

Dark Matter Halos

Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics

Feb 9: Disk Galaxies (I)

Feb 12: Disk Galaxies (II)

Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics

Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations

Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)

Mar 9: Elliptical Galaxies (II)

Mar 23: Dark Matter Halos



Mar 30: Large Scale Structure

Apr 13: Galaxy Stellar Populations

Apr 20: Lessons from Large Galaxy Samples at $z < 0.2$

Apr 23: Evolution of Galaxies with Redshift

May 4: Galaxy Evolution at $z = 1-7$

May 11: Galaxy Evolution at $z > 7$ / Review for Final Exam

Practical Sessions

Feb 19: Board Work + Problem Set 1

Mar 12: Board Work + Problem Set 2

Mar 26: Problem Set 3 / Paper Presentations (4 slots)



Apr 2: Paper Presentations (7 slots)

Apr 16: Problem Set 4 / Paper Presentations (4 slots)

Apr 30: Problem Set 5 / Paper Presentations (4 slots)

May 7: Problem Set 6 / Paper Presentations (4 slots)

March 26 Practical Session

(In 3 days)

Problem Set 3 - Problems 2, 3, and 5 (to be discussed)

Ids Nieuwstraten

Andreea Suta

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Sonal Garg

Sanne van Beek

Galaxies: Structure, Dynamics, and Evolution
Problem Set 3
Instructor: Dr. Bouwens

Here is Problem Set 3. The entire problem set will be due before class on Monday, April 6 (email them to Wout and include GSD in the subject line). Be sure to pay extra attention to problem 1, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. Consider the case of the homogeneous ellipsoid potential discussed on page 3 of this handout. For this potential, the motion of a star in the x , y , and z directions can be described as harmonic oscillators. Suppose that we represent the motion of each particle as $x = a_x \cos(\omega_x t + \phi_x)$, $y = a_y \cos(\omega_y t + \phi_y)$, $z = a_z \cos(\omega_z t + \phi_z)$ where ω_x does not equal ω_y does not equal ω_z and ω_x/ω_y , ω_y/ω_z , and ω_x/ω_z are not rational numbers. Particles in such a potential follow box orbits.

(a) Argue that a particle travels arbitrarily close to every spatial position in the entire volume $(-a_x, a_x) \times (-a_y, a_y) \times (-a_z, a_z)$. If you cannot prove it explicitly, demonstrate the plausibility of this statement by showing 2-D projections of the orbital tracks for two separate choices of $(\omega_x, \omega_y, \omega_z)$. Plot out the orbital tracks for varying integration times (short time, intermediate time, long time intervals).

(b) Write down a formula for the angular momentum of a particle in this potential. Is the angular momentum conserved? Why or why not? What is the average value of the angular momentum averaged over time?

(c) If a were equal to b (and hence ω_x were equal to ω_y), would the angular momentum be conserved? Why or why not? Would particles still travel on box orbits, or would the orbits be loop orbits?

2. Finding a solution to the collisionless Boltzmann equation using the Jeans theorem.

(a) Derive ρ and Ψ for a spherically-symmetric system with some distribution function f of the form $f(\epsilon) = \begin{cases} \epsilon^{n-3/2}, & \text{if } \epsilon > 0 \\ 0, & \text{if } \epsilon < 0 \end{cases}$. where $n = 1$, $\epsilon = -E + \Phi_0$ and E is the energy of a particle orbiting around the system. Adopt the standard definition that $\Psi = \epsilon + (1/2)v^2$. Show that the total mass of the model is $(1/2)\Psi_0 G^{-3/2} \sqrt{\pi}/c_1$ where c_1 is defined by equation (4-107b) from BT. Hint this is problem 4-16 from Binney & Tremaine (BT) and is discussed in some depth on BT 223-225.

(b) Derive ρ and Ψ for some spherically symmetric system with the distribution function f with the form $f(\epsilon) = \begin{cases} \epsilon^{n-3/2}, & \text{if } \epsilon > 0 \\ 0, & \text{if } \epsilon < 0 \end{cases}$. where $n = 5$.

This is the distribution function for a Plummer model. Find the expression for ρ and Ψ . Derive also the formula for the total mass of the system.

3. Derive the third Jeans equation by subtracting the second Jeans equation from the first Jeans equation multiplied by \bar{v}_j . See the supplementary reading posted on the course website for hints on how to do this.

4. How many integrals of motion does a particle have in a Kepler potential? What are they?

5. Dynamical Friction. Dynamical friction is an important mechanism which causes colliding galaxies to rapidly merge. How important would this mechanism be, if we consider the collision of a galaxy with an isolated star (wandering alone through the universe)? Make use of the following Chandrasekhar dynamical friction formula presented in class:

$$\frac{dv_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 (M+m) \rho_m}{v_M^2}$$

Assume that a galaxy is a 3 kpc x 3 kpc x 3 kpc cube with mass $3 \times 10^{10} M_\odot$ and is entirely composed of stars with $1 M_\odot$. Assume that a star with one solar mass $M = 1 M_\odot$ approaches the galaxy at velocity $v_M = 200$ km/s and at an angle perpendicular to the surface of the cube. How much will dynamical friction change the velocity of the star if it falls in from infinity and continues to infinity? Feel free to assume that the dynamical friction is constant throughout the entire passage of the star through the galaxy (i.e., that the slowing velocity of the star has no effect on the amplitude of the dynamical friction). How would the impact of dynamical friction change if the star (unrealistically) had a mass of $10^9 M_\odot$? We ignored the effect of dynamical friction in calculating the relaxation time for a star in lecture 5. Is this assumption justified?

March 26 Practical Session

(In 3 days)

Paper Presentations (12 + 3 minutes)

Gaia-Enceladus / Sausage Merger with Milky Way: Helmi+2018, Belokurov+2018

Marleen Besseling, Vinay Chakawri

Stellar Streams in Gaia Era: Antoja+2018, Bonaca+2025

Noah Kaiser, Eugenia Redondo Gonzalez

Gaia-era Inner-Halo Decomposition, Anisotropy/Metallicity Links

Ines Bercuk, George-Luca Iconaru

March 26 Practical Session

(In 3 days)

Suggested Presentation Structure (12 minutes total)

Overall Approach

- Treat as a short lecture segment
- Build on concepts from the course
- Focus on clarity and guiding the audience
- Avoid trying to cover everything from both papers

Time Allocation

- **~9 minutes:** Main paper
 - Core ideas
 - Methodology
 - Key figures/results
- **~3 minutes:** Second paper
 - Additional insights
 - Complementary results or perspectives

Division of Roles

- One presenter: main paper
- One presenter: second paper

March 26 Practical Session

(In 3 days)

Suggested Presentation Structure (12 minutes total)

Content Prioritization

- Emphasize:
 - Primary results
 - Core insights
- De-emphasize:
 - Secondary details
- Optional:
 - Final slide with points not covered in detail

Presentation Format

- Use a **small number of slides**
- **Do not scroll through PDFs**
- Select only the most important figures and concepts

Logistics

- Option to send presentation files in advance
- Or bring a **single laptop** for presenting

Supplementary Reading

Relevant Material for Lecture I

“Galaxies: Structure, Dynamics, and Evolution”

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-1>

2 Gravitational force and potential (BT 2 - 2.1)

The matter in galaxies (whether stars, gas, dark matter, etc) is kept from escaping by gravity. Before we study the motions of individual particles, we show how we can calculate the gravitation force and potential from an extended density distribution.

The gravitational force caused by a point mass M at \vec{x}_0 on a unit mass at position \vec{x} is

$$\vec{F}(\vec{x}) = GM \frac{\vec{x}_0 - \vec{x}}{|\vec{x}_0 - \vec{x}|^3}$$

In general, the gravitational force is related to the potential Φ by

$$\vec{F}(\vec{x}) = -\vec{\nabla}\Phi(\vec{x})$$

so that

$$\Phi(\vec{x}) = -\frac{GM}{|\vec{x}_0 - \vec{x}|}$$

The gravitational potential for extended density distribution $\rho(\vec{x})$ can be obtained by integrating over the density distribution

$$\Phi(\vec{x}) = -G \iiint \frac{\rho(\vec{x}_0) d^3x_0}{|\vec{x}_0 - \vec{x}|}$$

Note:

The triple integration is often expensive
Easier for special geometries, mass stratifications

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-2>

- Sphere $\rho = \rho(r)$
- Classical ellipsoid $\rho = \rho(m^2)$ where $m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$
- Thin disk

The density follows from the potential by Poisson's equation:

$$4\pi G\rho(\vec{x}) = \vec{\nabla}^2\Phi(\vec{x})$$

The mass in some volume can easily be derived from the force field: Integrate both sides of Poisson's equation over the volume enclosing a total mass M . For the left hand side we obtain:

$$4\pi G \int_V \rho d\vec{x} = 4\pi GM$$

Using the divergence theorem, we obtain for the right hand side:

$$\int_V \nabla^2\Phi d\vec{x} = \int_S \vec{\nabla}\Phi \cdot d^2S$$

Combining left and right side gives Gauss's theorem:

$$4\pi GM = \int_S \vec{\nabla}\Phi \cdot d^2S$$

→the integral of the normal component of $\vec{\nabla}\Phi$ over any closed surface equals $4\pi G$ times the mass contained within that surface

The potential energy can be shown to be:

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-3>

$$W = 1/2 \int \rho(\vec{x})\Phi(\vec{x})d\vec{x}$$

We derive this as follows. Assume that we "build" up the galaxy slowly. We have a galaxy with a density $f\rho$, with $0 < f < 1$. We add a tiny bit of density $\delta f\rho$, taking the mass from infinity to the galaxy. Ignoring the change in the potential, this costs an energy

$$\int \delta f\rho(\vec{x}) f\Phi(\vec{x})d\vec{x}$$

where $f\Phi$ is simply the potential of density $f\rho$, and the integral is the integral over the full galaxy volume. We now have to add all the contributions together to derive the full energy needed to "build" the full galaxy

$$\begin{aligned} W &= \int_0^1 \int \rho(\vec{x}) f\Phi(\vec{x})d\vec{x} df \\ &= \int \rho(\vec{x})\Phi(\vec{x})d\vec{x} \int_0^1 f df \\ &= 1/2 \int \rho(\vec{x})\Phi(\vec{x})d\vec{x} \end{aligned}$$

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-4>

3.1 Potential for spherical systems (BT 2.1, 2.2)

Newton's Theorems:

- First Theorem:
A body inside an infinitesimally thin spherical shell of matter experiences no net gravitational force from that shell

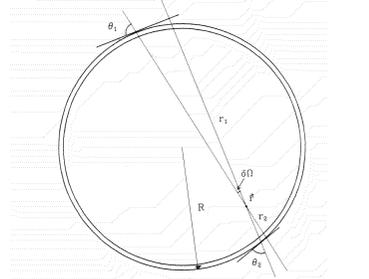


Figure 2-1. Proof of Newton's first theorem.

Consider contributions to the force at point \vec{r} , due to the matter in the shell in a very narrow cone $d\Omega$. The intersection angles at 1 and 2, Θ_1 and Θ_2 , are equal for infinitely small $d\Omega$. The relative masses in the cone δm_1 and δm_2 satisfy $\delta m_1/\delta m_2 = (r_1/r_2)^2$. The gravitational forces are

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-5>

proportional to $\delta m_1/r_1^2$ and $\delta m_2/r_2^2$, and therefore equal, but of opposite sign. Hence the matter in the cone does not contribute any net force at the location \vec{r} . If we sum over all cones, we find no net force!

- Newton's Second Theorem:
The gravitational force on a body outside a closed spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.

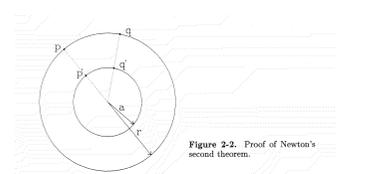


Figure 2-2. Proof of Newton's second theorem.

Calculate the potential at point \vec{r} at radius r from the center of an infinitesimally thin shell with mass M and radius a . Consider the contribution from the portion of the sphere with solid angle $\delta\Omega$ at q' :

$$\delta\Phi_1 = -\frac{GM}{|\vec{r} - \vec{q}'|} \frac{\delta\Omega}{4\pi}$$

Now take an infinitesimally thin shell with the same mass M , but radius r . Scale \vec{r} down to \vec{r}' , so that it lies at a radius a inside the shell. Scale \vec{q}' up, so that it lies on the shell. Calculate the potential at \vec{r}' . The contribution of the matter near \vec{q}'

13-9-07 see <http://www.strw.leidenuniv.nl/~frank/college/mf-sta-07-c2-6>

with the same solid angle $\delta\Omega$ is:

$$\delta\Phi_2 = -\frac{GM}{|\vec{r}' - \vec{q}'|} \frac{\delta\Omega}{4\pi}$$

Since $|\vec{r} - \vec{q}'| = |\vec{r}' - \vec{q}'|$, $\delta\Phi_1 = \delta\Phi_2$. Sum over all solid angles to obtain

$$\Phi_1 = \Phi_2$$

Since Φ_2 is the potential inside a sphere with mass M and radius r , it is equal to $\Phi_2 = -GM/r$, and this is equal to Φ_1 . This is the same as the potential at r if all the mass is concentrated at the center.

We can now calculate potential of spherical system with density $\rho(r)$. Divide system up into shells, and add contribution from each shell. Distinguish between shells with radius r' , $r' < r$ and shells with $r' > r$:
 $r' < r$: $\delta\Phi = -G\delta M/r$,
 $r' > r$: $\delta\Phi = -G\delta M/r'$.

Hence total potential:

$$\Phi = -4\pi G \left[\frac{1}{r} \int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r' dr' \right].$$

Hence only single integration! The force on the unit mass at radius r is determined by mass interior to r :

$$\vec{F}(r) = -\frac{d\Phi}{dr} \vec{e}_r = -\frac{GM(r)}{r^2} \vec{e}_r,$$

where

Additional Information on Website

GALAXIES: STRUCTURE, DYNAMICS, AND EVOLUTION (SPRING 2026)

SCHEDULE

Note: The schedule is subject to change. Check back for changes and updates



Semester Timeline

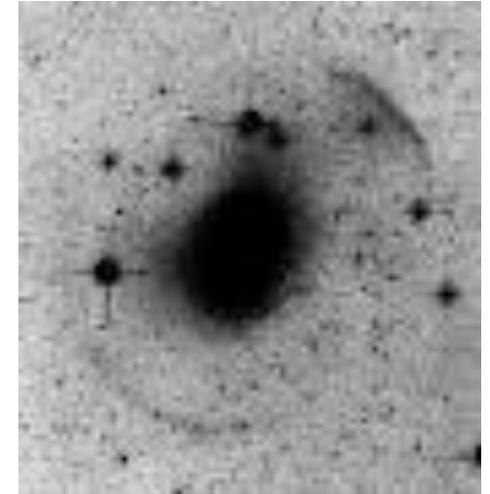
Date	Topic	pdf of lecture	supplemental reading	sections in book
Feb 2	Introduction and Galaxy Formation Basics	Here	Here	BT1.1,BT1.3
Feb 5	No Practical Session			
Feb 9	Disk Galaxies (I)	Here	Here	BT2.7
Feb 12	Disk Galaxies (II)	Here	Here	BT3.3,BT6.1,BT6.2
Feb 16	Disk Galaxies (III) and Collisionless Stellar Dynamics	Here	Here	BT6.2,BT6.3
Feb 19	Practical Session - Board Work + Problem Set 1			
Feb 23	Collisionless Stellar Dynamics	Here	Here	BT1.2,BT3.1,BT4.1,BT4.7
Feb 26	Solving Collisionless Boltzmann Equation / Jeans Equations	Here	Here	BT4.2,BT4.8
Mar 2	No Lecture			
Mar 5	No Practical Session			
Mar 9	Elliptical Galaxies (I)	Here	Here	BT8.1
Mar 12	Practical Session - Problem Set 1+2			
Mar 16	No Lecture			
Mar 19	No Practical Session			
Mar 23	Elliptical Galaxies (II)			

**First, let's review the important
material from last week**

What is the nature of elliptical galaxies?

End state of galaxy formation!

Dominated by the random motions of its component parts (stars, hot gas, dark matter)



While progenitors to elliptical galaxies experienced lots of star formation, elliptical galaxies themselves experience almost no star formation

Only way for an elliptical galaxy to transform into another type of galaxy (e.g., spiral) is if lots of cold gas cools onto it (but this may not happen!)

What we can learn from Images of Elliptical Galaxies?

Important clues can come from looking at the two dimensional surface brightness profiles of elliptical galaxies.

Two Types of Deviations from Elliptical Isophotes are Observed
(Few Percent)

Boxy



'boxy'

Disky

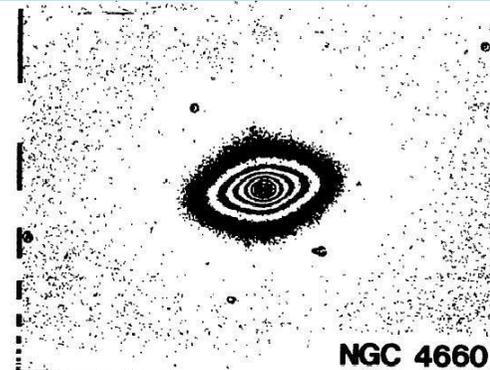


'disky'



NGC 5322

FIGURE 7. — R-image of NGC 5322, an elliptical galaxy with box-shaped isophotes ($a(4)/a \sim -0.01$).

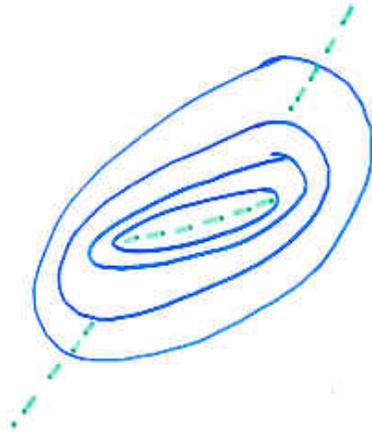


NGC 4660

FIGURE 6. — R-image of NGC 4660, an elliptical galaxy with a disk-component in the isophotes ($a(4)/a \sim +0.03$).

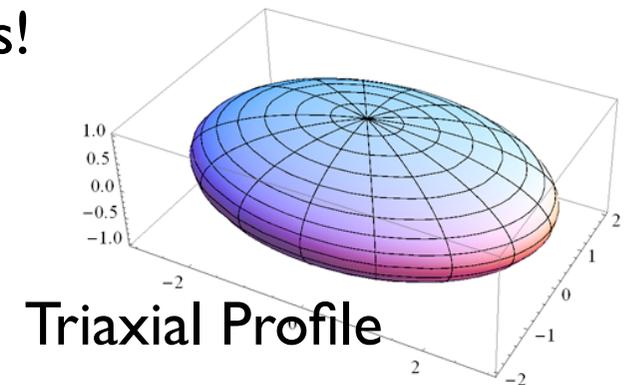
What we can learn from Images of Elliptical Galaxies?

In studying the elliptical galaxies, one quantifies the ellipticity and position angle of the galaxy at different surface brightness levels.



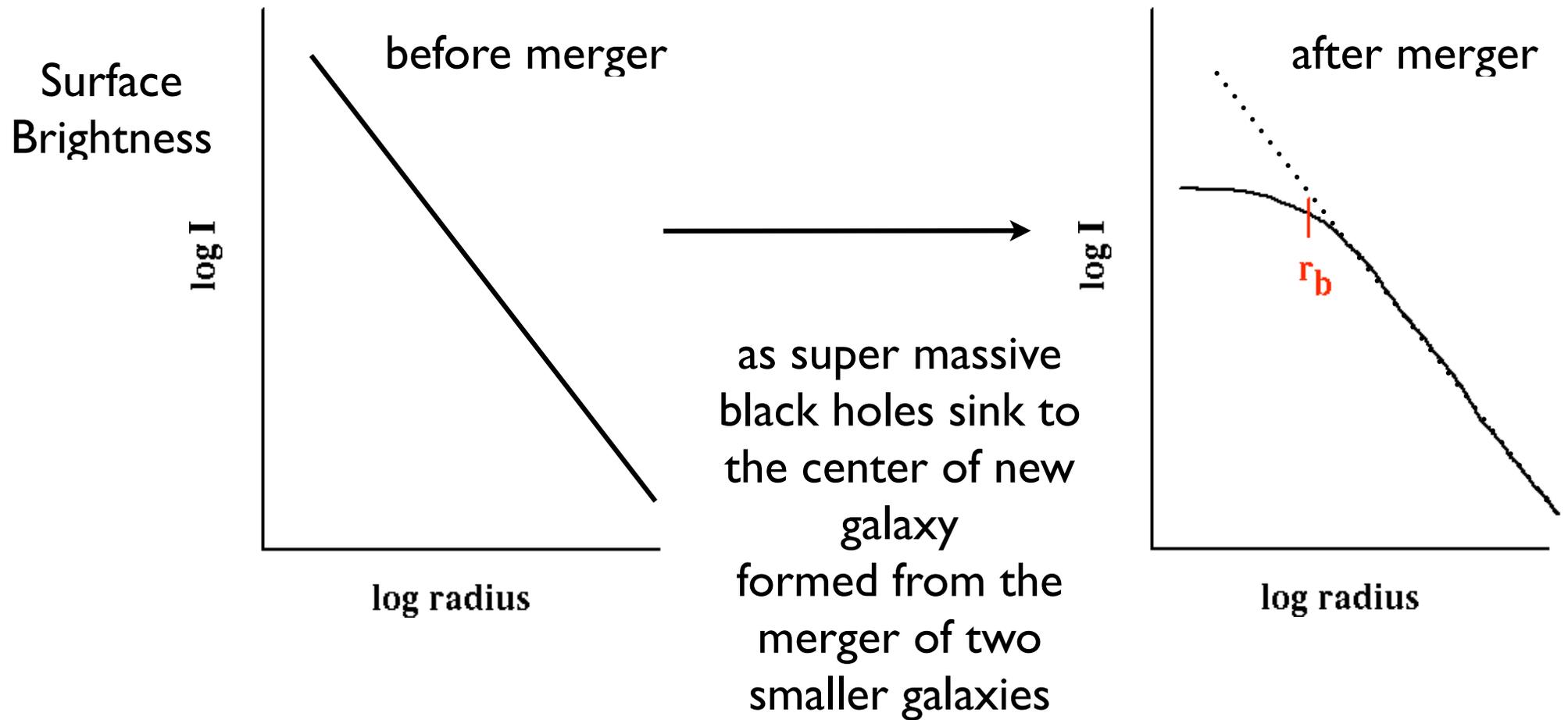
It is not uncommon to find that the position angle and ellipticity changes as a function of radius!

This twisting of the isophotes is expected for a triaxial profile from projection effects.



Center of Ellipticals: Core vs. Power-Law Galaxies

Core in centers of ellipticals likely created from Scouring by Super Massive Black Holes (10^9 solar masses) During Merging

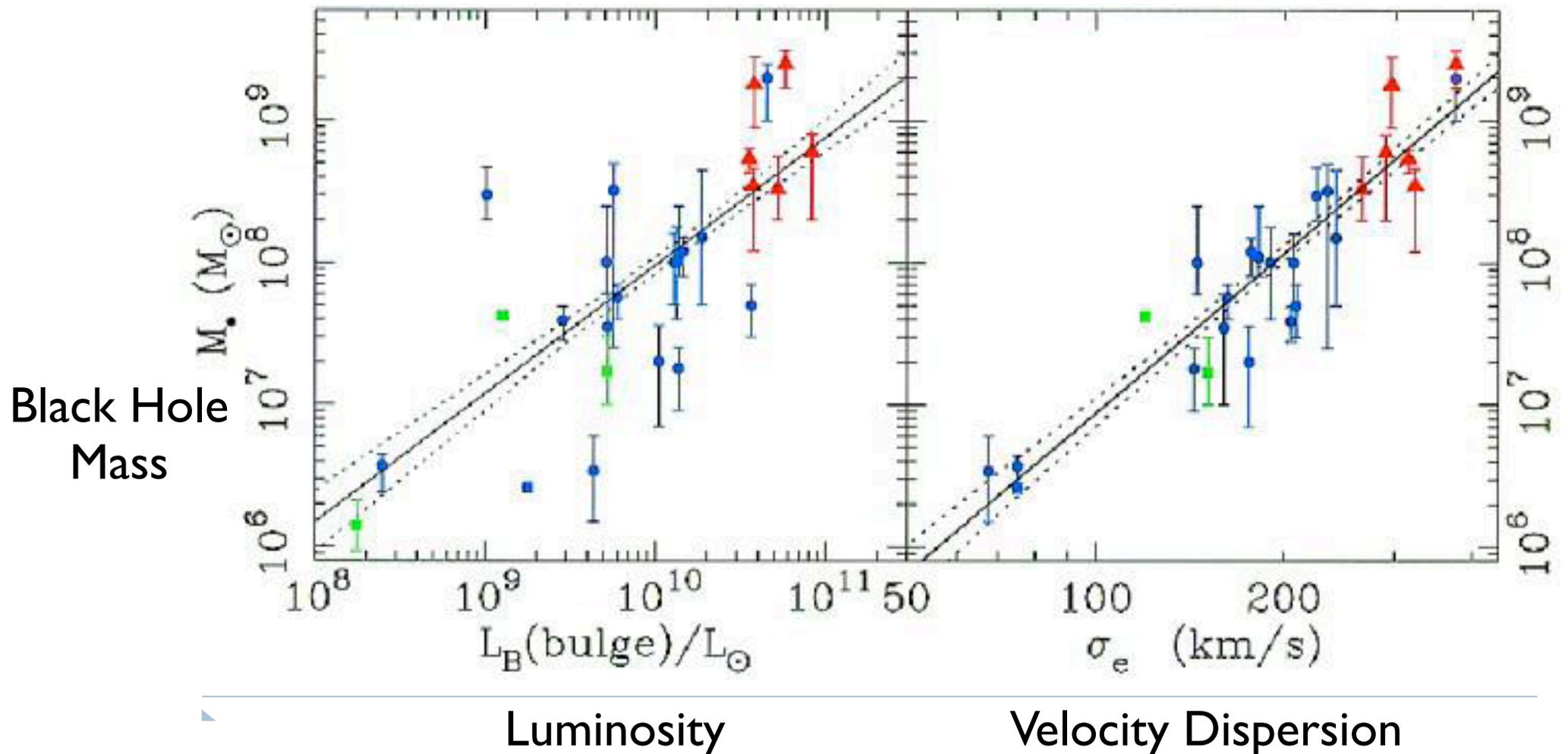


these black holes eject stars that they collide with gravitationally

Mass of ejected stars is similar to mass of supermassive black hole.

Super Massive Black Holes in Centers of Elliptical Galaxies

The mass of the black hole is $\sim 10^8$ to 10^9 solar masses and is strongly correlated with the luminosity and total mass of the elliptical galaxy. This is called the M- σ relation. There are a number of competing explanations for this trend.



There appear to be two different classes of elliptical galaxies.
They form in two different ways.

Case #1: “Wet” Mergers

(e.g., between two spiral galaxies)

(tends to occur more frequently for lower mass galaxies, when galaxy evolution less advanced)

Galaxy
(with gas)



Galaxy
(with gas)

Case #2: “Dry” Mergers

(e.g., between two elliptical galaxies)

(frequently occurs after many previous mergers, when the mass is higher)

Galaxy
(without gas)



Galaxy
(without gas)

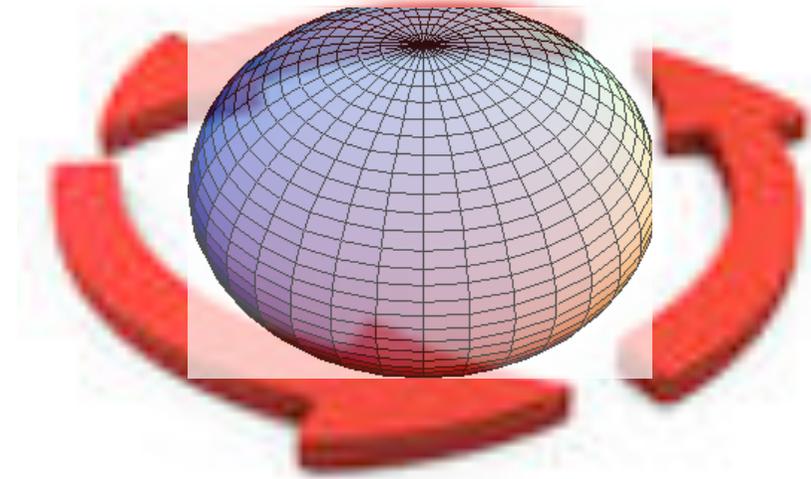
There appear to be two different classes of elliptical galaxies.
They form in two different ways.

Case #1: “Wet” Mergers

(e.g., between two spiral galaxies)

Appear to result in ellipticals that
appear consistent with being
rotationally flattened.

Rotationally supported



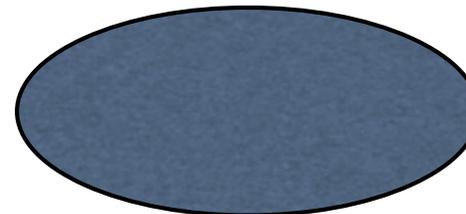
Case #2: “Dry” Mergers

(e.g., between two elliptical galaxies)

Appear to result in ellipticals who
ellipticity can be explained by
anisotropy in the velocity dispersion.

(Explained by anisotropy in velocity dispersion)

Random motions
greater in this direction



Random
motions less in
this direction

There appear to be two different classes of elliptical galaxies.
They form in two different ways.

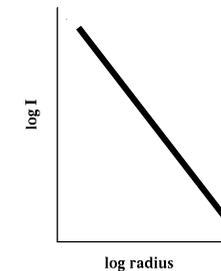
Case #1: “Wet” Mergers

(e.g., between two spiral galaxies)

Appear to result in ellipticals that
appear consistent with being
rotationally flattened.

Lower Luminosity

No core in
center of
elliptical



Disky Isophotes

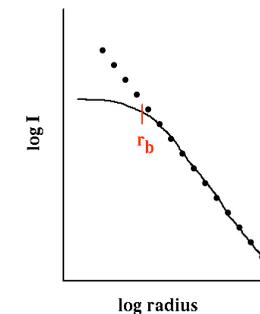
Case #2: “Dry” Mergers

(e.g., between two elliptical galaxies)

Appear to result in ellipticals who
ellipticity can be explained by
anisotropy in the velocity dispersion.

Higher Luminosity

Core in
center of
elliptical

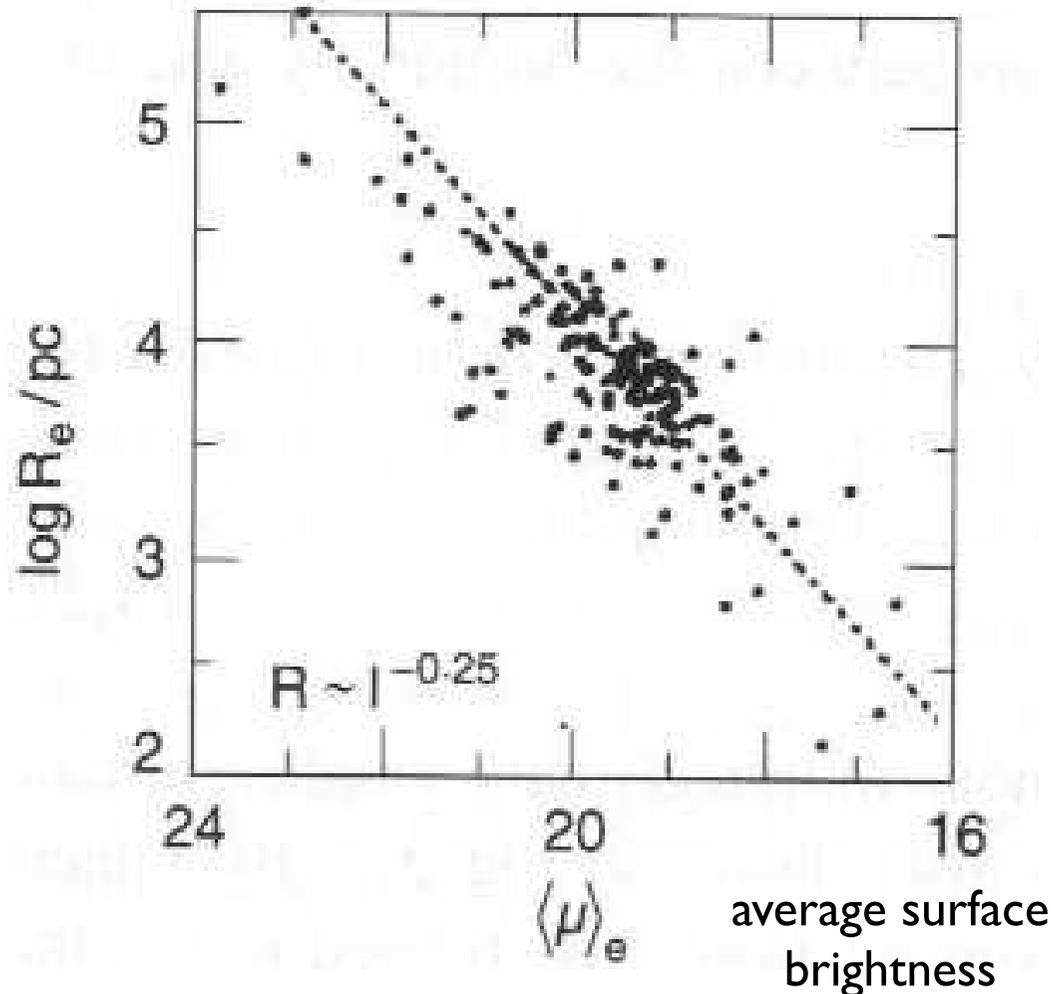


Boxy Isophotes

Intrinsic Correlations between Galaxy Properties

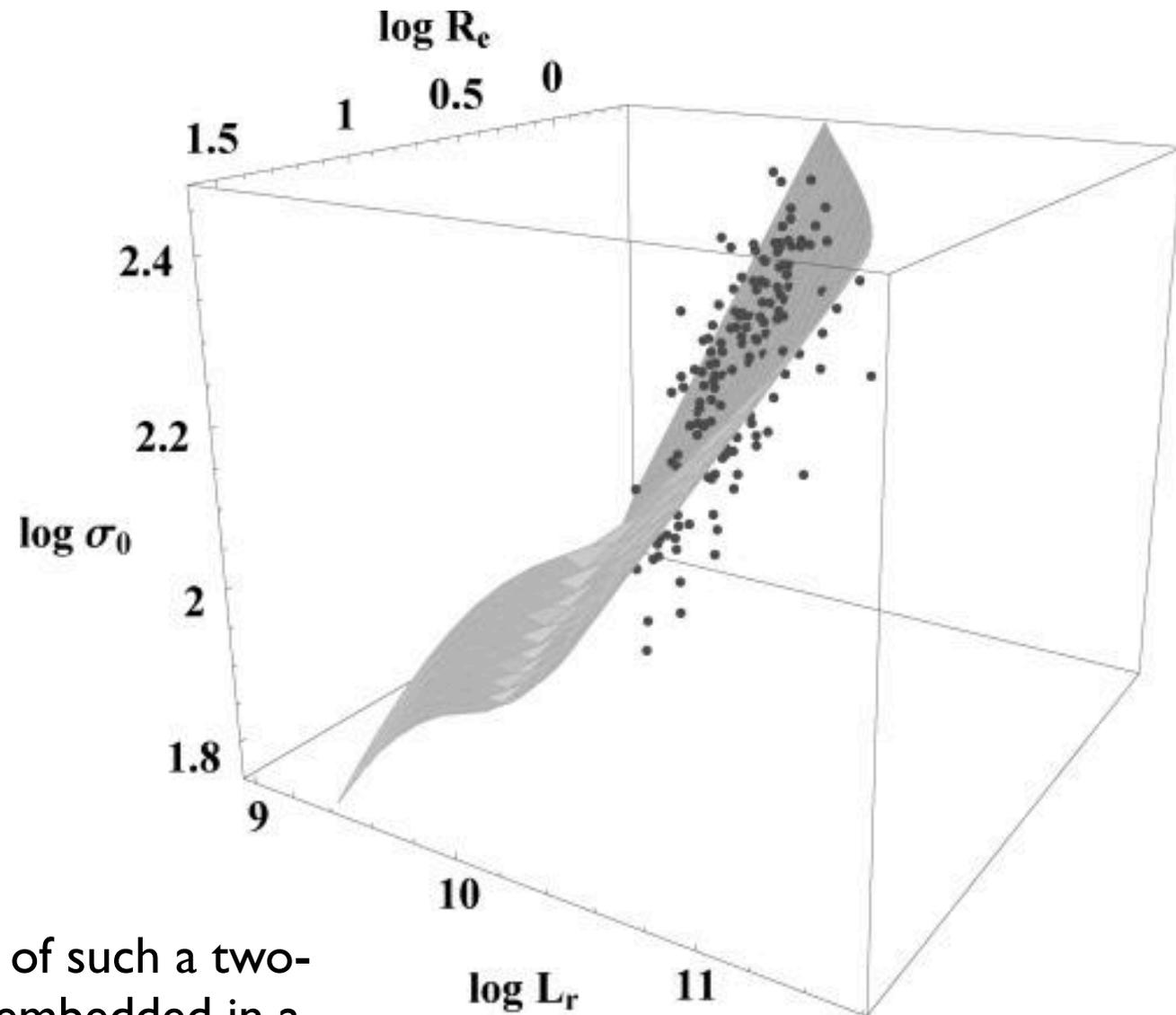
Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

half-light
radius
i.e., radius
containing
half of the
light



As we noted earlier, such correlations would not need to be present for a generic collisionless system and tell us something fundamental about the formation of galaxies themselves.

The properties of galaxies occupy a two dimensional plane in the three dimensional parameter space.

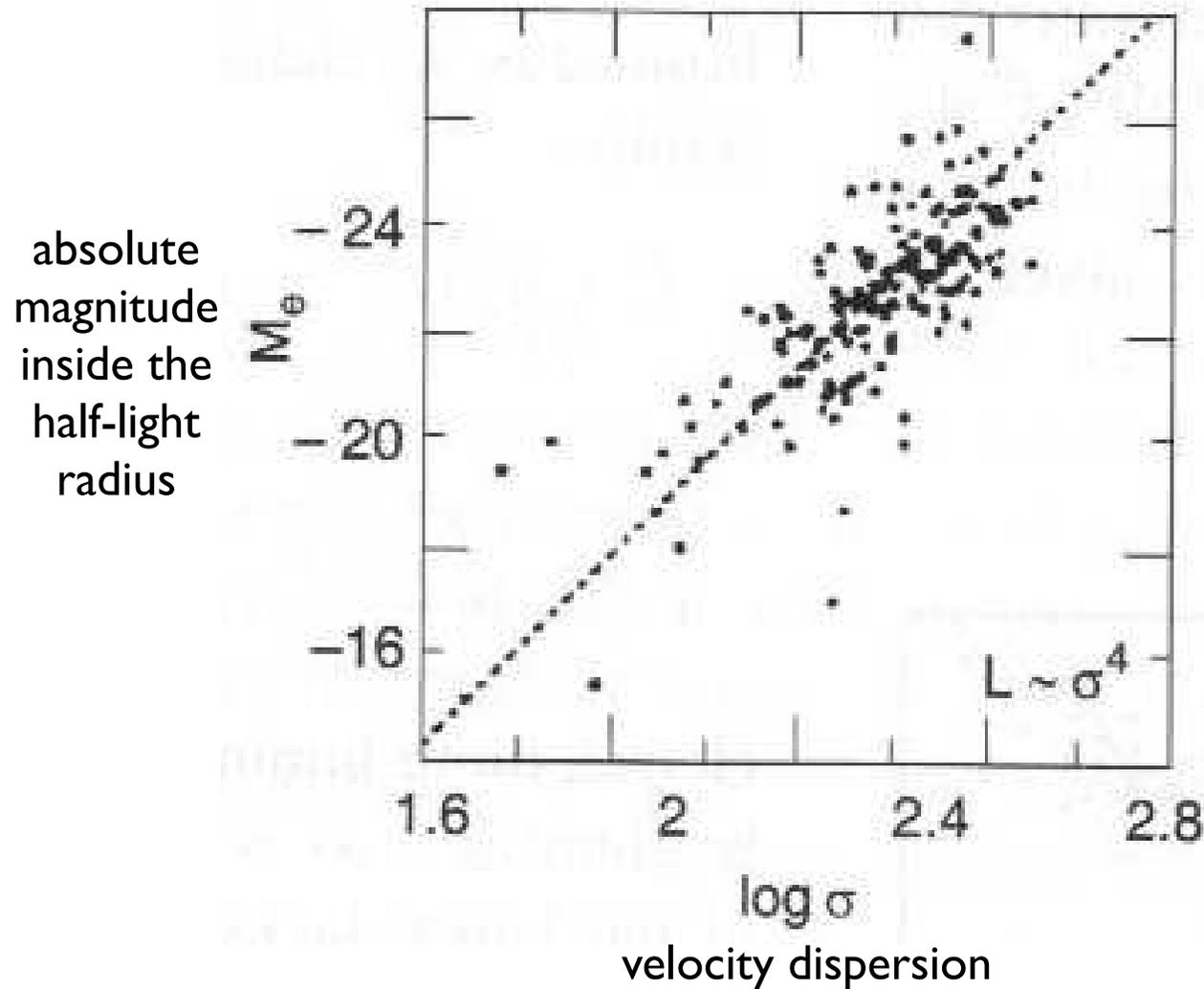


Here is an illustration of such a two-dimensional surface embedded in a three-dimensional space.

NOW new material for this
week

Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.



This is called
Faber-Jackson
Relation

And it is valuable as a
distance indicator in
Observational Cosmology

Velocity Dispersion Measurements

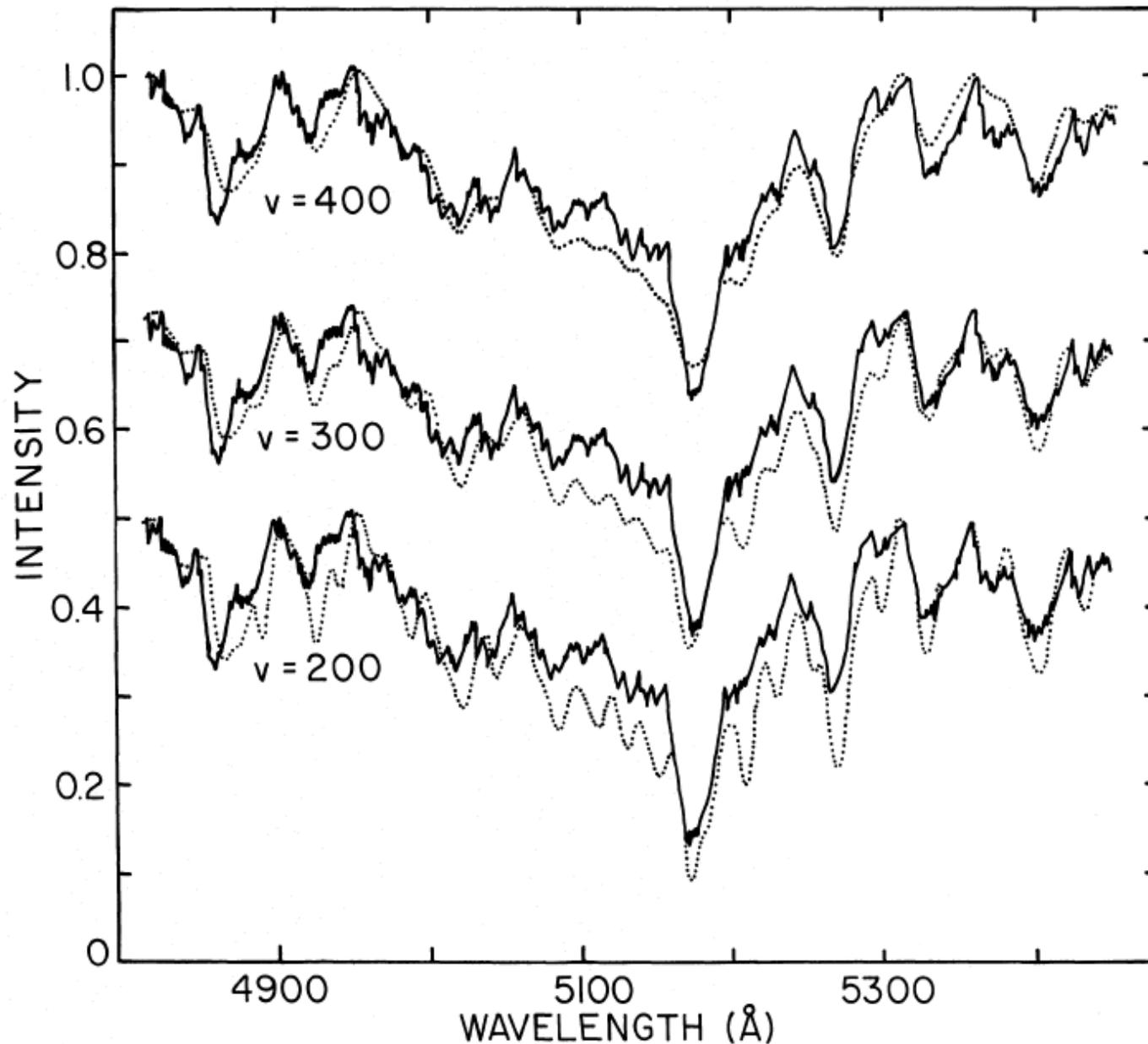
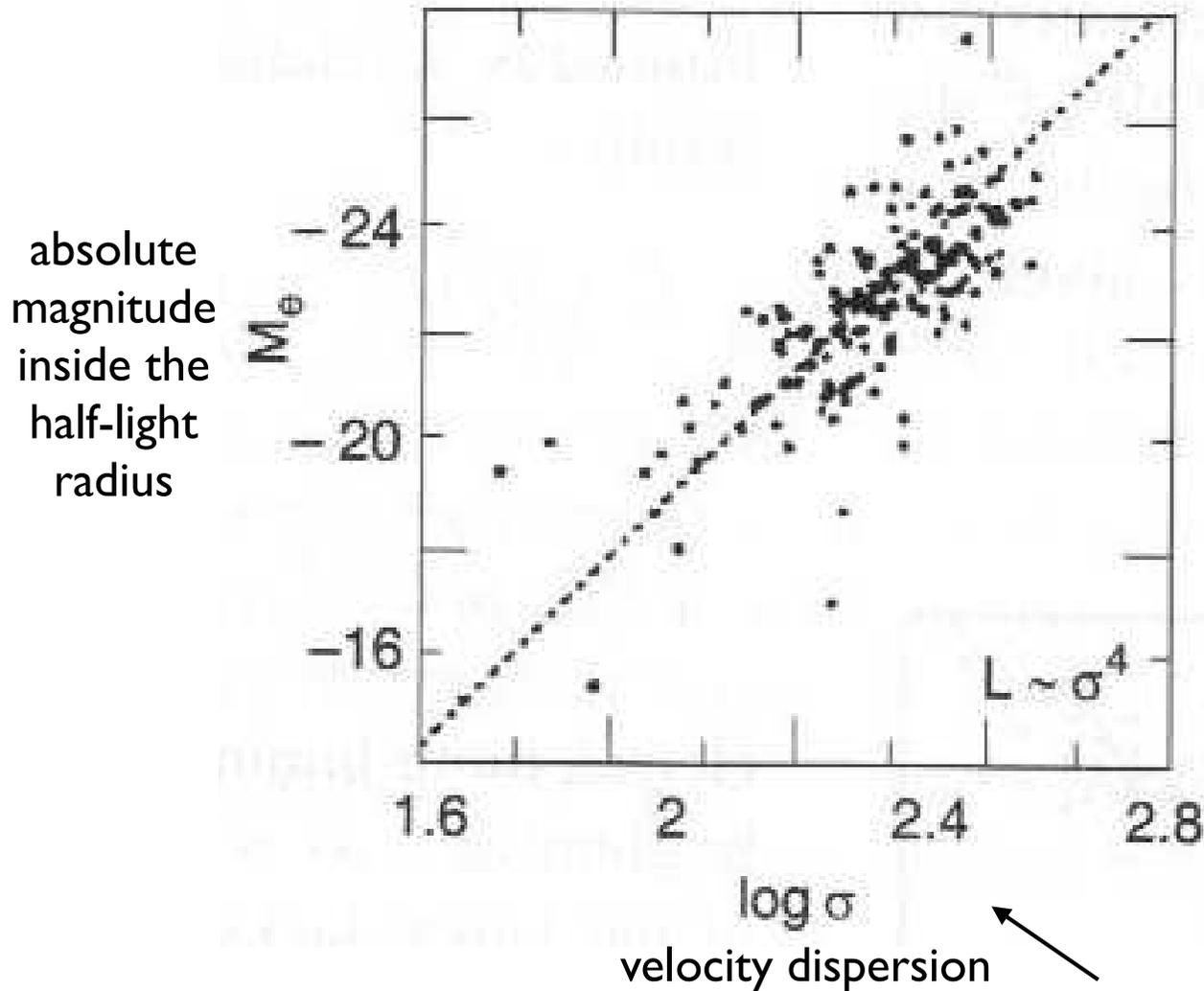


FIG. 3.—NGC 4472 compared with standard star HR 1805 (K3 III), broadened by various line-of-sight velocities (*dotted line*)

Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.



absolute
magnitude
inside the
half-light
radius

$\log \sigma$
velocity dispersion

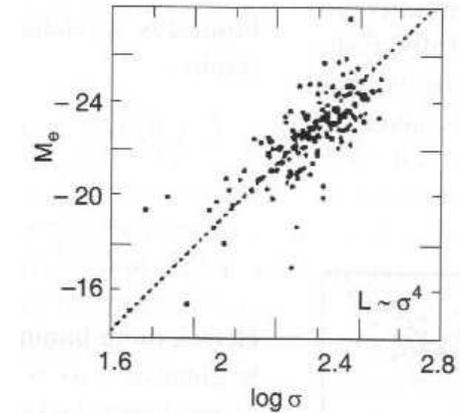
Directly
measurable

This is called
Faber-Jackson
Relation

And it is valuable as a
distance indicator in
Observational Cosmology

Intrinsic Correlations between Galaxy Properties

Note similarities between the Faber-Jackson relation:
 $L \propto \sigma^4$



And the Tully-Fisher relationship: $L \propto (v_c)^4$

From lecture 2 (for disk galaxies):

What type of scaling relations might we expect to hold?

If we assume that there is a fixed circular velocity at large radii (as is the case for many disk galaxies):

the mass enclosed in some radius is

$$M = Rv_c^2/G$$

Assuming that galaxies form at a fixed redshift, we would expect

$$M \propto \langle \rho \rangle R^3$$

Manipulating the second expression to derive an equation for R and substituting it in the first equation, we find

$$M \propto v_c^3$$

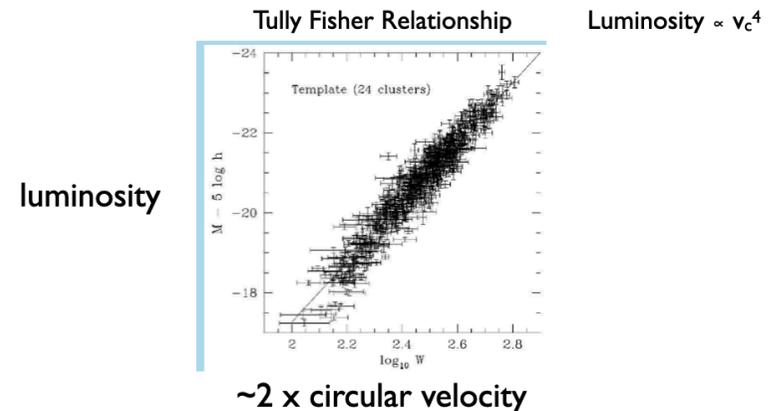
If the galaxy luminosity is proportional to mass, then

$$L \propto v_c^3$$

which isn't quite true, but is close to the observed scaling.

How do their structural parameters correlate?

The global properties of spiral galaxies are observed to correlate with each other:

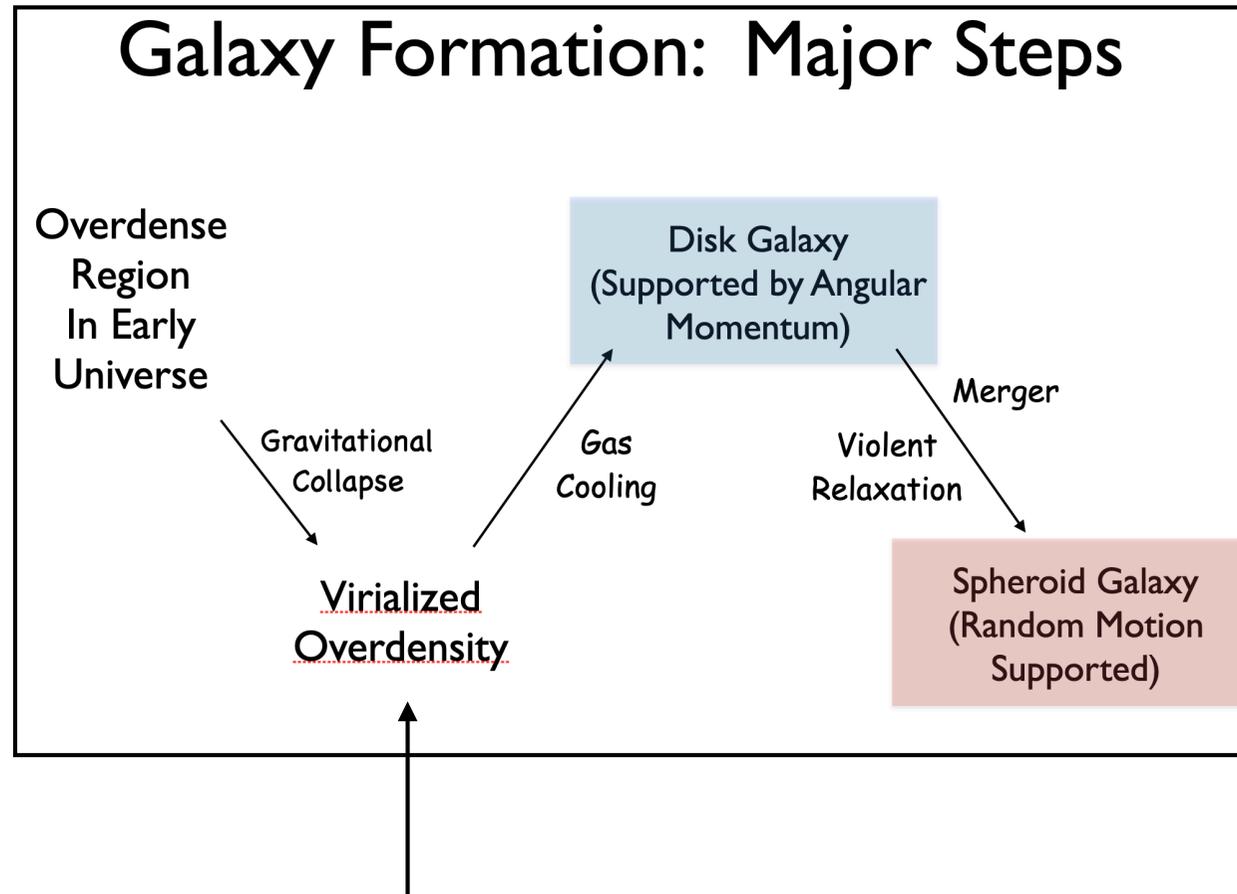


Slope of Tully-Fisher Relationship depends on which wavelength one measures the luminosity

Dark Matter Halos

Dark Matter Halos

Background:



Here we will focus on this step and how it impacts galaxy properties

Theory of Dark Matter Halos

Let's derive the key relations between the mass, radius, and velocity dispersion in halos

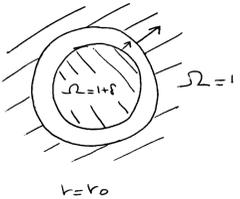
When a halo collapses --
how does ρ_{halo} compare to ρ_{critical} ?

Recall from the spherical collapse model (lecture 1):

FIRST STEP: Gravitational Collapse

Due to the proximity of Ω to 1 at early times, a small amount of extra matter could cause various regions of the universe to have a density $\rho > \rho_c$.

How would spherically symmetric regions of the universe at supercritical density $\rho > \rho_c$ evolve?



$\delta = \rho/\rho_c - 1$
 $\delta = \text{Overdensity relative to critical...}$

Because of spherical symmetries, can analyze force on outside of spherically symmetric region using only mass internal to it.

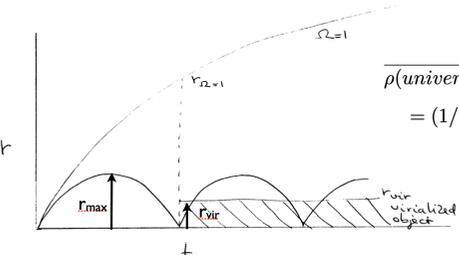
Can analyze the evolution of this region of the universe as if it were a "separate universe" with its own Ω . If Ω for this region is > 1 , then the region will eventually collapse.

FIRST STEP: Gravitational Collapse

Substituting $r = A(1 - \cos \theta)$, $t = B(\theta - \sin \theta)$ in the differential equation for the outer spherical shell, i.e.,

$$\ddot{r} = -\frac{GM(< r)}{r^2}$$

$$r_{\text{max}} = \frac{1}{\frac{1}{4}(12\pi)^{2/3}} r_{\Omega=1}(t_{\text{collapse}}) \quad (\approx 0.36 r_{\Omega=1}(t_{\text{collapse}}))$$

$$r_{\text{vir}} = \frac{1}{\frac{1}{2}(12\pi)^{2/3}} r_{\Omega=1}$$


$$\frac{\rho_{\text{vir}}}{\rho(\text{universe})(z = z_{\text{vir}})} = (r_{\Omega=1}/r_{\text{vir}})^3$$

$$= (1/2(12\pi)^{2/3})^3 = 18\pi^2 = 178$$


$$\rho_{\text{halo}} = 178 \rho_{\text{crit}}$$

$$\rho_{\text{halo}} \cong 200 \rho_{\text{crit}}$$

Theory of Dark Matter Halos

The total mass of a halo is given by the following:

$$M = (4\pi(r_{200})^3/3) 200 \rho_{\text{critical}}$$

where r_{200} is the radius of the halo

The ρ_{critical} is as follows:

$$\rho_{\text{critical}} = (3H(z)^2)/(8\pi G)$$

As such,

$$M = 100 H(z)^2 (r_{200})^3 / G$$

Theory of Dark Matter Halos

From the virial relationship:

$$(V_{200})^2 = GM/r_{200}$$

One can write expressions for the mass and radius of the halo in terms of the circular velocity and the Hubble constant:

$$M = (V_{200})^3 / 10GH(z)$$

$$r_{200} = V_{200}/10H(z)$$

The Hubble “constant” $H(z)$ increases towards higher redshifts, effectively scaling as $(1+z)^{3/2}$ at very high redshifts.

Halos of a given mass are more compact at high redshift.

Navarro-Frenk-White Density Profiles

Simulations show that collapsed halos approximately have the following density profile:

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

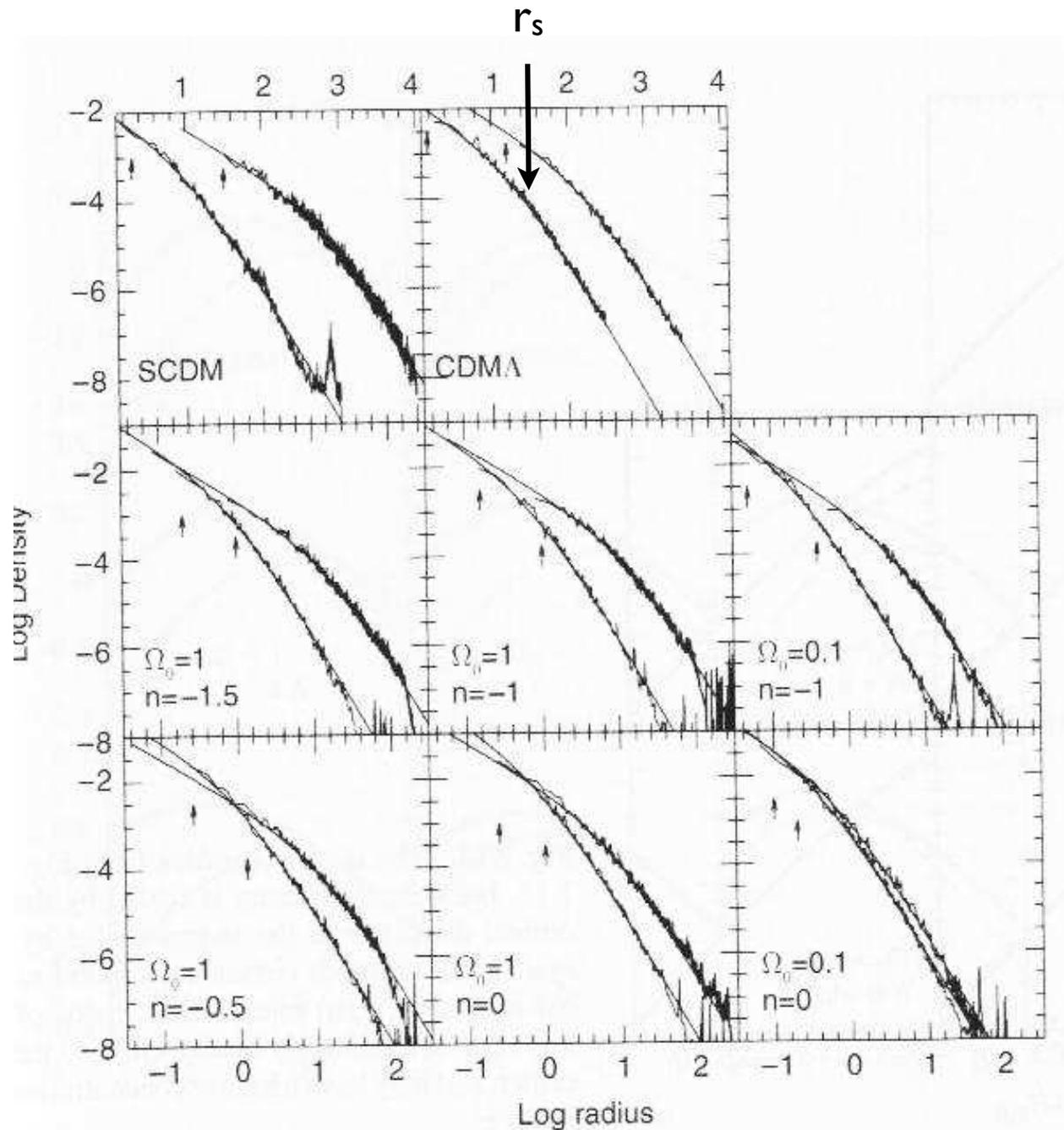
where r_s and ρ_s are some scaling parameters.

At small radii ($r < r_s$), the density profile ρ scales approximately as r^{-1}

and at large radii ($r > r_s$), the density profile ρ scales approximately as r^{-3}

At $r \sim r_s$, the density profile ρ changes slope

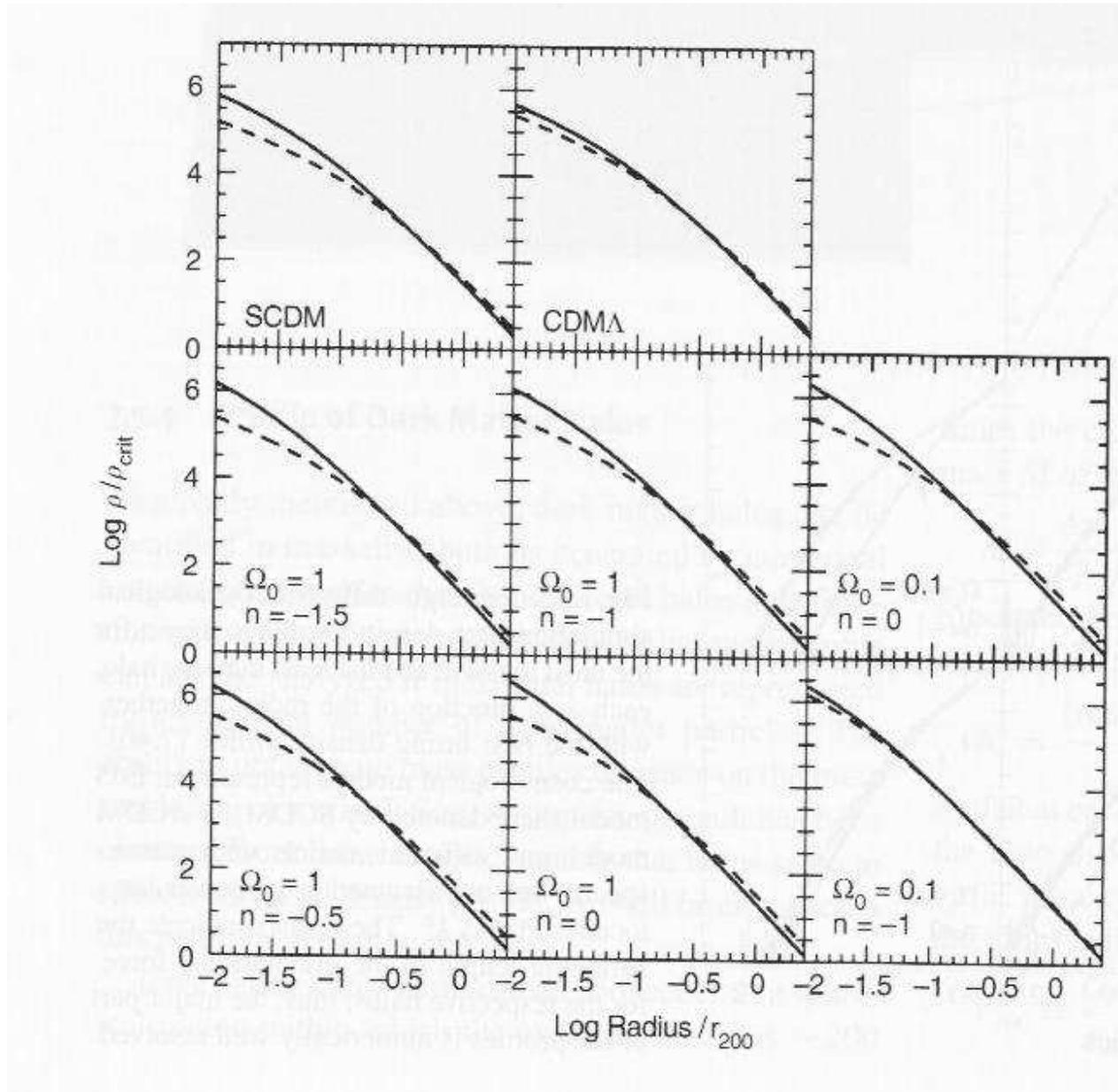
Here are some examples of the density profiles from simulations:



SCDM = standard cold
dark matter
(no dark energy)

Λ CDM = cold
dark matter with
dark energy

Here are some examples of the density profiles from simulations:



SCDM = standard cold
dark matter
(no dark energy)

Λ CDM = cold
dark matter with
dark energy

Navarro-Frenk-White Density Profiles

Define concentration parameter $c = r_{200} / r_s$

Requiring the total mass in the halo is equal to $200 \rho_{\text{crit}}(z)$
 $(4\pi/3 (r_{200})^3)$, one can show that

$$\rho_s = \frac{200}{3} \rho_{cr}(z) \frac{c^3}{\ln(1+c) - c/(1+c)}$$

so that the density profile of a halo is completely determined by its mass and concentration parameter c .

Navarro-Frenk-White Density Profiles

From simulations, one finds that the concentration parameter c for a halo is closely related to its formation redshift:

$$c \propto \frac{M^{-1/9}}{M_*} (1 + z)^{-1}$$

Lower mass halos have higher concentration parameters.

Navarro-Frenk-White Density Profiles

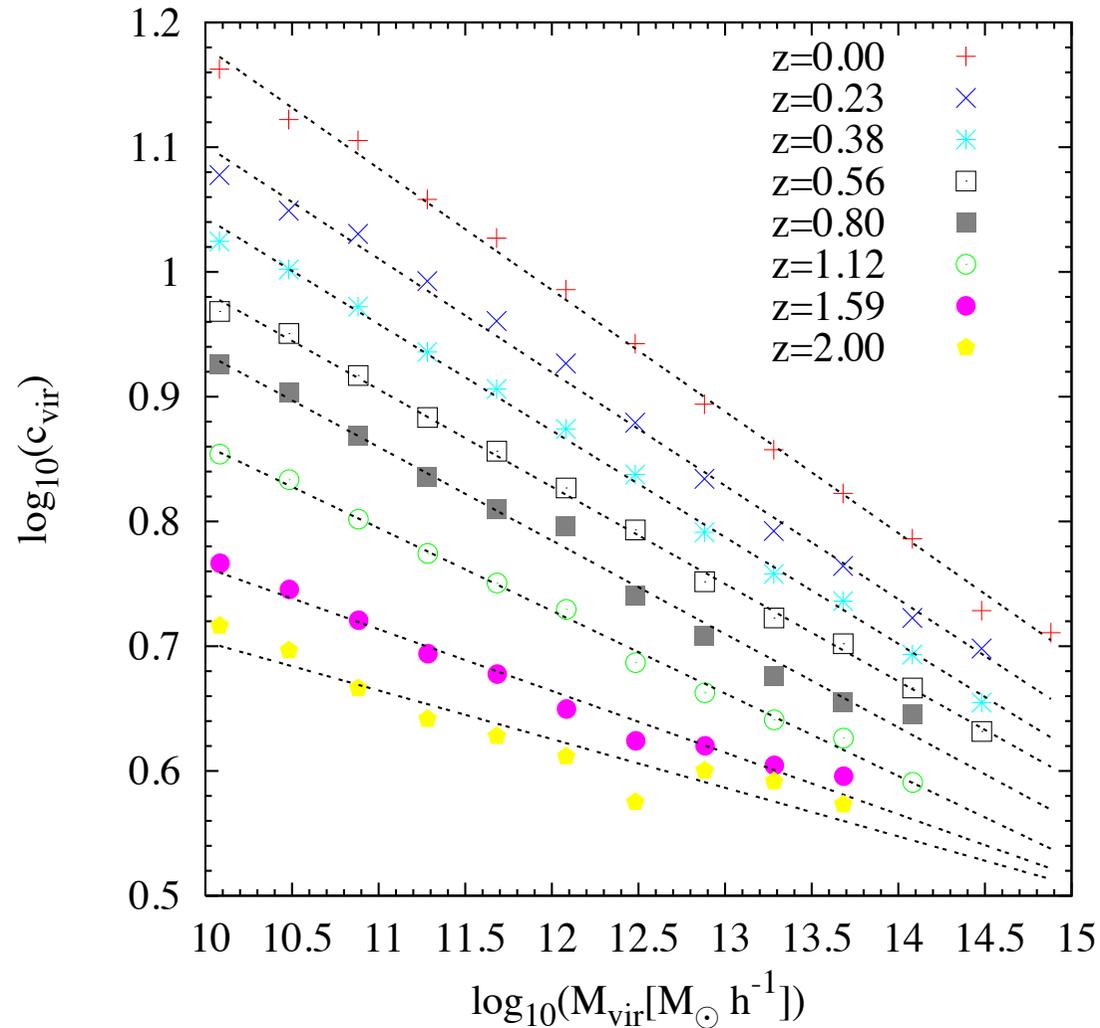
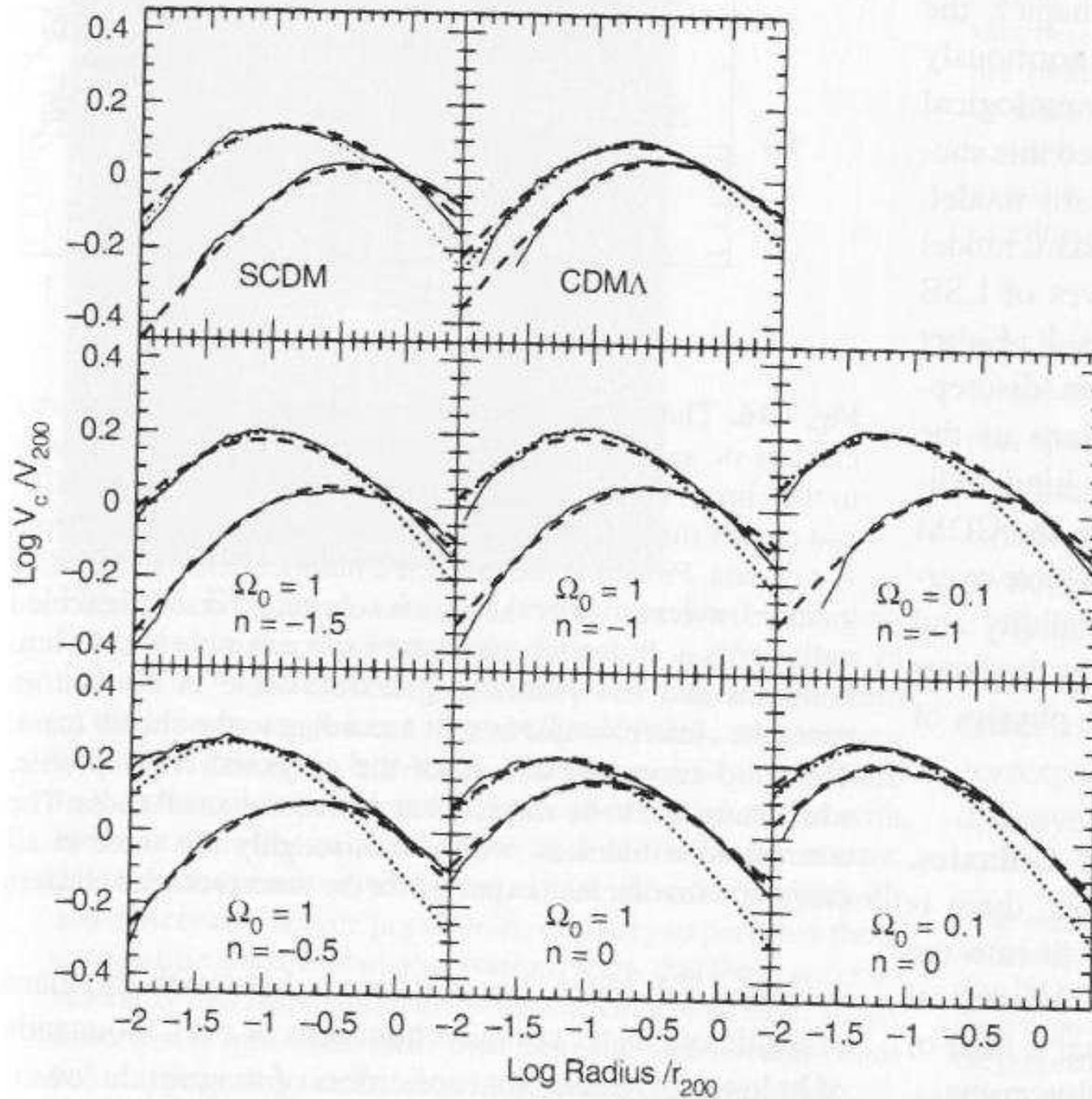


Figure 1. Mass and redshift dependence of the concentration parameter. The points show the median of the concentration as computed from the simulations, averaged for each mass bin. Lines show their respective linear fitting to eq. 5.

Let's also look at the rotational curves:



Does this make sense
in terms of
 $V^2 \sim GM(R)/R$?

Collapsed Halos: Comparison with the Observations

First -- we look at the structure of the dark matter halo
for very massive collapsed halo ($M \sim 10^{14} - 10^{15} M_{\text{solar}}$)

i.e., as appropriate for galaxy clusters

Halos of Galaxy Clusters: Comparison against the Observations

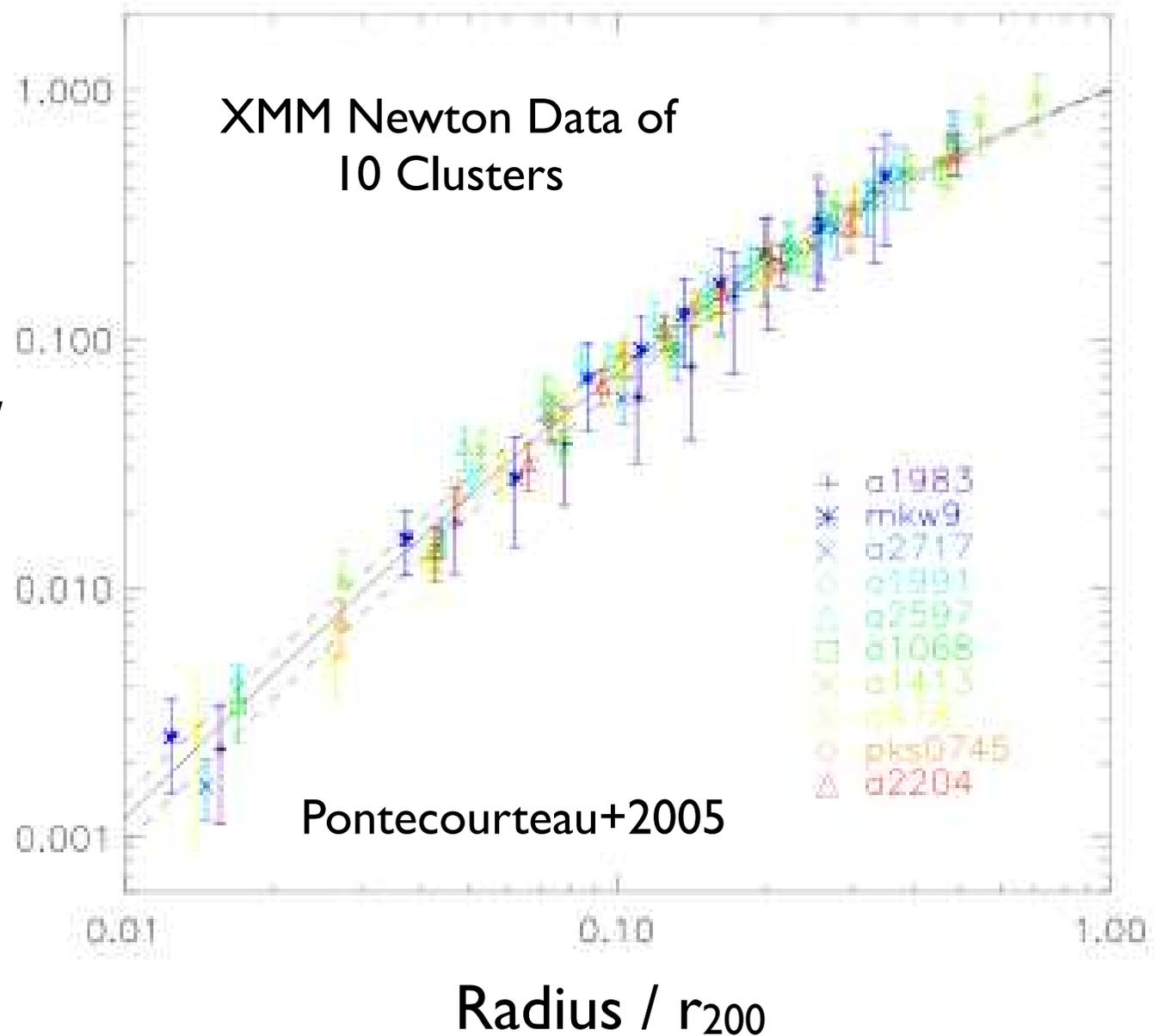
Radial Profile

Information from
x-ray profiles

Mass /
 M_{200}

Mass Profile comes assuming
hydrostatic equilibrium

Pressure gradient balances
gravitational forces!

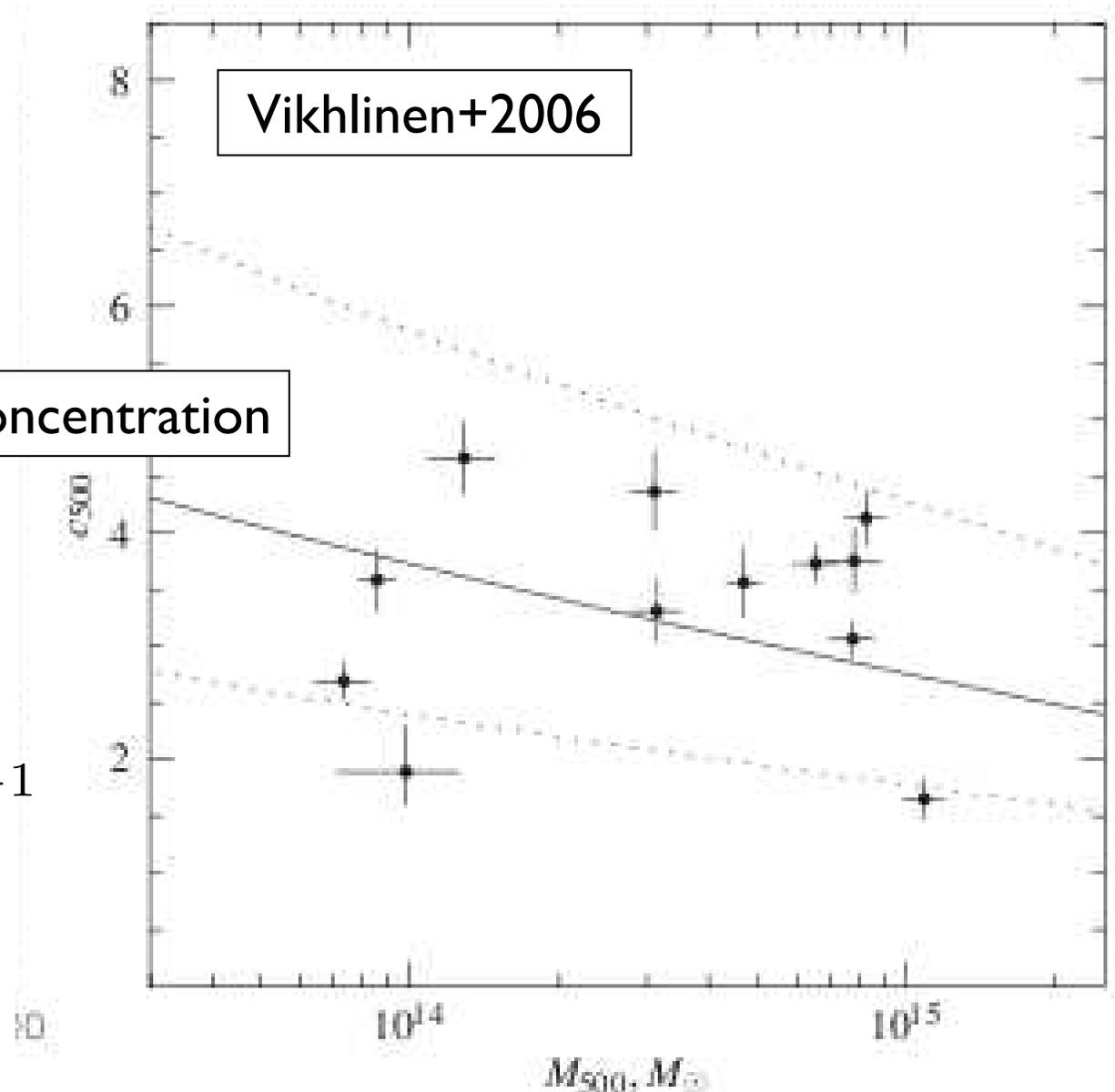


Halos of Galaxy Clusters: Comparison against the Observations

Do the 10 clusters from previous page fit the theoretical concentration vs. mass relationship?

$$c \propto \frac{M^{-1/9}}{M_*} (1+z)^{-1}$$

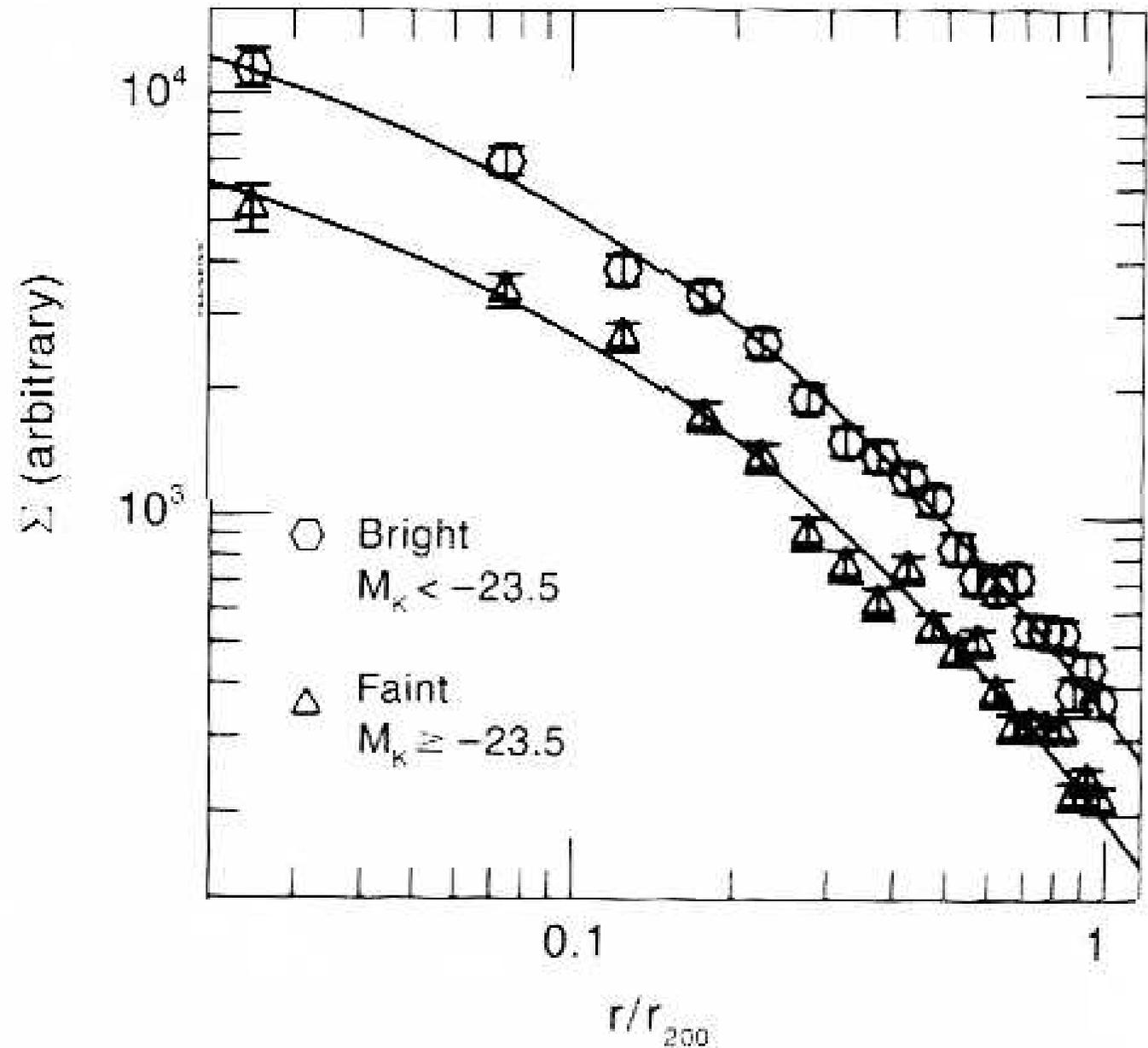
Concentration



Cluster Mass

Halos of Galaxy Clusters: Comparison against the Observations

We can also use the apparent surface density of galaxies within clusters vs. radius to probe the density profile.



Halos of Galaxies: Comparison against the Observations

We can also examine the mass profiles of galaxies to see if they follow the expected mass profile:

What are some techniques we can use?

1. Kinematics of satellite galaxies
2. Kinematics of distant stars
3. Gravitational Lensing by Galaxy

Kinematics of Satellite Galaxies:

One method is to take advantage of the satellite companions found around galaxies in the nearby local universe as a probe of kinematics.

Unfortunately, one only tends to find one such satellite galaxy per central galaxy (with a measured velocity along the line of sight), so it is not possible to make a reliable measurement using individual galaxies.

To make progress, one needs to treat these galaxies as orbiting around some “composite average” galaxy.

Results are in good agreement with an NFW profile

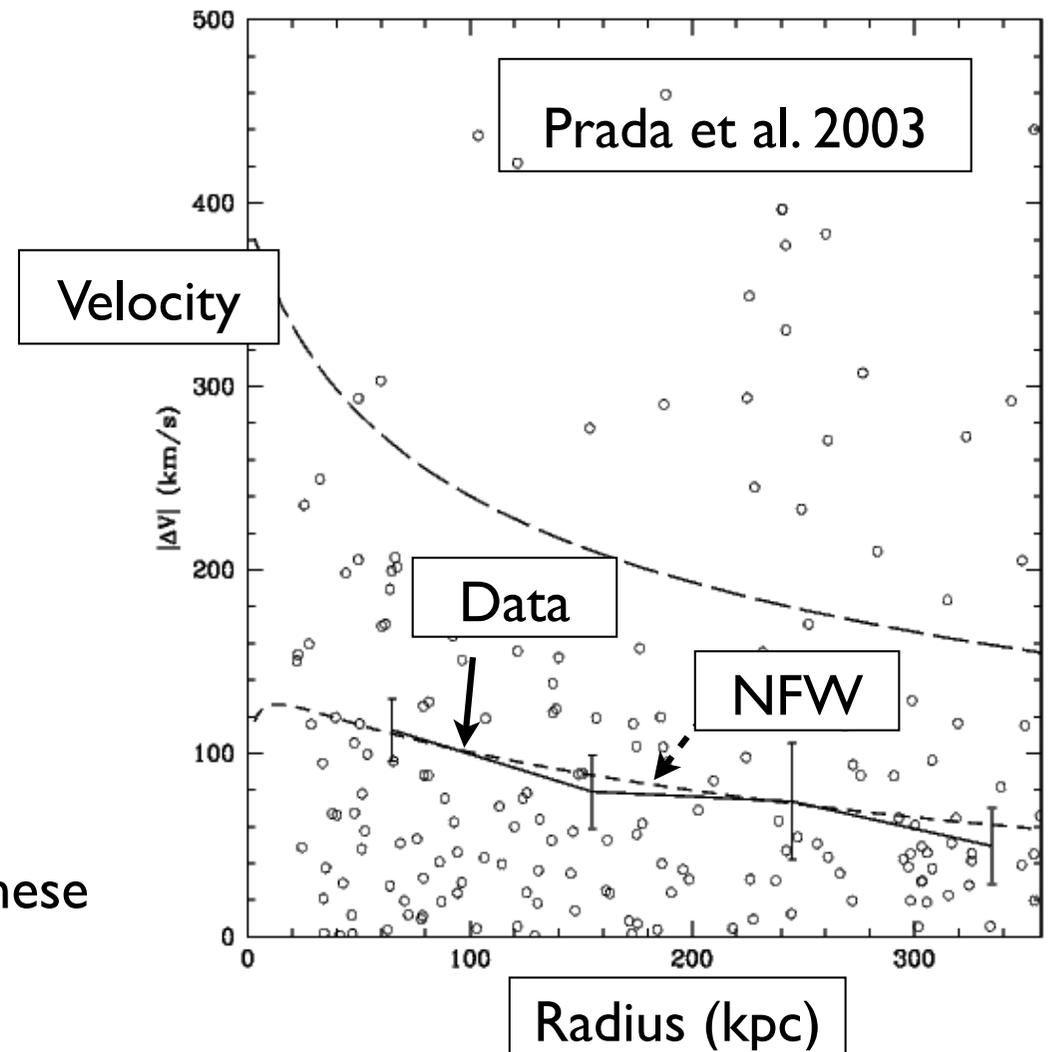
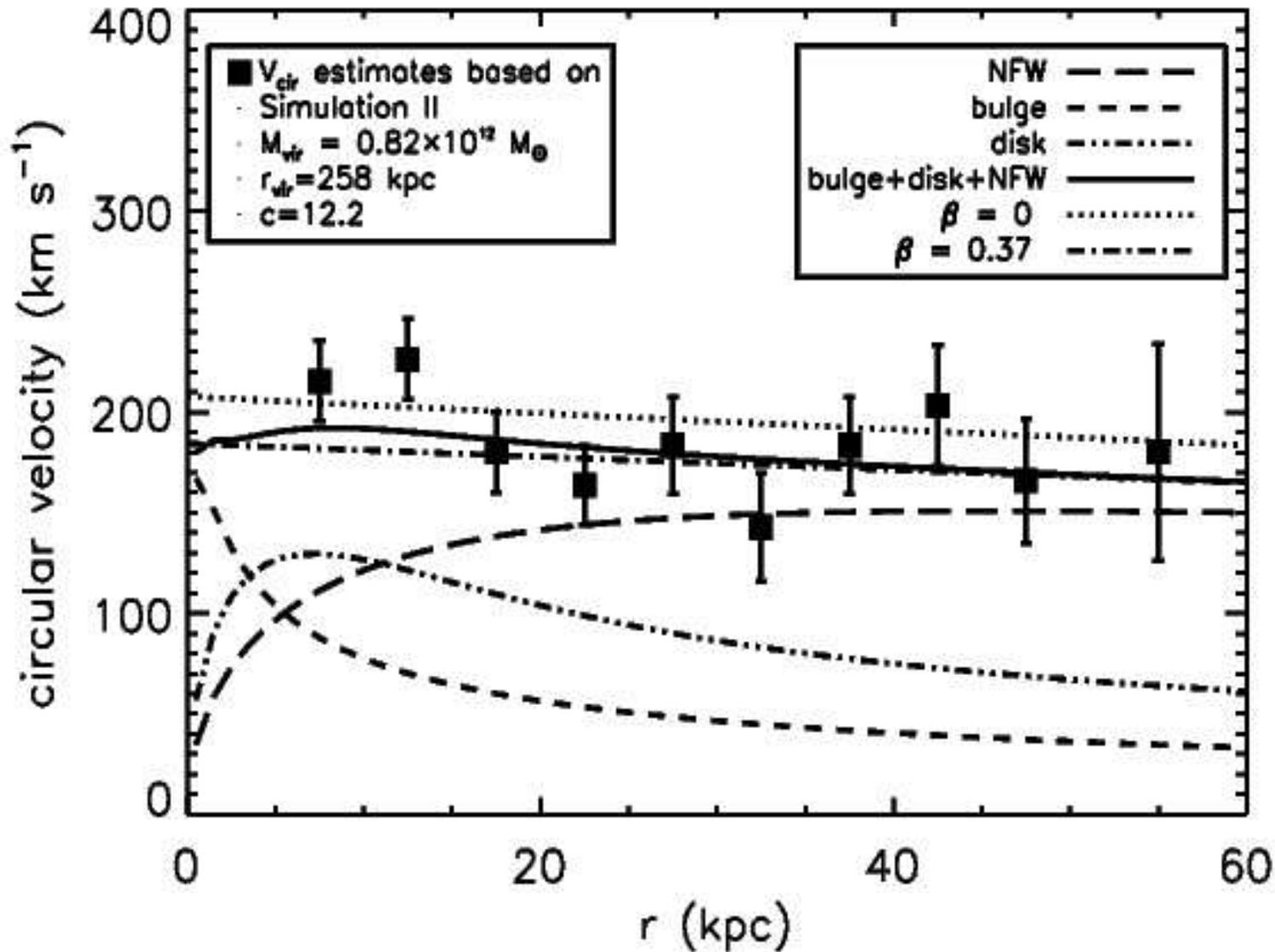


FIG. 9.—Same as in Fig. 7, but with the removal of the interlopers. The solid curve with the error bars shows the rms velocity after removal of interlopers. As in Fig. 8, the rms is clearly declining and consistent with the NFW profile (*short-dashed curve*) with $M_{\text{vir}} = 1.5 \times 10^{12} M_{\odot}$. The long-dashed curve shows the escape velocity from the NFW halo of this mass. All satellites above the escape velocity curve are interlopers.

Kinematics of Distant Stars

Most easily applied to our own galaxy.
One use of this method shown below
(Xue et al. 2008)

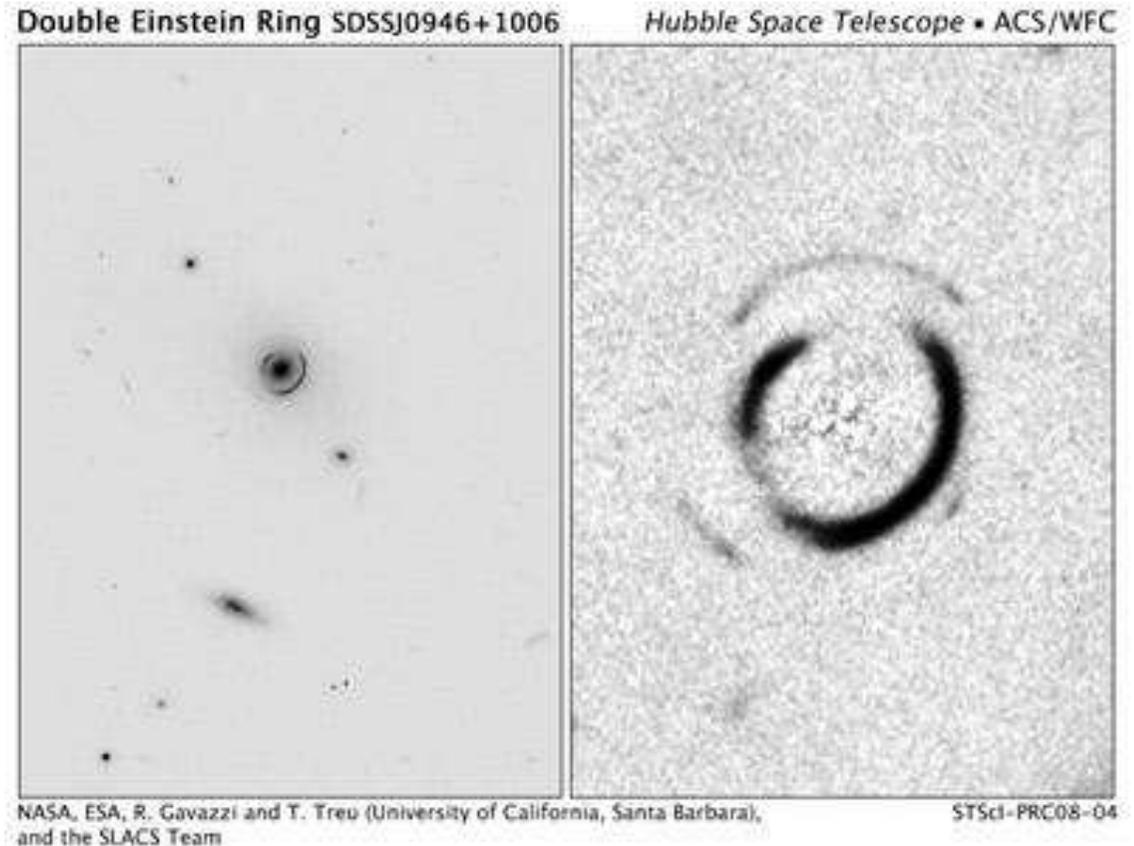
Applied to 60 kpc! (which is
nevertheless smaller than 200 kpc:
which is approximately the halo radius)



Gravitational Lensing by Galaxy

Gravitational Lensing also gives us a direct way of measuring the mass inside some radius.

For sources we can see an approximate Einstein ring, measuring the mass enclosed is easy -- since we can derive the mass enclosed at the ring radius just from the geometry.

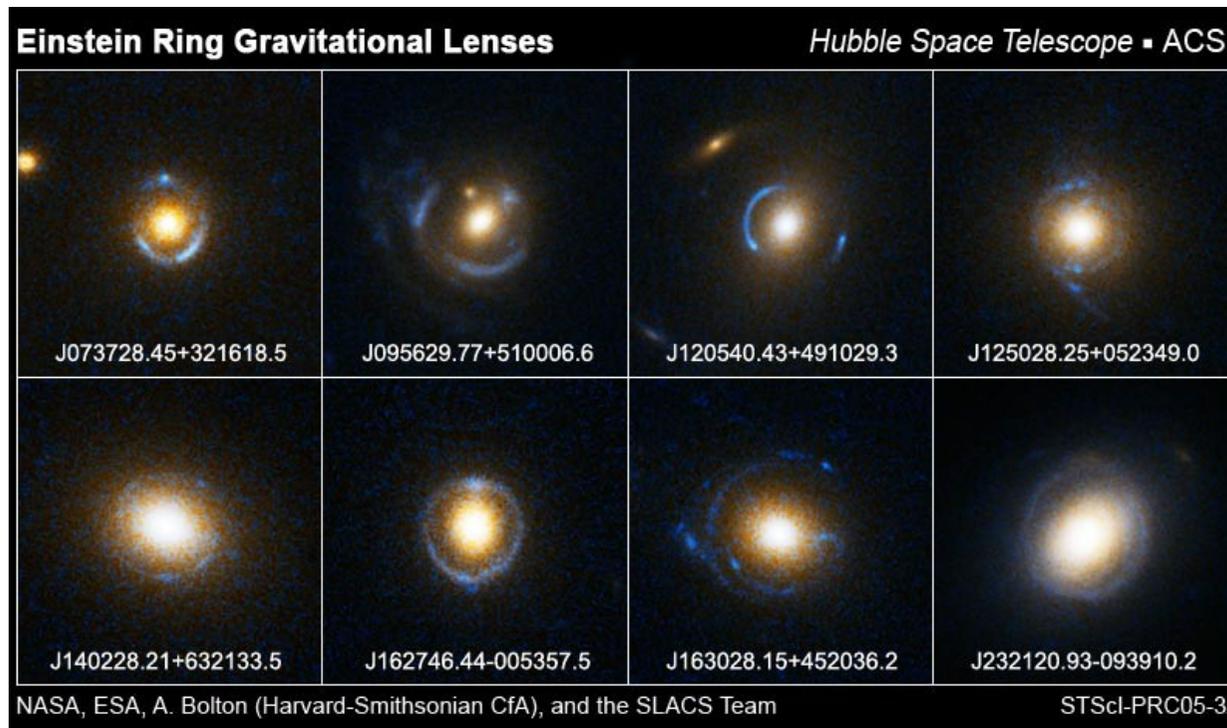


In fact, there are now large programs, i.e., SLACS, to use exactly these sort of techniques to measure the mass profile of elliptical galaxies.

Gravitational Lensing by Galaxy

The Sloan Lens ACS Survey (SLACS)

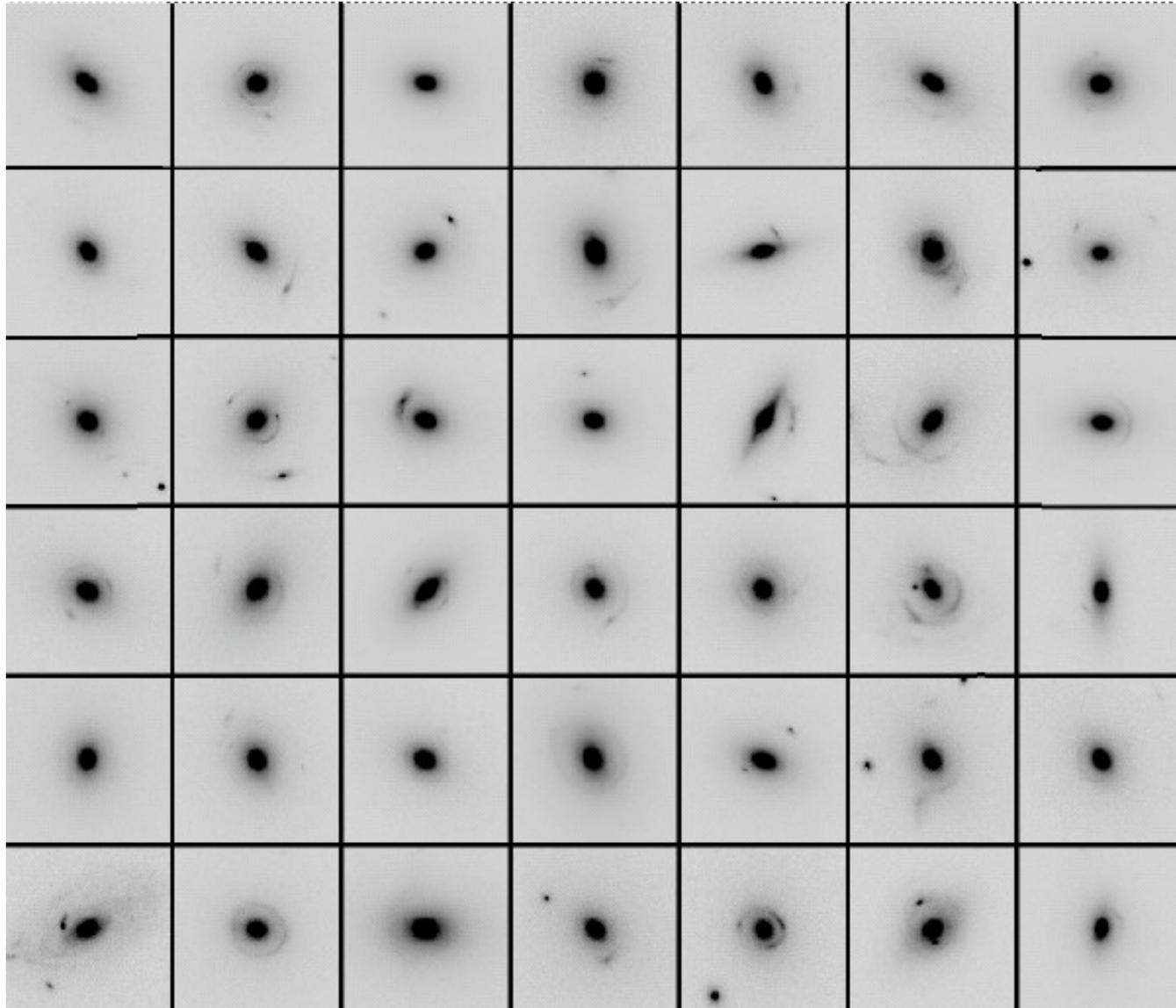
Cols: T. Treu (UCSB), L. Koopmans (Kapteyn),
A. Bolton (CfA), S. Burles (MIT), L. Moustakas (JPL)



Spectroscopic selection (spurious emission lines), then HST follow-up imaging for confirmation and for accurate modeling

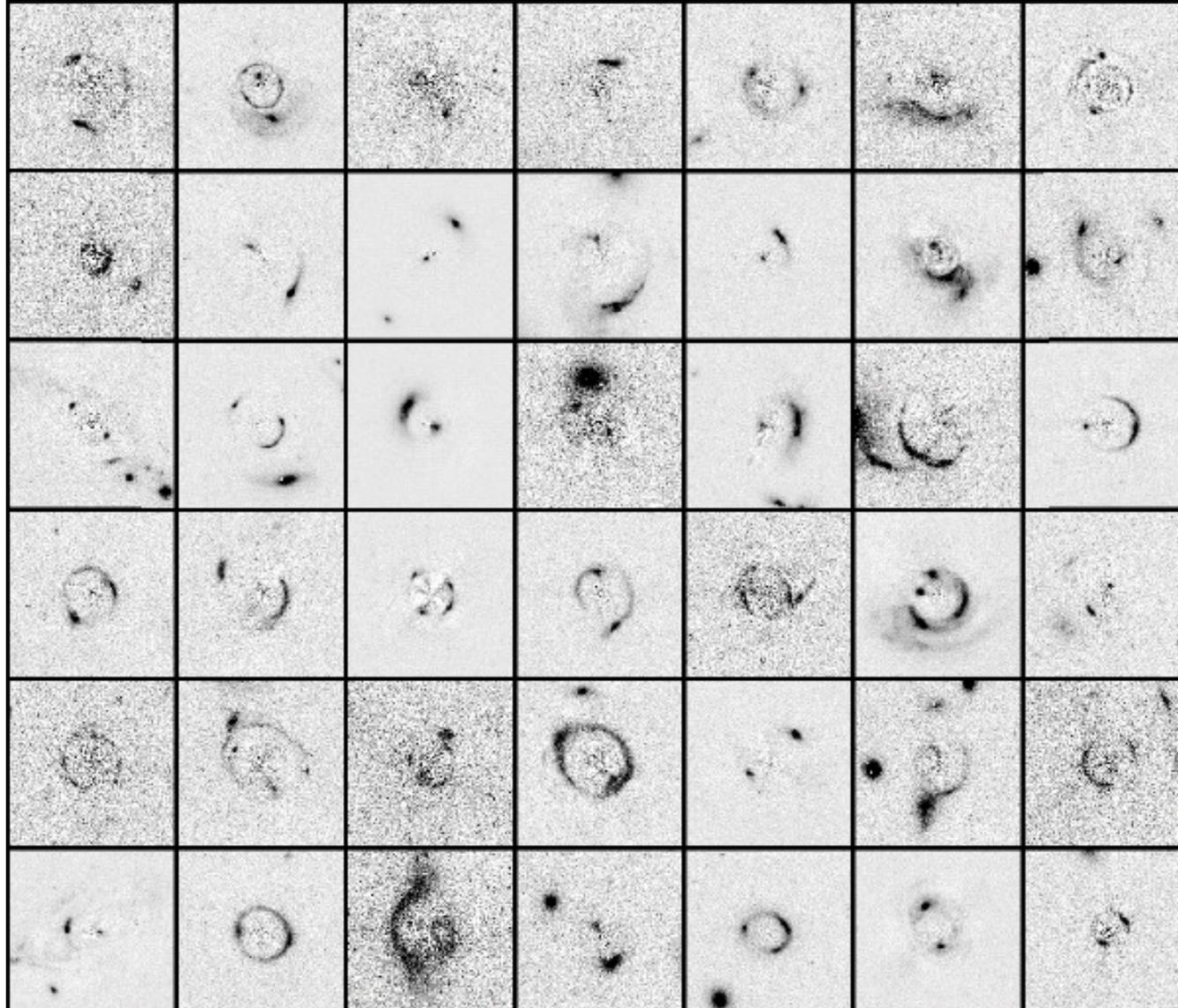
Gravitational Lensing by Galaxy

SLACS sample



Gravitational Lensing by Galaxy

SLACS sample

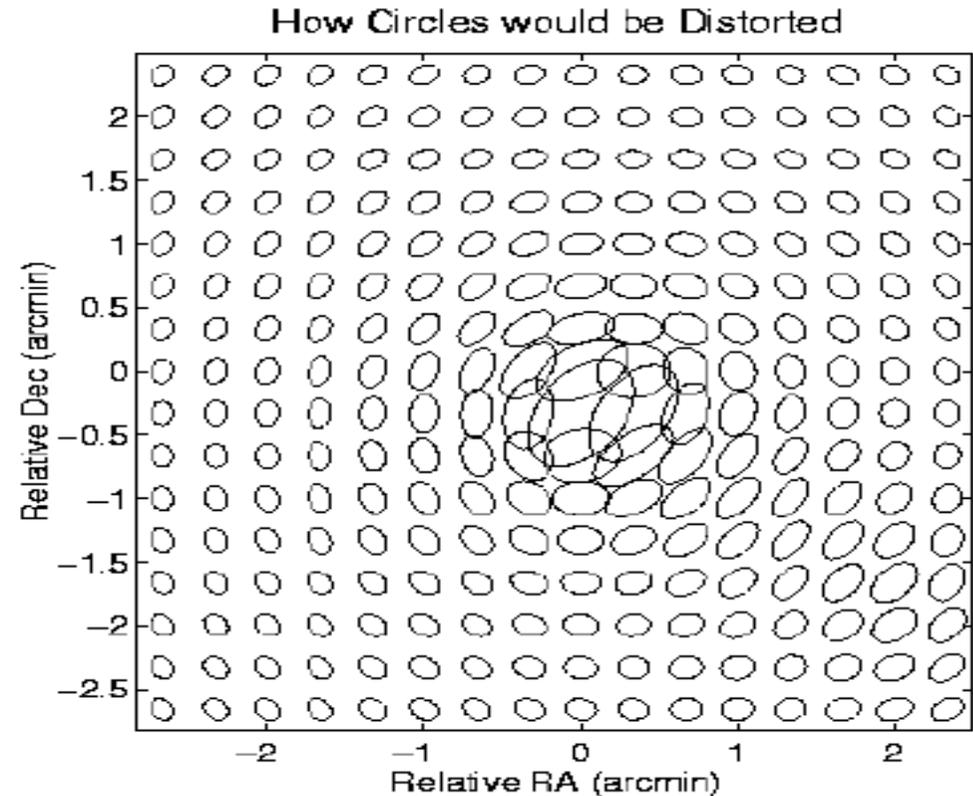


Gravitational Lensing by Galaxy

Despite some success in this regard, it is not always easy to find large numbers of galaxies with Einstein rings.

An alternate technique is to measure the average impact of gravitational lensing of the shapes of galaxies.

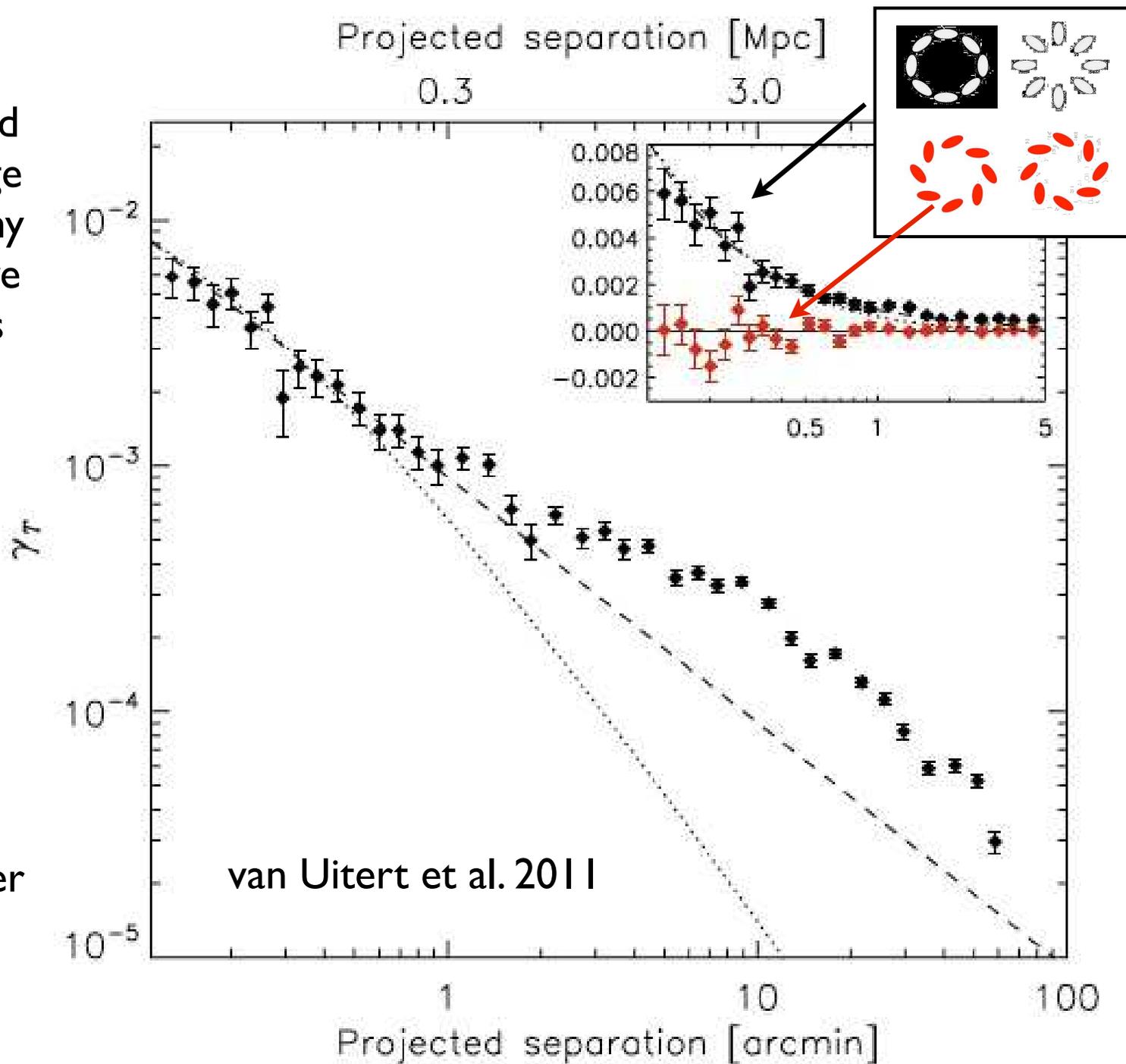
Impact of gravitational lensing is to introduce subtle change in the shape of galaxies on average (so elongated along an angle tangential to the galaxy)



By looking at many foreground galaxies and determining the average change in shape of many background galaxies, we can measure the mass profile of the galaxy.

Such averaging is required because background galaxies have random shapes and orientations.

However, averaging over enough galaxies, one can overcome the random component.



Challenges / Issues with our standard model for the
collapsed dark matter halos

Cores of Halos

What can we infer about the cores of dark matter halos from the observations?

Where should we look?

Almost all collapsed halos have baryons cooling and falling towards their centers. These baryons will change the mass profile of the original collapsed halo.

Basically, we have the following situation:

$$V(R)^2 R / G \sim M(R) \sim M_{\text{DARK}}(R) + M_{\text{COOL}}(R)$$

To obtain best constraints on $M_{\text{DARK}}(R)$, we need $M_{\text{COOL}}(R)$ to be as small as possible.

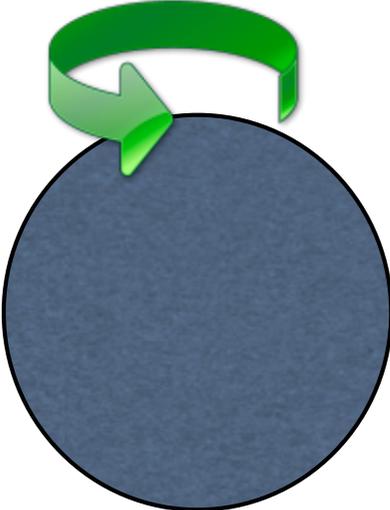
Cores of Halos

For which galaxies would we expect the halos to be the least affected by the baryons cooling to the centers?

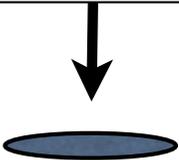
For which galaxies, will $M_{\text{COOL}}(R)$ be the smallest?

Here are two galaxies where the baryonic component has different angular momentum. For which case, can we get a cleaner look at the dark matter halo?

spinning moderate speed

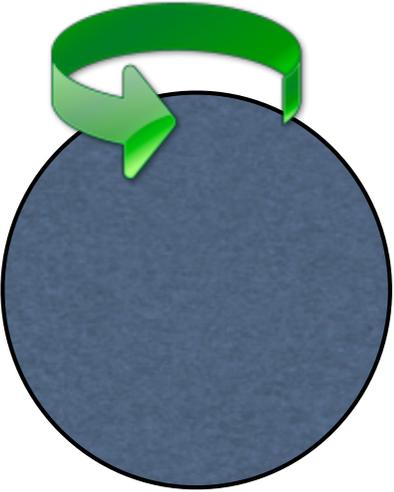


after cooling

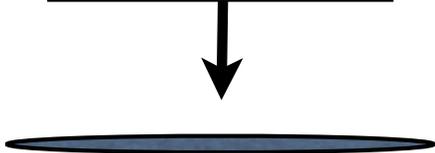


intermediate size,
more dense disk galaxy
higher surface brightness

spinning fast

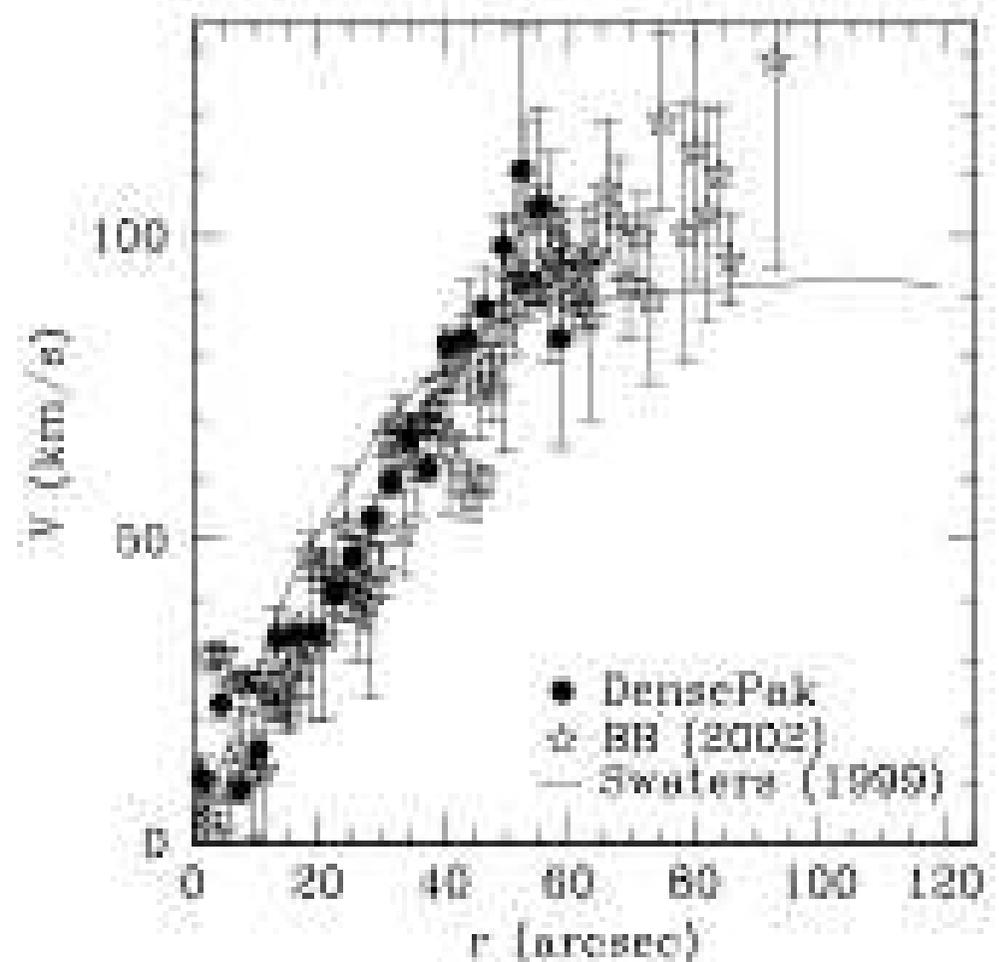
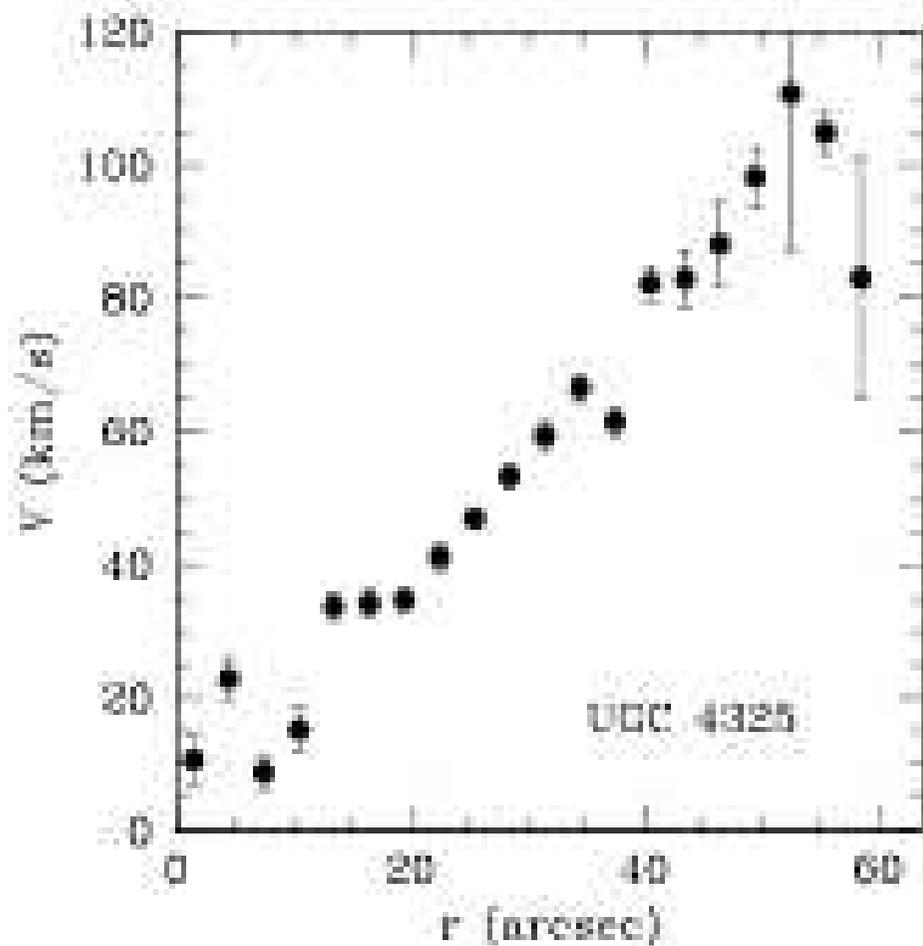


after cooling

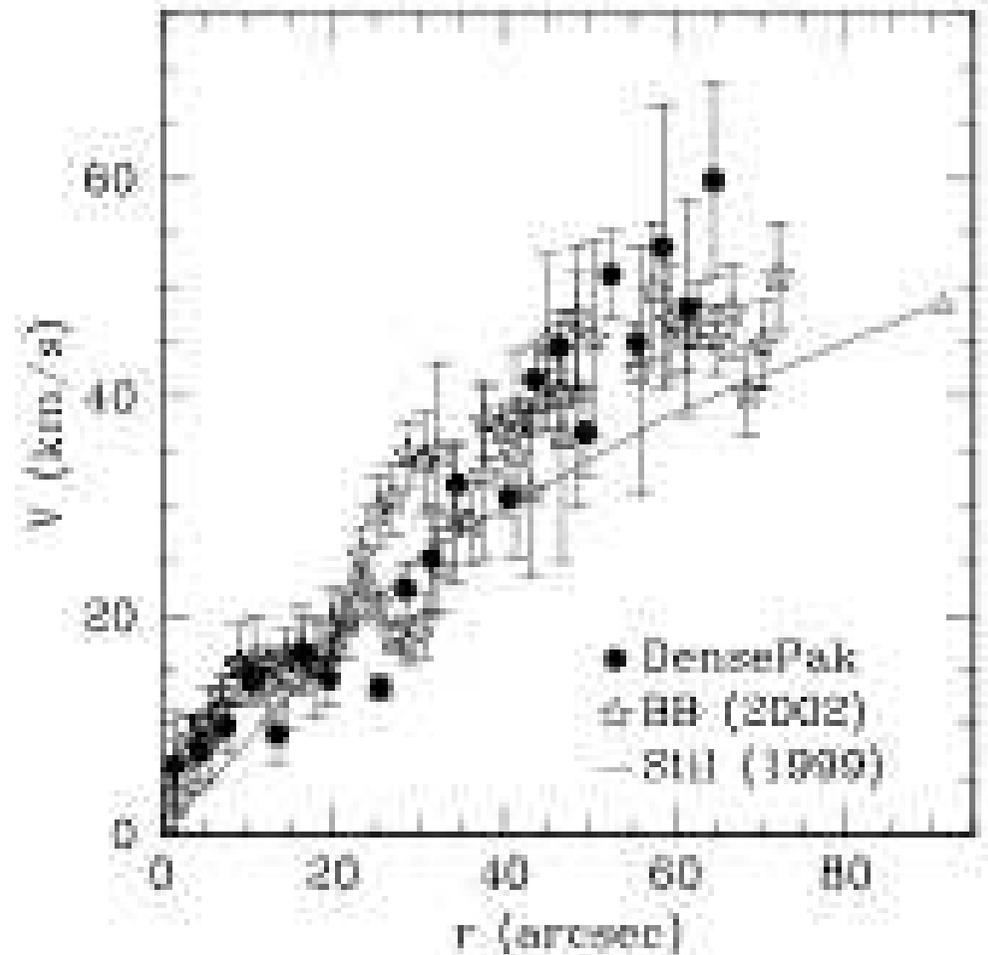
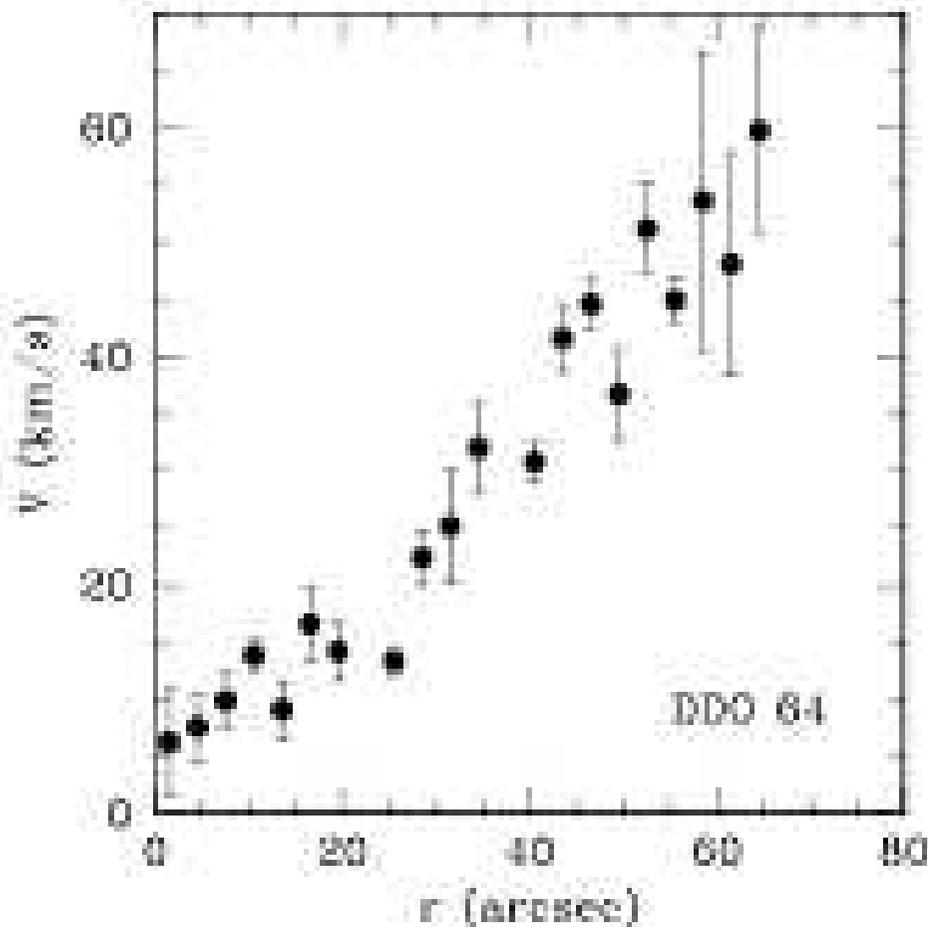


large,
lower density galaxy disk
lower surface brightness

Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:



Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:



Cores of Halos

What would infer about the mass profile of a halo where the circular velocity appears to rise linearly with radius?

Using the relation $M = V(R)^2 R / G$,

we would infer that $M \propto R^3$ or $\rho \sim$ independent of radius

But for NFW, we would expect $\rho \propto r^{-1}$ at small radii

This discrepancy is known as the core-cusp problem.

It is thought that baryonic feedback may be able to flatten the density profiles

Substructure

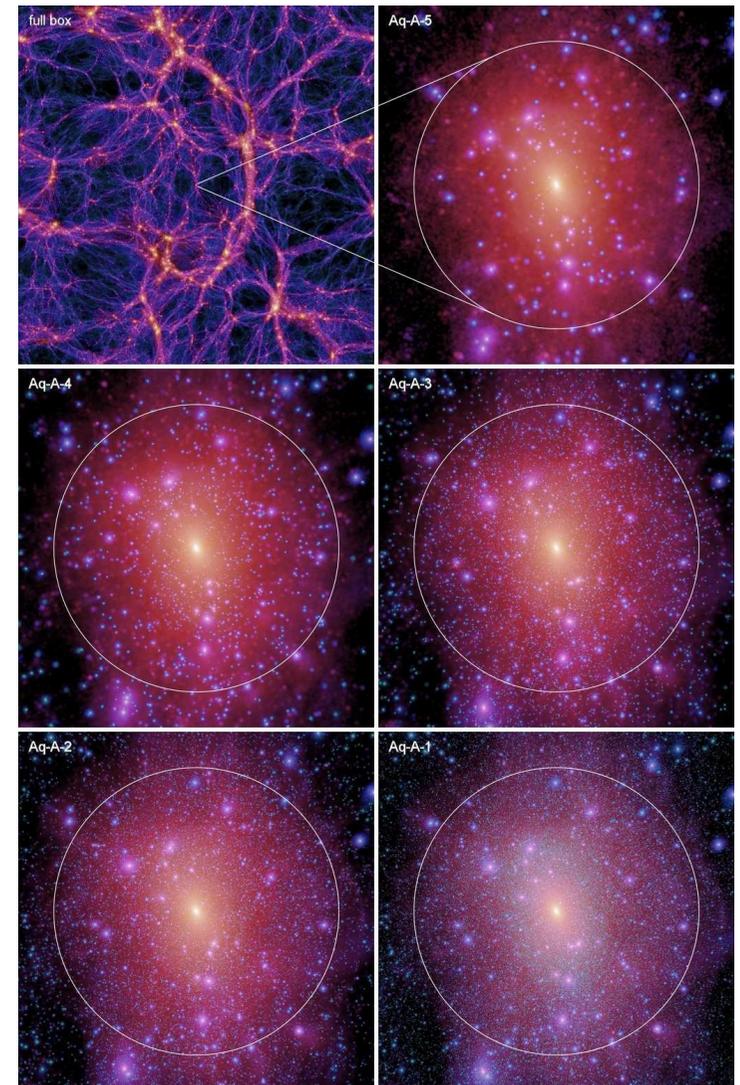
You might assume that the halos from the simulations are smooth.

But they are not perfectly smooth, as you can see to the right:

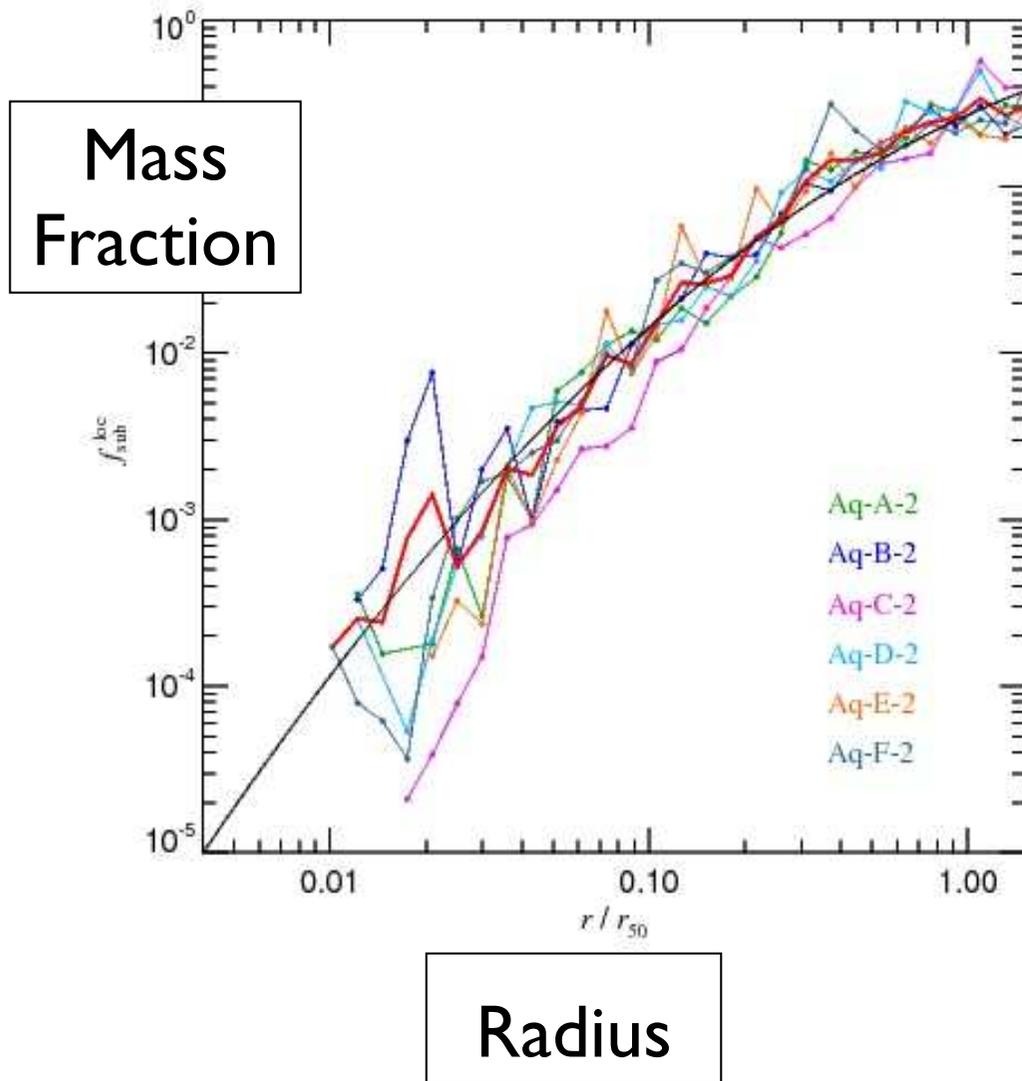
These halos include large numbers of subhalos.

What is the origin of these subhalos?

Previously, they were their own collapsed halos, but merged into a bigger halo



What fraction of the mass do the subhalos contain?



Subhalos contain only a small fraction of the mass at the centers of the main halo

But subhalos contain a large fraction of the mass at larger radii.

Why? Subhalos have a hard time remaining together in the center of the main halo, due to the higher densities there.

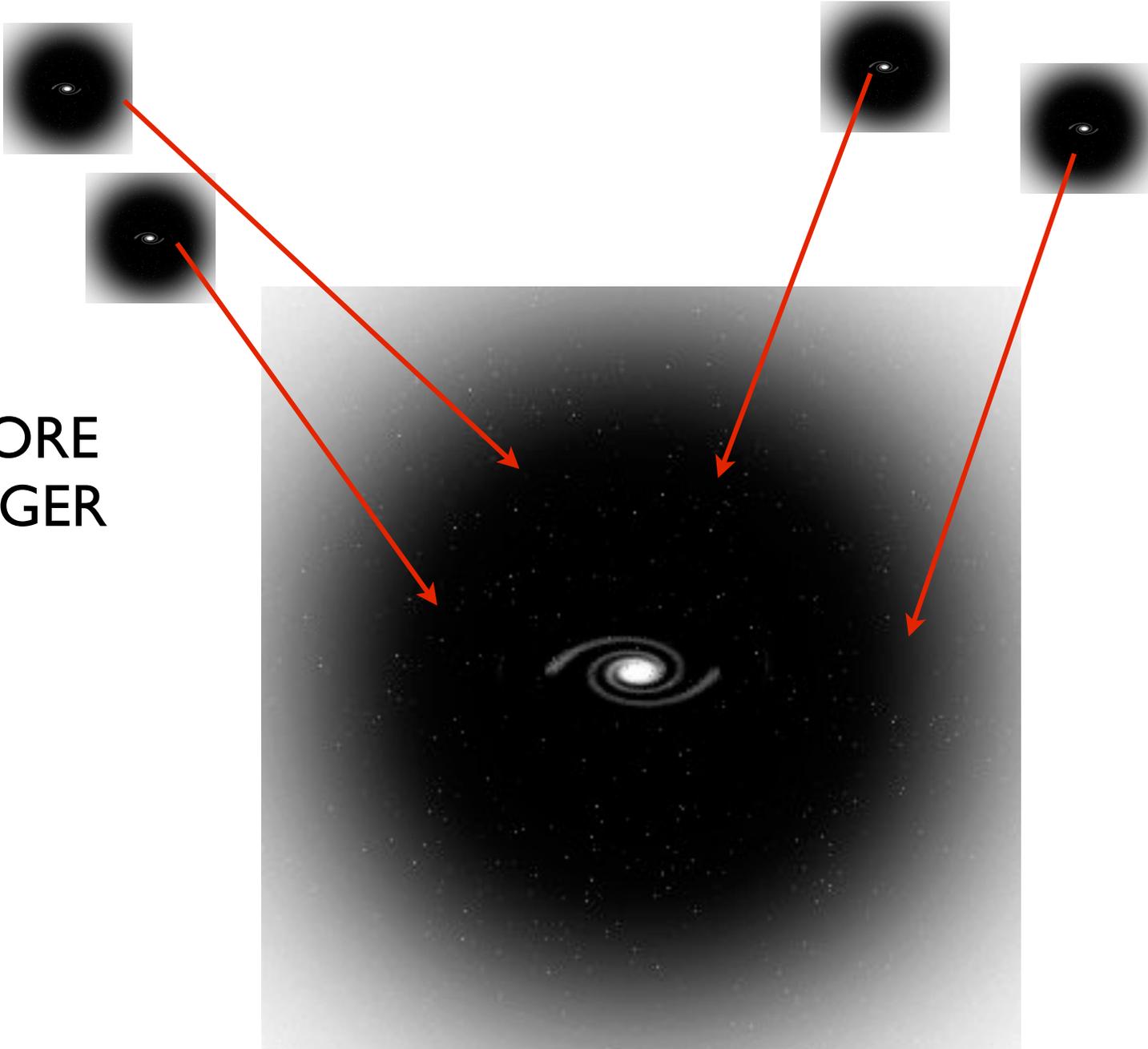
In most cases, you would expect most collapsed halos, even with masses of 10^8 - 10^{10} solar masses to have had some baryons to cool to the center of the halo and form stars.



As these halos will merge with a larger halo, they become subhalo and the luminous matter within these subhalos should become satellite galaxies.

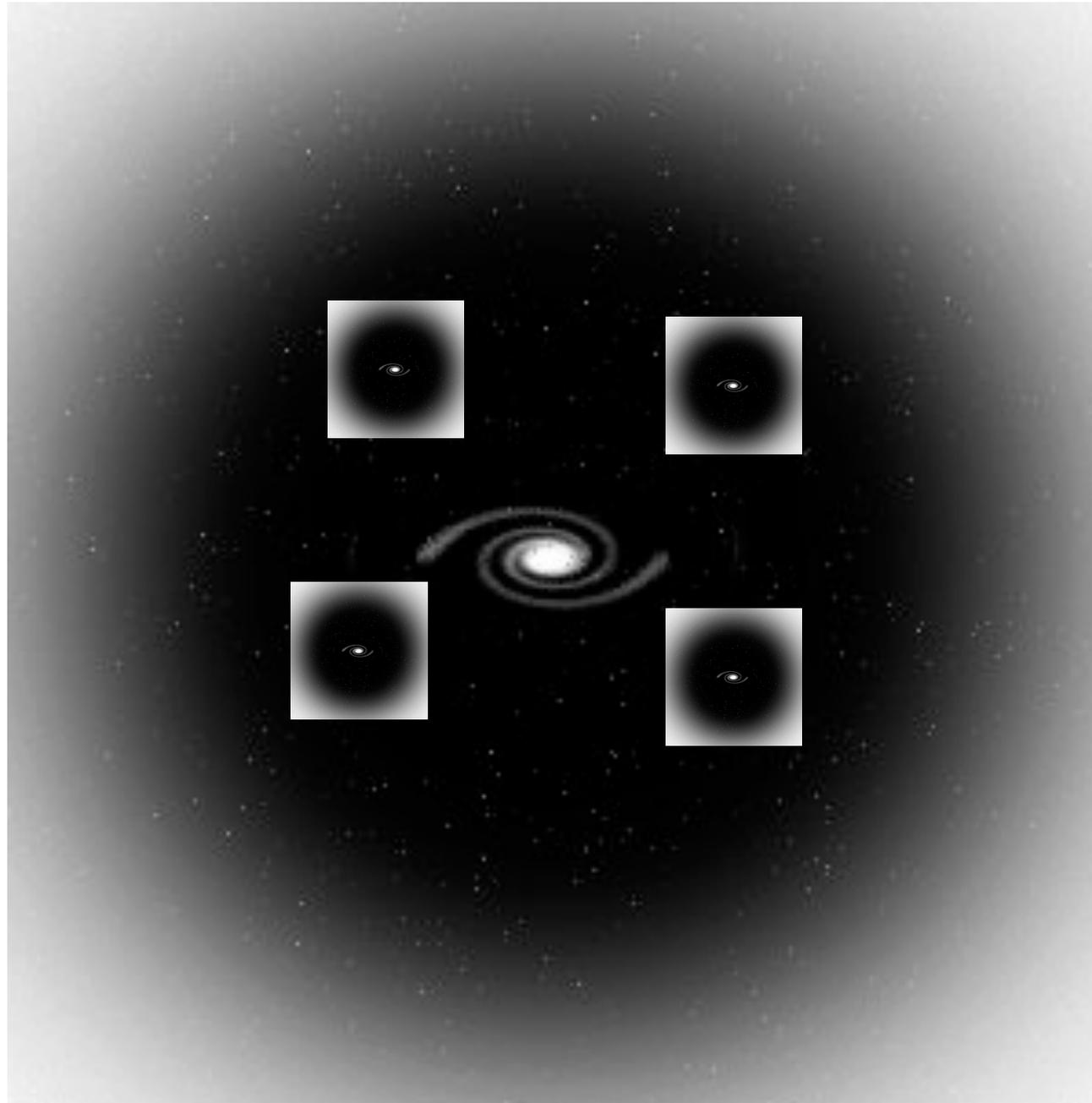
In pictures:

BEFORE
MERGER



In pictures:

AFTER
MERGER



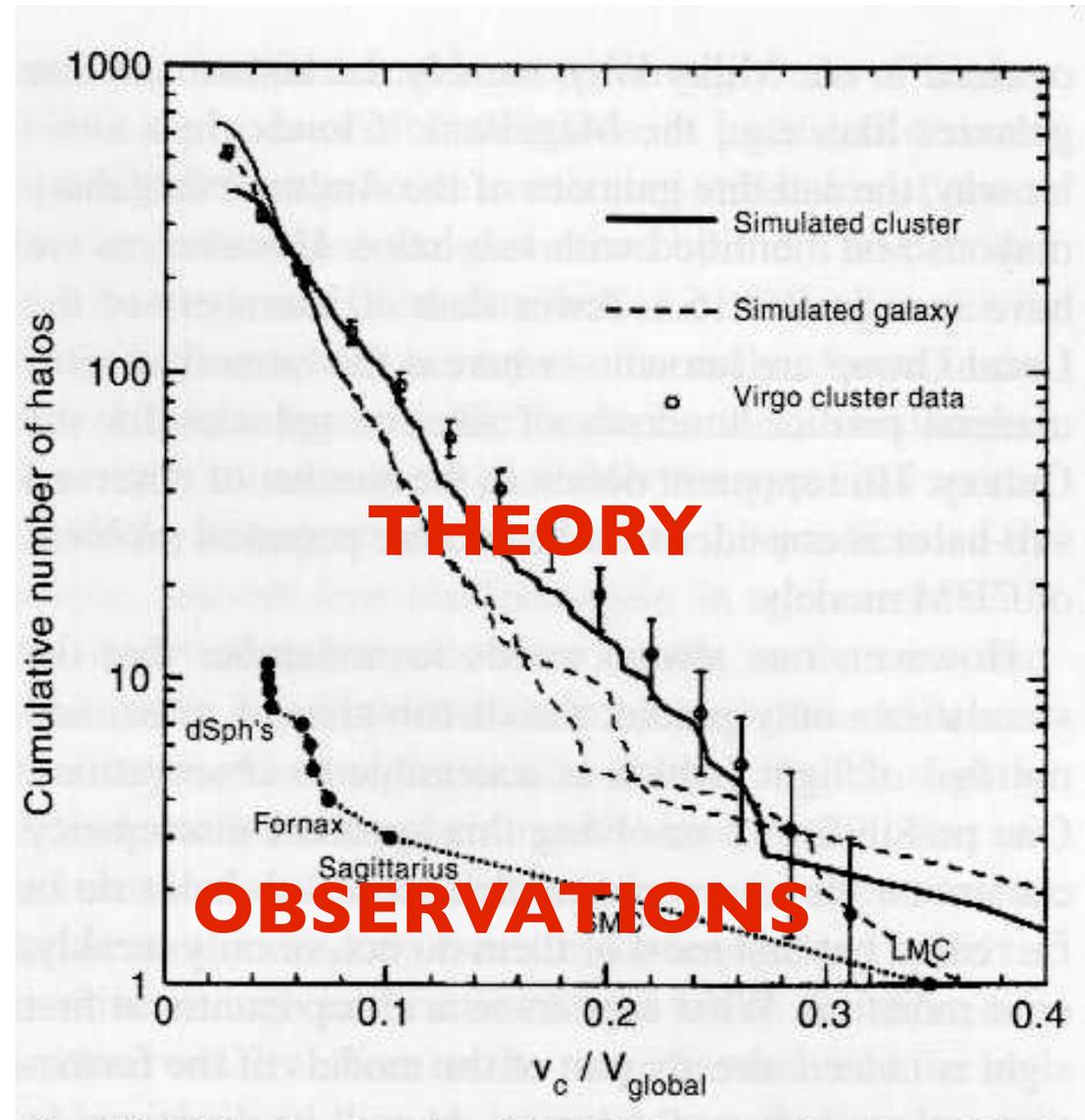
All these subhalos should indicate the position of dwarf galaxy!

Do we find these galaxies around our own Milky Way galaxy?

If we include only the most obvious satellites, the answer is no.

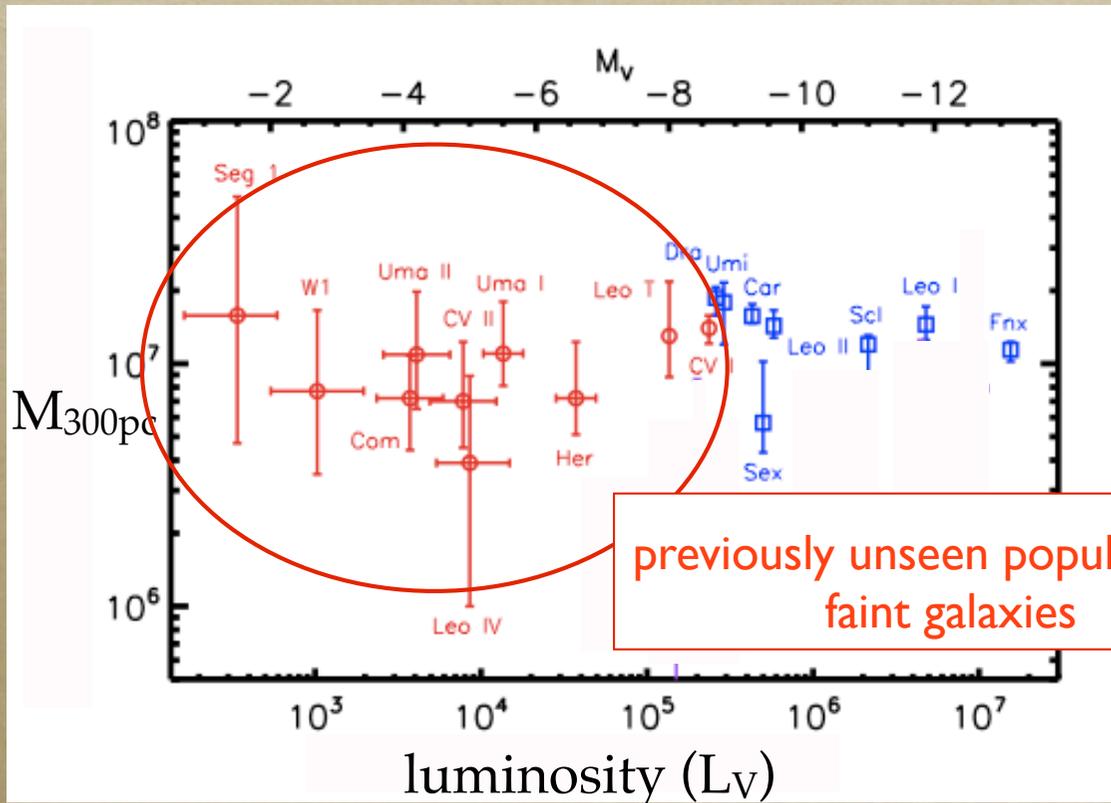
This is called the “missing satellites problem.”

One resolution may be subhalos each containing only a few stars

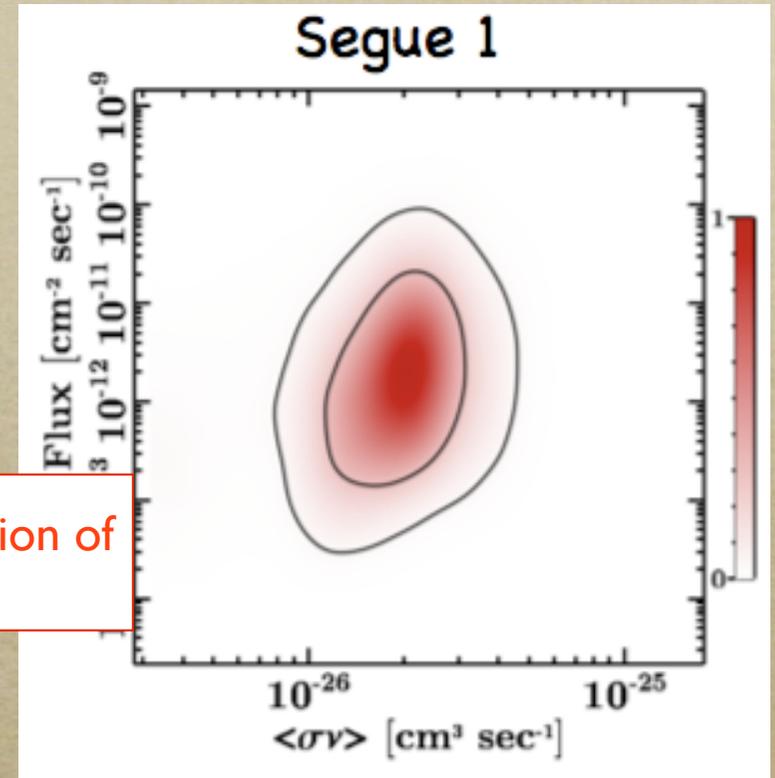


One resolution may be ultra-low luminosity galaxies populating subhalos

Kinematics of Ultra-Faint Galaxies



Strigari et al 2008



Martinez et al 2009

Mass/density profiles depend on assumption that measured velocities probe gravitational potential

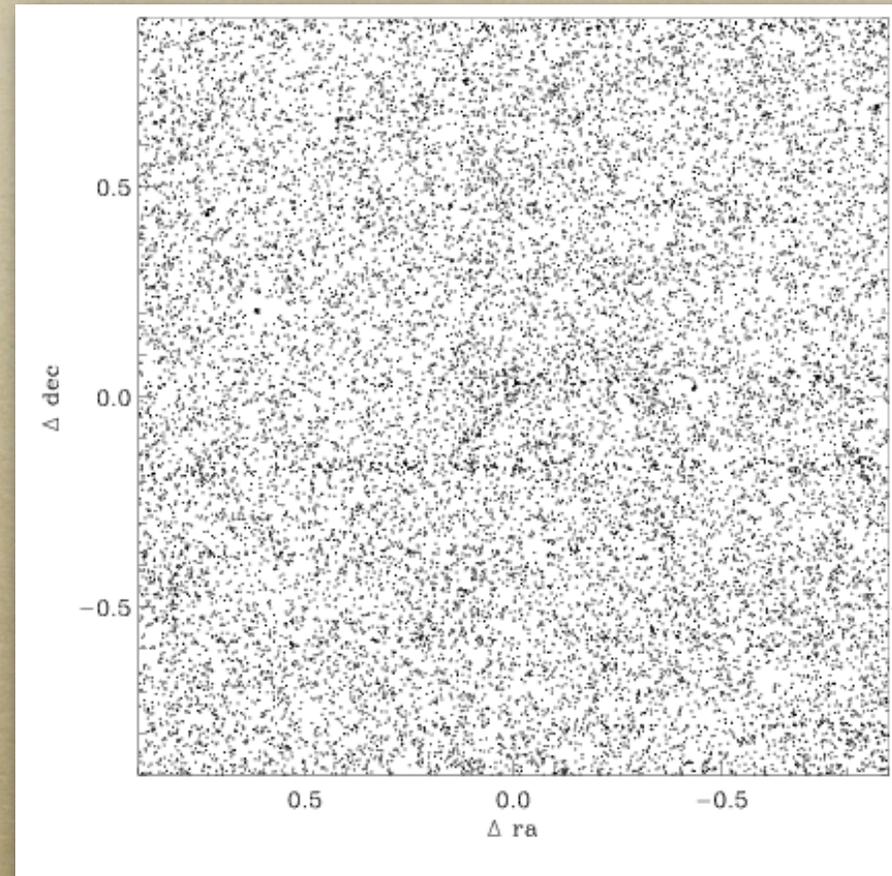
Credit: Geha 2009

How are Such Ultra-Faint Galaxies Found?

Finding the Milky Way Ultra-Faint Galaxies

The ultra-faint galaxies are found via over-densities of resolved stars.

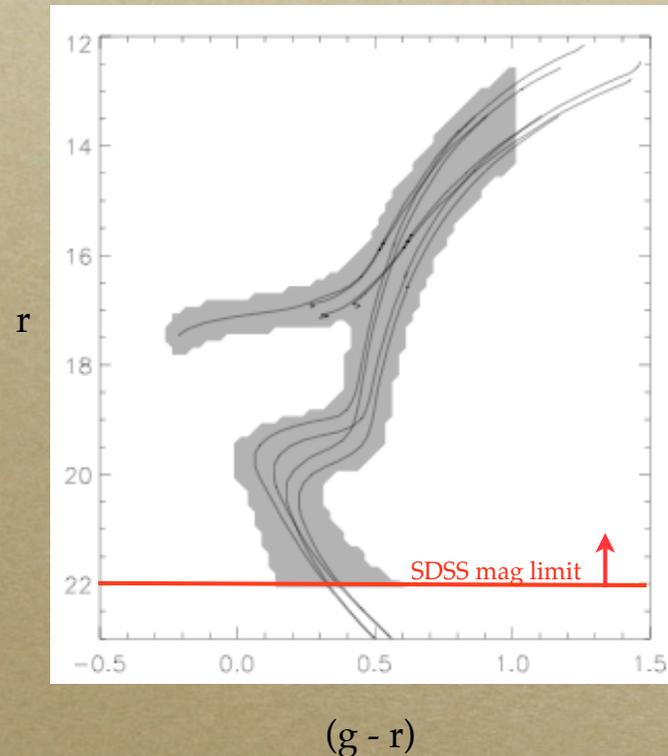
Milky Way stellar foreground overwhelms the dwarf galaxy.



Credit: Geha 2009

Finding the Milky Way Ultra-Faint Galaxies

Assume: Dwarf galaxies are old, metal-poor stellar populations, with typical size ~ 50 - 100 pc. This defines a narrow region in color-magnitude space.



Walsh, Willman & Jerjen (2008)

A generous definition
of old and metal-poor:
age = 8 to 14 Gyr
[Fe/H] = -1.5 to -2.3

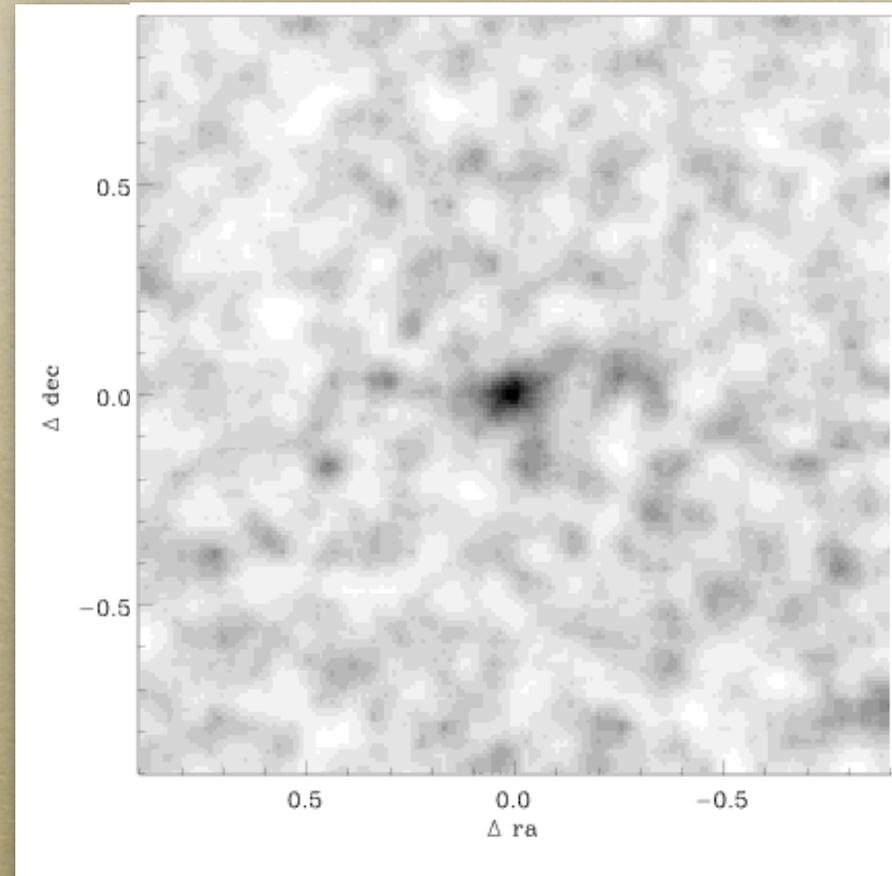
Distance = 20 kpc

Credit: Geha 2009

Finding the Milky Way Ultra-Faint Galaxies

Filtered+Smoothed

1. Assumed old/metal-poor stellar population
2. Assumed physical size



Credit: Geha 2009

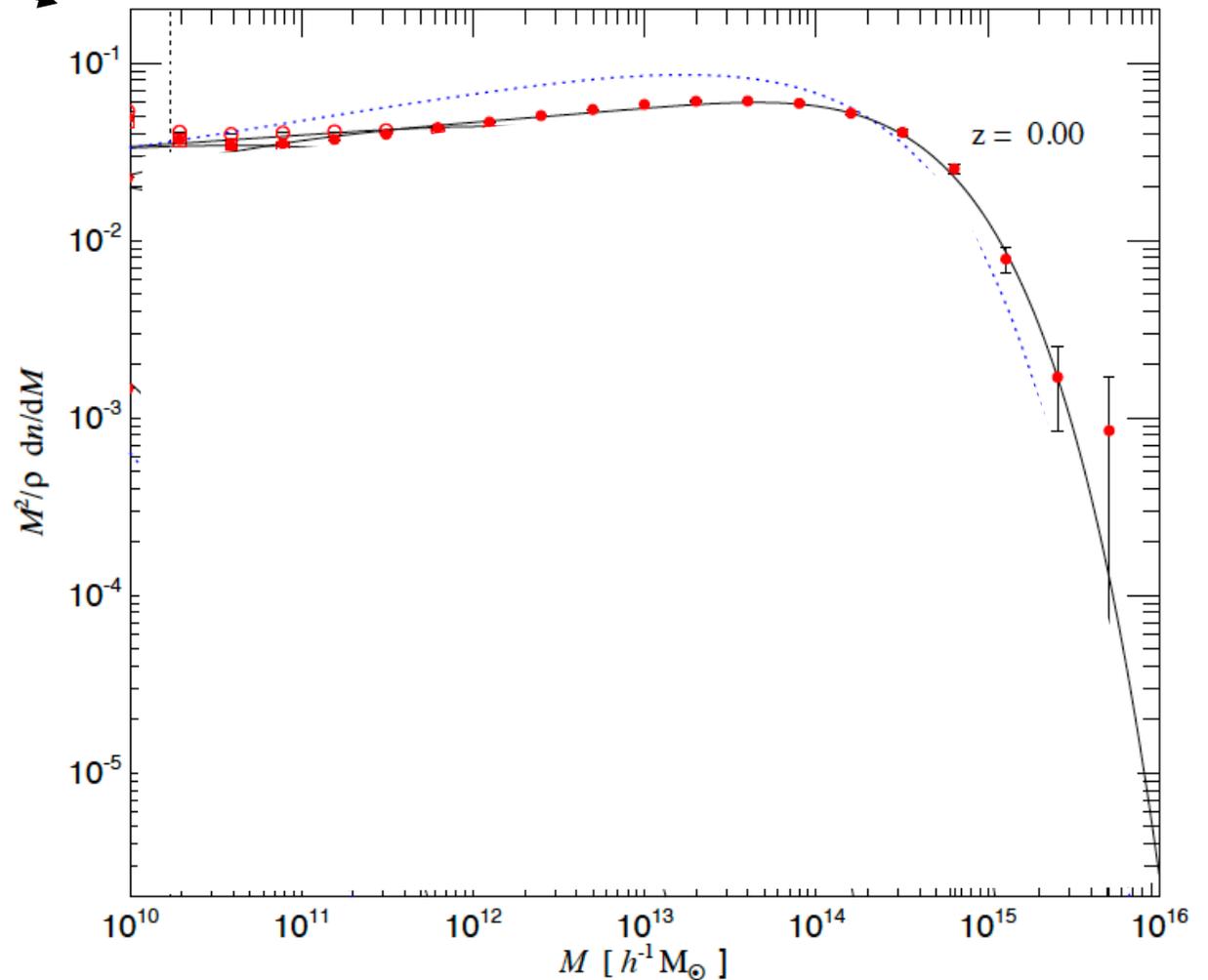
Impact Dark Matter Halo Have on the Density of Galaxies with Various Properties

Key Concept is



Volume
Density of
Dark Matter
Halos of a
Given Mass

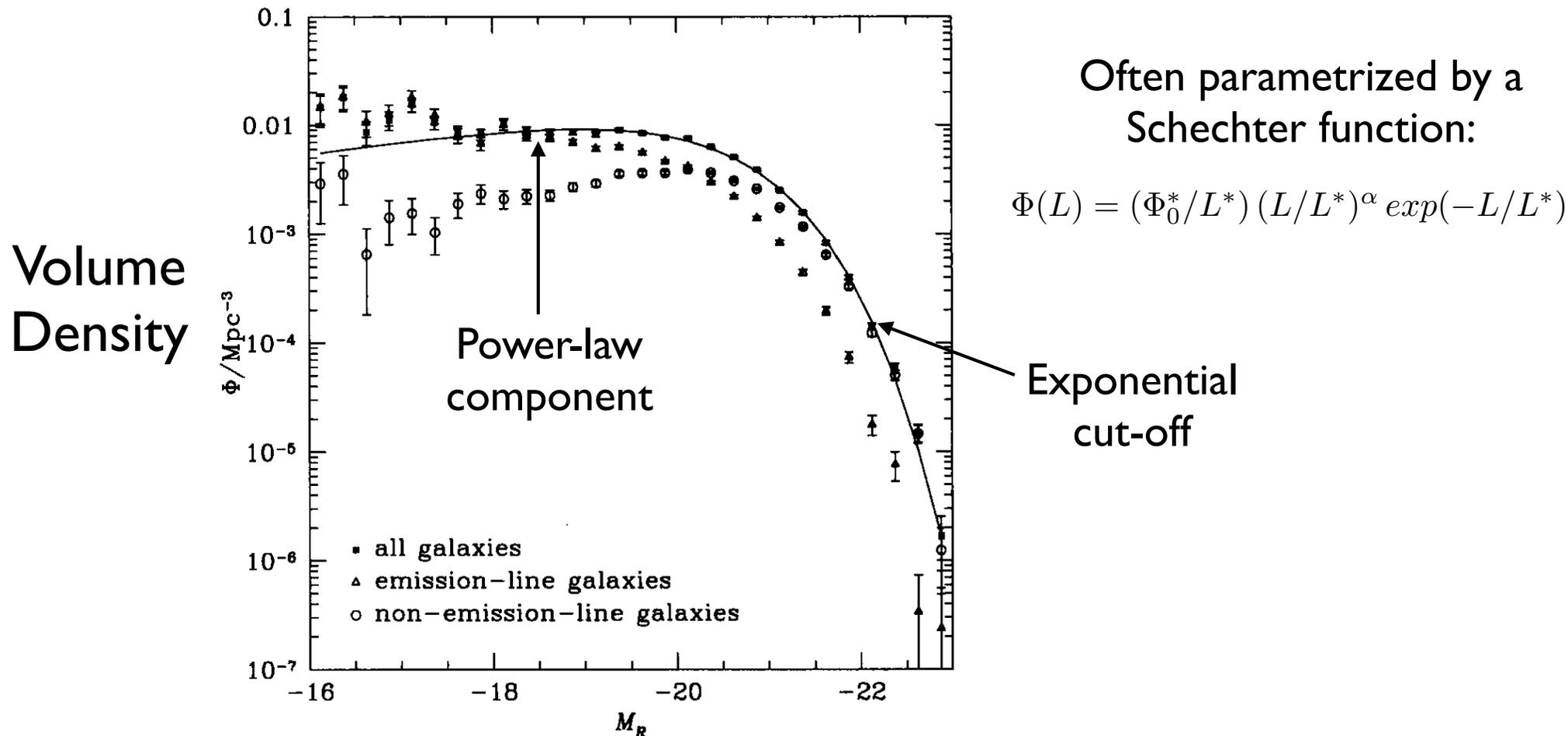
Halo Mass Function



Dark Matter Halo Mass

How do astronomers typically parameterize the volume density of galaxies with some property?

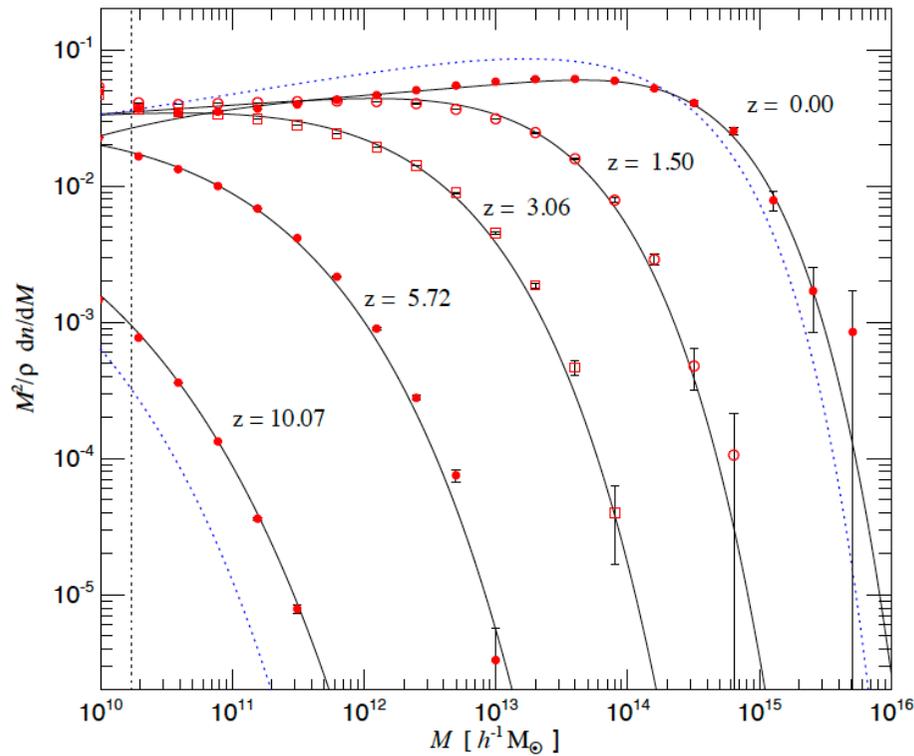
A standard approach uses a luminosity function:



Absolute Magnitude in R band (~600 nm)

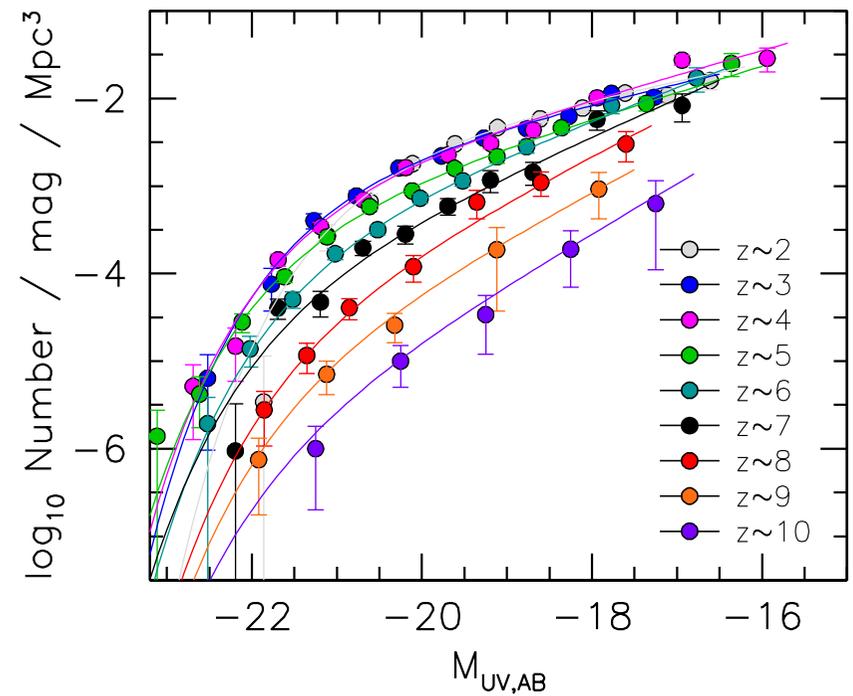
Impact Dark Matter Halo Have on the Density of Galaxies with Various Properties

Evolution of Dark Matter Halo Mass Function



Springel+2005

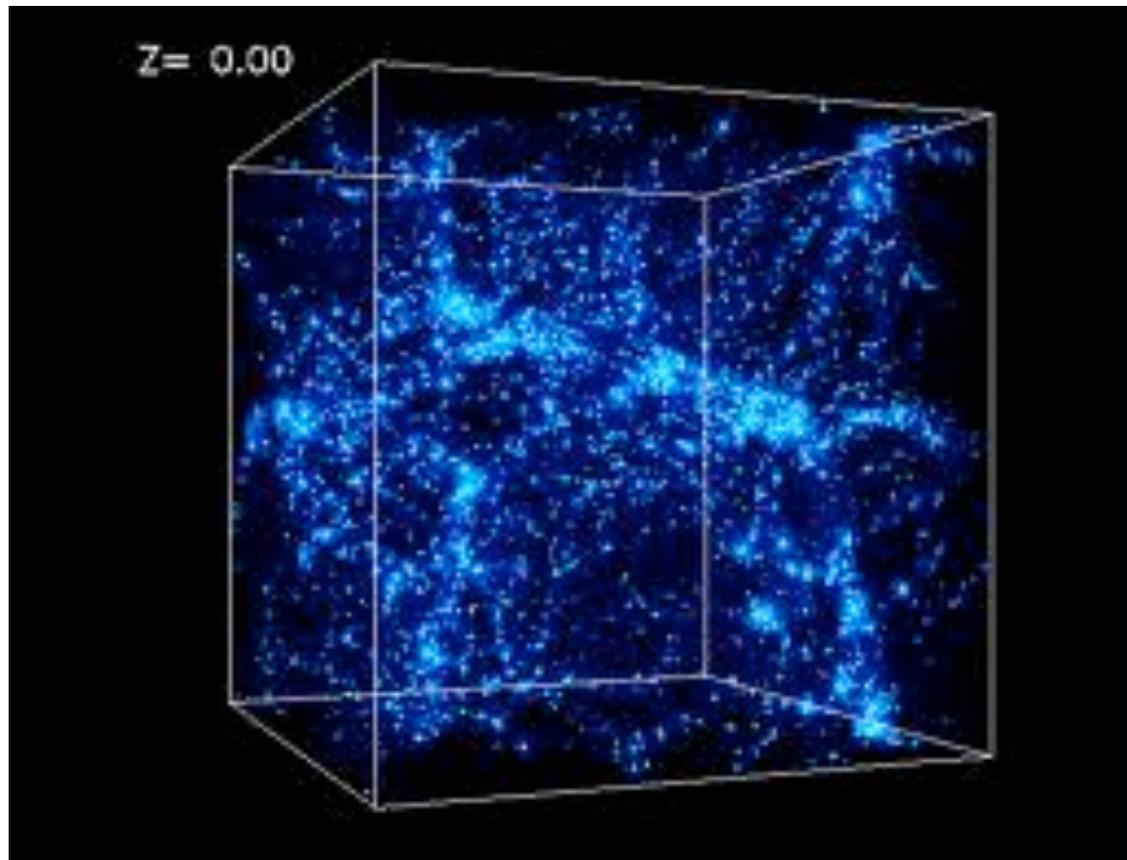
Evolution of Luminosity Function at UV Wavelengths



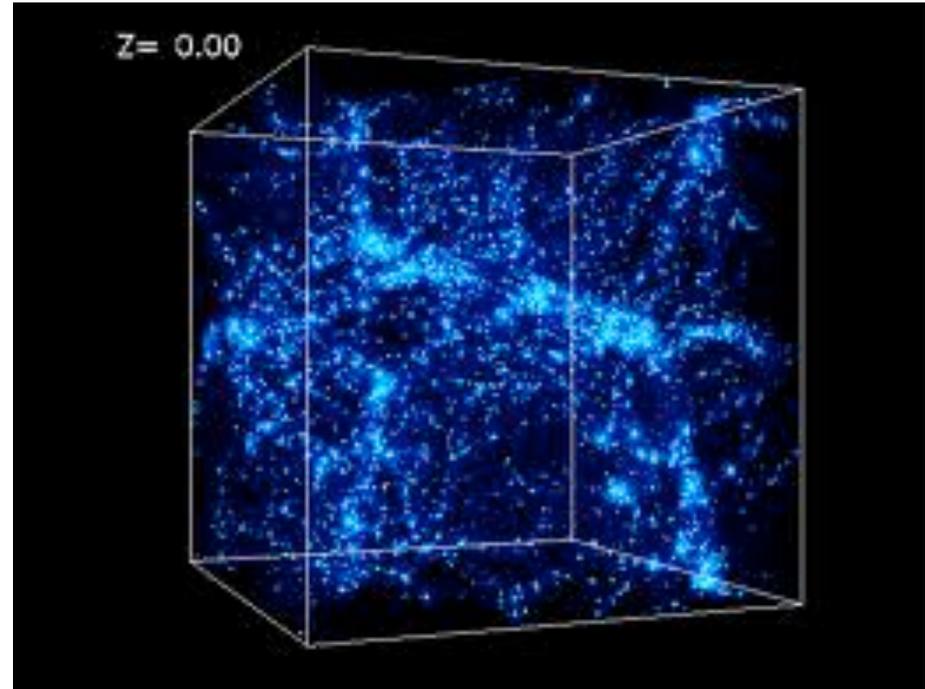
Bouwens+2021

How Do Galaxies Distribute Themselves in Space?

What does this teach us?



“How Do Galaxies Distribute Themselves in Space?”

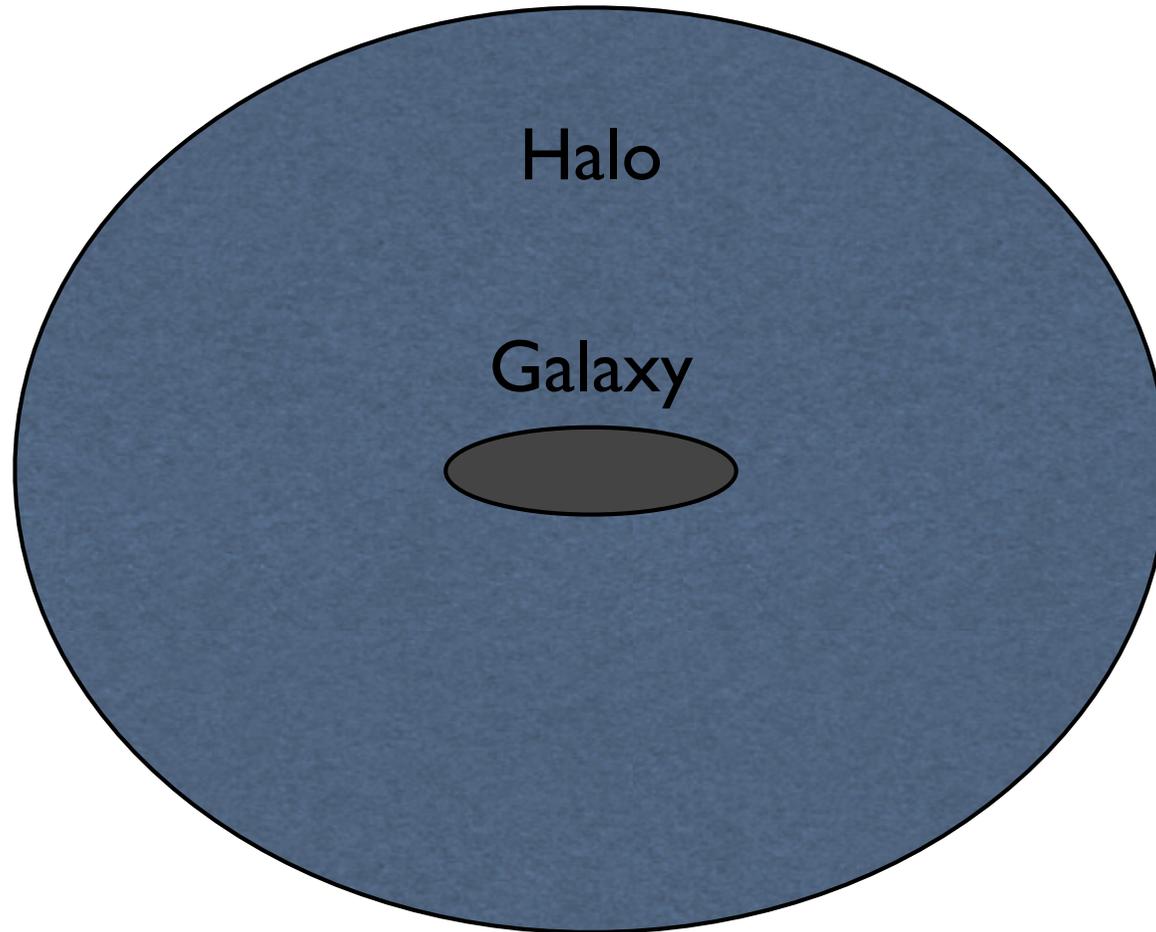


What does this teach us?”

It provides insight into the masses or properties of the collapsed dark matter halos in galaxies form and evolve.

(first a prologue)

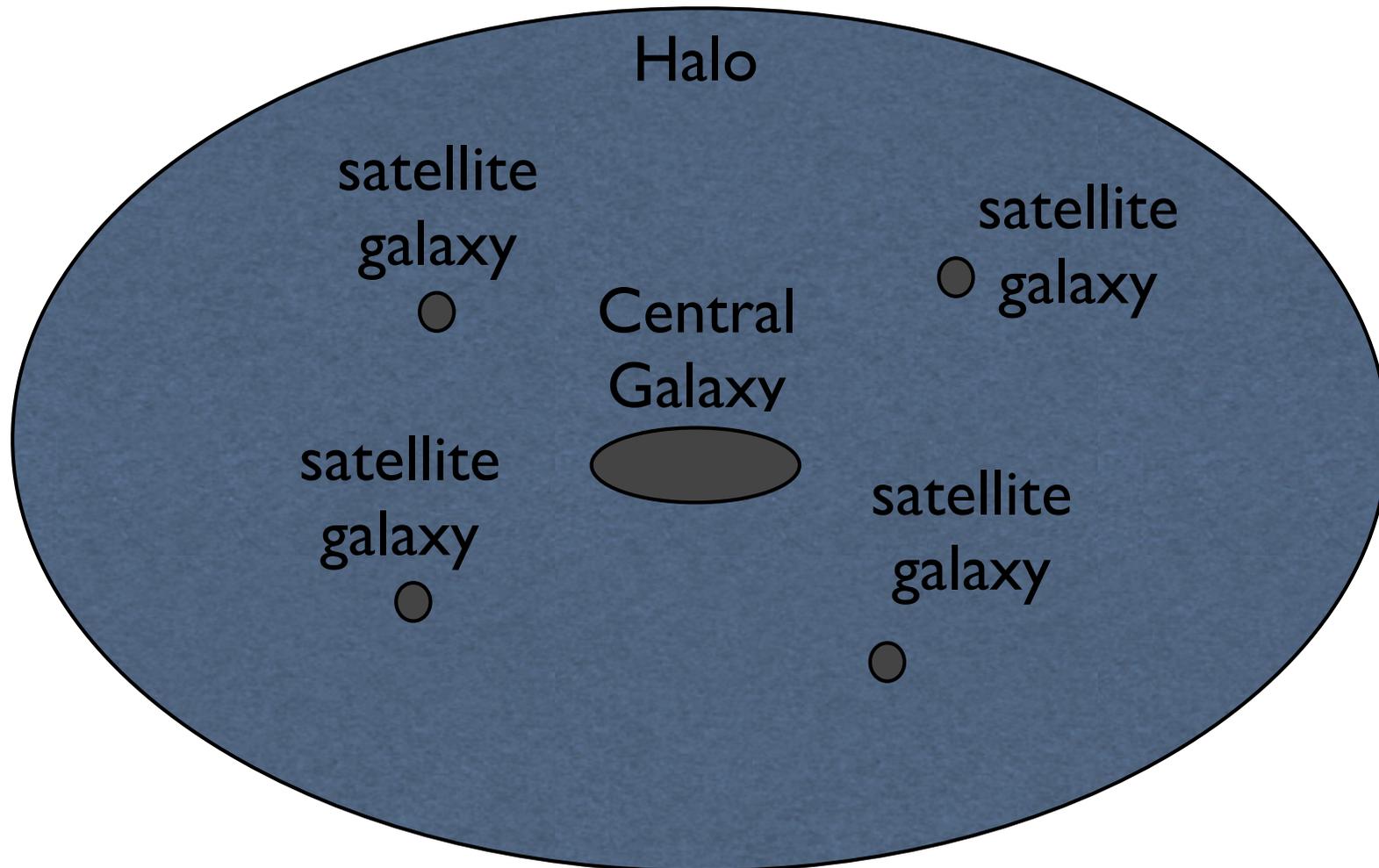
By now, you should be familiar with the idea that collapsed halos contain galaxies at their center.



these galaxies form from the cooling of gas onto the center of the halo and forming a gas disk

however, many dark matter halos contain more than one galaxy...

each halo almost always one most massive galaxy at the center (called the “central galaxy”) and any number of satellite galaxies (orbiting around the center)



You should all be quite familiar with halos that contain many satellite galaxies. An excellent example is a galaxy cluster:

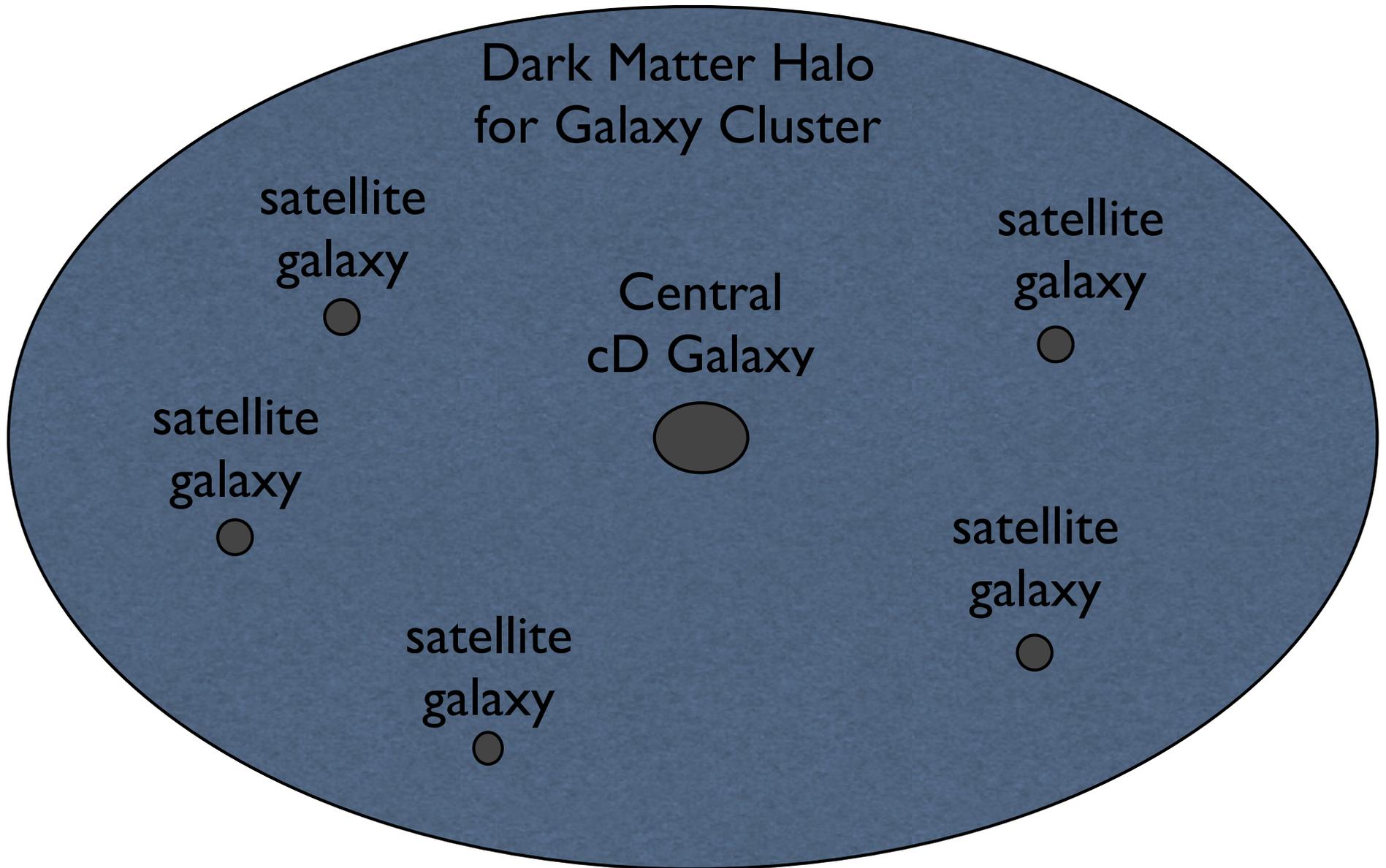


Table 4.3 Local Group members

Name	Alternate Name	Coordinates		Type	Distance (kpc)	M_V
		RA (1950)	Dec			
M31	NGC 224	00 40.0	+40 59	Sb	725	-21.1
Milky Way	Galaxy	17 42.4	-28 55	Sbc	8	-20.6
M33	NGC 598	01 31.1	+30 24	Sc	795	-18.9
LMC		05 24.0	-69 48	Irr	49	-18.1
IC 10		00 17.7	+59 01	Irr	1250	-17.6
NGC 6822	DDO 209	19 42.1	-14 56	Irr	540	-16.4
M32	NGC 221	00 40.0	+40 36	dE2	725	-16.4
NGC 205		00 37.6	+41 25	dE5	725	-16.3
SMC		00 51.0	-73 06	Irr	58	-16.2
NGC 3109	DDO 236	10 00.8	-25 55	Irr	1260	-15.8
NGC 185		00 36.2	+48 04	dE3	620	-15.3
IC 1613	DDO 8	01 02.2	+01 51	Irr	765	-14.9
NGC 147	DDO 3	00 30.5	+48 14	dE4	589	-14.8
Sextans A	DDO 75	10 08.6	-04 28	Irr	1450	-14.4
Sextans B	DDO 70	09 57.4	+05 34	Irr	1300	-14.3
WLM	DDO 221	23 59.4	-15 45	Irr	940	-14.0
Sagittarius		18 51.9	-30 30	dSph/E7	24	-14.0
Fornax		02 37.8	-34 44	dSph/E3	131	-13.0
Pegasus	DDO 216	23 26.1	+14 28	Irr	759	-12.7
Leo I	DDO 74	10 05.8	+12 33	dSph/E3	270	-12.0
Leo A	DDO 69	09 56.5	+30 59	Irr	692	-11.7
And II		01 13.5	+33 09	dSph/E3	587	-11.7
And I		00 43.0	+37 44	dSph/E0	790	-11.7
SagDIG		19 27.9	-17 47	Irr	1150	-11.0
Antlia		10 01.8	-27 05	dSph/E3	1150	-10.7
Sculptor		00 57.6	-33 58	dSph/E3	78	-10.7
And III		00 32.6	+36 12	dSph/E6	790	-10.2
Leo II	DDO 93	11 10.8	+22 26	dSph/E0	230	-10.2
Sextans		10 10.6	-01 24	dSph/E4	90	-10.0
Phoenix		01 49.0	-44 42	Irr	390	-9.9
LGS 3		01 01.2	+21 37	Irr	760	-9.7
Tucana		22 38.5	-64 41	dSph/E5	900	-9.6
Carina		06 40.4	-50 55	dSph/E4	87	-9.2
Ursa Minor	DDO 199	15 08.2	+67 23	dSph/E5	69	-8.9
Draco	DDO 208	17 19.2	+57 58	dSph/E3	76	-8.6

SOURCE: From data kindly provided by M. Irwin.

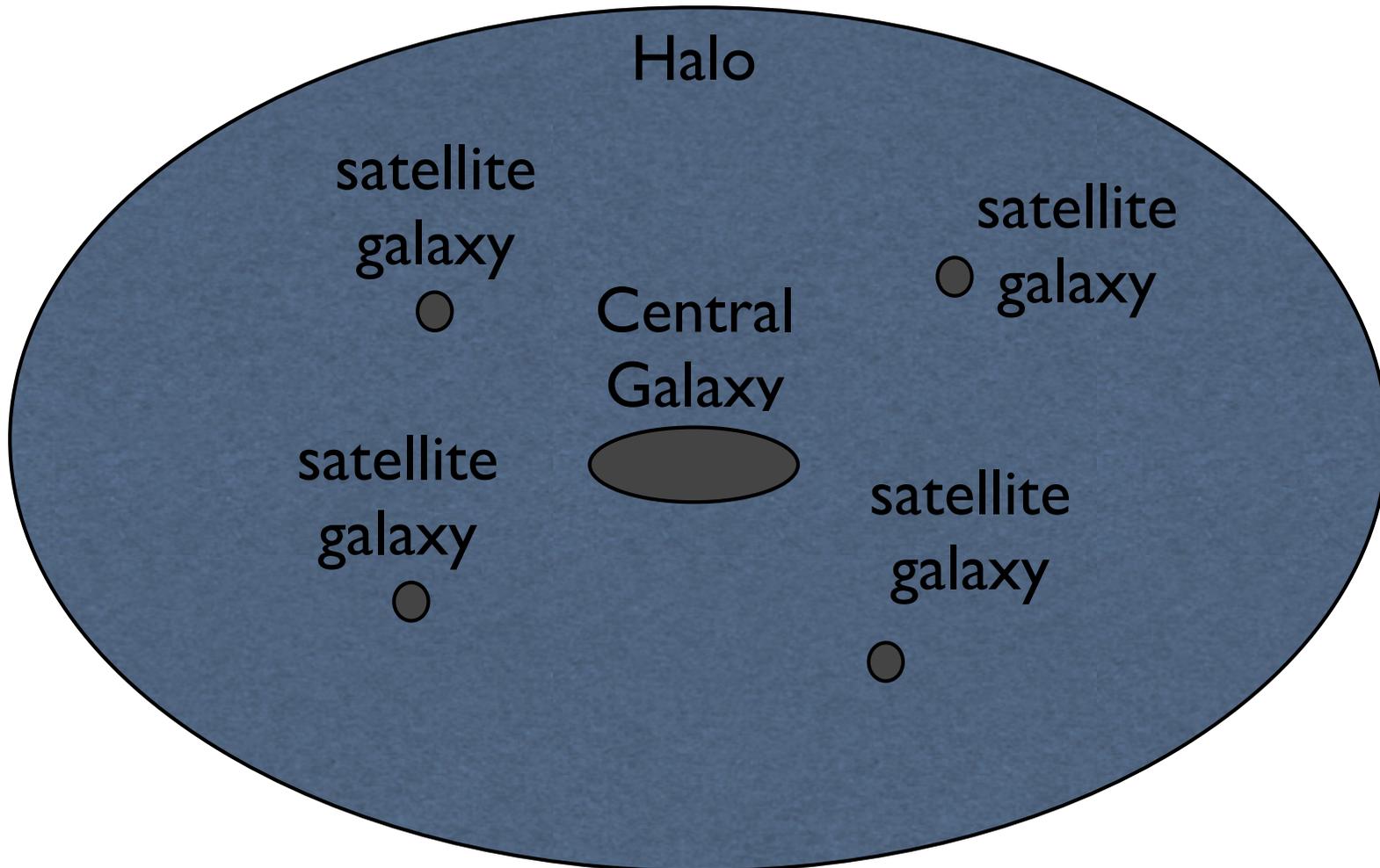
Of course, this list may be incomplete as the Milky Way itself blocks our field of view!

New Gaia mission may help in this regard!

how can we explain the origin of the satellite galaxies?

can we explain them as a result of gas cooling?

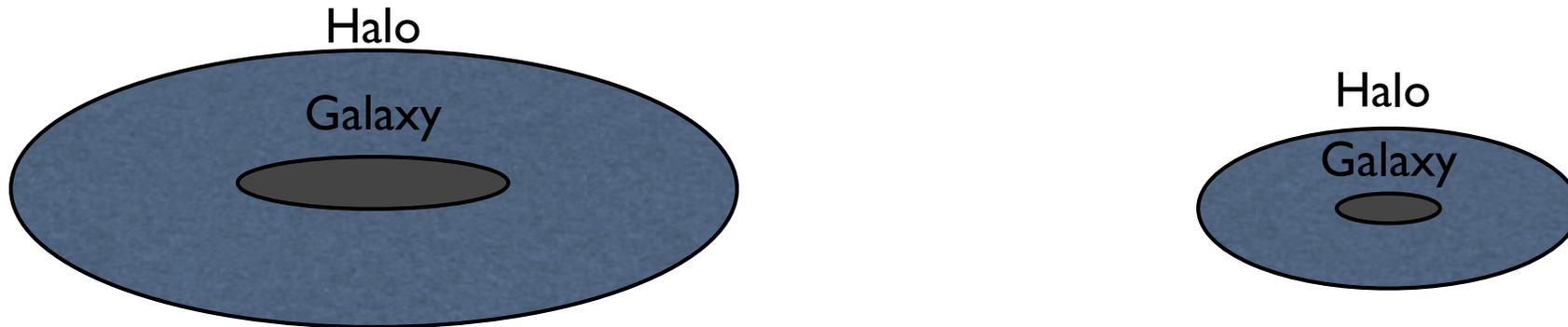
no -- since gas only efficiently cools onto the central position in the dark matter halo (gravitational potential)



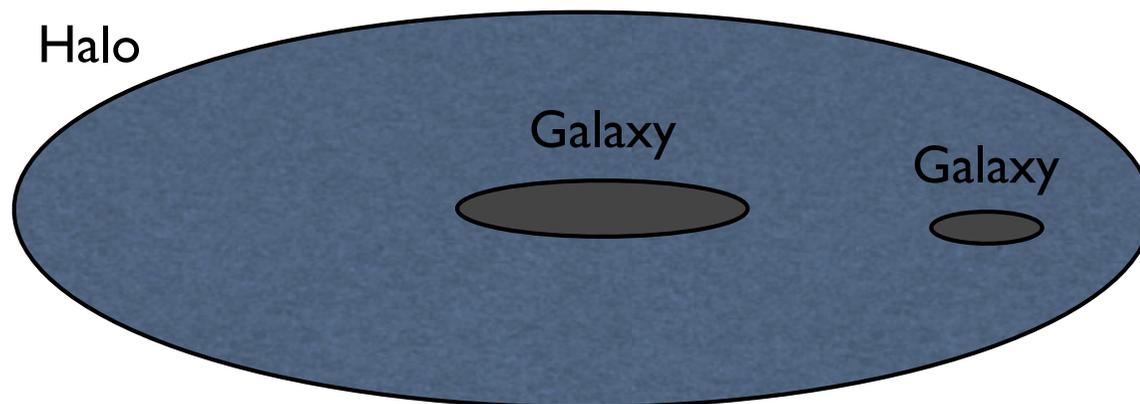
How can we explain the origin of the satellite galaxies?

A better explanation is through merging:

Step #1: We have two dark matter halos. Gas cools to the center of each halo to form disk galaxies in each:

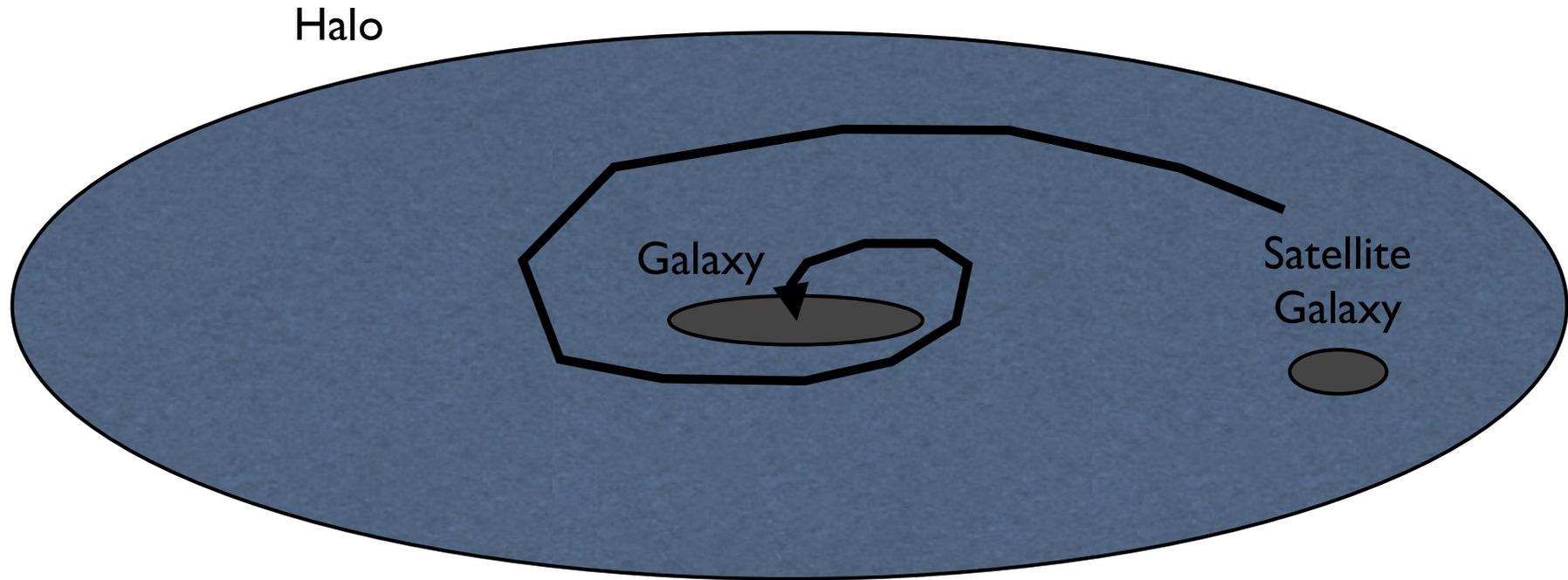


Step #2: These two dark matter halos merge to form a single more massive halo



The most massive halos contain large numbers of satellite galaxies, while less massive galaxies contain less. The merger origin of satellite galaxies explains why this is the case.

NOTA BENE: In many cases, satellite galaxies merge with the central galaxy as a result of dynamical friction.



The Local Group is not in equilibrium!

M31 and the Milky Way galaxy are traveling towards each other at
120 km/s!

Since M31 and the Milky Way are 700 kpc away from each other,
these two galaxies will collide in perhaps ~4 billion years!

Can we use this information to estimate the mass of the Milky Way
galaxy? Yes!

The Local Group is not in equilibrium!

First, consider the fact that the Milky Way and M31 will be initially flying away from each other with the Hubble flow.

Due to the self gravity of the mass within the local group, the Milky Way and M31 will stop expanding with the Hubble flow and start to fall towards each other.

Assuming that it takes the two main galaxies in the local group 14 Gyr to start falling towards each other at 120 km/s and have a distance of 700 kpc, we can calculate the mass of the local group to be $3 \times 10^{12} M_{\text{solar}}$

Comparing this mass to the total luminosity of the local group, one derives $70 M_{\text{sol}} / L_{\text{sol}}$.

To put these numbers in context, I remind you of the mass-to-light ratios presented in lecture #1

Table 10-2. Estimates of the density parameter

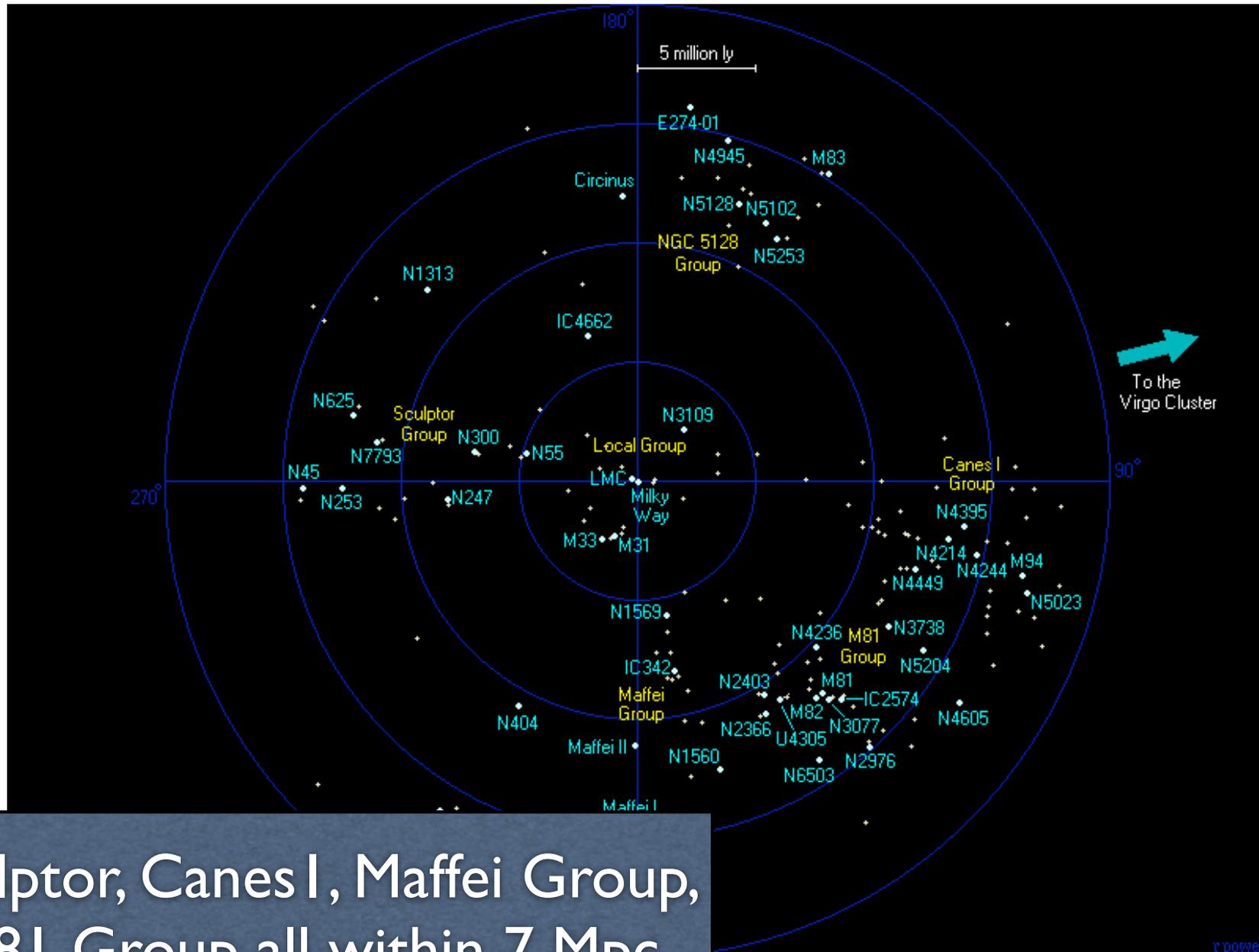
Method	$\Upsilon_V/\Upsilon_\odot$	Ω_0
Solar neighborhood	5	$0.003h^{-1}$
Elliptical galaxy cores	$12h$	0.007
Local escape speed	30	$0.018h^{-1}$
Satellite galaxies	30	$0.018h^{-1}$
Magellanic Stream	> 80	$> 0.05h^{-1}$
Rotation curve of NGC 3198	$> 28h$	> 0.017
X-ray halo of M87	> 750	$> 0.46h^{-1}$
Local Group timing	100	$0.06h^{-1}$
Groups of galaxies	$260h$	0.16
Clusters of galaxies	$400h$	0.25
Virgocentric flow		0.25
Nucleosynthesis		$(0.01 - 0.05)h^{-2}$
Inflation		1

NOTES: All lines except the last three are based on the luminosity density (10-24). Nucleosynthesis estimate omits density in non-baryonic matter. Several methods, such as Local Group timing and X-ray halo of M87, depend on h in complicated ways, and this dependence has been suppressed. See text for further detail.

Mass-to-Light
Ratio

As we probe larger spatial scales, the mass-to-light ratio increases!

What collapsed clusters or groups are nearby the local group?

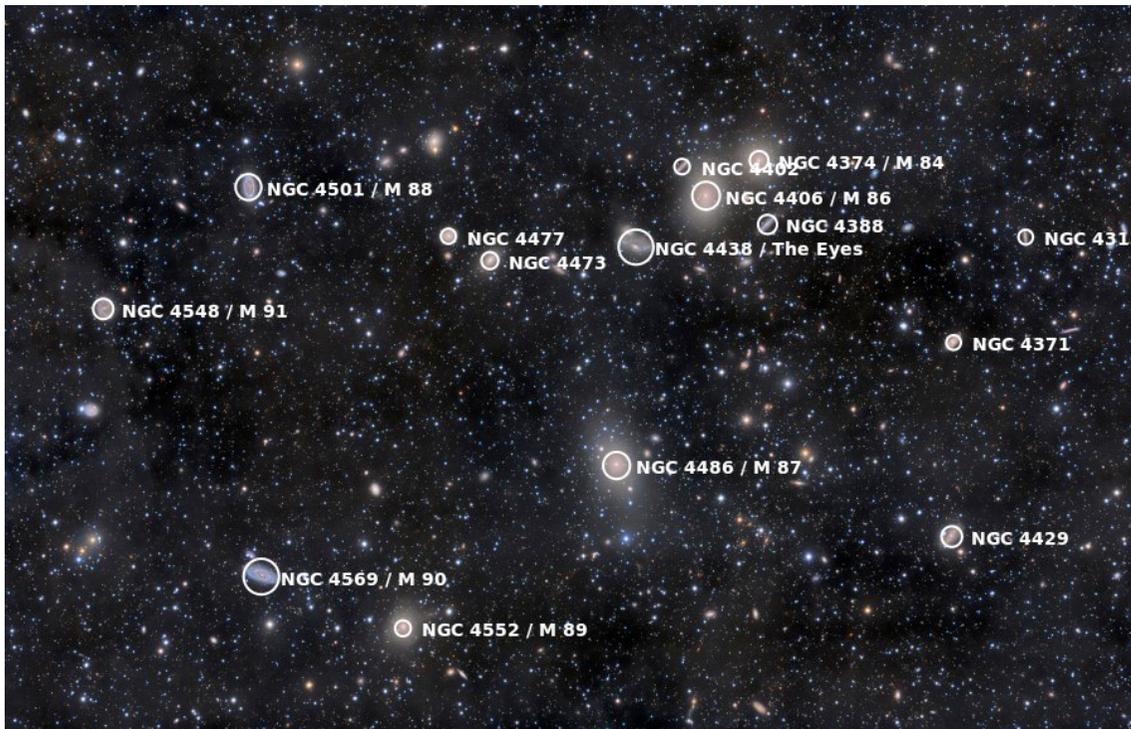


Sculptor, Canes I, Maffei Group, M81 Group all within 7 Mpc

What collapsed clusters or groups are nearby the local group?

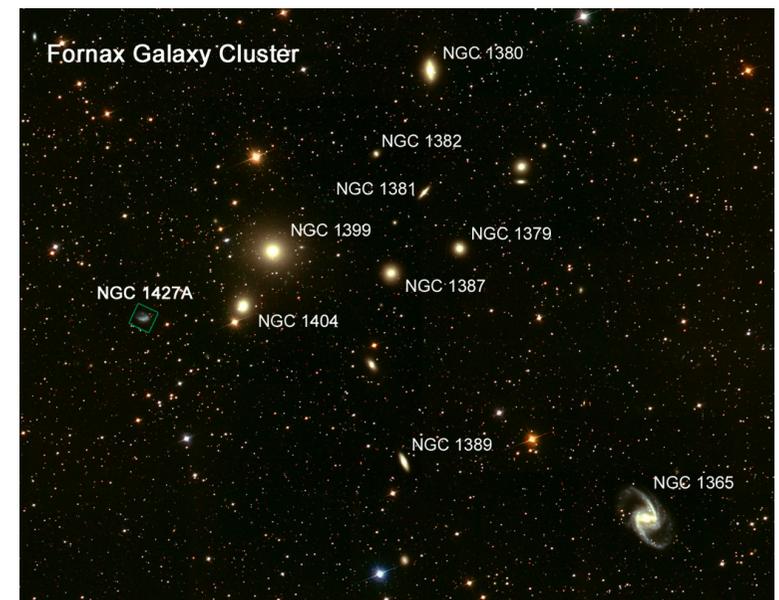
Virgo Cluster

- contains >250 large galaxies
- contains 2000 smaller galaxies
- covers 10 x 10 degrees on sky
 - 18 Mpc away
 - 3 Mpc diameter



Fornax Cluster

- contains >50 large galaxies
 - 19 Mpc away
 - less massive than Virgo



What collapsed clusters or groups are nearby the local group?

Coma cluster

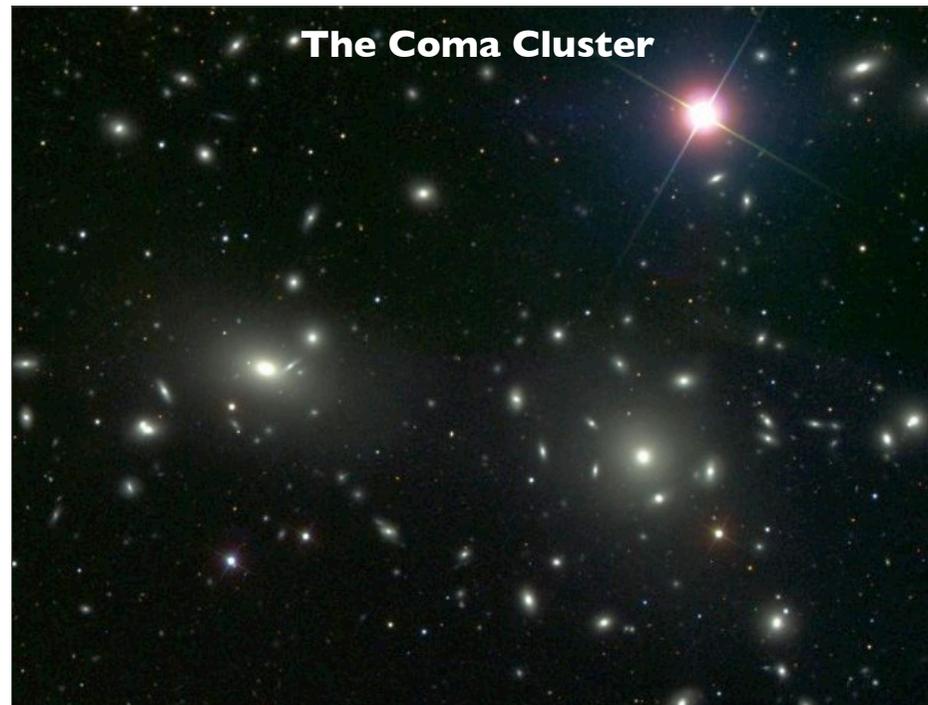
contains >1000 large galaxies

contains 10000 smaller galaxies

~100 Mpc away

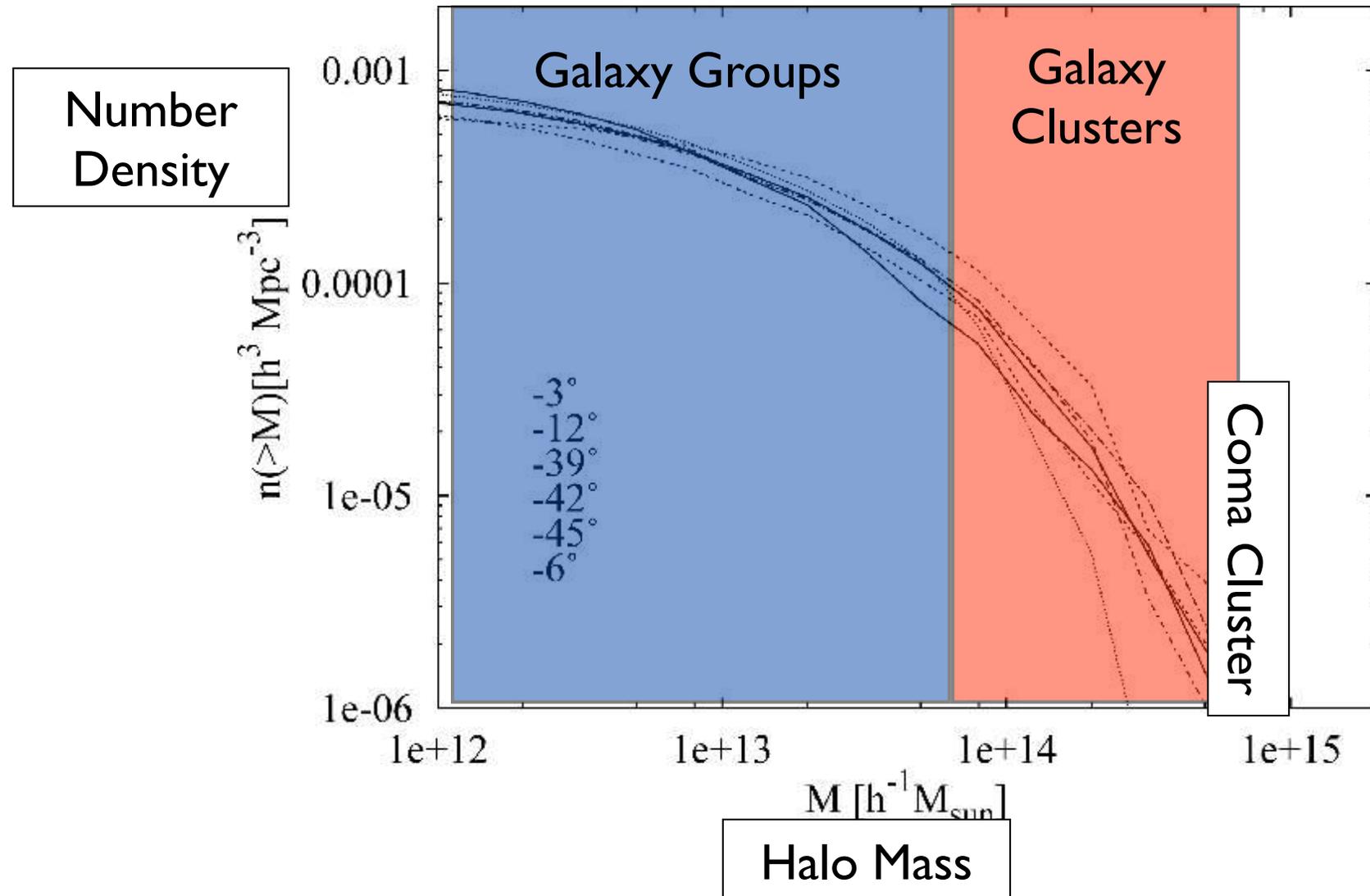
6 Mpc diameter

largest galaxies are giant ellipticals



How common are galaxy groups or clusters of various masses?

from Heinamaki et al (A&A 397, 63)



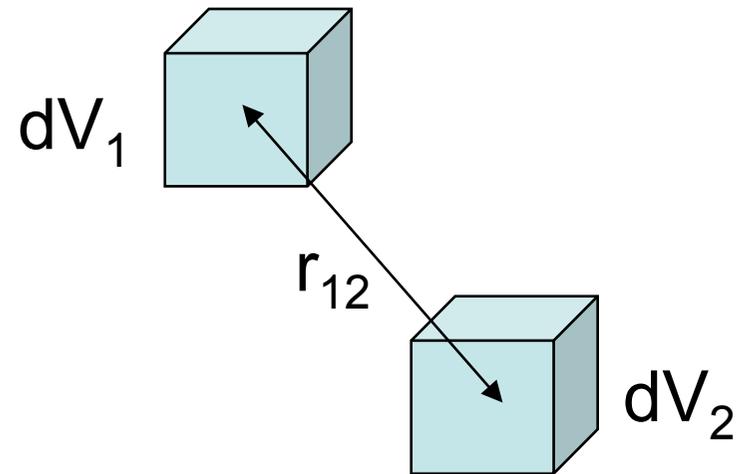
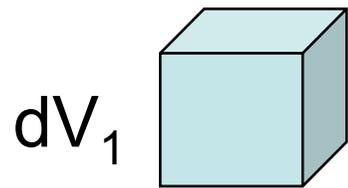
Gravity does not simply result in the production of collapsed systems. It can also cause galaxies or other collapsed structures to be “close to each other” or clustered

We can quantify the non-uniform distribution of galaxies on the sky through a measurement of the clustering signal.

This clustering signal ultimately helps us learn about the dark matter halos in which galaxies live.

We quantify clustering in terms of correlation functions

The Correlation function ξ is not equal to zero -- since the presence of a galaxy at some place in space makes it more likely another one will be close by....



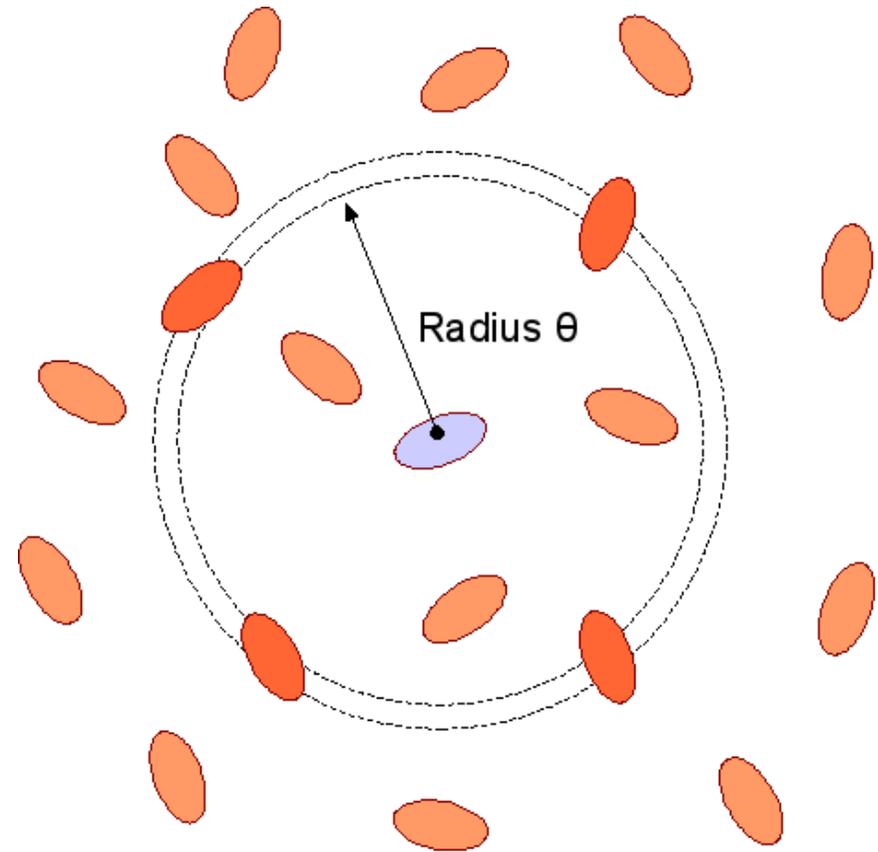
$$dP_1 = n dV_1$$

$$dP_{12} = n^2 (1 + \xi(r_{12})) dV_1 dV_2$$

n = average density of galaxies

How do we quantify the correlation function?

The Correlation function ξ is calculated by examining the distances between every pair of galaxies in a survey and comparing it to a random distribution



Correlations between points can be determined by counting pairs.

How do we quantify the correlation function?

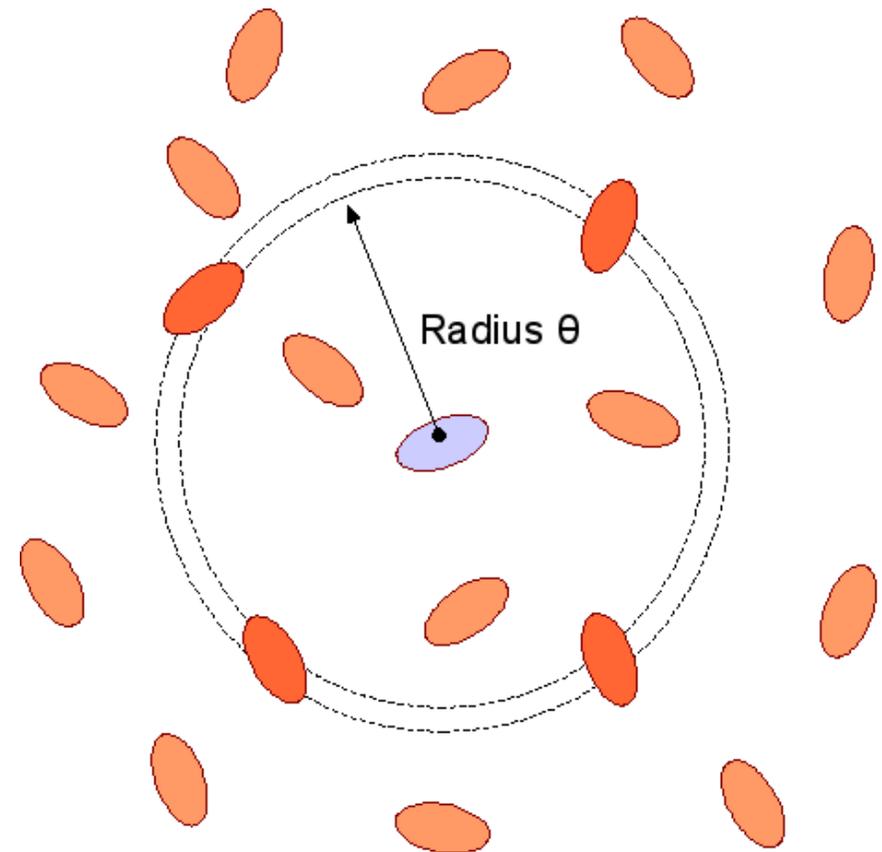
Fundamentally, this involves counting the number of galaxies at a certain distance from each other on the sky and then comparing that with a random distribution

$$\xi(r) = DD / RR - 1$$

DD = number of pairs in the data at a distance r

RR = number of pairs in some mock data set at a distance r

(in mock data sets pairs laid down randomly with uniform distribution)



Correlations between points can be determined by counting pairs.

How do we typically express the correlation function?

The Correlation function ξ is typically parametrized as a power-law in radius:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Typical values for γ are 1.8. r_0 is known as the correlation length and it tells us the typical distance from a source we can expect a large enhancement in neighboring sources

How do we compute the power spectrum of galaxies from the observed clustering?

The power spectrum is the Fourier transform of the correlation function ξ

$$P(k) = \int \xi(r) e^{ik \cdot r} d^3 r \equiv \int \xi(r) \frac{\sin(kr)}{kr} r^2 dr$$