Galaxies: Structure, Dynamics, and Evolution

Dark Matter Halos and Large Scale Structure
Layout of the Course

Feb 5:  Review: Galaxies and Cosmology
Feb 12:  Review: Disk Galaxies and Galaxy Formation Basics
Feb 19:  Disk Galaxies (I)
Feb 26:  Disk Galaxies (II)
Mar 5:  Disk Galaxies (III) / Review: Vlasov Equations
Mar 12:  Review: Solving Vlasov Equations
Mar 19:  Elliptical Galaxies (I)
Mar 26:  Elliptical Galaxies (II)
Apr 2:  (No Class)
Apr 9:  Dark Matter Halos / Large Scale Structure
Apr 16:  Large Scale Structure
Apr 23:  (No Class)
Apr 30: Analysis of Galaxy Stellar Populations
May 7:  Lessons from Large Galaxy Samples at z<0.2
May 14:  (No Class)
May 21: Evolution of Galaxies with Redshift
May 28: Galaxy Evolution at z>1.5 / Review for Final Exam
June 4:  Final Exam
First, let’s review the important material from last week
First: Elliptical Galaxies
There appear to be two different classes of elliptical galaxies. They form in two different ways.

**Case #1:** “Wet” Mergers
(e.g., between two spiral galaxies)

Galaxy (with gas) →

(tends to occur more frequently for lower mass galaxies, when galaxy evolution less advanced)

Galaxy (with gas)

**Case #2:** “Dry” Mergers
(e.g., between two elliptical galaxies)

Galaxy (without gas) →

(frequently occurs after many previous mergers, when the mass is higher)

Galaxy (without gas)
There appear to be two different classes of elliptical galaxies. They form in two different ways.

Case #1: “Wet” Mergers  
(e.g., between two spiral galaxies)

Appear to result in ellipticals that appear consistent with being rotationally flattened.

Case #2: “Dry” Mergers  
(e.g., between two elliptical galaxies)

(Explained by anisotropy in velocity dispersion)  
Random motions greater in this direction

Random motions less in this direction
There appear to be two different classes of elliptical galaxies. They form in two different ways.

**Case #1:** “Wet” Mergers
(e.g., between two spiral galaxies)

Appear to result in ellipticals that appear consistent with being rotationally flattened.

**Case #2:** “Dry” Mergers
(e.g., between two elliptical galaxies)

Appear to result in ellipticals who ellipticity can be explained by anisotropy in the velocity dispersion.

**Lower Luminosity**

No core in center of elliptical

Disky Isophotes

**Higher Luminosity**

Core in center of elliptical

Boxy Isophotes
As we just illustrated, this dichotomy between these two apparent classes of elliptical galaxies is seen across a wide variety of physical properties.

<table>
<thead>
<tr>
<th>Property</th>
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<th>Low</th>
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<tr>
<td>Luminosity</td>
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<td>Low</td>
</tr>
<tr>
<td>Rotation Rate</td>
<td>Slow</td>
<td>Faster</td>
</tr>
<tr>
<td>Flattening</td>
<td>Anisotropy</td>
<td>Rotation</td>
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<tr>
<td>Isophotes</td>
<td>Boxy</td>
<td>Disky</td>
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<td>Shape</td>
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<tr>
<td>X-ray/radio</td>
<td>Loud</td>
<td>Quiet</td>
</tr>
</tbody>
</table>
Dramatic changes in the gravitational potential that occur during mergers between galaxies -- or the collapse of a static cloud -- can cause very rapid changes in the energy of individual stars in a galaxy. This process is known as violent relaxation.

Violent relaxation drives the phase space distribution of stars in galaxies, so that the radial profile of a galaxy has a $r^{1/4}$ structure.
Fundamental Plane for Elliptical Galaxies

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

These correlations are such that galaxies populate a fundamental plane in these parameters, so that if you know two of the following three variables for a galaxy, you can determine the third.

\[ R \propto \sigma^{1.4} \mu_e^{-0.9} \]

where \( R \) is the size (radius), \( \sigma \) is the velocity dispersion, and \( \mu \) is the galaxy surface brightness.
How shall we interpret this fundamental plane relation?

Let us interpret in terms of the mass to light ratio of galaxies:

\[
\frac{M}{L} = \frac{M}{\mu_e R_e^2} = \frac{M}{\sigma^{1.5} R_e^{-1.1} R_e^2} = \frac{M}{M^{0.75} R_e^{-0.75} R_e^{-1.1} R_e^2} = M^{0.25} R_e^{-0.15}
\]

\[\mu_e \sim \sigma^{1.5} R_e^{-1.1}\]

From virial relation:

\[M \sim \sigma^2 R_e\]

\[M^{0.75} \sim \sigma^{1.5} R_e^{0.75}\]

The fundamental plane relation implies that the mass-to-light ratio scales as \(M^{0.25} R_e^{-0.15}\).

It is unclear what causes this mass dependence. It could be due to the stellar populations (i.e., that the baryonic component in massive galaxies have higher mass-to-light ratios) or due to dark matter playing a larger dynamical role in more massive galaxies.
Second: Dark Matter Halos
Navarro-Frenk-White Density Profiles

One of the most important steps in galaxy formation is the collapse of overdensities early in the universe. The density profile of the collapsed objects can have important impact on the formation of galaxies.

Simulations show that collapsed halos approximately have the following density profile:

\[
\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}
\]

where \( r_s \) and \( \rho_s \) are some scaling parameters.

At small radii \( (r < r_s) \), the density profile \( \rho \) scales approximately as \( r^{-1} \)

and at large radii \( (r > r_s) \), the density profile \( \rho \) scales approximately as \( r^{-3} \)

At \( r \sim r_s \), the density profile \( \rho \) changes slope

Since \( \rho_s \) is completely determined by the total mass and concentration parameter \( c = r_{200} / r_s \), all of the properties of dark matter halos can be expressed in terms of two variables (1) the mass of the dark matter halo and (2) the concentration parameter.
How does the median concentration parameters depend on the mass of the halo and redshift of the galaxy we are examining?

**Figure 1.** Mass and redshift dependence of the concentration parameter. The points show the median of the concentration as computed from the simulations, averaged for each mass bin. Lines show their respective linear fitting to eq. 5.
NOW new material for this week
Collapsed Halos: Comparison with the Observations

First -- we look at the structure of the dark matter halo for very massive collapsed halo \((M \sim 10^{14} - 10^{15} M_{\text{solar}})\) i.e., as appropriate for galaxy clusters
Halos of Galaxy Clusters: Comparison against the Observations

Information from x-ray profiles

Radial Profile

XMM Newton Data of 10 Clusters

Mass / $M_{200}$

Mass Profile comes assuming hydrostatic equilibrium

Pressure gradient balances gravitational forces!

Radius / $r_{200}$
Halos of Galaxy Clusters: Comparison against the Observations

Do the 10 clusters from previous page fit the theoretical concentration vs. mass relationship?

\[ c \propto \frac{M^{-1/9}}{M_*} (1 + z)^{-1} \]
Halos of Galaxy Clusters: Comparison against the Observations

We can also use the apparent surface density of galaxies within clusters vs. radius to probe the density profile.

![Graph showing comparison of mass profiles derived from X-ray data to those of NFW profiles. The fit is remarkable, and the concentration index is well within the limits expected for the clusters. Another (simpler) way to do this is by looking at the distribution of the galaxies within the clusters. Again, a good fit is found.](http://www.strw.leidenuniv.nl/~franx/college/galaxies12/c04-6)
We can also examine the mass profiles of galaxies to see if they follow the expected mass profile:

What are some techniques we can use?

1. Kinematics of satellite galaxies
2. Kinematics of distant stars
3. Gravitational Lensing by Galaxy
Kinematics of Satellite Galaxies:

One method is to take advantage of the satellite companions found around galaxies in the nearby local universe as a probe of kinematics.

Unfortunately, one only tends to find one such satellite galaxy per central galaxy (with a measured velocity along the line of sight), so it is not possible to make a reliable measurement using individual galaxies.

To make progress, one needs to treat these galaxies as orbitting around some “composite average” galaxy.

Results are in good agreement with an NFW profile.

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**Prada et al. 2003**

<table>
<thead>
<tr>
<th>Velocity (km/s)</th>
<th>Radius (kpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>NFW</td>
</tr>
</tbody>
</table>

Fig. 9.—Same as in Fig. 7, but with the removal of the interlopers. The solid curve with the error bars shows the rms velocity after removal of interlopers. As in Fig. 8, the rms is clearly declining and consistent with the NFW profile (*short-dashed curve*) with $M_{vir} = 1.5 \times 10^{12} \ M_\odot$. The long-dashed curve shows the escape velocity from the NFW halo of this mass. All satellites above the escape velocity curve are interlopers.
Kinematics of Distant Stars

Most easily applied to our own galaxy.

One use of this method shown below (Xue et al. 2008)  

Applied to 60 kpc! (which is nevertheless smaller than 200 kpc: which is approximately the halo radius)
Gravitational Lensing also gives us a direct way of measuring the mass inside some radius.

For sources we can see an approximate Einstein ring, measuring the mass enclosed is easy -- since we can derive the mass enclosed at the ring radius just from the geometry.

In fact, there are now large programs, i.e., SLACS, to use exactly these sort of techniques to measure the mass profile of elliptical galaxies.
Gravitational Lensing by Galaxy

The Sloan Lens ACS Survey (SLACS)

Cols: T. Treu (UCSB), L. Koopmans (Kapteyn), A. Bolton (CfA), S. Burles (MIT), L. Moustakas (JPL)

Spectroscopic selection (spurious emission lines), then HST follow-up imaging for confirmation and for accurate modeling

R. Gavazzi, SBastro07
Gravitational Lensing by Galaxy

SLACS sample
Gravitational Lensing by Galaxy

SLACS sample
Gravitational Lensing by Galaxy

Despite some success in this regard, it is not always easy to find large numbers of galaxies with Einstein rings.

An alternate technique is to measure the average impact of gravitational lensing of the shapes of galaxies.

Impact of gravitational lensing is to introduce subtle change in the shape of galaxies on average (so elongated along an angle tangential to the galaxy)
Problems with dark matter halos and real galaxies

Generally, two problems exist: some galaxies may have rather large central cores (larger than expected in the NFW profiles), and the dark matter halos in the simulations have too many “sub-halos”.

• Cores

Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:

Fits with NFW profiles are hard. The rise of the rotation curve is too slow:

By looking at many foreground galaxies and determining the average change in shape of many background galaxies, we can measure the mass profile of the galaxy.

Such averaging is required because background galaxies have random shapes and orientations.

However, averaging over enough galaxies, one can overcome the random component.

van Uitert et al. 2011
Challenges / Issues with our standard model for the collapsed dark matter halos
Cores of Halos

What can we infer about the cores of dark matter halos from the observations?

Where should we look?

Almost all collapsed halos have baryons cooling and falling towards their centers. These baryons will change the mass profile of the original collapsed halo.

Basically, we have the following situation:

\[ \frac{V(R)^2 R}{G} \sim M(R) \sim M_{\text{DARK}}(R) + M_{\text{COOL}}(R) \]

To obtain best constraints on \( M_{\text{DARK}}(R) \), we need \( M_{\text{COOL}}(R) \) to be as small as possible.
Cores of Halos

For which galaxies would we expect the halos to be the least affected by the baryons cooling to the centers?

For which galaxies, will $M_{\text{COOL}}(R)$ be the smallest?
Here are two galaxies where the baryonic component has different angular momentum. For which case, can we get a cleaner look at the dark matter halo?

- Spinning moderate speed:
  - After cooling
  - Intermediate size, more dense disk galaxy
  - Higher surface brightness

- Spinning fast:
  - After cooling
  - Large, lower density galaxy disk
  - Lower surface brightness
Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:
Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:
What would infer about the mass profile of a halo where the circular velocity appears to rise linearly with radius?

Using the relation $M = V(R)^2 R / G$, we would infer that $M \propto R^3$ or $\rho \sim$ independent of radius.

But for NFW, we would expect $\rho \propto r^{-1}$ at small radii.
Substructure

You might assume that the halos from the simulations are smooth.

But they are not perfectly smooth, as you can see to the right:

These halos include large numbers of subhalos.

What is the origin of these subhalos?

Previously, they were their own collapsed halos, but merged into a bigger halo.
What fraction of the mass do the subhalos contain?

Subhalos contain only a small fraction of the mass at the centers of the main halo.

But subhalos contain a large fraction of the mass at larger radii.

Why? Subhalos have a hard time remaining together in the center of the main halo, due to the higher densities there.
In most cases, you would expect most collapsed halos, even with masses of \(10^8 - 10^{10}\) solar masses to have had some baryons to cool to the center of the halo and form stars.

As these halos will merge with a larger halo, they become subhalo and the luminous matter within these subhalos should become satellite galaxies.
In pictures:

AFTER MERGER
All these subhalos should indicate the position of dwarf galaxy!

Do we find these galaxies around our own Milky Way galaxy?

If we include only the most obvious satellites, the answer is no.

This is called the “missing satellites problem.”

One resolution may be subhalos each containing only a few stars.

![Graph showing cumulative number of halos versus v_c / V_global with data from Simulated cluster, Simulated galaxy, Virgo cluster data, dSph's, Fornax, Sagittarius, LMC, SMC.](image)
Kinematics of Ultra-Faint Galaxies

One resolution may be ultra-low luminosity galaxies populating subhalos.

Mass/density profiles depend on assumption that measured velocities probe gravitational potential.

Credit: Geha 2009

Previously unseen population of faint galaxies

Strigari et al 2008
Martinez et al 2009
Finding the Milky Way Ultra-Faint Galaxies

The ultra-faint galaxies are found via over-densities of resolved stars.

Milky Way stellar foreground overwhelms the dwarf galaxy.

Credit: Geha 2009
Finding the Milky Way Ultra-Faint Galaxies

Assume: Dwarf galaxies are old, metal-poor stellar populations, with typical size ~ 50-100pc. This defines a narrow region in color-magnitude space.

Walsh, Willman & Jerjen (2008)

A generous definition of old and metal-poor:
age = 8 to 14 Gyr
[Fe/H] = -1.5 to -2.3
Distance = 20 kpc

Credit: Geha 2009
Finding the Milky Way Ultra-Faint Galaxies

1. Assumed old/metal-poor stellar population
2. Assumed physical size

Credit: Geha 2009
How Do Galaxies Distribute Themselves in Space?

What does this teach us?
“How Do Galaxies Distribute Themselves in Space?

What does this teach us?”

It provides insight into the masses or properties of the collapsed dark matter halos in galaxies form and evolve.
By now, you should be familiar with the idea that collapsed halos contain galaxies at their center. These galaxies form from the cooling of gas onto the center of the halo and forming a gas disk.
however, many dark matter halos contain more than one galaxy...

each halo almost always one most massive galaxy at the center (called the “central galaxy”) and any number of satellite galaxies (orbitting around the center)
You should all be quite familiar with halos that contain many satellite galaxies. An excellent example is a galaxy cluster:
A less extreme version of a galaxy cluster is a galaxy group. Our own local group is a good example:

The 3D distribution is

Lots of satellite galaxies in the M31 and Milky Way Halos!
Structure on larger scales

5.1 The content of massive halos: groups, clusters

Many halos contain more than one galaxy. Obvious examples are clusters - which can contain hundreds of galaxies. More mundane clusters are those with lower masses, and we call those groups. Most "normal" galaxies are member of a group. Take for example our Milky Way, which is part of the Local Group. This group contains 2 massive galaxies (MW, Andromeda), and several lower mass galaxies (Magellanic Clouds, M33, M32, NGC 205, ...), and many more lower mass galaxies. The local group is shown below:

Of course, this list may be incomplete as the Milky Way itself blocks our field of view!

New Gaia mission may help in this regard!

Table 4.3  Local Group members

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<th>Name</th>
<th>Alternate Name</th>
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</tbody>
</table>

Source: From data kindly provided by M. Irwin.
how can we explain the origin of the satellite galaxies?

can we explain them as a result of gas cooling?

no -- since gas only efficiently cools onto the central position in the dark matter halo (gravitational potential)
How can we explain the origin of the satellite galaxies?

A better explanation is through merging:

Step #1: We have two dark matter halos. Gas cools to the center of each halo to form disk galaxies in each:

Step #2: These two dark matter halos merge to form a single more massive halo

The most massive halos contain large numbers of satellite galaxies, while less massive galaxies contain less. The merger origin of satellite galaxies explains why this is the case.
NOTA BENE: In many cases, satellite galaxies merge with the central galaxy as a result of dynamical friction.
The Local Group is not in equilibrium!

M31 and the Milky Way galaxy are traveling towards each other at 120 km/s!

Since M31 and the Milky Way are 700 kpc away from each other, these two galaxies will collide in perhaps ~4 billion years!

Can we use this information to estimate the mass of the Milky Way galaxy? Yes!
The Local Group is not in equilibrium!

First, consider the fact that the Milky Way and M31 will be initially flying away from each other with the Hubble flow.

Due to the self gravity of the mass within the local group, the Milky Way and M31 will stop expanding with the Hubble flow and start to fall towards each other.

Assuming that it takes the two main galaxies in the local group 14 Gyr to start falling towards each other at 120 km/s and have a distance of 700 kpc, we can calculate the mass of the local group to be $3 \times 10^{12} M_{\text{solar}}$

Comparing this mass to the total luminosity of the local group, one derives $70 M_{\text{sol}} / L_{\text{sol}}$. 
To put these numbers in context, I remind you of the mass-to-light ratios presented in lecture #1

Table 10-2. Estimates of the density parameter

<table>
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<tr>
<th>Method</th>
<th>( \Upsilon_V / \Upsilon_\odot )</th>
<th>( \Omega_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar neighborhood</td>
<td>5</td>
<td>0.003h(^{-1})</td>
</tr>
<tr>
<td>Elliptical galaxy cores</td>
<td>12h</td>
<td>0.007</td>
</tr>
<tr>
<td>Local escape speed</td>
<td>30</td>
<td>0.018h(^{-1})</td>
</tr>
<tr>
<td>Satellite galaxies</td>
<td>30</td>
<td>0.018h(^{-1})</td>
</tr>
<tr>
<td>Magellanic Stream</td>
<td>&gt; 80</td>
<td>&gt; 0.05h(^{-1})</td>
</tr>
<tr>
<td>Rotation curve of NGC 3198</td>
<td>&gt; 28h</td>
<td>&gt; 0.017</td>
</tr>
<tr>
<td>X-ray halo of M87</td>
<td>&gt; 750</td>
<td>&gt; 0.46h(^{-1})</td>
</tr>
<tr>
<td>Local Group timing</td>
<td>100</td>
<td>0.06h(^{-1})</td>
</tr>
<tr>
<td>Groups of galaxies</td>
<td>260h</td>
<td>0.16</td>
</tr>
<tr>
<td>Clusters of galaxies</td>
<td>400h</td>
<td>0.25</td>
</tr>
<tr>
<td>Virgocentric flow</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Nucleosynthesis</td>
<td></td>
<td>(0.01 - 0.05)h(^{-2})</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

NOTES: All lines except the last three are based on the luminosity density (10-24). Nucleosynthesis estimate omits density in non-baryonic matter. Several methods, such as Local Group timing and X-ray halo of M87, depend on \( h \) in complicated ways, and this dependence has been suppressed. See text for further detail.

Mass-to-Light Ratio

As we probe larger spatial scales, the mass-to-light ratio increases!
What collapsed clusters or groups are nearby the local group?

Sculptor, Canes I, Maffei Group, M81 Group all within 7 Mpc
What collapsed clusters or groups are nearby the local group?

**Virgo Cluster**
- contains >250 large galaxies
- contains 2000 smaller galaxies
- covers 10 x 10 degrees on sky
  - 18 Mpc away
  - 3 Mpc diameter

**Fornax Cluster**
- contains >50 large galaxies
  - 19 Mpc away
  - less massive than Virgo
What collapsed clusters or groups are nearby the local group?

**Coma cluster**
- contains >1000 large galaxies
- contains 10000 smaller galaxies
- ~100 Mpc away
- 6 Mpc diameter
- largest galaxies are giant ellipticals
How common are galaxy groups or clusters of various masses?

An additional complication is that some catalogues are incomplete. The SDSS spectroscopic sample is incomplete for galaxies with nearby neighbors. This is due to the fact that the fibers cannot be close together. This makes it harder to define groups, and careful corrections are needed for the catalogues.

This probably makes it clear that the definition of groups and clusters is never easy from observational material. This has never stopped anybody from making such catalogues.

Once a catalogue of groups/clusters has been built, one can construct the "mass function", the number density as a function of mass. Below is an example from Heinamaki et al (A&A 397, 63)

This graph shows the cumulative density \( n(>M) \), the number density of groups larger than \( M \), as a function of \( M \). It is clear that groups have very large masses, from \( 10^{12} \) to \( 10^{15} \) \( M_\odot \). Notice that clusters like the Coma cluster (with \( M \approx 10^{15} M_\odot \)) are rare (\( 10^{-6} - 3 \) Mpc\(^{-3} \)), or \( \approx 1 \) every 100 Mpc.

The velocity dispersions correlate well with mass as shown by the same authors:

Groups and clusters are the largest collapsed systems, but structure on larger scales exist. This is measured statistically. We express this by the correlation function. The correlation function describes how the probability of finding another galaxy in the neighborhood of a galaxy increases with decreasing distance.
Gravity does not simply result in the production of collapsed systems. It can also cause galaxies or other collapsed structures to be “close to each other” or clustered.

We can quantify the non-uniform distribution of galaxies on the sky through a measurement of the clustering signal.

This clustering signal ultimately helps us learn about the dark matter halos in which galaxies live.
We quantify clustering in terms of correlation functions

The Correlation function $\xi$ is not equal to zero -- since the presence of a galaxy at some place in space makes it more likely another one will be close by...

\[ dP_1 = n \, dV_1 \]
\[ dP_{12} = n^2 \left( 1 + \xi(r_{12}) \right) \, dV_1 \, dV_2 \]

$n = \text{average density of galaxies}$
The Correlation function $\xi$ is calculated by examining the distances between every pair of galaxies in a survey and comparing it to a random distribution.

Correlations between points can be determined by counting pairs.

Symbolically, $\bar{n} + \xi = n_D \hat{D} + n_R \hat{R}$, where $n_D$ and $n_R$ represent the data and the random point set, respectively.

Several other ways of measuring $\xi$ have been proposed, such as $\bar{n} + \xi^2 = n_D \hat{D} + n_R \hat{R}$, $\bar{n} + \xi^3 = n_D \hat{D} + n_R \hat{R} \hat{R}$, and $\bar{n} + \xi^4 = \bar{n} + \hat{D} \hat{R}$, which are all equivalent in the ideal situation of an infinitely extended point distribution. For finite point sets, $\xi^3$ and $\xi^4$ are superior to $\bar{n}$ and $\xi^2$ due to their better noise properties.

How do we quantify the correlation function?
How do we quantify the correlation function?

Fundamentally, this involves counting the number of galaxies at a certain distance from each other on the sky and then comparing that with a random distribution.

\[ \xi(r) = \frac{DD}{RR} - 1 \]

- **DD** = number of pairs in the data at a distance \( r \)
- **RR** = number of pairs in some mock data set at a distance \( r \) (in mock data sets pairs laid down randomly with uniform distribution)

Correlations between points can be determined by counting pairs.
How do we typically express the correlation function?

The Correlation function \( \xi \) is typically parametrized as a power-law in radius:

\[
\xi_g(r) = \left( \frac{r}{r_0} \right)^{-\gamma}
\]

Typical values for \( \gamma \) are 1.8. \( r_0 \) is known as the correlation length and it tells us the typical distance from a source we can expect a large enhancement in neighboring sources.
How do we compute the power spectrum of galaxies from the observed clustering?

The power spectrum is the Fourier transform of the correlation function $\xi$

$$P(k) = \int \xi(r) e^{i k \cdot r} \, d^3 r = \int \xi(r) \frac{\sin(kr)}{kr} \, r^2 \, dr$$
What can we learn about galaxies from the observed clustering / power spectrum?

It provides us with fundamental information about the dark matter halos in which galaxies live...

How does it provide this information?

To understand this, we should review the basics of the growth of density perturbations in the universe.

(which we will discuss in a few slides...)
How can we can insight into the collapsed structures in which galaxies live?

We assume that galaxies are found in collapsed dark matter halos and we look at which mass halos have the same power spectrum as galaxies do in the observations:

\[
P(k)_{\text{galaxies}} = P(k)_{\text{DMHalo}} = b^2 P(k)_{\text{DM}}
\]

- \(P(k)_{\text{galaxies}}\) can be computed from the observed correlation function \(\xi(r)\) for galaxies.
- \(P(k)_{\text{DMHalo}}\) is the power spectrum in dark matter halo distribution.
- \(P(k)_{\text{DM}}\) is the matter power spectrum.
- \(b\) is the bias factor (\(\geq 1\) in general).
How are the dark matter power spectrum and power spectrum for dark matter halos calculated?
How can we describe dark matter fluctuations?

Convenient to express it in terms of Fourier modes and power spectrum:

- **Long wavelength (large scales)**
- **Short wavelength (small scales)**
- **High Power (large amplitude)**
- **Low Power (small amplitude)**

Credit: Pearson
How can then we can calculate the power spectrum?

Subtract off mean density:

\[ \delta(\vec{r}) = \frac{\rho - \overline{\rho}}{\overline{\rho}} = \frac{\Delta \rho}{\overline{\rho}} \]

Fourier Transform:

\[ \delta_k = \sum \delta(\vec{r}) e^{-ik \cdot \vec{r}} \]

Power Spectrum:

\[ P(k) = \left\langle |\delta_k|^2 \right\rangle \]
What happens to density perturbations over time?

They grow through gravitational forces.

How does this operate in the current universe?
What is the primordial power spectrum?

The initial power spectrum of fluctuations is the following:

\[ P_0(k) = A k^{n_s} \]

\[ n_s \sim 1 : \text{"Harrison-Zeldovich Spectrum"} \]
How fast does the power spectrum grow?

**z>3500: Radiation-dominated Epoch:**
No significant growth in structure occurs -- except at scales larger than the horizon, where the growth goes as $R^2$ ($R =$ scale of universe)

For causally connected regions (i.e., within the horizon), there is no growth

For regions not causally connected (i.e., super horizon scale), growth occurs as $R^2$

**z<3500: Matter-Dominated Epoch:**
growth goes as $R$ ($R =$ scale of universe)
Most of the growth in the power spectrum will occur on large scales!

identical growth of structure at large scales

less growth of structure at small scales as universe becomes casually connected

\[ P(k) = k^2 \]

\[ \text{lg}(M) = \frac{4}{3} \left( \frac{\text{lg}(k)}{2} \right)^2 \]

\[ \text{Average mass contained with a sphere of radius } \frac{200}{k} \]

\[ \text{Mean squared mass density within sphere} \]

\[ M(M) = M(3) + M(M) \]

Instead of simply \[ P(k) \] often plot \[ (k^3 P(k))^{1/2} \] the root mean square mass fluctuations
Evolution of the Matter Power Spectrum

Large Scales

Small Scales

Credit: Bohringer
Evolution of the Matter Power Spectrum

$P(k)$

$K$ (wavevector)

H.-Z. spectrum

Horizon scale

Large Scales

Small Scales

Credit: Bohringer
Evolution of the Matter Power Spectrum

Credit: Bohringer
How does one calculate the power spectrum for matter fluctuations in early universe?

Formally, one utilizes a transfer function to include these physics:

\[ P_{0}(k) = A \ k^{n_s} \ T^2(k) \]

where \( T(k) \) is the transfer function.

The transfer function \( T(k) \) depends on the cosmological model and in particular on the quantity \( \Gamma = \Omega_{m} h \). \( \Gamma \) is called the shape parameter.
How does the power spectrum grow after epoch of radiation-matter equality and recombination?

To simplest approximation, it can be expressed in terms of growing modes...

\[ P(k, t) = D_+^2(t) P(k, t_0) =: D_+^2(t) P_0(k) \]

where \( P(k,t) \) is the power spectrum at some later time and \( D_+(t) \) is the growth factor.

In the linear growth regime (before modes start turning around and collapsing and virializing), the time \( t \) and mode \( k \) are totally separable in the above equation.
What does the matter power spectrum look like when all of these effects are included?

Position of turn-over determined by horizon size @ matter-radiation equality

$l \ll 1/L_0$

$\left(\frac{c}{H_0}\right)^3 P(k)$
This is very similar to the observed power spectrum!
What is the power spectrum of collapsed halos of a given mass? How does it compare with the power spectrum of underlying matter?

Halos only form in overdense regions of the universe which have had time to collapse.

Collapsed halos with particularly high masses would only form in regions with the most significant overdensities.

This would lead to a spatial distribution of halos that shows considerably more structure than the underlying dark matter.
Why dark matter halos show a less uniform distribution than the underlying matter distribution?

Because dark matter halos overestimate fluctuations in the underlying density distribution, they are said to be biased.
The most massive galaxy halos are expected to be distributed in the least uniform way relative to the underlying dark matter density distribution and be the most biased.

In summary, in terms of the bias factor $b$, 

$$ b^2 P(k)_\text{DM} = P(k)_\text{DMHalos} $$

Less massive galaxies, on the other hand, are less biased tracers of the underlying dark matter distribution.
The classic demonstration of this result (i.e., relationship between the bias/clustering properties and halo mass) is found in Mo & White (1996).

Here are some results from that paper:

Correlation functions of clusters, in a simulation. As can be seen, the low mass clusters (left panel) are less strongly clustered than the high mass clusters.