Galaxies: Structure, Dynamics, and Evolution

Elliptical Galaxies (III) / Dark Matter Halos
# Layout of the Course

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep 9</td>
<td>Review: Galaxies and Cosmology</td>
</tr>
<tr>
<td>Sep 16</td>
<td>No Class — Science Day</td>
</tr>
<tr>
<td>Sep 23</td>
<td>Review: Disk Galaxies and Galaxy Formation Basics</td>
</tr>
<tr>
<td>Sep 30</td>
<td>Disk Galaxies (II)</td>
</tr>
<tr>
<td>Oct 7</td>
<td>Disk Galaxies (III)</td>
</tr>
<tr>
<td>Oct 14</td>
<td>No Class</td>
</tr>
<tr>
<td>Oct 28</td>
<td>Elliptical Galaxies (I)</td>
</tr>
<tr>
<td>Nov 4</td>
<td>Elliptical Galaxies (II)</td>
</tr>
<tr>
<td>Nov 11</td>
<td>Elliptical Galaxies (III) / Dark Matter Halos</td>
</tr>
<tr>
<td>Nov 18</td>
<td>Large Scale Structure</td>
</tr>
<tr>
<td>Nov 25</td>
<td>Analysis of Galaxy Stellar Populations</td>
</tr>
<tr>
<td>Dec 2</td>
<td>Lessons from Large Galaxy Samples at $z&lt;0.2$</td>
</tr>
<tr>
<td>Dec 9</td>
<td>Evolution of Galaxies with Redshift</td>
</tr>
<tr>
<td>Dec 16</td>
<td>Galaxy Evolution at $z&gt;1.5$ / Review for Final Exam</td>
</tr>
<tr>
<td>Jan 13</td>
<td>Final Exam</td>
</tr>
</tbody>
</table>

*This lecture*
First, let’s review the important material from last week
There appear to be two different classes of elliptical galaxies. They form in two different ways.

**Case #1:** “Wet” Mergers
(e.g., between two spiral galaxies)

Galaxy (with gas) \(\rightarrow\) Galaxy (with gas)
(tends to occur more frequently for lower mass galaxies, when galaxy evolution less advanced)

**Case #2:** “Dry” Mergers
(e.g., between two elliptical galaxies)

Galaxy (without gas) \(\rightarrow\) Galaxy (without gas)
(frequently occurs after many previous mergers, when the mass is higher)
There appear to be two different classes of elliptical galaxies. They form in two different ways.

**Case #1:** “Wet” Mergers  
(e.g., between two spiral galaxies)

Appear to result in ellipticals that appear consistent with being rotationally flattened.

**Case #2:** “Dry” Mergers  
(e.g., between two elliptical galaxies)

Appear to result in ellipticals who ellipticity can be explained by anisotropy in the velocity dispersion.

(Explained by anisotropy in velocity dispersion)

Random motions greater in this direction

Random motions less in this direction
There appear to be two different classes of elliptical galaxies. They form in two different ways.

**Case #1: “Wet” Mergers**  
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Appear to result in ellipticals whose ellipticity can be explained by anisotropy in the velocity dispersion.

**Lower Luminosity**
- No core in center of elliptical
- Disky Isophotes

**Higher Luminosity**
- Core in center of elliptical
- Boxy Isophotes
NOW new material for this week
Why do Elliptical Galaxies Look So Similar?
Why do Elliptical Galaxies All Look So Much Alike?

i.e., why do their surface brightness profiles approximately satisfy a de-Vaucouleur law?

As we have seen from the first few lectures, there is a huge freedom in constructing collisionless systems that are in equilibrium.

So, these profiles must arise in some other way!
Violent Relaxation

One way to set up the equilibrium phase-space structure in elliptical galaxies is through violent relaxation!

Under normal conditions, for equilibrium systems, we would expect minimal changes in phase space structure of galaxies.

However, what would happen if there was a collision between two galaxies?

In this case, individual stars would experience a rapidly changing gravitational potential.
How would the time-varying gravitational potential affect individual particles?

Individual particles can gain or lose energy (due to the changing potential)

Credit: van den Bosch
As a result, while the total energy of the colliding galaxies does not change, there is a considerable energy exchange between individual stars in the colliding galaxies.

The redistribution of energy between particles rapidly moves the phase space distribution to being more Maxwellian in form (and its density profile more like an isothermal sphere)

The time scale for violent relaxation is equal to

\[ t_{\text{vr}} = \left\langle \frac{(dE/dt)^2}{E^2} \right\rangle^{-1/2} = \left\langle \frac{(\partial \Phi/\partial t)^2}{E^2} \right\rangle^{-1/2} = \frac{3}{4} \left\langle \dot{\Phi}^2 / \Phi^2 \right\rangle^{-1/2} \]

where the last equation comes from the time-dependent virial theorem (Lynden-Bell 1967)

As such, the time scale for violent relaxation is basically the time scale for the merger itself -- which is a few dynamical times. It is fast!
The redistribution of energy between particles proceeds in such a way as to be independent of the mass of stars in galaxies (since it only occurs due to time variations in the potential).

It therefore very different from what happens during collisional relaxation where there is a dependence on the mass of the colliding stars (e.g., as in globular clusters where the most massive stars tend to collect in the center).
Let us consider one concrete example of a system undergoing violent relaxation.

One such system is a cloud of particles that start out locally at rest.

Gravity will cause this cloud of particles to collapse onto the center of the cloud...

How does this system evolve with time?

This problem was initially considered by van Albada while working in Groningen (1982).
How does such a system evolve with time?

van Albada (1982)
Here is a short movie illustrating such a collapsing cloud:

Note that the flow is initially very uniform and ordered! After collapse, individual stars have a wide range of velocities and energies!
How does this re-distribution of energy look for this collapsing cloud?
Remarkably, it was found that the final density profile of the equilibrium system that formed from the collapsing cloud followed a $R^{1/4}$ law.

What is the final density profile for this collapsing cloud?
Such density distributions that result from such a collapse are also naturally triaxial in form.

Here are projections of the density distribution of stars in the xy, xz, and yz planes.
Additional smoothing out of the phase space distribution is provided by the process of phase mixing:

\[
\text{Phase Mixing}
\]

Consider circular motion in a disk with \( V_c = \frac{V_0}{R} = \text{constant} \). The frequency of a circular orbit at radius \( R \) is then \( \omega = \frac{1}{T} = \frac{V_0}{2 \pi R} \). Thus, points in the disk that are initially close will separate according to \( \Delta \phi(t) = \Delta \left( \frac{V_0}{R} t \right) = 2 \pi \Delta \omega t \).

We thus see that the timescale on which the points are mixed over their accessible volume in phase-space is of the order of \( t_{\text{mix}} \approx \Delta \omega^{-1} \). Where due to differences in the oscillation frequencies for different stars in a collisionless system, stars spread out to more completely fill phase space.
What about the merger between two galaxies?

Amazingly, one finds very similar density profiles to what one finds for the collapsing clouds. Here are some results from Hopkins et al. (2008):

These merging galaxies also largely follow $R^{1/4}$ density profiles.
Interestingly, mergers also produce profiles which are seen as down as far as the equilibrium laws are concerned. In merger simulations, one finds very similar density profiles to what one finds for the collapsing clouds. Here are some results from Hopkins et al. (2008):

![Graph showing density profiles for different models: Pre-Starburst Stars, Sersic + Cusp, Sersic Only, Cored Sersic.](image)

These merging galaxies also largely follow $R^{1/4}$ density profiles.
Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

As we noted earlier, such correlations would not need to be present for a generic collisionless system and tell us something fundamental about the formation of galaxies themselves.
Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.
Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

![Graph showing correlations between average surface brightness $(\mu)_e$ and velocity dispersion $\sigma$](image)

- **Average Surface Brightness**: $(\mu)_e$
- **Velocity Dispersion**: $\sigma$

The graph illustrates the distribution of galaxies based on these properties, with a particular focus on the relationship between $(\mu)_e$ and $\log \sigma$.
Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

Notice how much tighter the relationship can become if one combines the quantities in the appropriate fashion!
Intrinsic Correlations between Galaxy Properties

These correlations are such that galaxies populate a fundamental plane in these parameters, so that if you know two of the following three variables for a galaxy, you can determine the third.

\[ R \propto \sigma^{1.4} \mu_e^{-0.9} \]

where \( R \) is the size (radius), \( \sigma \) is the velocity dispersion, and \( \mu \) is the galaxy surface brightness.
The properties of galaxies occupy a two dimensional plane in the three dimensional parameter space.

Here is an illustration of such a two-dimensional surface embedded in a three-dimensional space.
How shall we interpret this fundamental plane relation?

Let us interpret in terms of the mass to light ratio of galaxies:

\[
\frac{M}{L} = \frac{M}{\mu_e R_e^2} = \frac{M}{\sigma^{1.5} R_e^{-1.1} R_e^2} = \frac{M}{M^{0.75} R_e^{-0.75} R_e^{-1.1} R_e^2} = M^{0.25} R_e^{-0.15}
\]

\[\mu_e \sim \sigma^{1.5} R_e^{-1.1}\]

From virial relation:

\[M \sim \sigma^2 R_e\]

\[M^{0.75} \sim \sigma^{1.5} R_e^{0.75}\]

The fundamental plane relation implies that the mass-to-light ratio scales as \(M^{0.25} R_e^{-0.15}\).

It is unclear what causes this mass dependence. It could be due to the stellar populations (i.e., that the baryonic component in massive galaxies have higher mass-to-light ratios) or due to dark matter playing a larger dynamical role in more massive galaxies.
What about Baryonic Gas in Ellipticals? Do they have any?

Yes -- lots of hot ionized gas!

Luminosity of the hot ionized gas scales approximately as the optical luminosity squared!

Total gas mass ranges from $10^9$ to $10^{11}$ solar masses

Hot ionized gas appears to originate partially from mass loss from AGB stars.

Heating is from SNe and movement of stars through gas.
What about Baryonic Gas in Ellipticals? Do they have any?

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Heating is from SNe and movement of stars through gas
What about Baryonic Gas in Ellipticals? Do they have any?

Occasionally there is cool gas

However, most of the time it is outside the center of a gas.

It is expected to be rare, since elliptical galaxies do not undergo much star formation.
Dark Matter Halos
Dark Matter Halos

Two Key Steps in Galaxy Formation

1. Gravitational Collapse
2. Cooling of Baryons to Center

Form of the Collapsed Dark Matter Halo (step #1 in galaxy formation process) can have an important impact on the structure and properties of galaxies...
Let’s derive the key relations between the mass, radius, and velocity dispersion in halos

When a halo collapses -- how does $\rho_{\text{halo}}$ compare to $\rho_{\text{critical}}$?
The total mass of a halo is given by the following:

\[ M = (4\pi (r_{200})^3/3) \times 200 \rho_{\text{critical}} \]

where \( r_{200} \) is the radius of the halo.

The \( \rho_{\text{critical}} \) is as follows:

\[ \rho_{\text{critical}} = \frac{3H(z)^2}{8\pi G} \]

As such,

\[ M = 100 H(z)^2 (r_{\text{halo}})^3 / G \]
Theory of Dark Matter Halos

From the virial relationship:

\[(V_{200})^2 = GM/r_{200}\]

One can write expressions for the mass and radius of the halo in terms of the circular velocity and the Hubble constant:

\[M = (V_{200})^3 / 10GH(z)\]

\[r_{200} = V_{200}/10H(z)\]

The Hubble “constant” \(H(z)\) increases towards higher redshifts, effectively scaling as \((1+z)^{3/2}\) at very high redshifts.

Halos of a given mass are more compact at high redshift.
Navarro-Frenk-White Density Profiles

Simulations show that collapsed halos approximately have the following density profile:

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

where $r_s$ and $\rho_s$ are some scaling parameters.

At small radii ($r < r_s$), the density profile $\rho$ scales approximately as $r^{-1}$

and at large radii ($r > r_s$), the density profile $\rho$ scales approximately as $r^{-3}$

At $r \sim r_s$, the density profile $\rho$ changes slope
Here are some examples of the density profiles from simulations:

SCDM = standard cold dark matter (no dark energy)

ΛCDM = cold dark matter with dark energy
Here are some examples of the density profiles from simulations:

SCDM = standard cold dark matter (no dark energy)

ΛCDM = cold dark matter with dark energy
Define concentration parameter $c = r_{200} / r_s$

Requiring the total mass in the halo is equal to $200 \rho_{\text{crit}}(z) (4\pi/3 (r_{200})^3)$, one can show that

$$\rho_s = \frac{200}{3} \rho_{\text{crit}}(z) \frac{c^3}{\ln(1 + c) - c/(1 + c)}$$

so that the density profile of a halo is completely determined by its mass and concentration parameter $c$. 

Navarro-Frenk-White Density Profiles
Navarro-Frenk-White Density Profiles

From simulations, one finds that the concentration parameter $c$ for a halo is closely related to its formation redshift:

$$c \propto \frac{M^{-1/9}}{M_*} (1 + z)^{-1}$$

Lower mass halos have higher concentration parameters.
Figure 1. Mass and redshift dependence of the concentration parameter. The points show the median of the concentration as computed from the simulations, averaged for each mass bin. Lines show their respective linear fitting to eq. 5.
Let’s also look at the rotational curves:

Simulations show that the concentration index is strongly correlated with the mass and the redshift. Approximately

$$c \propto M^{-1/9} M^* (1 + z)^{-1}$$

where $M^*$ is the characteristic halo mass at a given mass (similar to the Schechter $L^*$ for the luminosity function of galaxies, and $z$ is the redshift of the halo. Hence low mass halos have higher concentration.

Does this make sense in terms of $V^2 \sim GM(R)/R$ ?
Collapsed Halos: Comparison with the Observations

First -- we look at the structure of the dark matter halo for very massive collapsed halo \((M \sim 10^{14} - 10^{15} M_{\text{sol}})\) i.e., as appropriate for galaxy clusters
4.2 Comparison to observations

The easiest test to make is to compare the density profiles of clusters to those of the simulations (the NFW profiles, and variants).

Below we show the comparison of mass profiles derived from X-ray data to those of NFW profiles:

It is remarkable how good the fit is, and the concentration index is well within the limits expected for the clusters.

Another (simpler) way to do this is by looking at the distribution of the galaxies within the clusters. Again, a good fit is found:

Radial Profile

Information from x-ray profiles

XMM Newton Data of 10 Clusters

Mass / \( M_{200} \)

Mass Profile comes assuming hydrostatic equilibrium

Pressure gradient balances gravitational forces!
Halos of Galaxy Clusters: Comparison against the Observations

Do the 10 clusters from previous page fit the theoretical concentration vs. mass relationship?

\[ c \propto \frac{M^{-1/9}}{M_*} (1 + z)^{-1} \]
Halos of Galaxy Clusters: Comparison against the Observations

We can also use the apparent surface density of galaxies within clusters vs. radius to probe the density profile.
We can also examine the mass profiles of galaxies to see if they follow the expected mass profile:

What are some techniques we can use?

1. Kinematics of satellite galaxies
2. Kinematics of distant stars
3. Gravitational Lensing by Galaxy
One method is to take advantage of the satellite companions found around galaxies in the nearby local universe as a probe of kinematics.

Unfortunately, one only tends to find one such satellite galaxy per central galaxy (with a measured velocity along the line of sight), so it is not possible to make a reliable measurement using individual galaxies.

To make progress, one needs to treat these galaxies as orbitting around some “composite average” galaxy.

Results are in good agreement with an NFW profile.
Kinematics of Distant Stars

Most easily applied to our own galaxy. One use of this method shown below (Xue et al. 2008)

Applied to 60 kpc! (which is nevertheless smaller than 200 kpc: which is approximately the halo radius)
Gravitational Lensing also gives us a direct way of measuring the mass inside some radius.

For sources we can see an approximate Einstein ring, measuring the mass enclosed is easy -- since we can derive the mass enclosed at the ring radius just from the geometry.

In fact, there are now large programs, i.e., SLACS, to use exactly these sort of techniques to measure the mass profile of elliptical galaxies.
Gravitational Lensing by Galaxy

The Sloan Lens ACS Survey (SLACS)

Cols: T. Treu (UCSB), L. Koopmans (Kapteyn), A. Bolton (CfA), S. Burles (MIT), L. Moustakas (JPL)

Spectroscopic selection (spurious emission lines), then HST follow-up imaging for confirmation and for accurate modeling

R. Gavazzi, SBastro07
Gravitational Lensing by Galaxy

SLACS sample
Gravitational Lensing by Galaxy

SLACS sample
Despite some success in this regard, it is not always easy to find large numbers of galaxies with Einstein rings.

An alternate technique is to measure the average impact of gravitational lensing of the shapes of galaxies.

Impact of gravitational lensing is to introduce subtle change in the shape of galaxies on average (so elongated along an angle tangential to the galaxy)
Problems with dark matter halos and real galaxies

Generally, two problems exist: some galaxies may have rather large central cores (larger than expected in the NFW profiles), and the dark matter halos in the simulations have too many "sub-halos".

• Cores

Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:

Fits with NFW profiles are hard. The rise of the rotation curve is too slow:

By looking at many foreground galaxies and determining the average change in shape of many background galaxies, we can measure the mass profile of the galaxy.

Such averaging is required because background galaxies have random shapes and orientations.

However, averaging over enough galaxies, one can overcome the random component.

van Uitert et al. 2011
Challenges / Issues with our standard model for the collapsed dark matter halos
Cores of Halos

What can we infer about the cores of dark matter halos from the observations?

Where should we look?

Almost all collapsed halos have baryons cooling and falling towards their centers. These baryons will change the mass profile of the original collapsed halo.

Basically, we have the following situation:

$$V(R)^2 \frac{R}{G} \sim M(R) \sim M_{DARK}(R) + M_{COOL}(R)$$

To obtain best constraints on $M_{DARK}(R)$, we need $M_{COOL}(R)$ to be as small as possible.
Cores of Halos

For which galaxies would we expect the halos to be the least affected by the baryons cooling to the centers?

For which galaxies, will $M_{\text{COOL}}(R)$ be the smallest?
Here are two galaxies where the baryonic component has different angular momentum. For which case, can we get a cleaner look at the dark matter halo?

- Spinning fast:
  - Intermediate size, more dense disk galaxy
  - Higher surface brightness
  - After cooling

- Spinning moderate speed:
  - Large, lower density galaxy disk
  - Lower surface brightness
  - After cooling
Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:
Low surface brightness galaxies have rotation curves which rise slowly. This is unexpected for galaxies with NFW halos. Naray et al 2007 (arxiv: 0712.0860) show this result:
Cores of Halos

What would infer about the mass profile of a halo where the circular velocity appears to rise linearly with radius?

Using the relation $M = V(R)^2 R / G$,

we would infer that $M \propto R^3$ or $\rho \sim$ independent of radius

But for NFW, we would expect $\rho \propto r^{-1}$ at small radii