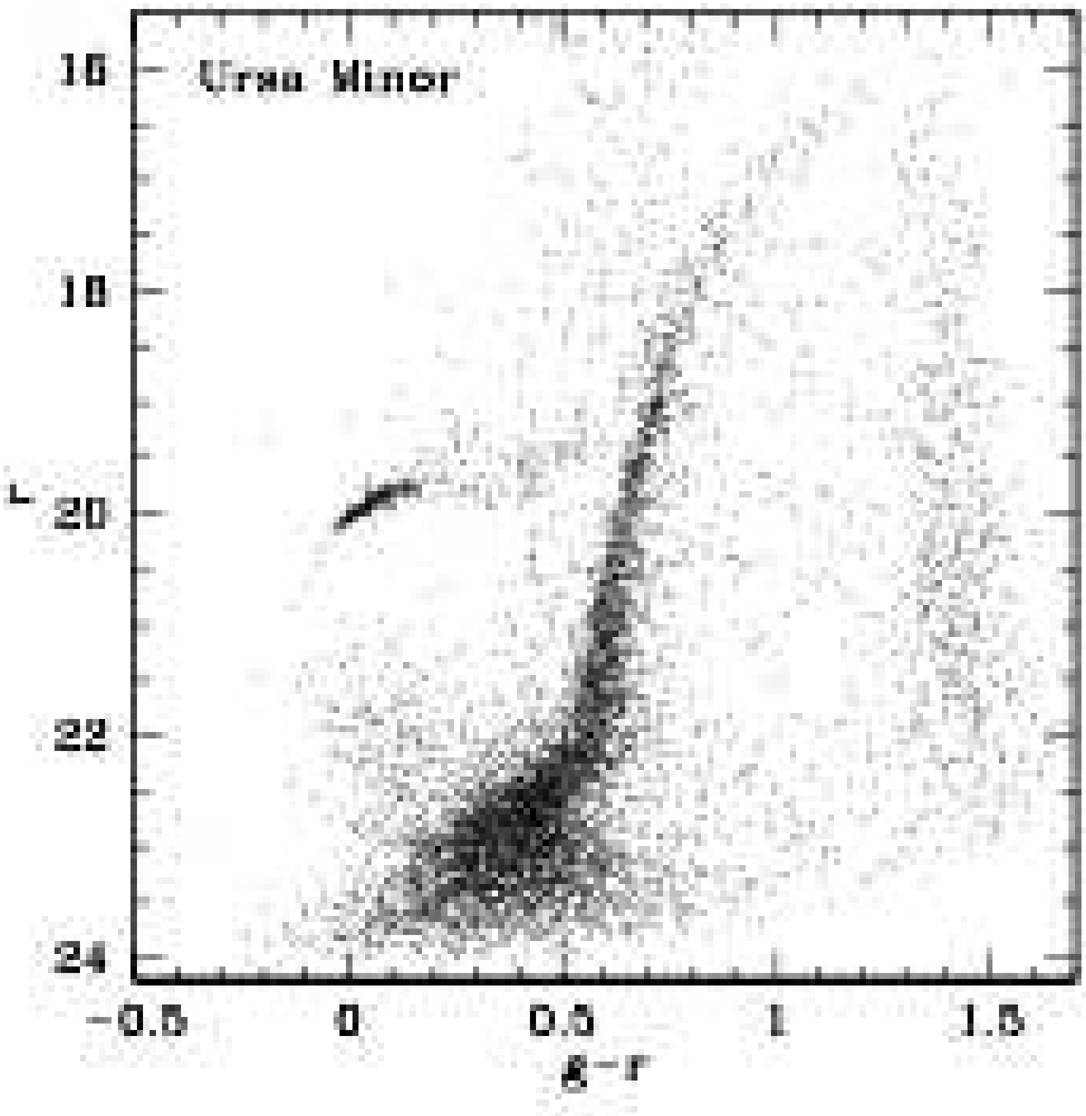
As you can, it has a well-defined giant branch and well-defined horizontal branch.

This galaxy is clearly old and consistent with having stars of the same age!



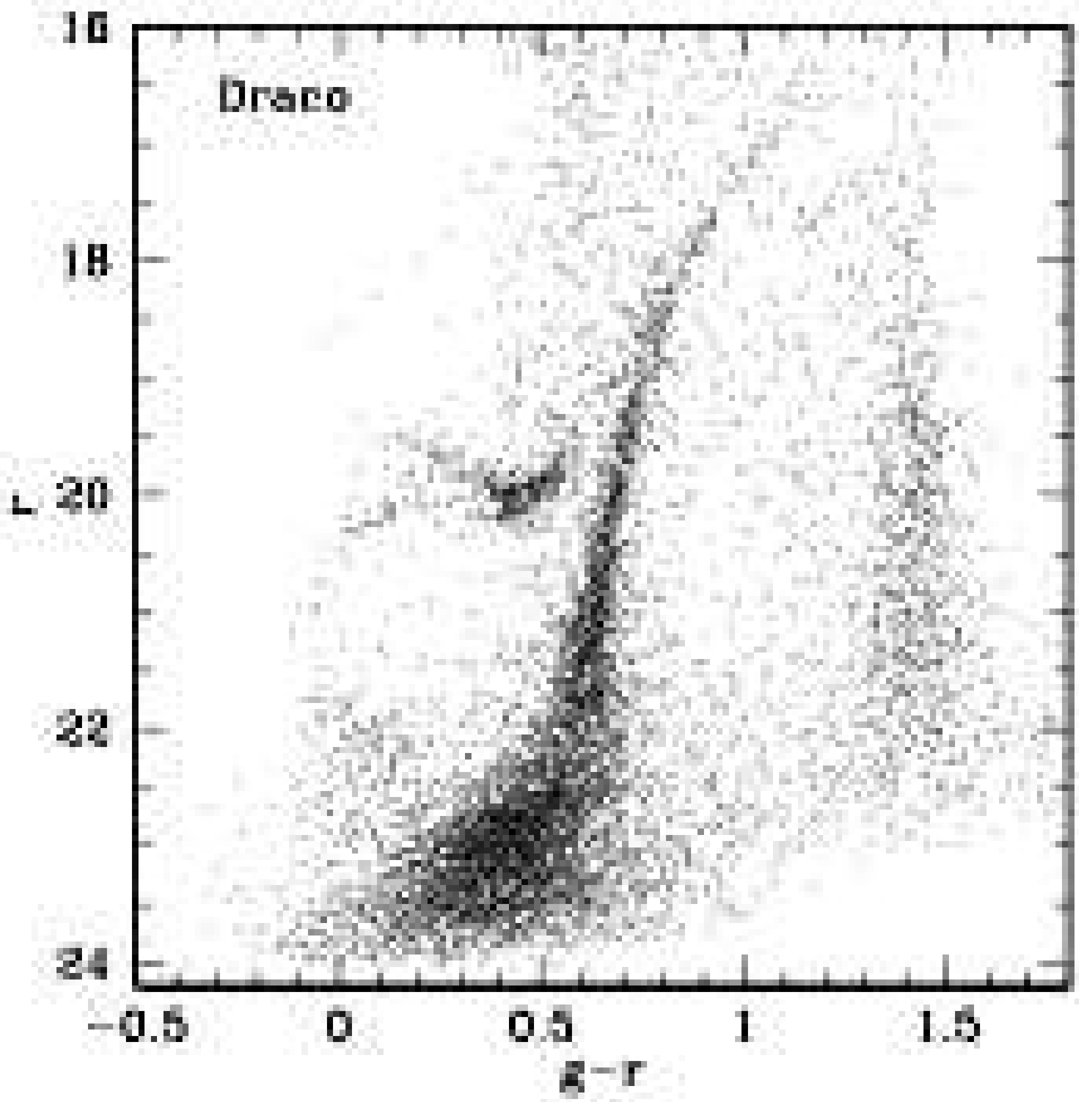
1) Resolved Stellar Population Analyses

Nearby Dwarf Galaxies



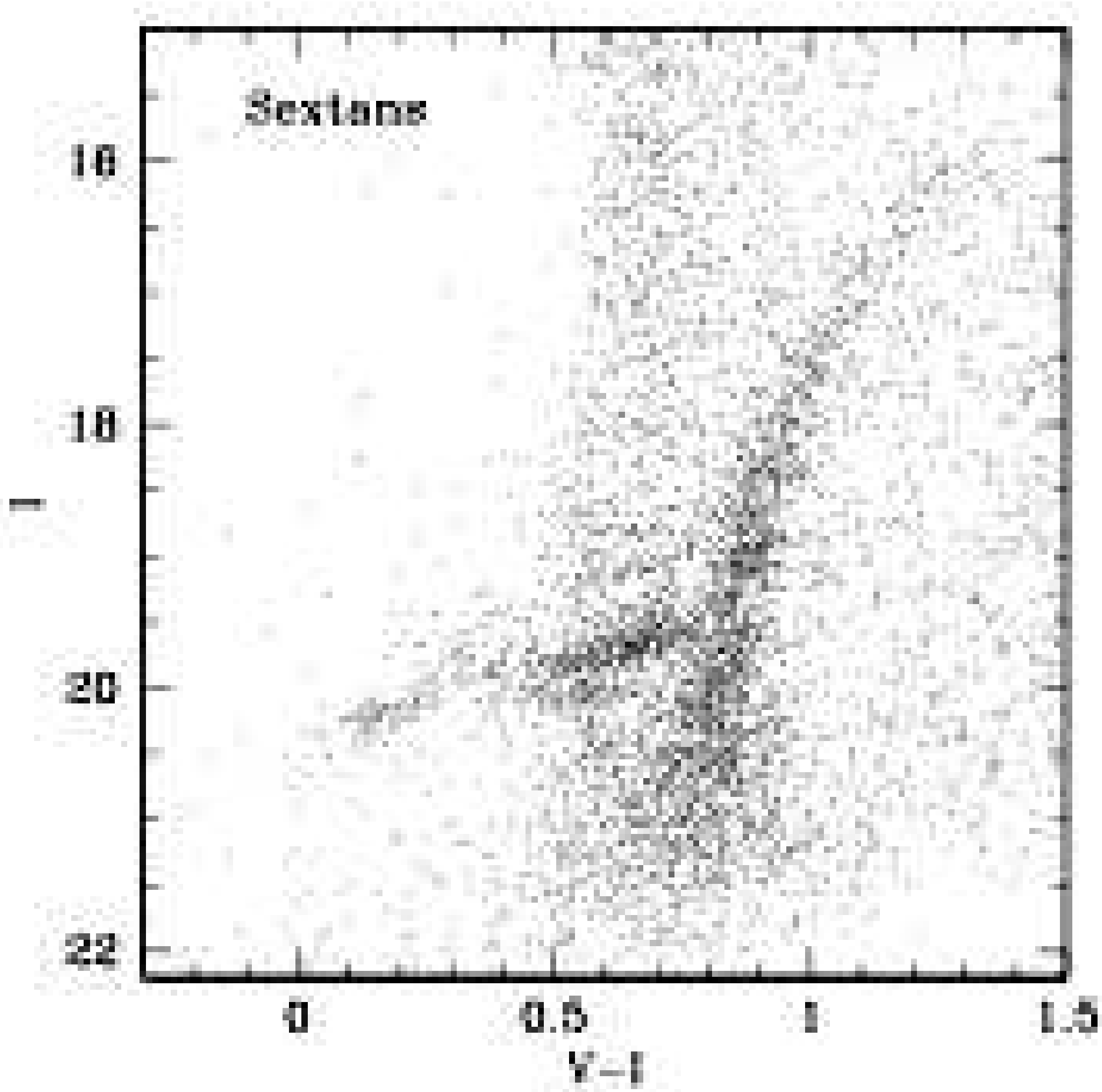
1) Resolved Stellar Population Analyses

Nearby Dwarf Galaxies



1) Resolved Stellar Population Analyses

Nearby Dwarf Galaxies



1) Resolved Stellar Population Analyses

What do you think is going on with this dwarf galaxy?

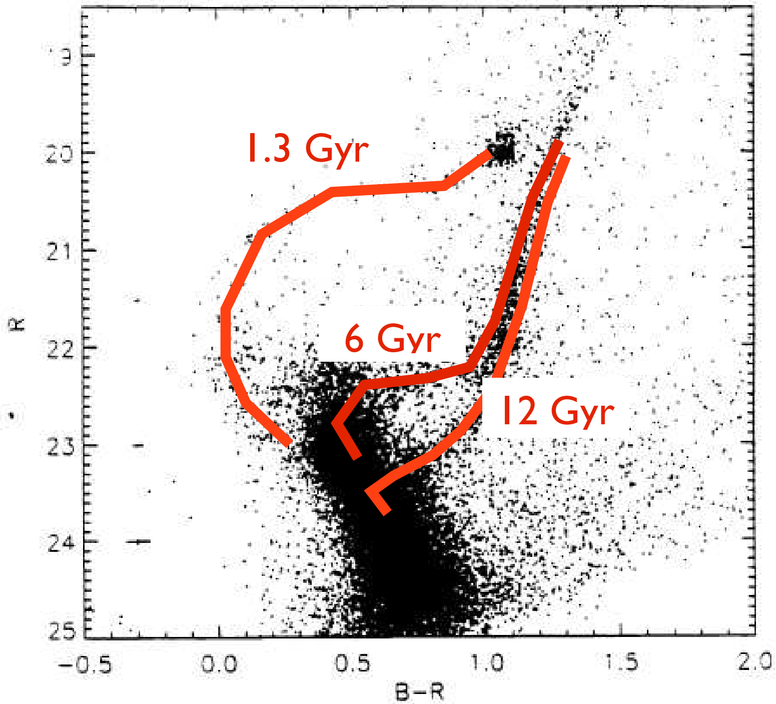


Figure 1. The CMD of the Carina dSph. The median internal photometric error at magnitudes $R = 21.5$, 23, and 24 are shown as on the left-hand side.

consider the expected tracks for stars forming at different ages:

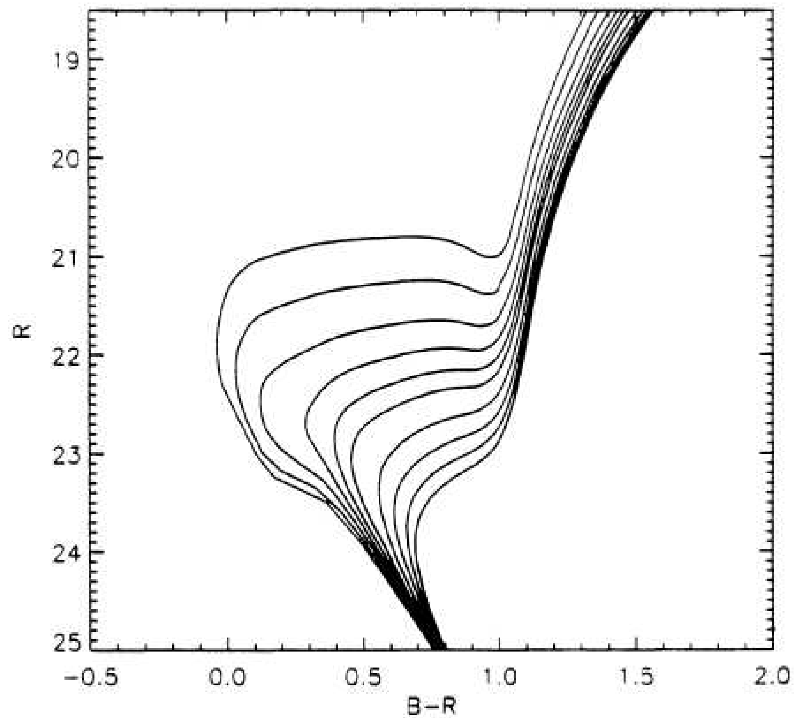


Figure 2. New theoretical isochrones from VandenBerg *et al.* (1996) for [Fe/H]= −1.84 and ages of 1.3, 2, 3, 4, 5, 6, 8, 10, 12, and 14 Gyr at the distance and reddening of the Carina dSph.

1) Resolved Stellar Population Analyses

What do you think is going on with this dwarf galaxy?

Fraction of
Stars Formed
as a function of
Time

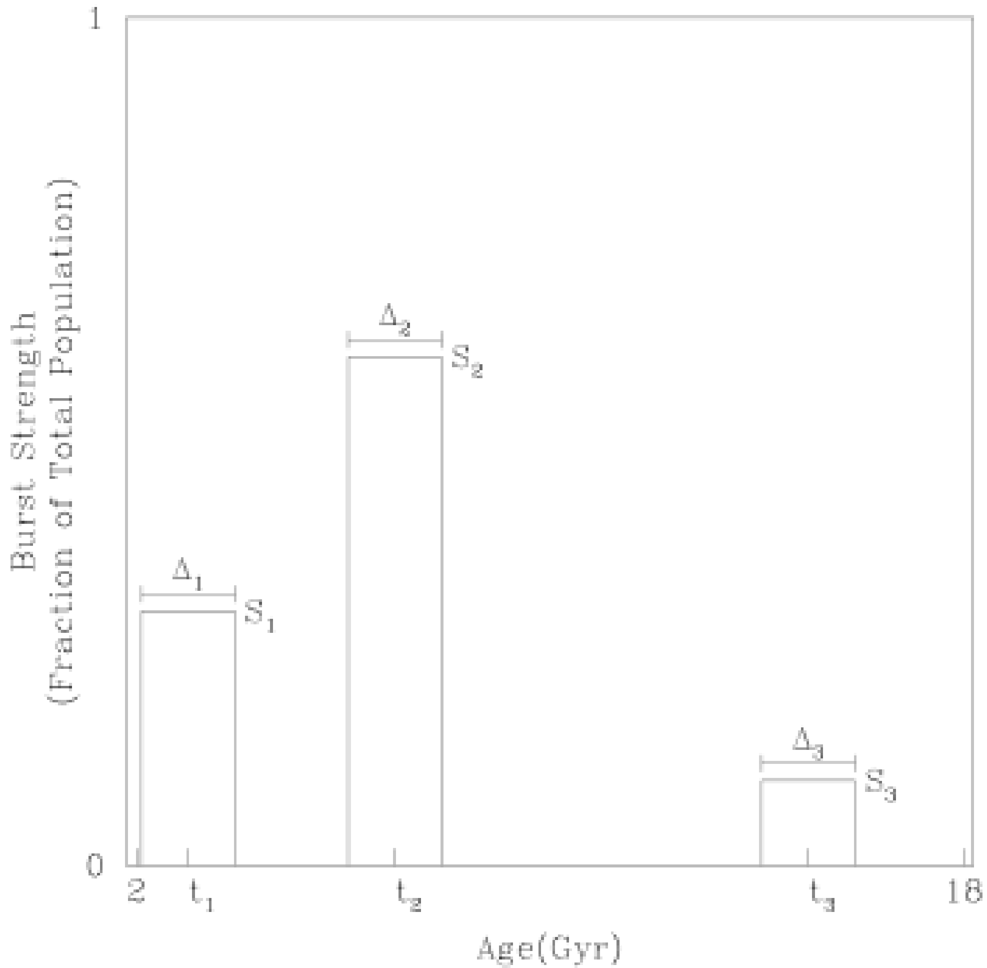
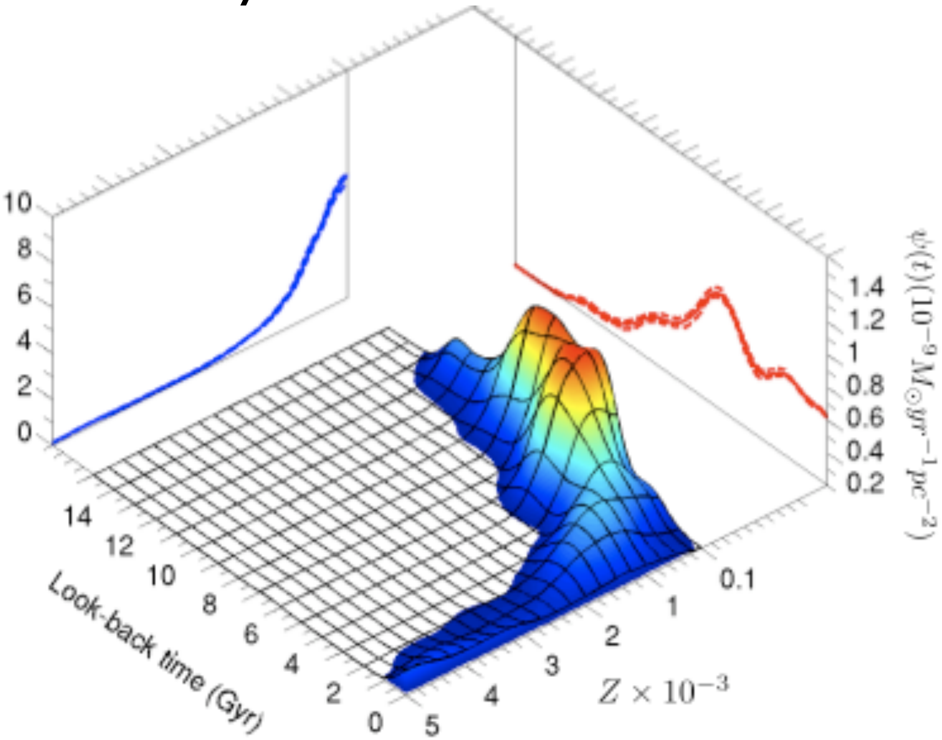


FIG. 13.—Parameterization of Carina's star formation history. We assumed three major episodes of star formation: t_i is the age of the episode in Gyr, Δ_i is the duration of the episode in Gyr, and S_i is the strength of the episode in a fraction of the total population. For reasons explained in the text, t_1, t_3, and Δ_3 are fixed at 3, 15, and 1 Gyr, respectively.

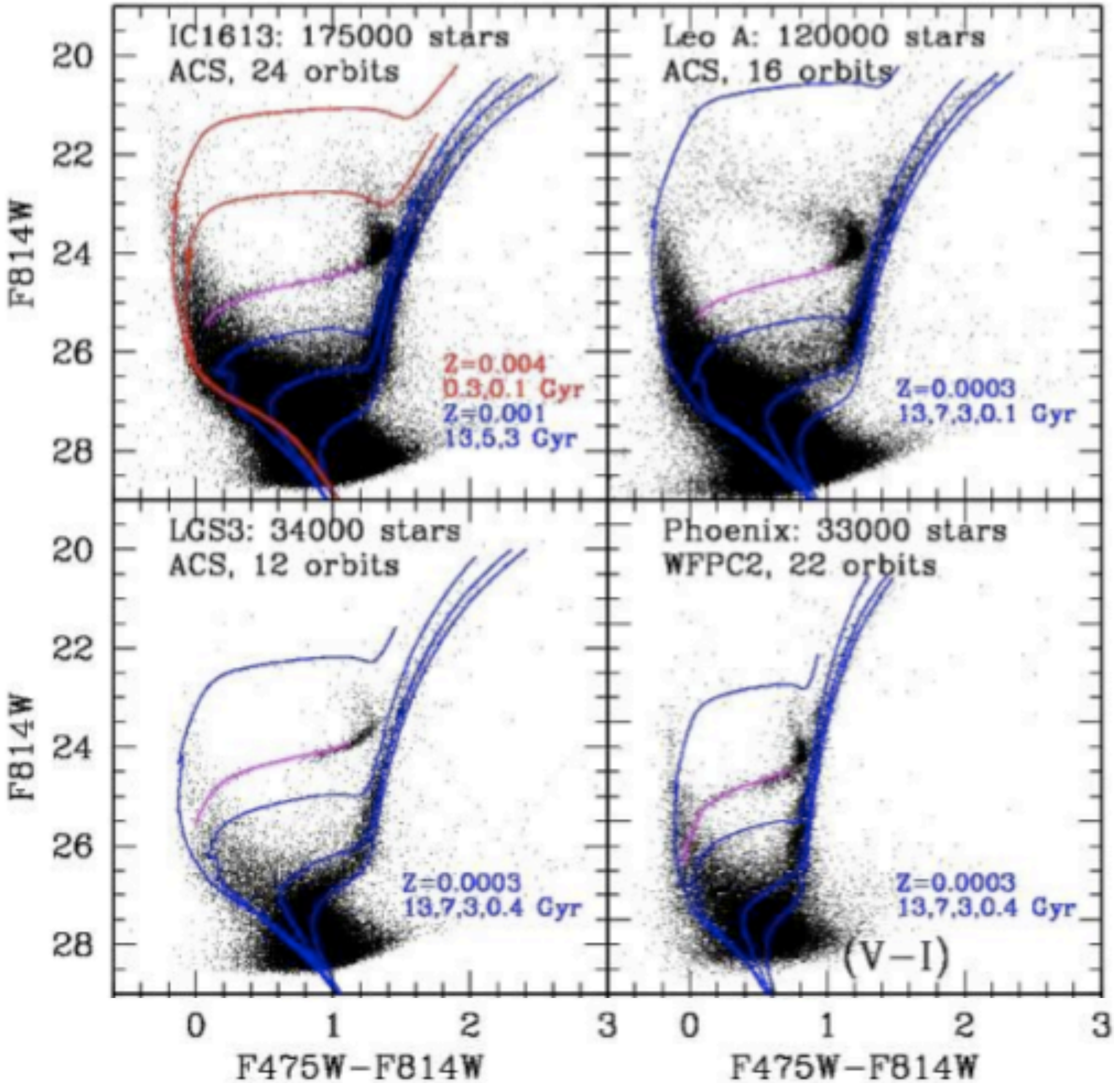
More complicated CMDs

General methods for fitting multiple populations in resolved populations.

Can aim to recover fraction of stars formed on a grid of age and metallicity.



LEO-A

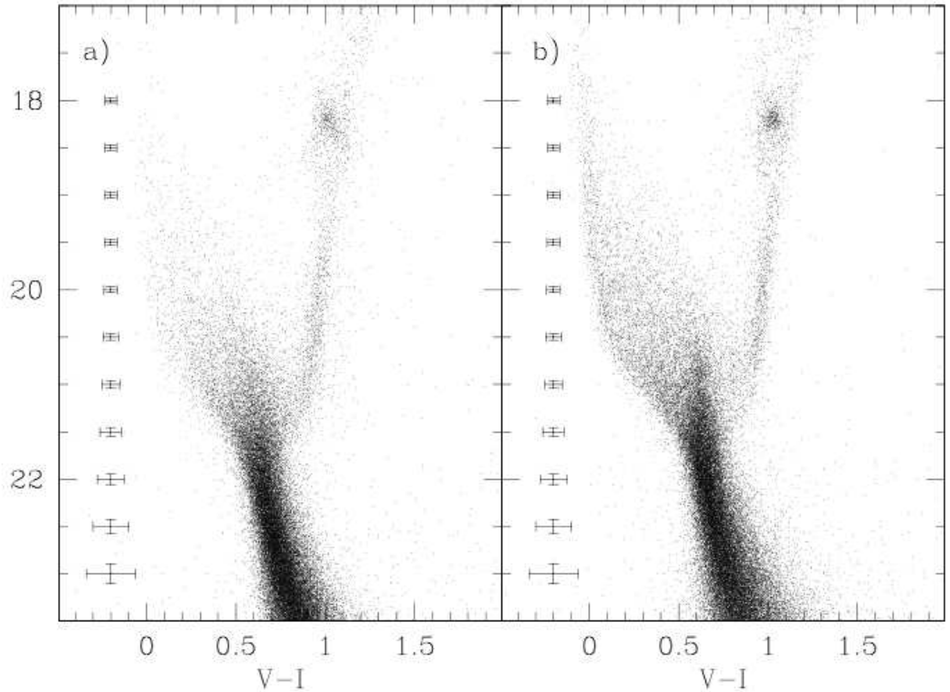


Credit: Russell Smith

Gallart et al. (2007)
Local Group Dwarf Galaxies

1) Resolved Stellar Population Analyses

One can also try the same sort of analysis on the Magellanic Clouds:



Evidence for lots of young stars, but also many older stars

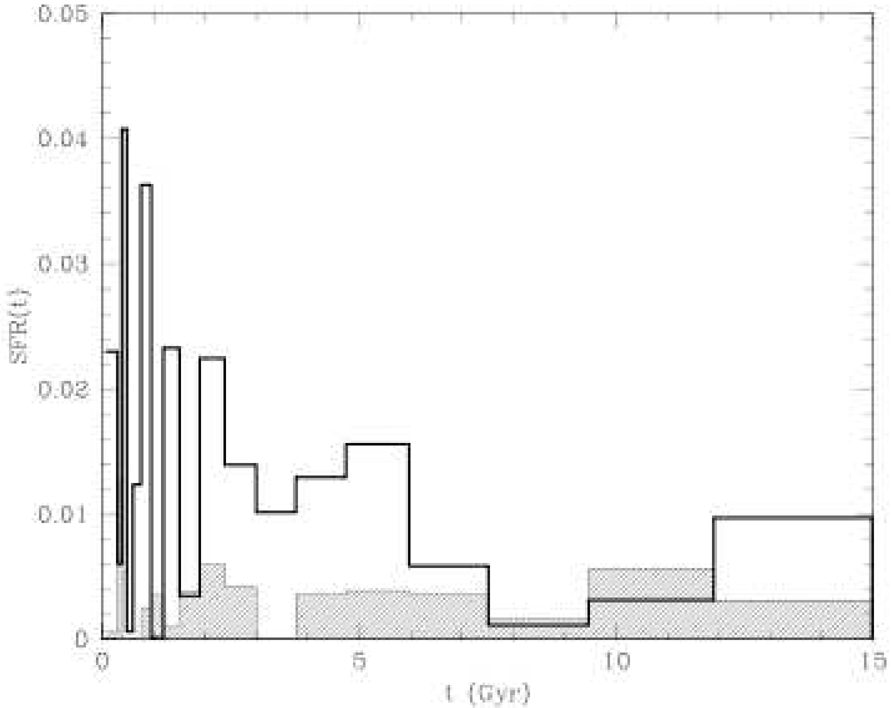


FIG. 5.—SFHs derived from the main-sequence LFs. *Thick line*: SFH bar; *shaded line*: 10 × SFH Disk 1. Units are $M_\odot$ yr^{-1} square degrees; the errors in each age bin are ≈ ±15%.

1) Resolved Stellar Population Analyses

Examining galaxies at even greater distances, one must use the red giant branch stars...

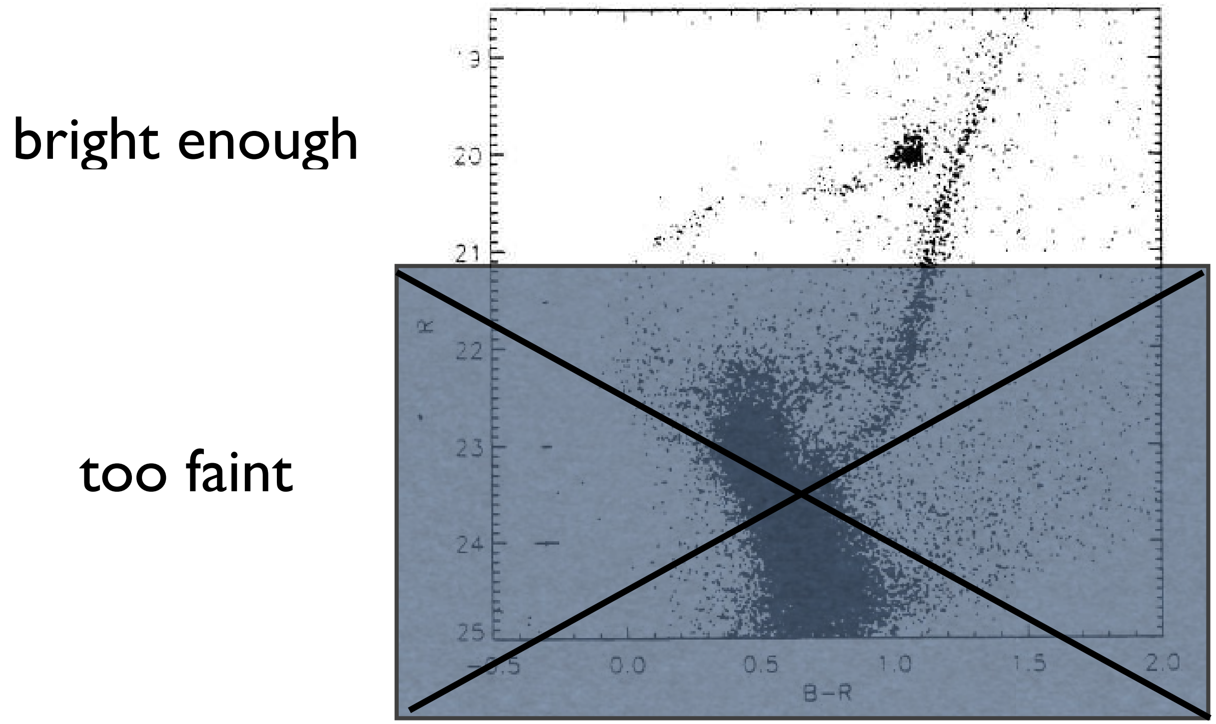


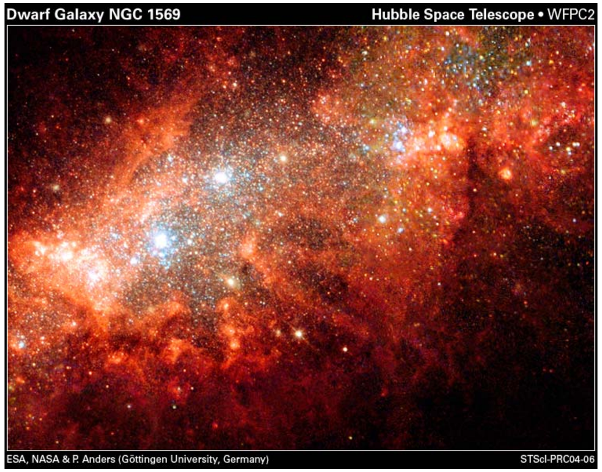
Figure 1. The CMD of the Carina dSph. The median internal photometric error at magnitudes $R = 21.5$, 23, and 24 are shown as on the left-hand side.

However, it is almost impossible to make use of individual stars in galaxies greater than 10-15 Mpc away...

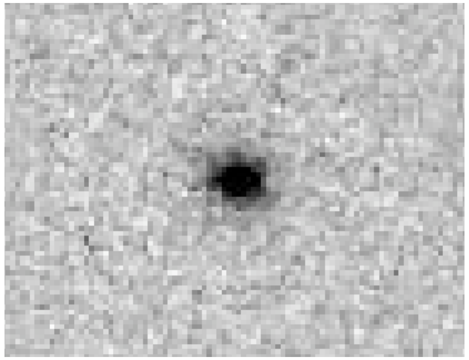
2) Integrated Stellar Population Analyses

If one cannot resolve out the individual stars, one must look at the total spectrum coming from all stars in a galaxy.

The Problem



"What you'd like to get"



"What you get"

(George Hau)

WHAT CAN WE MEASURE?

Broad-band colours (B-V, J-K, etc).

Surface brightness fluctuations (sometimes)

Spectroscopic features (absorption lines)

Credit: Russell Smith

2) Integrated Stellar Population Analyses

One attempts to model the observed spectrum using the following inputs:

1) Stellar Initial Mass Function ϕ(m): Defines the Fraction of Forming Stars as a function of the stellar mass m. Appears to be well described by the Salpeter power-law function $m^{-2.35}$ at intermediate to high masses.

At low-to-intermediate stellar masses, the number of stars formed are lower that one would predict based on the Salpeter power law.

One IMF that takes this into account is the Chabrier IMF:

$$m\phi(m) = exp\left[-\frac{(\log(m) - log(m_c))^2}{2\sigma^2}\right] \text{ for m } \leq 1\ M_\odot$$

$$= m^{-1.3} \text{for m } \geq 1 M_\odot$$

with $m_c = 0.08 M_\odot$ and $\sigma = 0.69$

Tracks & Isochrones

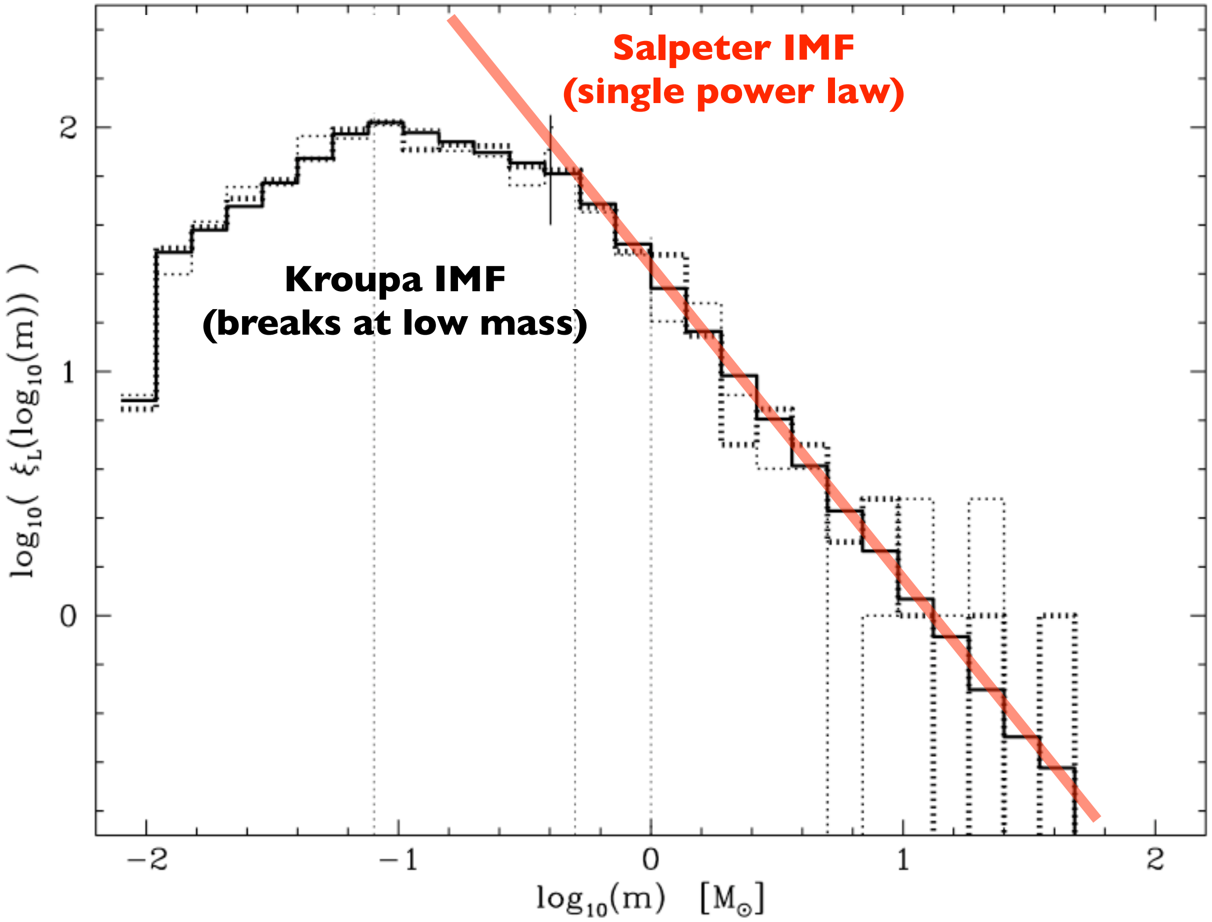
INITIAL MASS FUNCTION

How many stars formed per unit mass?

Determines number of stars expected at each point along isochrone.

Constrained from detailed observations of star-forming regions in the MW.

Applied to wide range of environments beyond MW!



Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

2) Metallicity and abundance ratios of the stars which are forming:

Metallicity can have a significant impact on the light emitted by stars, making them appear redder. It can also slow down the evolution of stars somewhat.

Aside: What Exactly is Metallicity?

"METAL" CONTENT OF THE GAS CLOUD FROM WHICH THE STARS FORMED

Usually assume the surface composition reflects the original composition.

REMEMBER: ASTRONOMERS THINK CARBON IS A METAL....

Stellar modellers express chemical mixture of material as mass fractions

X or H = mass fraction of H , Y = mass fraction of He

Z = mass fraction of everything else = "metals"

EMPIRICAL NOTATION

For measurements of metallicity in stellar atmospheres, we usually express abundances in terms of number density (not mass fractions). Total metallicity is often expressed as $[Z/H] = \log(N_Z/N_H) - \log(N_Z/N_H)_{sun}$. Then:

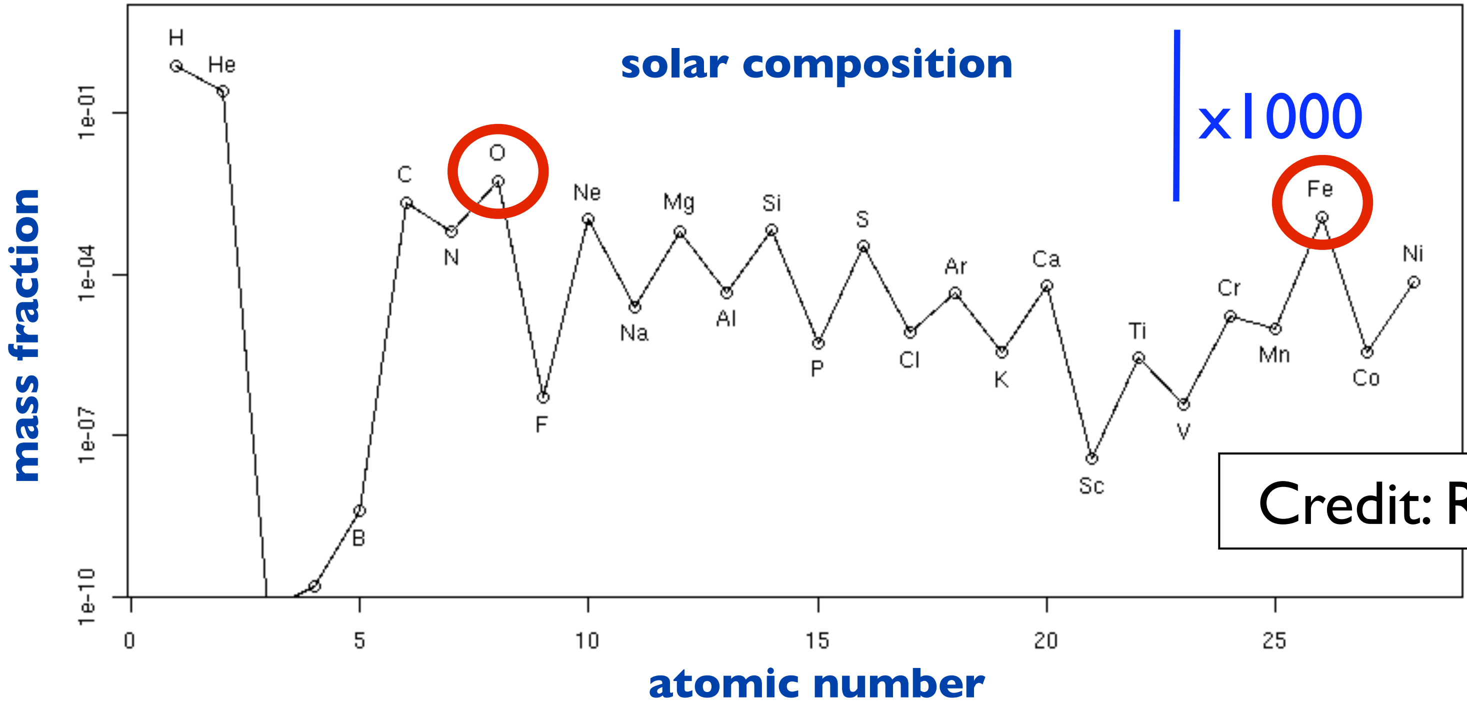
[Z/H] = 0 is "solar metallicity",

[Z/H]=+0.3 is "twice-solar",

[Z/H] = -1 is "one tenth solar", etc.

Credit: Russell Smith

More on Metallicity



Credit: Russell Smith

BUT WHAT IS COMPOSITION OF "Z" ?

Note that O is the most important element for stellar evolution: it is abundant and a big contributor to the opacities.

But unfortunately it is very hard actually to measure O from stellar spectra!

Much easier to measure Fe which has lots of absorption lines in the optical.

So we often talk about [Fe/H] instead. These are equivalent if [O/Fe] is solar, i.e. mixture between metals always the same. (We'll come back to this!)

Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

3) Detailed Stellar Evolution Models:

While most phases of stellar evolution seem to be well understood, other rarer phases of stellar evolution like the horizontal branch evolution or the asymptotic giant branch evolution are less well understood. This can make the predictions of the models uncertain.

Tracks & Isochrones

TRACKS

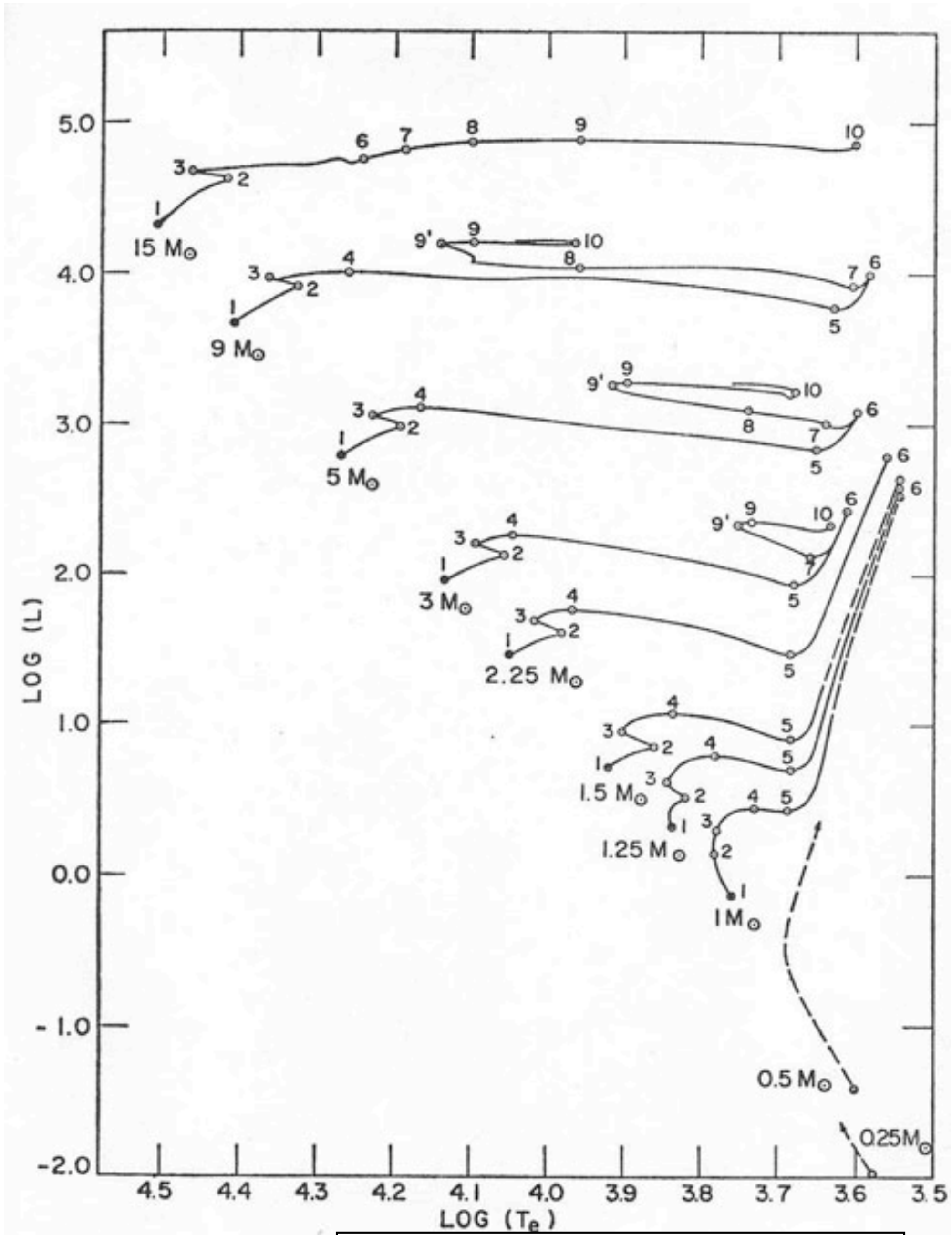
“Tracks” are trajectories of individual stars in the HRD.

Stellar evolutionary tracks are tables describing the evolving properties:

- luminosity,
- temperature,
- evolving mass, etc

as a function of initial mass (and initial chemical composition).

In detail, the tracks are computed from stellar evolution models (Padova, Geneva, BaSTI etc).



Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

4) Spectra of Stars at a given temperature, metallicity, surface gravity.

One can calculate the spectra of stars theoretically from stellar atmosphere modelling. However, the predictions from theory often differ from observations -- suggesting that one may want to use real spectra of actual stars. The challenge with using real

Empirical Spectral Libraries

OBSERVED SPECTRA OF STARS

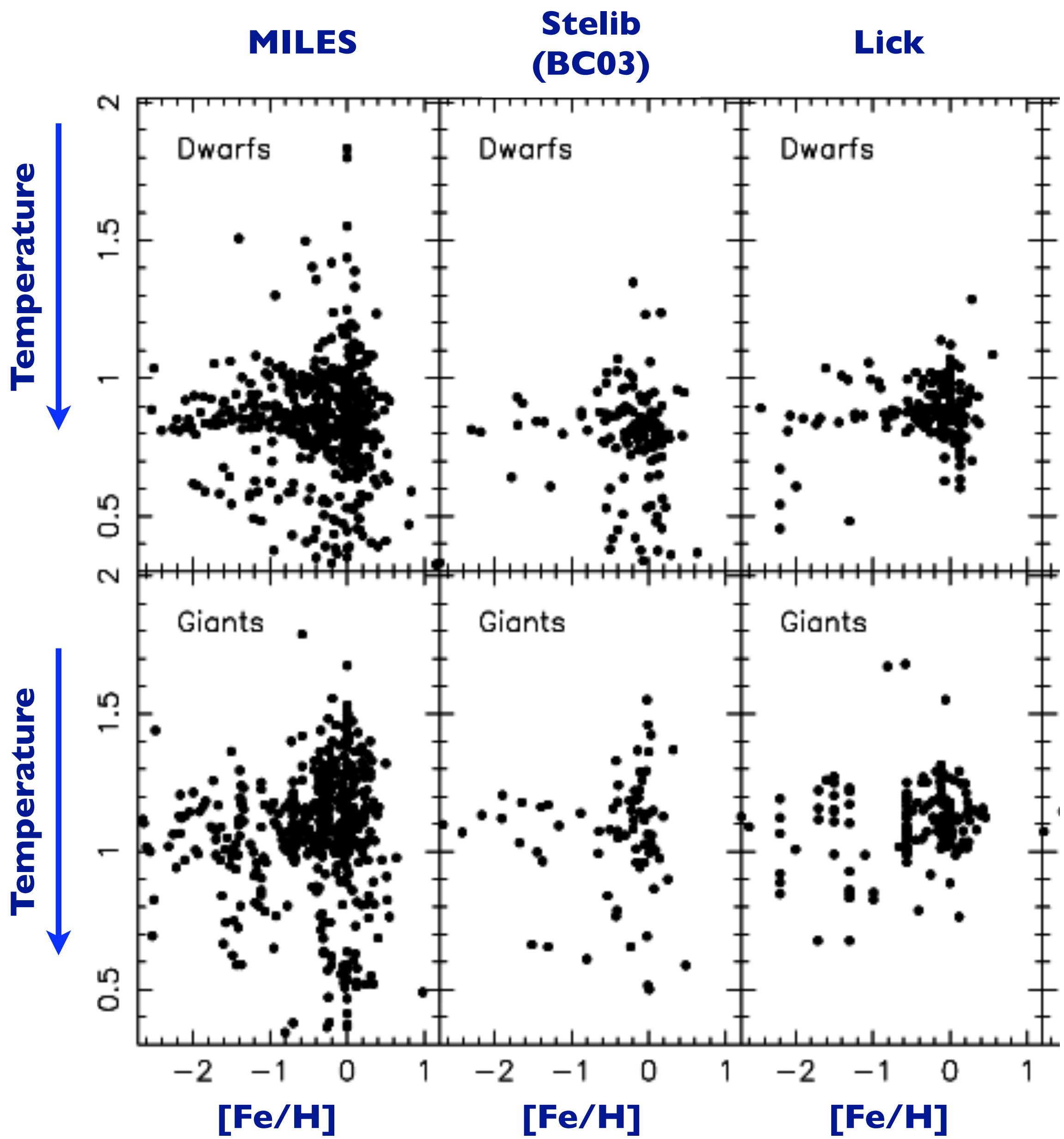
Desirable to cover large range in T_{eff}, log g (="gravity" i.e. dwarf vs giant) and Fe/H.

And to know the atmospheric parameters of the stars (difficult for the coolest stars.)

PROBLEMS

All the stars in empirical libraries are in our galaxy, which limits parameter coverage.

Valid application of the models implicitly restricted to systems with stars "like" those in our galaxy.



Sanchez-Blazquez et al. (2006)

Credit: Russell Smith

Theoretical Spectral Libraries

THEORETICAL LIBRARIES

Based on stellar atmosphere models, e.g. ATLAS9 (Castelli & Kurucz 2003) MARCS (Gustafsson et al. 2003)

ADVANTAGES

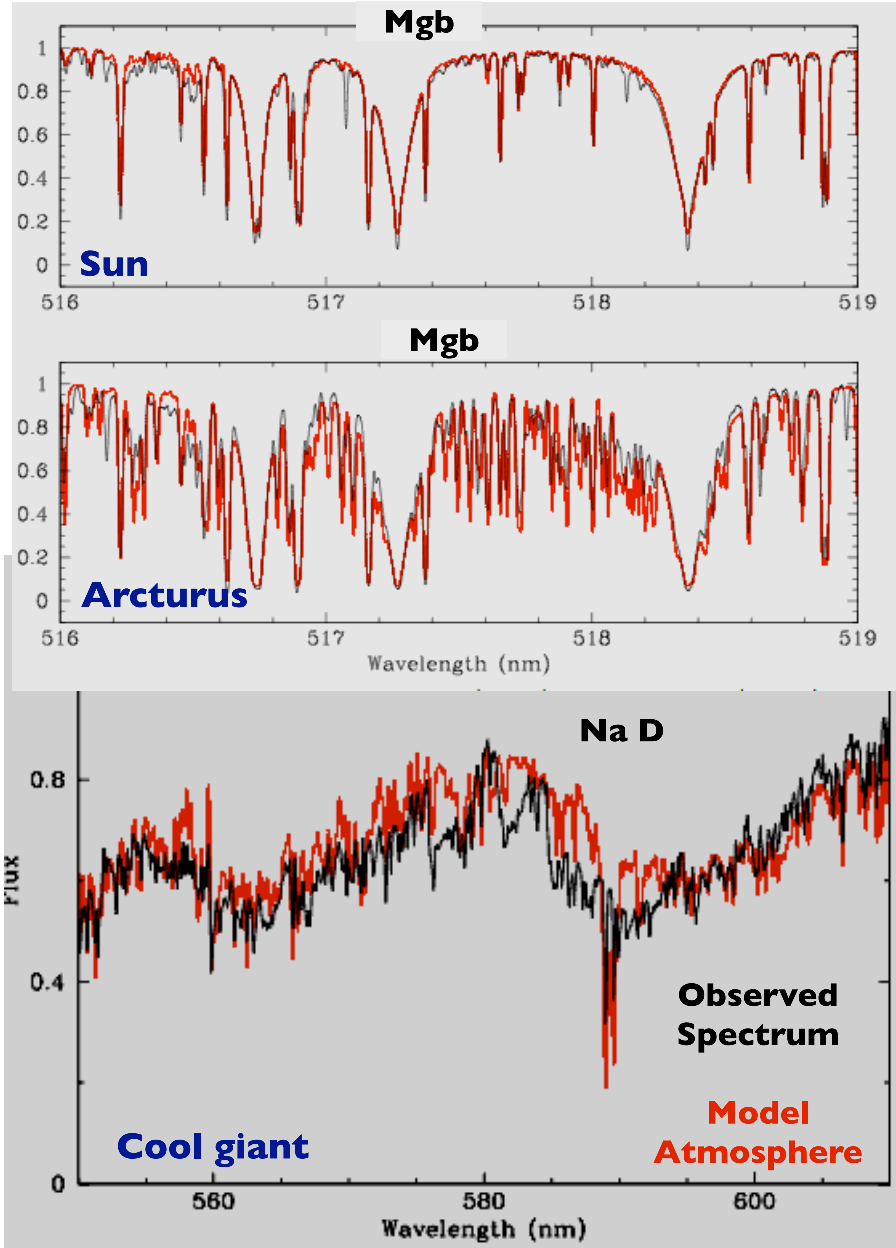
Much more flexible than empirical libraries: In principle can obtain spectra for any value of Teff, log g and Fe/H (and any chemical mixture).

CHALLENGES

Complex atmosphere physics (especially for hottest and coolest stars) may not be adequately modelled.

Empirical atomic and (especially) molecular line-lists may not be complete enough.

Include QM-predicted lines not verified in lab? These can be badly wrong in detail, but needed for accurate colours (Coelho et al.)



Coelho et al. (2007)

Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

5) Star Formation History of Galaxies

This is the typical input that people assume changes from galaxy to galaxy.

The simplest model is single burst stellar population models... where one assumes all the stars in a galaxy formed at a single point in the past.

Such models can work well for describing the stellar populations of elliptical galaxies and globular clusters, where most of the stars were first formed long ago in the past.

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

5) Star Formation History of Galaxies

Another simple model is to assume that stars in a galaxy formed at a fixed constant rate with time.

Such models can work well for describing the stellar populations of late spiral and irregular galaxies.

One can try to parameterize all star formation histories between a constant star formation model and a fixed burst in the past adopting an exponentially declining star formation rate:

$$e^{-t/\tau}$$

where τ is the time scale on which the star-formation rate of some galaxy declines with time.

Note that $\tau = 0$ corresponds to a simple stellar population (all stars formed at some time in the past)
while $\tau = \infty$ corresponds to a constant star formation model.

Interpolating between $\tau = 0$ and $\tau = \infty$ gives SFR history for Hubble sequence galaxies in between ellipticals and irregular galaxies

2) Integrated Stellar Population Analyses

From the models, we can calculate the integrated spectrum of a galaxy, if all the stars formed at certain times in the past.

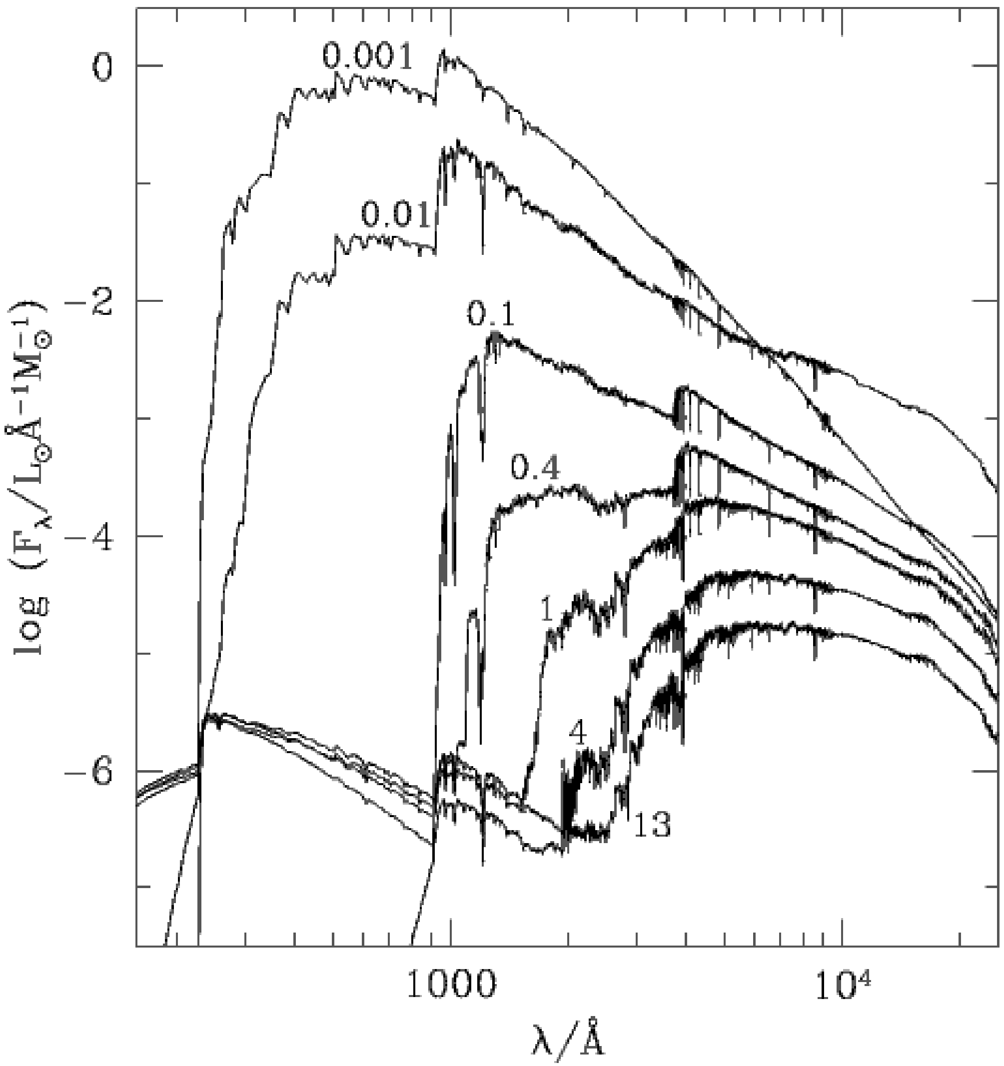


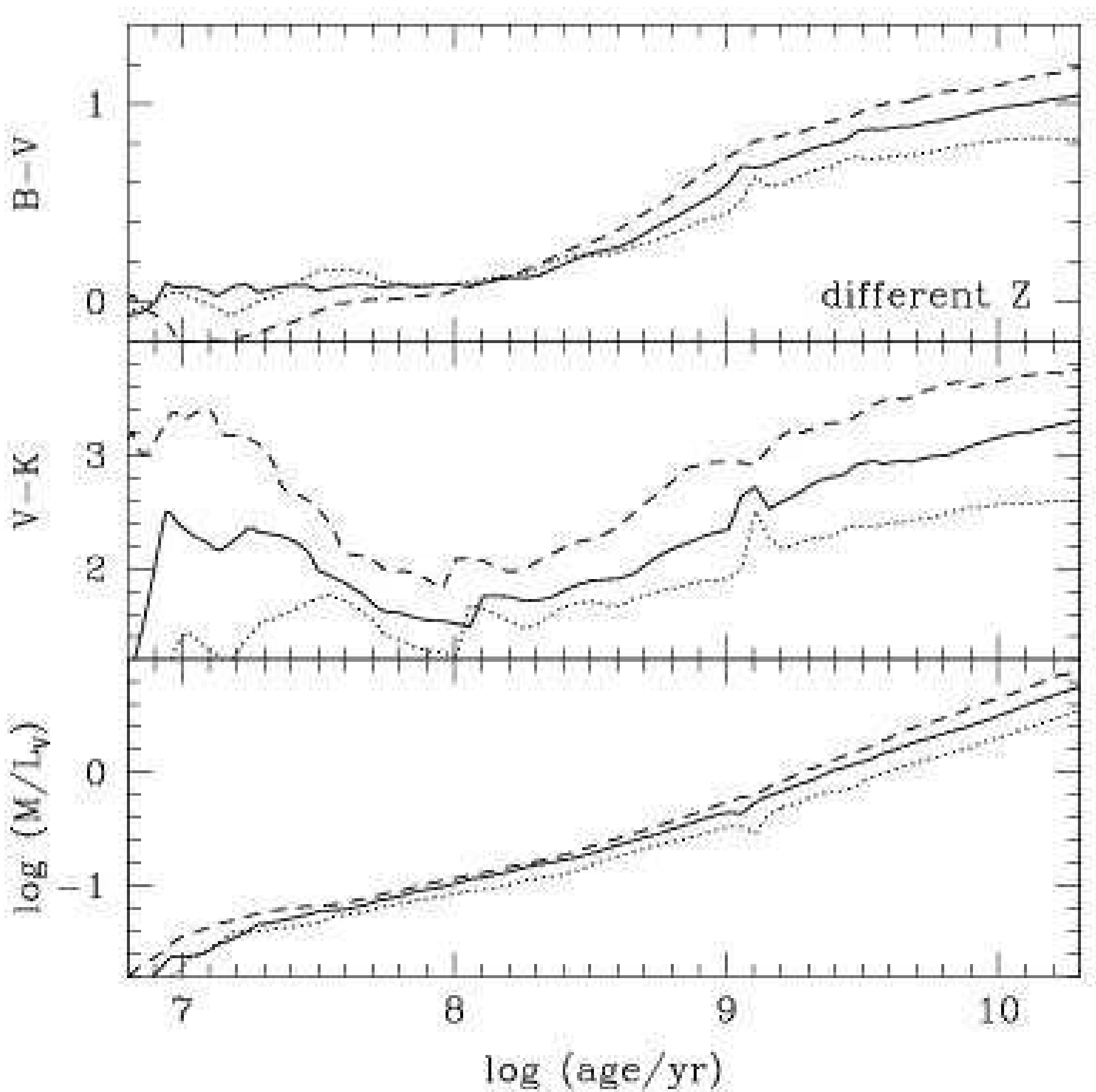
Figure 9. Spectral evolution of the standard SSP model of Section 3 for the solar metallicity. The STELIB/BaSeL 3.1 spectra have been extended blueward of 3200 Å and redward of 9500 Å using the Pickles medium-resolution library. Ages are indicated next to the spectra (in Gyr).

2) Integrated Stellar Population Analyses

How would we expect the colors or mass-to-light ratios to change with time for a simple stellar population?

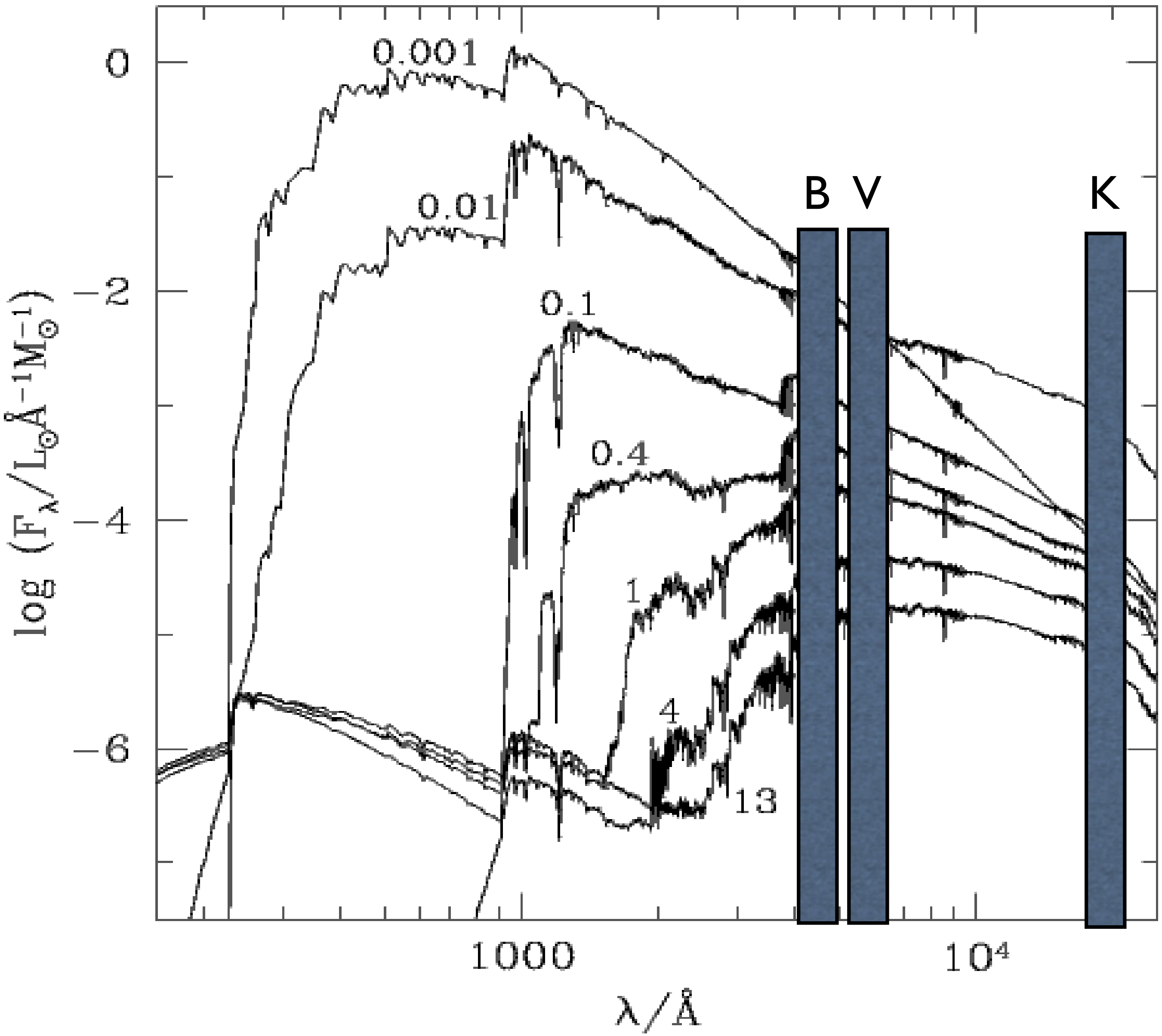
Here are the result for a few different metallicities:

0.2 Z_{solar}
1 Z_{solar}
2.5 Z_{solar}



2) Integrated Stellar Population Analyses

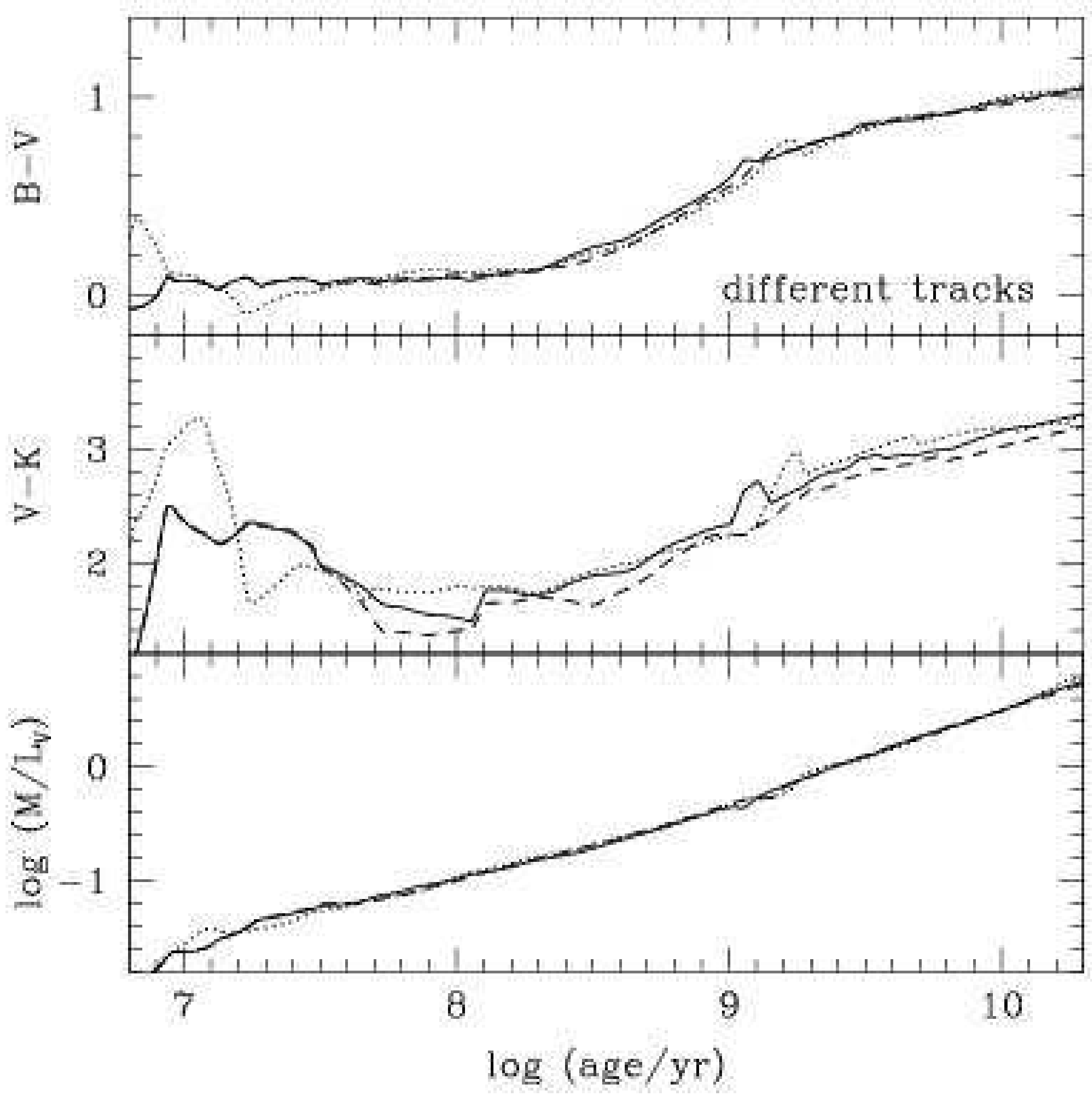
BACKGROUND INFORMATION (for previous slide): At which wavelengths do the B, V, and K bands refer to?



2) Integrated Stellar Population Analyses

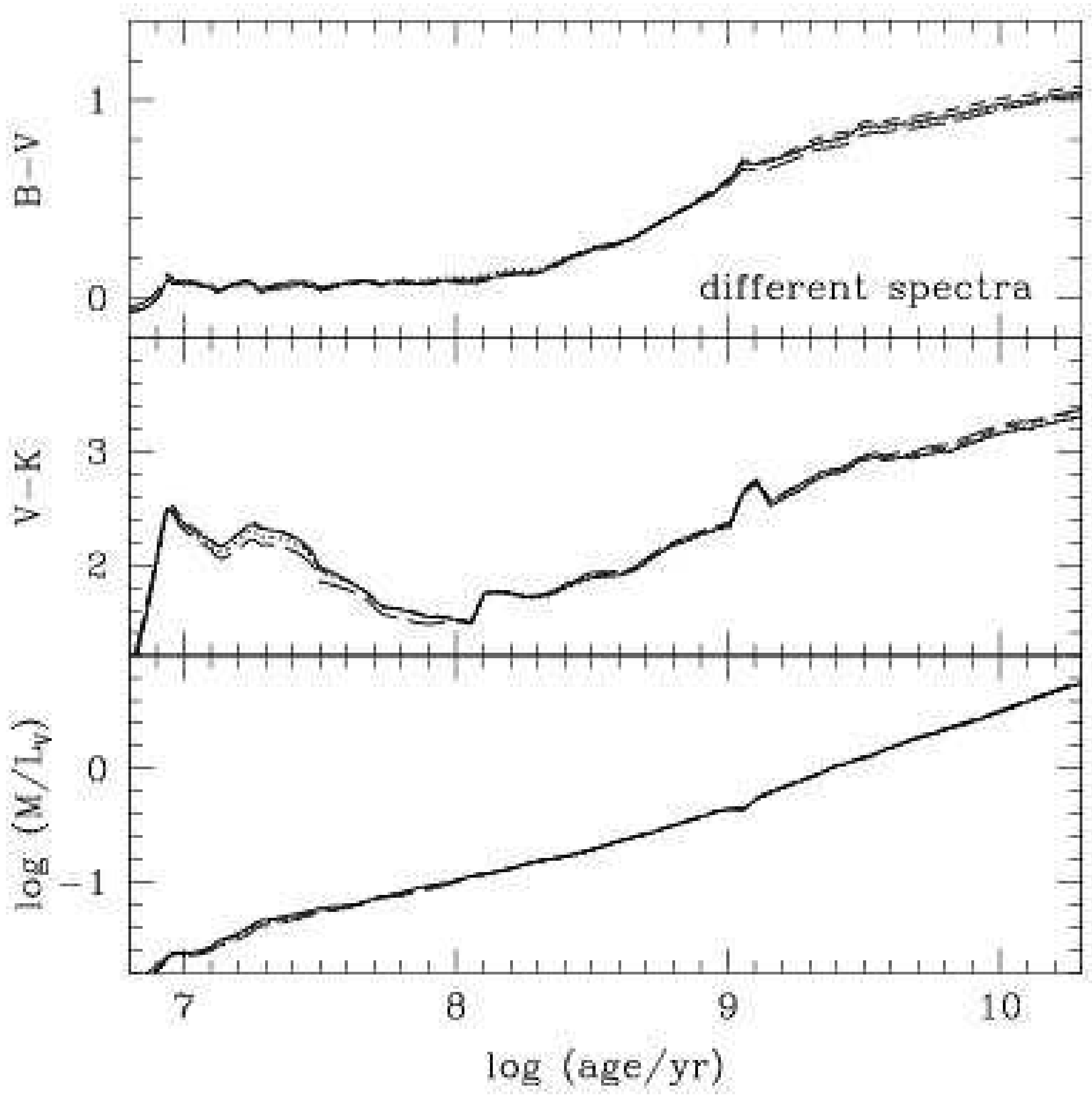
How do these results depend on the stellar tracks calculated for a simple stellar population?

Recall that the stellar tracks tell us how the luminosity, temperature, etc., of a star changes with time



2) Integrated Stellar Population Analyses

How do these results depend on the theoretical or empirical spectra one puts together with the stellar tracks?



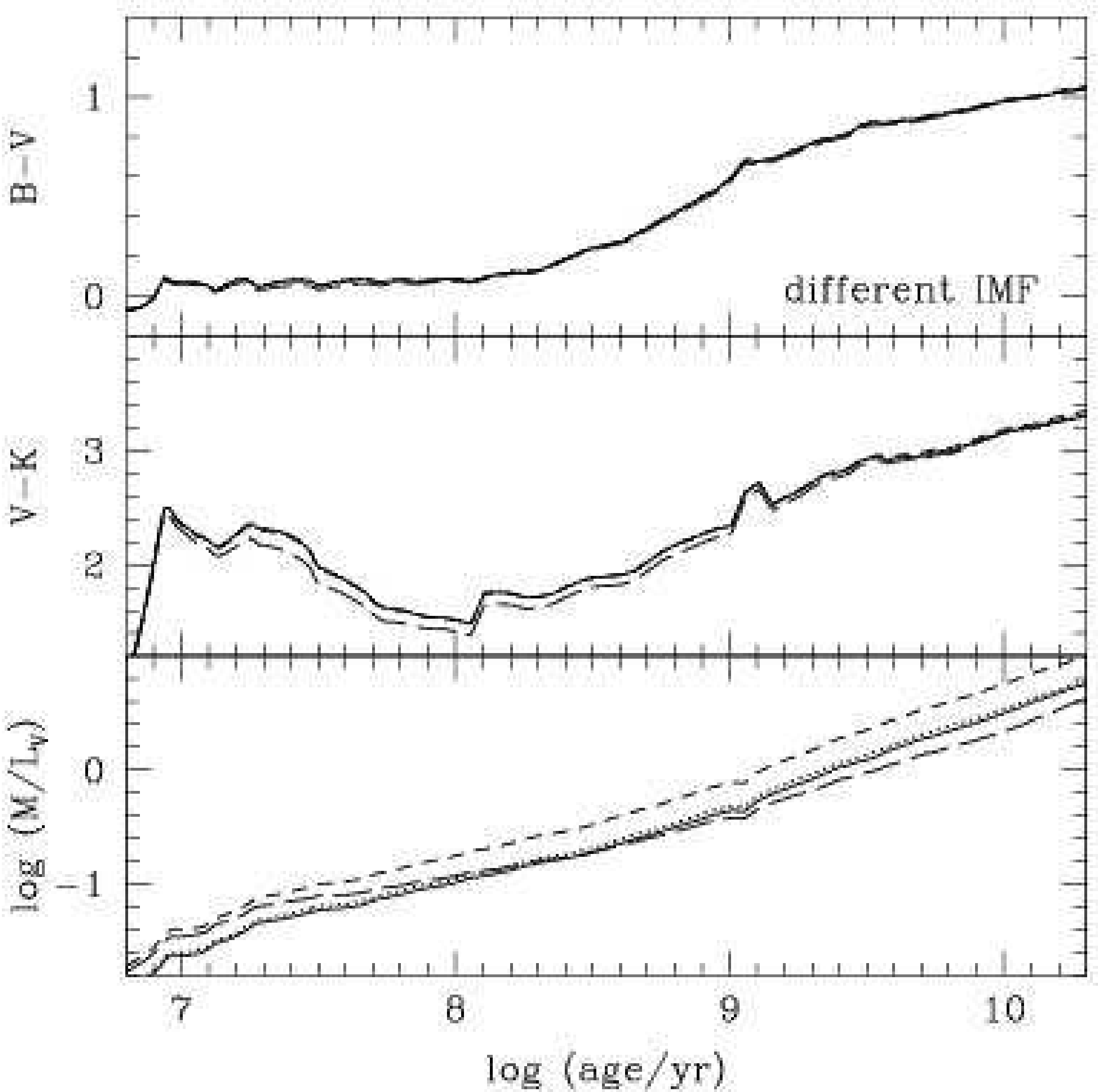
2) Integrated Stellar Population Analyses

Results shown for different IMFs

Salpeter,

Kroupa,
Scalo,
Chabrier

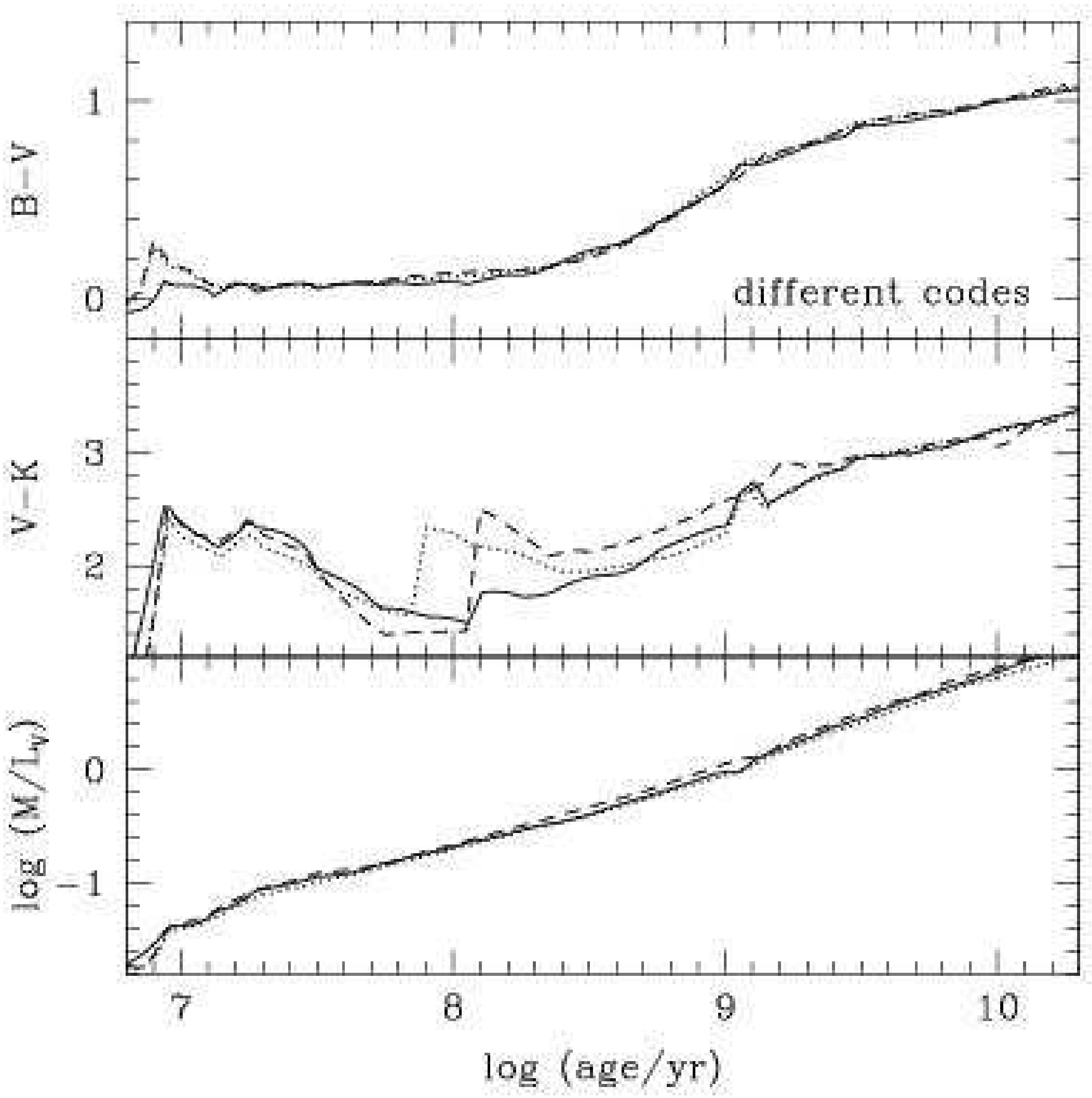
The lower three IMFs all have fewer low mass stars than Salpeter



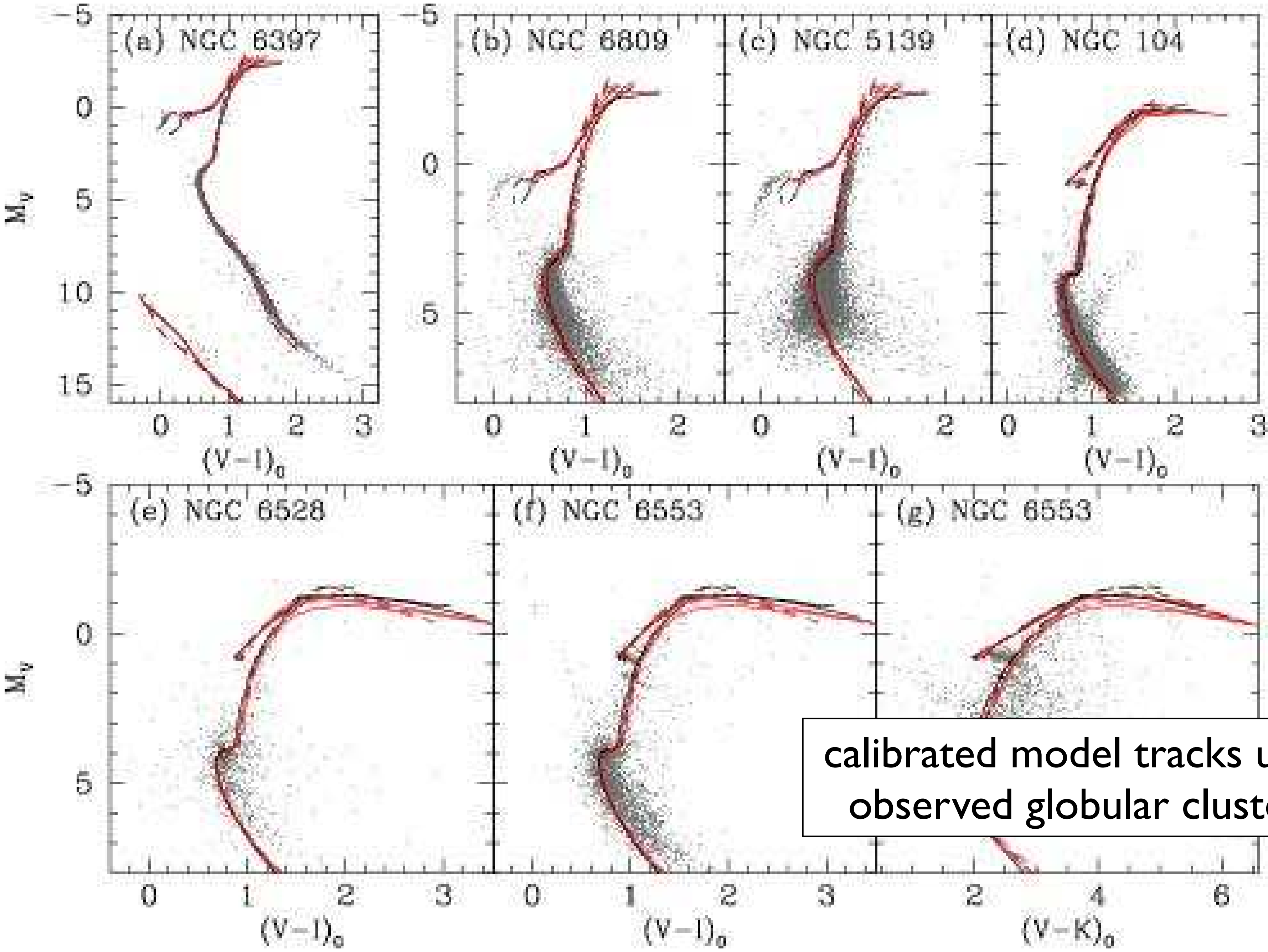
2) Integrated Stellar Population Analyses

How do the results depend on the code which puts together the results?

Differences generally occur depending on how one treats the asymptotic giant branch stars.



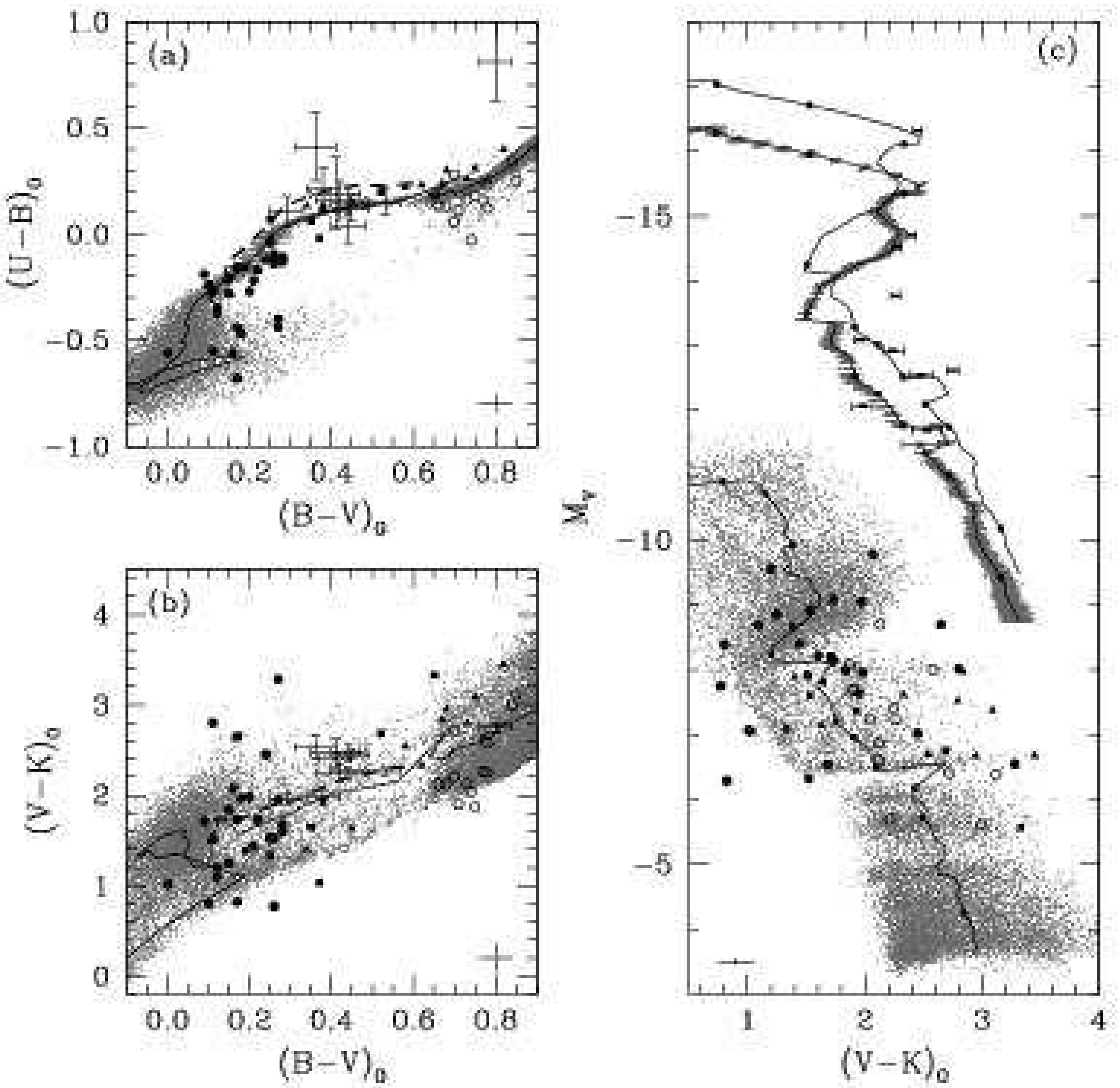
2) Integrated Stellar Population Analyses



calibrated model tracks using observed globular clusters

2) Integrated Stellar Population Analyses

calibrate using colors from observed star clusters



Age-Metallicity Degeneracy

A wide variety of different ages and metallicities for a stellar population produce approximately the same integrated spectrum:

It can therefore be quite challenging to determine both the age and metallicity of a galaxy uniquely.

There are subtle differences between these spectra.

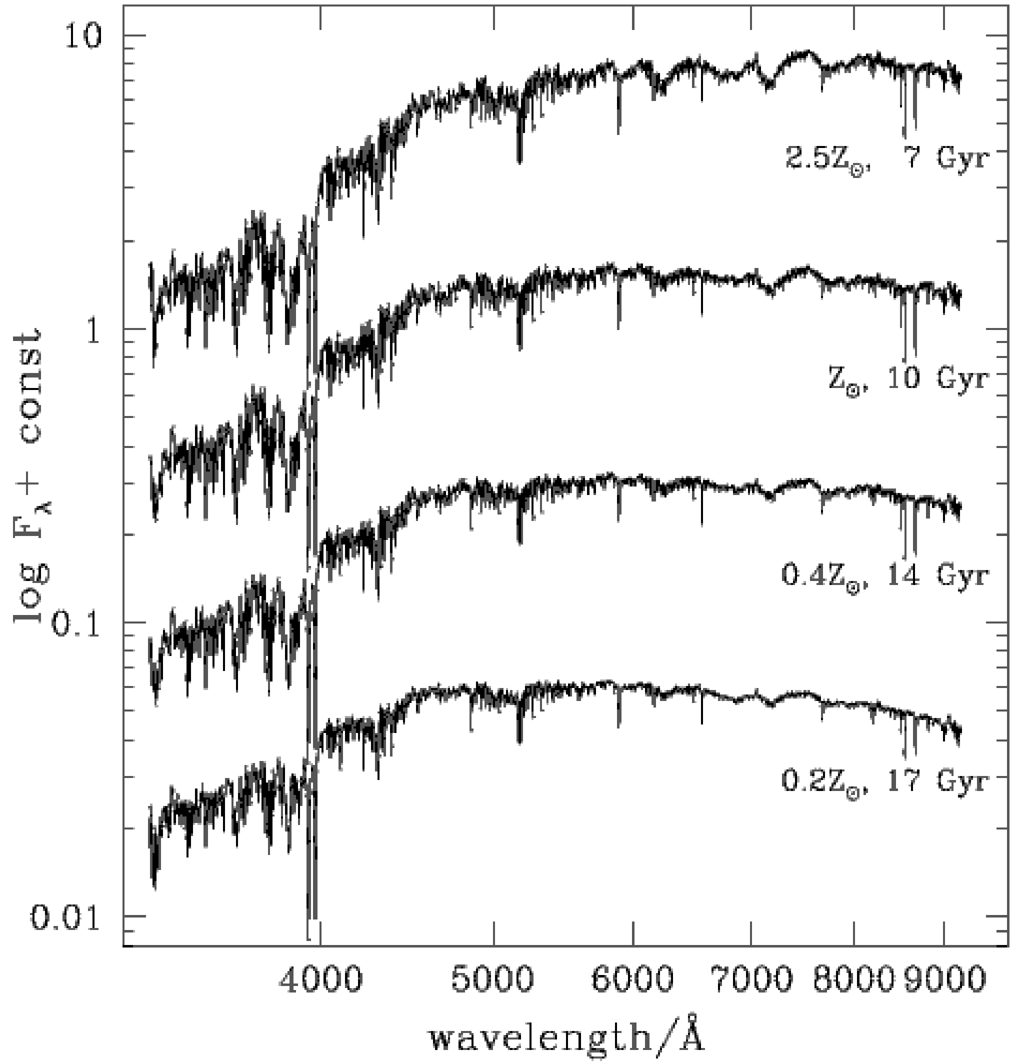


Figure 10. Spectra of the standard SSP model of Section 3 at different ages for different metallicities, as indicated. The prominent metallic features show a clear strengthening from the most metal-poor to the most metal-rich models, even though the shape of the spectral continuum is roughly similar in all models.

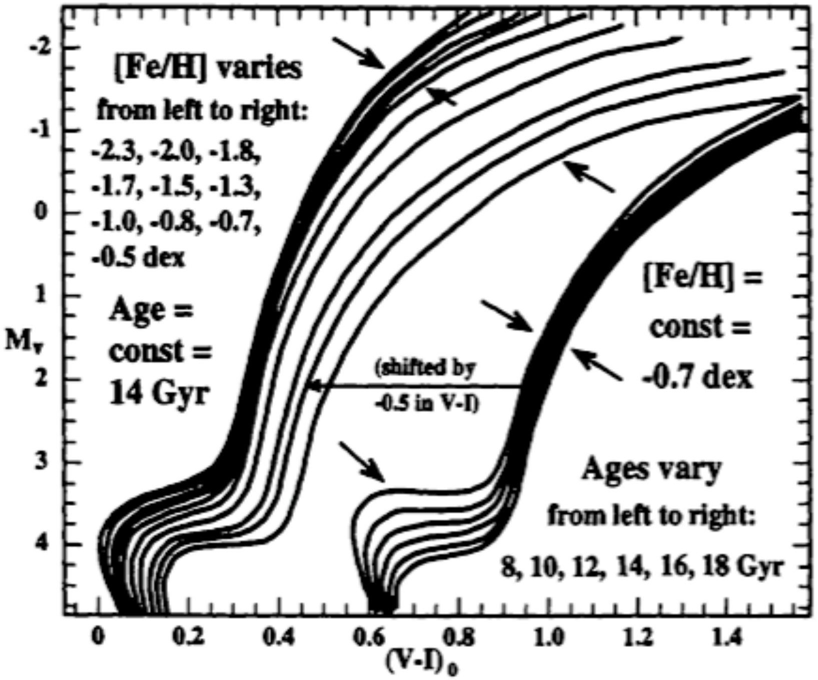
The Age-Metallicity Degeneracy

THE DREADED DEGENERACY

AGE -- Increasing age reddens the population by adding more luminosity to the RGB, removing hot stars from the MS.

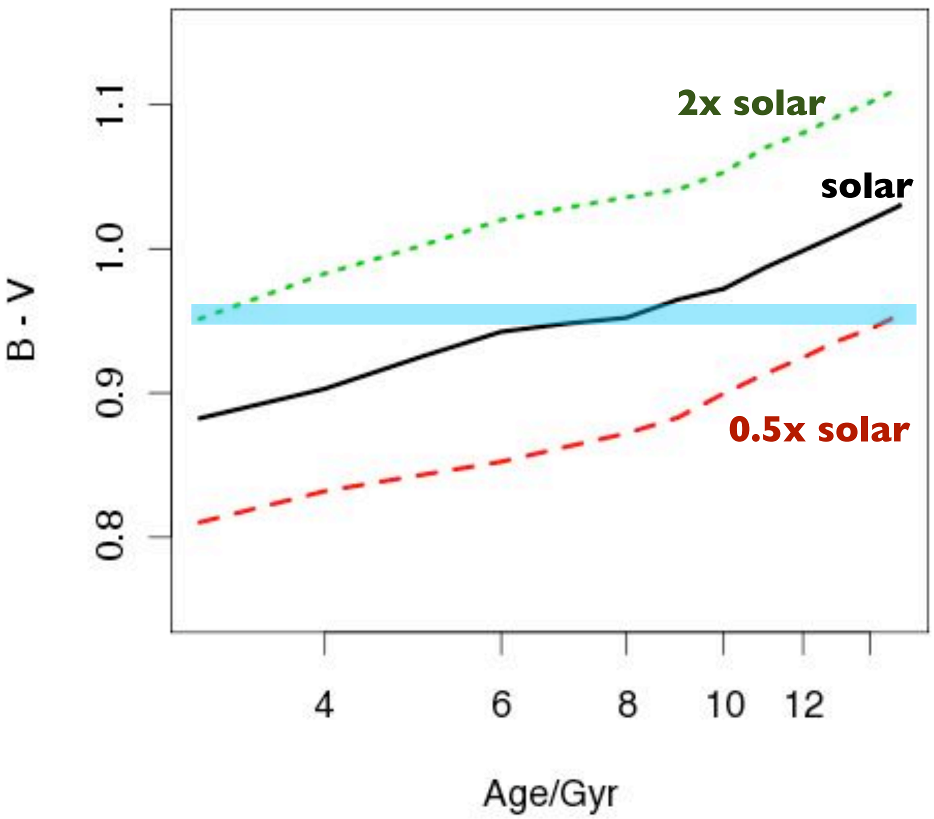
METALLICITY -- Increasing Metallicity reddens the population by changing the high-temperature opacities.

(Metallicity also reddens the population through increased line blanketing in cool phases.)



Credit: Russell Smith

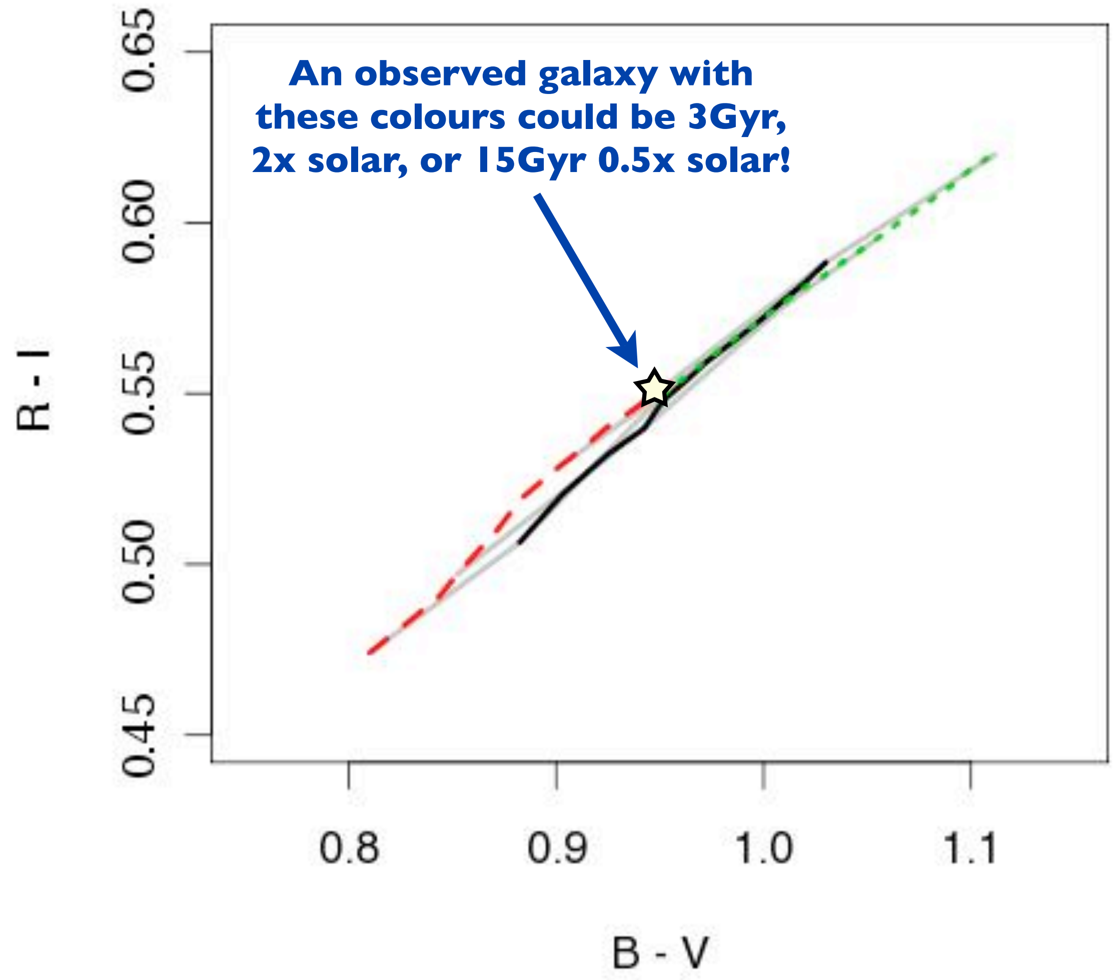
Age-Metallicity Degeneracy



Because both age and metallicity cause the population to redden, a single colour is not enough to disentangle the parameters.

Credit: Russell Smith

Age-Metallicity Degeneracy



A pair of (optical) colours is no help either!

Credit: Russell Smith

Disentangling age and metallicity

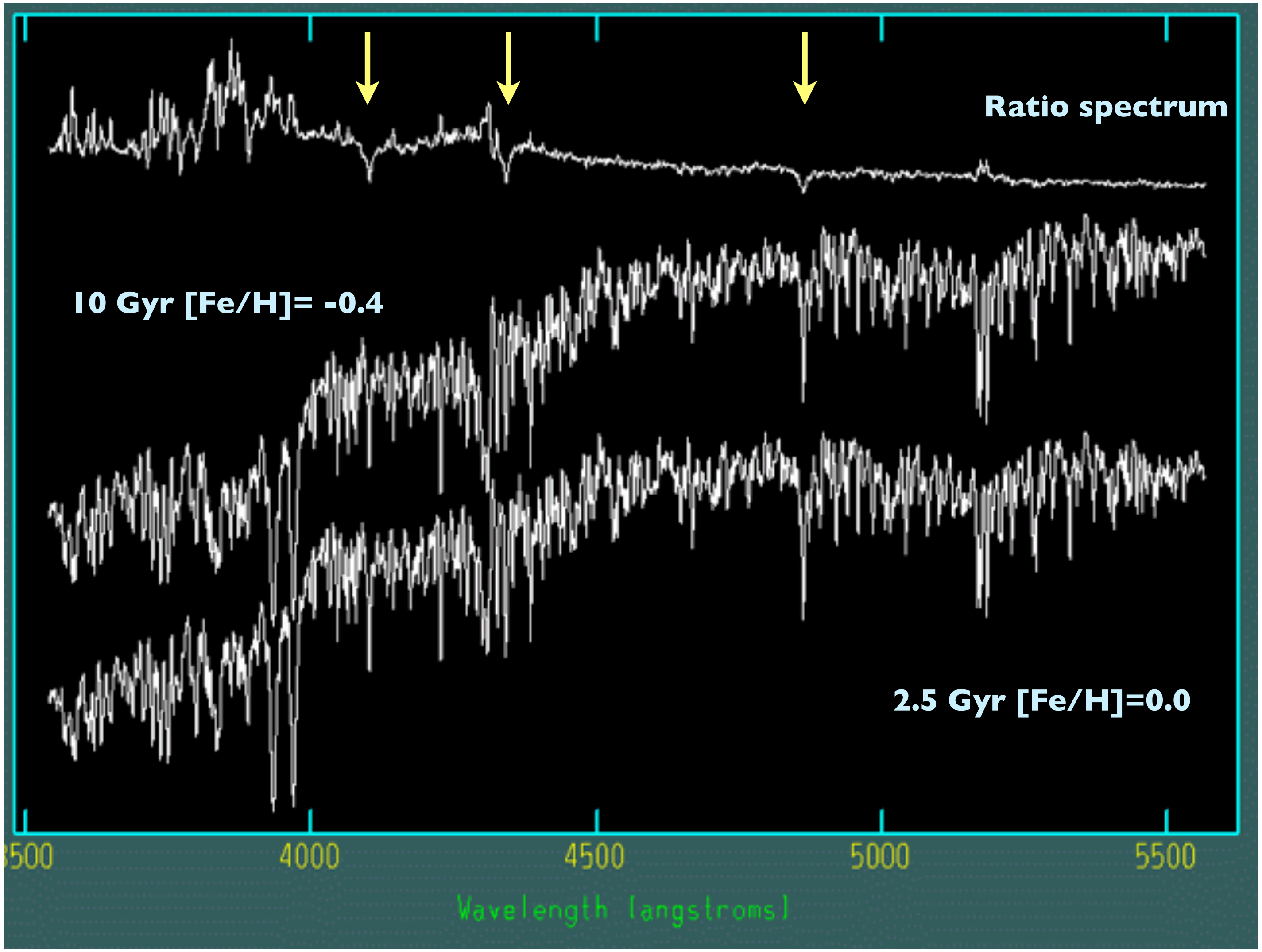
BREAKING THE DEGENERACY WITH SPECTRA

Two spectra with age and metallicity chosen to produce same broad-band colours.

Similar spectra, but differences in detail at the Balmer lines. Also differences at λ<4000Å.

We can exploit this localized spectral information to beat the age-metallicity degeneracy.

But how?



Vazdekis et al. (2007) models from MILES library

Credit: Russell Smith