Elliptical Galaxies (II)

March 9

Layout of the Course

Lectures

- Feb 2: Course Introduction, Overview, and Galaxy Formation Basics
- Feb 9: Disk Galaxies (I)
- Feb 12: Disk Galaxies (II)
- Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics
- Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations
- Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)
- **Mar 9: Elliptical Galaxies (II)** ←
- Mar 23: Elliptical Galaxies (III)
- Mar 30: Dark Matter Halos
- Apr 13: Large Scale Structure
- Apr 20: Galaxy Stellar Populations
- Apr 23: Lessons from Large Galaxy Samples at z<0.2
- May 4: Evolution of Galaxies with Redshift
- May 11: Galaxy Evolution at z>1.5 / Review for Final Exam

Problem Set 2

(Due March 16)

Galaxies: Structure, Dynamics, and Evolution
Problem Set 2
Instructor: Dr. Bouwens

Here is Problem Set 2. The entire problem set will be due before class on Monday, March 16 (email them to Wout). Be sure to pay extra attention to problem 4, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. How does the epicyclic (radial) frequency κ for both an isothermal potential ($v_c =$ constant) and a potential with the form $\phi(r) = r^{-3/4}$ compare with the azimuthal frequency Ω? How many radial/epicyclic oscillations will a star undergo for each orbit around a galaxy?

2. Calculate the epicyclic frequency for both an isothermal potential ($v_c =$ constant) and a potential with the form $\phi = -1/(1+r^2)$. Draw the behaviour of $\Omega - n\kappa/m$ as in figure 6.11.

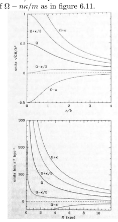

Figure 6.11 Behavior of $\Omega - n\kappa/m$ in: (top) the isochrone potential (eq. 2.47) (bottom) Model I for our Galaxy, described in §2.7.

1

3. In class, we calculated the relaxation time for a star in some dynamical system (e.g., a galaxy) by considering the effect of its interactions with other stars in a galaxy. In calculating the relaxation time, we only considered impact parameters b from b_{min} to the size of the galaxy R (where b_{min} was the minimum impact parameter where our simple formula for the velocity kick was approximately valid). What about the effect of impact parameters $b = 0$ to b_{min}? How does this impact the relaxation time?
(a) Calculate the probability that a star will pass by another star with impact parameter b less than or equal to b_{min}? (Ignore the curvature of the orbit.) Adopt the standard variables N, v, m, and G used in the derivation during lecture.

(b) If any star passes by another star with impact parameter $b < b_{min}$, its velocity will be so perturbed that it "will lose all memory of its initial orbit." Let us say it relaxes with just one encounter. Calculate the relaxation time assuming that $b < b_{min}$ encounters are the only meaningful relaxation process. Calculate the relaxation time for the same choice of N, m, G, and v considered in class for a galaxy (i.e., $N = 10^{10}$, $v = 100$ km s^{-1}, $r = 10$ kpc, $t_{cross} = 10^8$ yr). How does this compare with the relaxation time derived in class considering only $b > b_{min}$ encounters?

4. To help yourself visualize the Inner and Outer Lindblad Resonances work, as well as the corotational radius, I will ask you to sketch out a number of snapshots of the movement of a spiral arm around a galaxy. Assume that a galaxy has two spiral arms and that it is an isothermal sphere.
(a) Suppose that circular velocity of the galaxy is 200 km/s and that the inner Lindblad resonance for a galaxy is at a radius of 2.5 kpc. What is the pattern speed (or frequency) Ω_p?
(b) What is the period of epicyclic motion at the Inner Lindblad resonance?
(c) What is the corotation radius and radius of the Outer Lindblad resonance for the pattern speed you computed in part (a)?
(d) In multiples of the epicyclic period of stars at the inner Lindblad resonance (consider multiples out to 10), please sketch out the motion of stars in a spiral galaxy (similar to what I do in lecture, but now moving indicative stars at the Inner and Outer Lindblad resonances and corotation radius self consistently). Please indicate the position of the spiral arms, a star at the radius of the Inner Lindblad resonance, a star at the corotation radius, and a star at the radius of the outer Lindblad resonance. Assume that the position angle of the spiral arms and the stars at all three radii are all 0 at time t = 0. ←

2

This will be the graded problem

5. How many integrals of motion are there for a particle with the following force law $F(r) = -\hat{r}/r^2$ where r is the radius?

6. Show that the distribution function $f(\epsilon, L)$

$$f(\epsilon, L) = \begin{cases} F\delta(L^2)(\epsilon - \epsilon_0)^{-1/2} & \text{for } \epsilon > \epsilon_0, \\ 0 & \text{otherwise.} \end{cases}$$

where F and ϵ_0 are constants and δ is the familiar delta function. Show that this distribution function self-consistently generates a model with density

$$\rho(r) = \begin{cases} Cr^{-2} & \text{for } r < r_0 \\ 0 & \text{otherwise.} \end{cases}$$

where C is a constant and the relative potential at r_0 satisfies $\Psi(r_0) = \epsilon_0$. This is the only analytic stellar system known to us in which all stars are on perfectly radial orbits.

March 12 Practical Session

(In 3 days)

Problems 2, 3, and 6 (to be discussed)

Ana Tejero	Arianna Zirotti
Dimitria Tsioutsia	Ottavia Zanello

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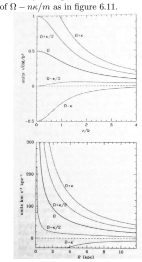

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3

Problem Set 2

(Distributed earlier, due on March 19)

Practical Sessions

Feb 19: Board Work + Problem Set 1
Mar 12: Board Work + Problem Set 2 ←
Mar 26: Problem Set 3 / Paper Presentations (4 slots)
Apr 2: Paper Presentations (7 slots)
Apr 16: Problem Set 4 / Paper Presentations (4 slots)
Apr 30: Problem Set 5 / Paper Presentations (4 slots)
May 7: Problem Set 6 / Paper Presentations (4 slots)

February 19 Practical Session

(In the past)

Problems 5 and 6

Eugenia Rodendo Gonzalez, Noah Kaijser, Andrea Gibilaro, Susana Carneiro

5. In lecture, we examined an arbitrary dynamical system and determined how that dynamical system can be scaled in position, mass, and velocity and still maintain the same qualitative form.
(a) Show explicitly that the virial theorem produces the same result for the scaling relations.
(b) Derive Kepler's Third Law using the scaling relations found in class.
(c) Do the same sort of scaling relations exist for stars? Is it possible to scale the position, velocity, and mass for particles in a star in the same way – and have a system with the same qualitative form? Which equilibrium is retained and which is lost?

6. Prove that $M \propto T^{3/2}/n^{1/2}$. Use the fact that $\sigma^2 \propto T$ and $n \propto M/R^3$. Comment on the importance of this scaling relative to the T vs. n diagram used to understand for which mass sources $T_{cool} < T_{dyn}$ (i.e., where galaxy formation is efficient).

→ Excellent solutions. Will receive full credit!

Homework Grade — What goes into It?

50% —> Graded Problem Sets (~6) → 12.5% of total
(Only One Problem from Each Set Graded)

30% —> Oral Presentation of a Solution During Practical Class → 8.75% of total

20% —> Attendance / Participation in Practical Classes → 6.25% of total
- 100% Score, if attend 71% of Classes
- 80% Score, if attend 57% of Classes
- 60% Score, if attend 43% of Classes
- 30% Score, if attend 29% of Classes

If you provide consistent feedback on problems to discuss in class, credit for 1 additional class attended.

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What about the Paper Presentations?

To connect with the material from the course with a more contemporary, in-depth analysis, part of your grade will be based on a 10-minute presentation on manuscript from the literature.

I will provide a list of 40 papers from which to choose to present — which will be extensions of the material covered in this course.

Depending on the paper you choose, it will be suggested that you give a presentations towards the mid or end point of the course.

It will be first come, first serve, for papers, so by choosing early, you will have more choice regarding the paper/topic to cover.

What about the Paper Presentations?

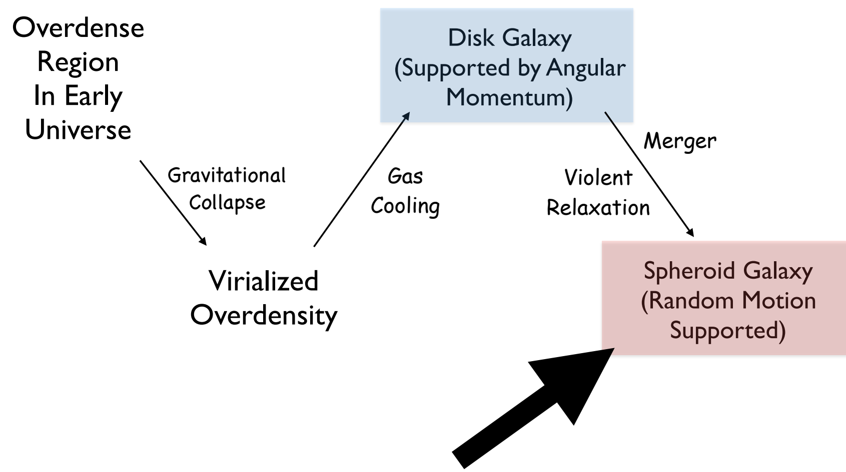
You will be sent a list later today!

Please sign up by Friday, otherwise you will be assigned some papers randomly.

Papers focusing on cooling and disk galaxy characteristics (spiral density wave, halo properties) will go first.

First, let's review the important material from last week

Galaxy Formation: Major Steps



Next topic is elliptical galaxies...

Elliptical galaxies consist of large numbers of stars on diverse orbits.

While spiral galaxies are rotation supported, elliptical galaxies are supported by the random motions of stars they contain

Their behavior can largely be described using collisionless dynamics.

REVIEW point from Bachelor course: The time evolution of the distribution function is defined by the distribution function at that time, spatial derivatives, and the gradients of the potential (Vlasov-Equation). This follows directly from a conservation equation on the stars.

At any time t, one can describe the collective positions and velocities for stars in a dynamical system by a distribution function f(**x**,**v**,t)

For the distribution function, one very useful equation is the Collisionless Boltzmann equaton which guarantees stars are conserved in moving through phase space.

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

or

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f - \vec{\nabla}\Phi.\frac{\partial f}{\partial \vec{v}} = 0$$

also can write as $\frac{df}{dt} = 0$

Setting up equilibrium models for a collisionless system.

It is not necessarily an easy thing to do

We must set up a self-consistent system whereby each of the following steps imply the next:

(1) given density distribution ρ(r), calculate the potential Φ(r) the density distribution would imply

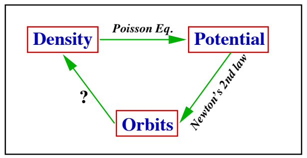
(2) given some potential Φ, determine the set of orbits that stars would undergo

(3) calculate the density distribution that would result from the collective orbits of all the stars in a system

The Density Distribution derived in step #3 must be the same as assumed in step #1

The relevant equations are:

$$\begin{aligned} \rho(\vec{x}) &= \int f(\vec{x},\vec{v})\, d^3\vec{v} \\ \nabla^2\Phi(\vec{x}) &= 4\pi G\rho(\vec{x}) \\ \frac{df}{dt} &= 0 \end{aligned}$$



Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

There is a huge amount of freedom in finding a distribution function that satisfies the collisionless boltzmann equation for some equilibrium model.

How can we find a solution?

One approach is to make the distribution function a function of the integrals of motion themselves.

By definition, the first time derivative of an integral of motion is zero. Since this is the same requirement the collisionless boltzmann equation makes on the distribution function, we know that any arbitrary function of the integrals of motion satisfies the collisionless boltzmann equation.

↕ Jean's theorem

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

By constructing the distribution function f as such, we ensure that we already solve one of the 3 equations needed to construct a self-consistent equilibrium system:

The equations we need to satisfy:

$$\begin{aligned} \boldsymbol{\rho(\vec{x})} &= \int \boldsymbol{f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v}} \\ \boldsymbol{\nabla^2\Phi(\vec{x})} &= \boldsymbol{4\pi G\rho(\vec{x})} \\ \boldsymbol{\frac{\mathrm{d}f}{\mathrm{d}t}} &= \boldsymbol{0} \end{aligned}$$

Using Jeans theorem to construct a self-consistent equilibirum, we automatically satisfy this equation

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

Now we move onto solving this one:

$$\begin{aligned} \boldsymbol{\rho(\vec{x})} &= \int \boldsymbol{f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v}} \\ \boldsymbol{\nabla^2\Phi(\vec{x})} &= \boldsymbol{4\pi G\rho(\vec{x})} \\ \boldsymbol{\frac{\mathrm{d}f}{\mathrm{d}t}} &= \boldsymbol{0} \end{aligned}$$

We showed in class that

$$\rho(r) = 4\pi \int_0^{\psi} f(\varepsilon)\sqrt{2(\psi-\varepsilon)}\,d\varepsilon$$

where ψ is the depth of a potential relative to the highest energy particles.

Finally, all that remains to be done to set an equilibrium system satisying Poisson's equation is the following:

$$\boldsymbol{\nabla^2\Phi(\vec{x})} = \boldsymbol{4\pi G\rho(\vec{x})}$$

REVIEW Point from Bachelor Studies. Another way of constructing equilibrium models is the Schwardschild method. It involves making a library of orbits, calculate their spatial densities, and calculate the weight function which reproduces the density distribution for which the orbits were calculated.

What is the Schwarzschild method for constructing equilibrium models?

1. Define density
2. Derive gravitational potential
3. Derive orbit families for this gravitational potential
   Determine the densities that result from different orbits
4. Combine the different orbital families in such a way to produce the defined density.

REVIEW Point from Bachelor Studies. Another way of constructing equilibrium models is the Schwardschild method. It involves making a library of orbits, calculate their spatial densities, and calculate the weight function which reproduces the density distribution for which the orbits were calculated.

In considering different starting positions in phase space (spatial position and velocities), can get a wide variety of different orbits, e.g.,

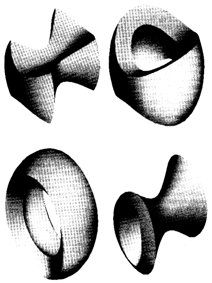


Figure 3-20. Orbits in a nonrotating triaxial potential. Clockwise from top left: (a) box orbit; (b) short-axis tube orbit; (c) inner long-axis tube orbit; (d) outer long-axis tube orbit. [Courtesy of T. Statler; see Statler (1986).]

Potential Density Distributions Defined by Different Orbits

One needs to figure out what combination of density distributions defined by different orbit families allows one to recover the originally assumed density distribution (step 1 in Schwarzschild method)

REVIEW point from Bachelor course. An important method for deriving the total mass of individual galaxy involves the Jeans equations and moments of the stellar velocity.

First define several velocity moments of the distribution function:

0. Spatial density of stars (0th moment):

$$\nu(\vec{x}) = \int f(\vec{x}, \vec{v})\, \mathrm{d}^3\vec{v}$$

1. Mean velocity of stars (1st moment):

$$\overline{v_i}\,(\vec{x}) \equiv \frac{1}{\nu}\int v_i f(\vec{x}, \vec{v})\, \mathrm{d}^3\vec{v}, \quad i = 1, 2, 3$$

2. Second moment of stars (2nd moment):

$$\overline{v_i v_j}\,(\vec{x}) \equiv \frac{1}{\nu}\int v_i v_j f(\vec{x}, \vec{v})\, \mathrm{d}^3\vec{v}, \quad j = 1, 2, 3$$

Velocity Dispersion Tensor:

$$\sigma_{ij}^2 \equiv \overline{(v_i - \overline{v_i})(v_j - \overline{v_j})} = \overline{v_i v_j} - \overline{v_i}\;\overline{v_j}$$

REVIEW point from Bachelor course. An important method for deriving the total mass of individual galaxy involves the Jeans equations and moments of the stellar velocity.

Jeans Equation 1 (Continuity equation):

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu\, \overline{v_i} = 0$$

Jeans Equation 2 (The Force Equation):

$$\frac{\partial(\nu\, \overline{v_j}\,)}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\nu\, \overline{v_i v_j}\,) + \nu \frac{\partial \Phi}{\partial x_j} = 0$$

Jeans Equation 3 (Rewrite of Equation 2)

$$\nu \frac{\partial\, \overline{v_j}}{\partial t} + \sum_{i=1}^{3} \nu\, \overline{v_i}\, \frac{\partial\, \overline{v_j}}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu \sigma_{ij}^2$$

Total Enclosed Mass and Rotation Curve

Assuming a particle is on a circular orbit at radius r, there is a relationship between the circular velocity v_c and $\frac{d\Phi}{dr}$

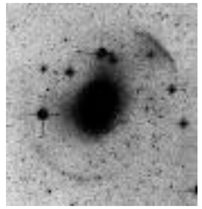
$$\frac{d\Phi}{dr} = \frac{GM(<r)}{r^2} = \frac{v_c^2}{r}$$

Using this relation, the Jeans Equation can be written as

$$v_c^2 = \frac{GM(<r)}{r} = -\overline{v_r^2}\left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\overline{v_r^2}}{d\ln r} + 2\beta\right)$$

From this expression, we see that if we can measure the density of stars ν, the velocity dispersion in the radial direction, and anisotropy function, we can determine the enclosed mass inside some radius

What is the nature of elliptical galaxies?



End state of galaxy formation!

Dominated by the random motions of its component parts (stars, hot gas, dark matter)

While progenitors to elliptical galaxies experienced lots of star formation, elliptical galaxies themselves experience almost no star formation

Only way for an elliptical galaxy to transform into another type of galaxy (e.g., spiral) is if lots of cold gas cools onto it (but this may not happen!)

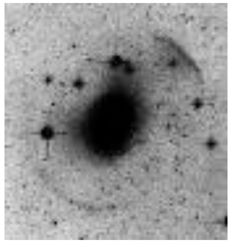
What is the nature of elliptical galaxies?

What can we learn about elliptical galaxies from their structure?

(from Photometry)

We begin by looking at the two dimensional surface brightness profiles of elliptical galaxies.

Not even necessary to use a telescope now to investigate -- as there is a lot of data from surveys like the Sloan Digital Sky Survey



What we can learn from Images of Elliptical Galaxies?

Important clues can come from looking at the two dimensional surface brightness profiles of elliptical galaxies.

Two Types of Deviations from Elliptical Isophotes are Observed (Few Percent)

Boxy

Disky

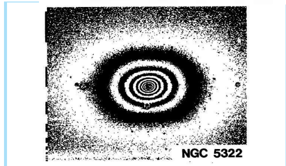


FIGURE 7. — R-image of NGC 5322, an elliptical galaxy with box-shaped isophotes $(a(4)/a \sim -0.01)$.

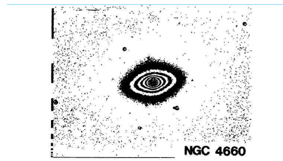
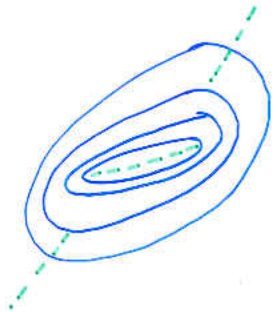


FIGURE 6. — R-image of NGC 4660, an elliptical galaxy with a disk-component in the isophotes $(a(4)/a \sim +0.03)$.

New Material

Structure of Elliptical Galaxies

Light is almost constant on elliptical "isophotes"



Position Angles and Ellipticities vary slightly (or twist) with radius

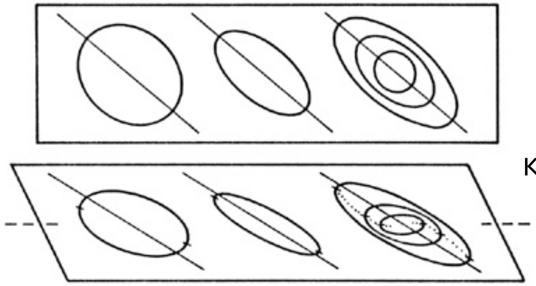
Such twisting of the position angles of the isophotes can easily result from projection effects if an elliptical galaxy has a triaxial shape.

Structure of Elliptical Galaxies

Let's see if we can't understand how this "isophote twisting" works:

"Original"

"After Projection"



Credit: Kormendy

Notice that the position angle of major axis of the ellipses change, after projection. Depending on the initial ellipticity of the original ellipse, the change in position angle can be quite large.

Such twists in the position angle of the major axis is expected, if elliptical galaxies have triaxial profiles.

Twisting isophotes in real galaxies

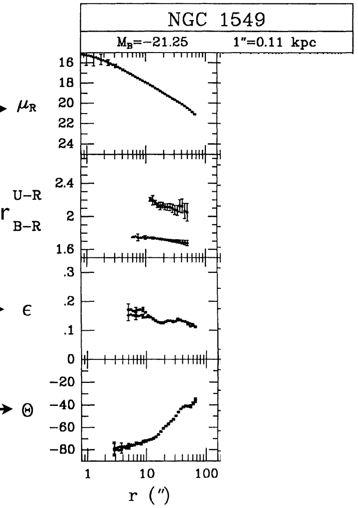
Example galaxy!

Surface Brightness →

Color

Note best-fit ellipticity varies with radius slightly →

Also position angle! →



Twisting isophotes in real galaxies

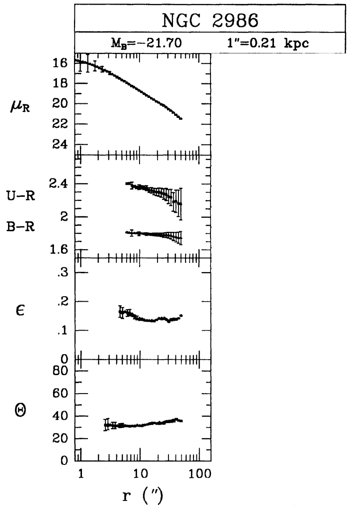
Another galaxy!

Surface Brightness →

Color

Note best-fit ellipticity varies with radius slightly →

Also position angle! →



Surface Brightness vs. Radius

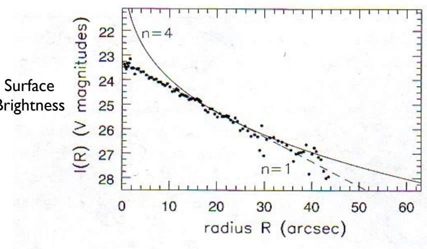
Intensity Profiles in Galaxies Well Described by Sersic Profile:

$$I(R) = I_e 10^{b_n[(R/R_e)^{1/n}-1]}$$

where R_e is the half-light Radius and n is the Sersic Index (from 2 to 5 for most Elliptical Galaxies)

For large n, there is a lot of light at very large radii and very small radii -- i.e., extended wings to light profile and bright center

For small n, the light is less concentrated in center. Also the light profile cuts off at large radii.



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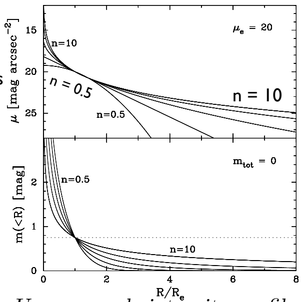
Sersic Profiles

Surface Brightness

How measured magnitude depends on radius of aperture in which measurement is made



Upper panel: intensity profiles astroph-0503176v1). The n pa

Structure of Elliptical Galaxies

Typical value is n=4
"de-Vaucouleur Profile"

Surface Brightness

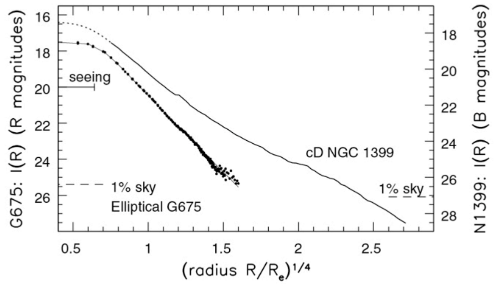


Fig 6.3 (Saglia, Caon) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Note that it is very challenging to trace the surface brightness profile of ellipticals to large radii, due to their low surface brightness there and high sky brightness

Seeing by the atmosphere can also affect the measurement of the profile at small radii

Center of Ellipticals: Cusp vs. Core

Some deviation from a Sersic profile can be seen in some galaxies.

In their centers, some elliptical galaxies show cusps and some show cores.

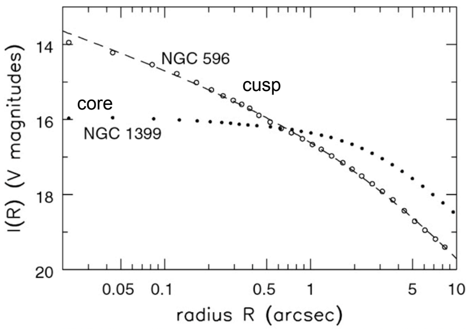


Fig 6.7 (T. Lauer) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

elliptical galaxies with cusps are also called "power-law" galaxies

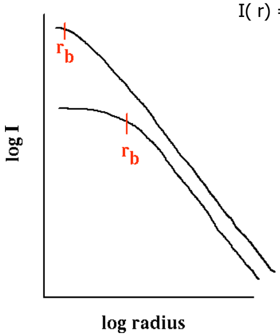
elliptical galaxies with core are also called "break" galaxies

Definition of Break Radius

The radius where galaxies show this break from the power law (or cuspy) profile is called the "break" radius.

$I(r) = I_b 2^{(\beta-\gamma)/\alpha} (r_b/r)^{\gamma} [1+(r/r_b)^{\alpha}]^{(\gamma-\beta)/\alpha}$ $r_b)^{\alpha}]^{(\gamma-\beta)/\alpha}$

Surface Brightness



r_b = radius where power-law changes shape
I_b = surface brightness where power-law changes shape
β = power-law at large radii
γ = power-law at small radii
α = sharpness of transition between power-law slopes

can be a five parameter fit!

deficit of stars at the center of ellipticals is thought to be due to scouring by the central super massive black hole

Shells and Ripples

Elliptical galaxies also show a significant number of shells and ripples
(presumably indicative of mergers between galaxies in the past)

Shells and Ripples

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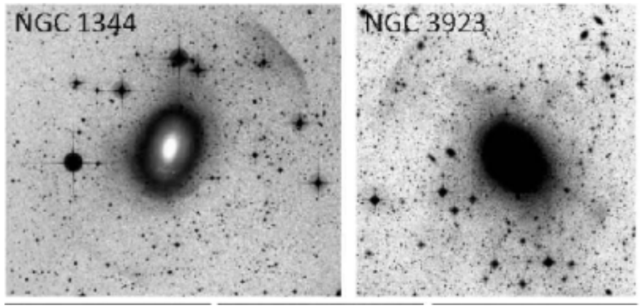
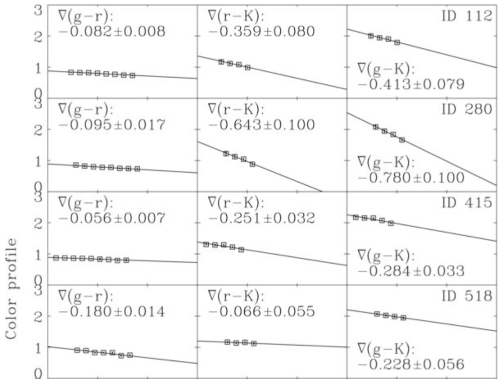
(presumably indicative of mergers between galaxies in the past)



Color Gradients in Ellipticals



Kim & Im
(2013)

Weak Colors Gradients -- Inner Parts Redder

Likely due to systematic differences in the
metallicities of the stars

Kinematics of Elliptical Galaxies

We can also learn things from the kinematic information
we can derive from elliptical galaxies.

While there are far too many stars to observe the signatures of stars individually, one can consider the velocity distribution of stars along the line of sight:

$F(v_{los})$

Typically this function is assumed to be very close to a Gaussian:

$$F(v_{los}) = exp\left[\frac{-(v_{los} - \overline{v_{los}})^2}{2\sigma_{los}^2}\right]$$

In reality, one cannot describe the velocity distribution as a perfect Gaussian, so one allows for a third or fourth moment in the distribution.

Make use a complete orthogonal polynomial series ("Gauss Hermite") multiplied by an exponential:

$$e^{-k} [1 + \Sigma_{k=3,n} h_k H_k (w)]$$

h_3: Gauss-Hermite information $(2w^3-3w)/3^{1/2}$

measures "skewness" or deviation from symmetry. Large h_3 represents a secondary bump at v > <v>, so the peak of the line is at v < <v>

h_4: Gauss-Hermite information $(4w^4-12w^2+1)/24^{1/2}$

measures "kurtosis" or symmetric departures from a Gaussian. Large h_4 indicates a boxy profile centered on <v>.

Deriving Kinematic Information

We can determine the mean line of sight velocity and the dispersion of a galaxy (and other higher order velocity components) by taking a high signal to noise spectrum of this galaxy and then comparing its spectrum with a star.

Model Spectral Energy Distribution:

$$\mathrm{Model(L)} = \mathrm{convolved\ star\ spectrum}(L) =$$

$$= \int S(L - v_{los}/c) F(v_{los}) dv_{los}$$

where S is the spectrum of the stars that dominate the light in a galaxy.

The integral is over the many different velocities v_{los} that stars in a galaxy could have.

Fitting the Velocity Dispersion

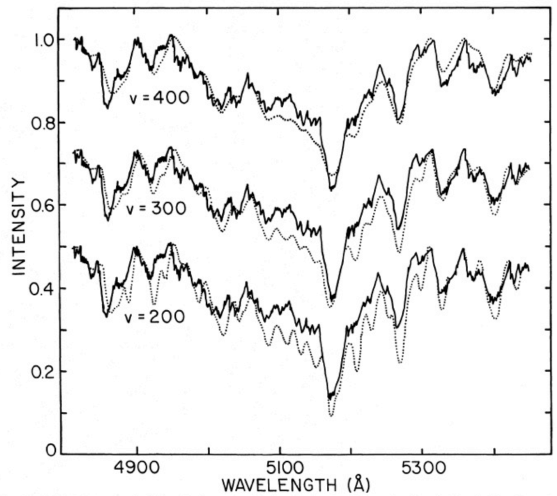


FIG. 3.—NGC 4472 compared with standard star HR 1805 (K3 III), broadened by various line-of-sight velocities (*dotted line*)

Deriving Kinematic Information

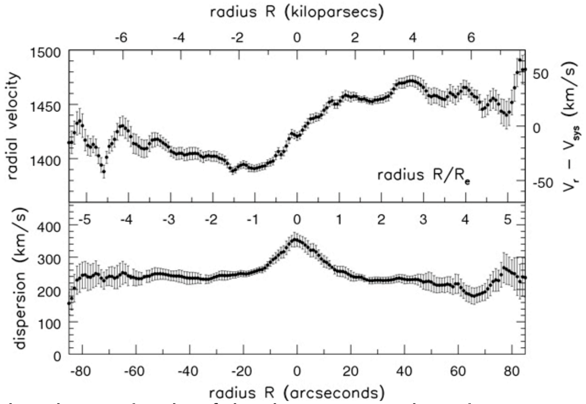
Find the model that gives the minimum X^2 with respect to the observed spectrum:

$$\chi^2 = \Sigma \left(\frac{(G(L) - Model(L))^2}{error(L)^2} \right)$$

In practice, the model spectrum never fits the observed spectral energy distribution exactly (due to the fact that the stars in the model never have exactly the same distribution of metallicities or temperatures as in the observations), so one needs to include some freedom in the models to overcome this effect.

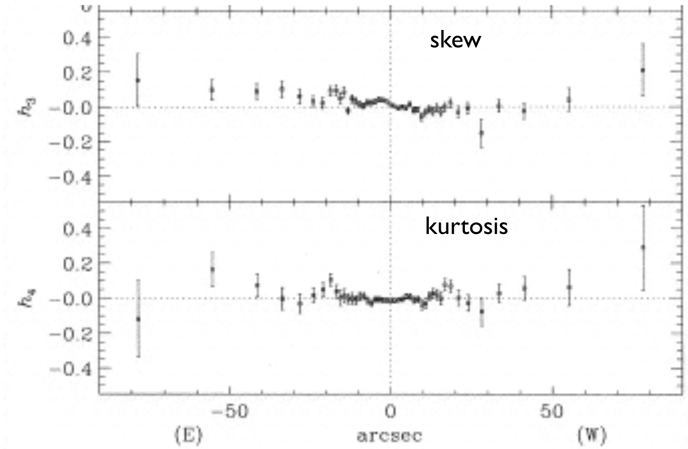
How does the mean line of sight velocity and velocity dispersion depend on the radius of an elliptical galaxy?

One Example:

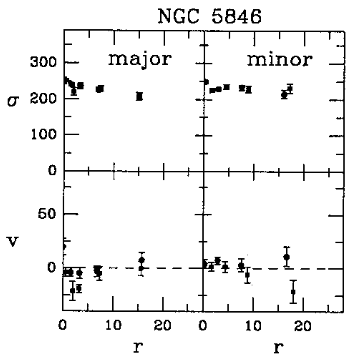


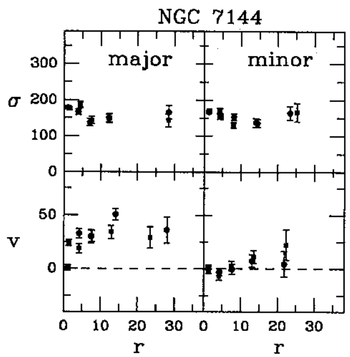
Notice that the amplitude of the dispersion in the velocity is much higher in general than the rotational velocities in these galaxies.

Can also attempt to quantify to the skew and the kurtosis of the velocity distribution as a function of radius in galaxies



Other examples of results for individual galaxies:





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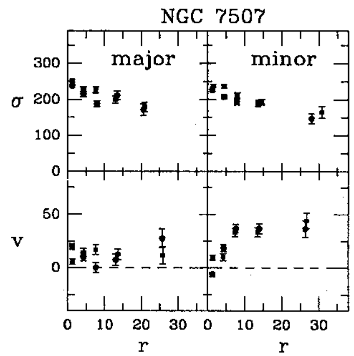
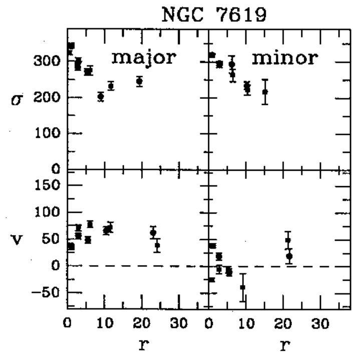
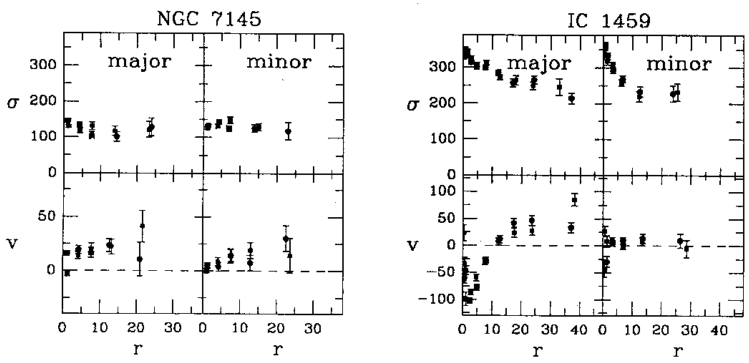


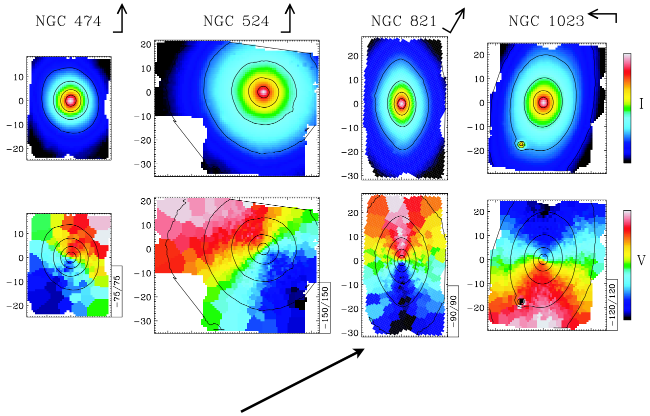


Other examples of results for individual galaxies:



Can also measure the velocity distribution of galaxies as a function of both coordinates on the sky:

(this work done with an IFU: Integrated Field Unit)

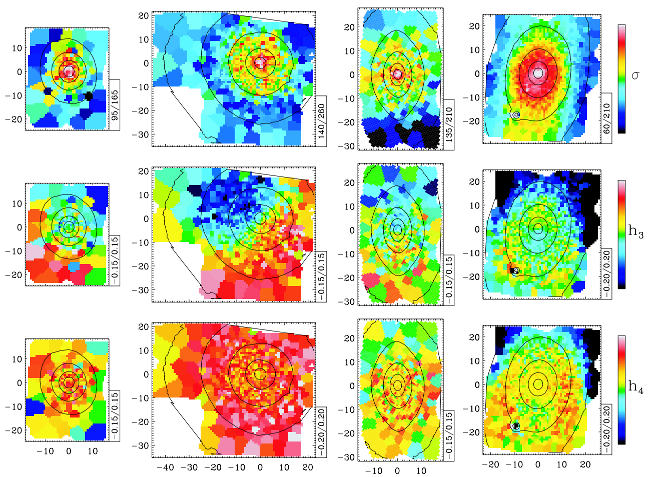


Surface Brightness

Average velocity (line of sight)

Dipole-like structure indicates a net rotation about the center

Can also measure the velocity distribution of galaxies as a function of both coordinates on the sky:



Velocity Dispersion

“Skew” in Velocity Distribution

“Kurtosis” in Velocity Distribution

General Conclusions from Kinematic Studies

Elliptical Galaxies Have Dispersions between 100 and 300 km/s

Rotation Velocity of galaxies is smaller ~50 km/s

Ratio of Rotation Velocity v to velocity dispersion σ, i.e., v/σ, is generally between 0 and 1

Most of Rotation on Major Axis, but also on Minor Axis

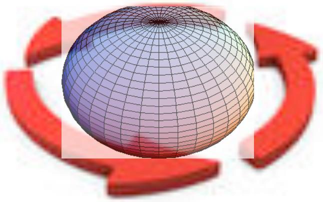
From examples we have just given -- NGC7144, NGC7145, IC1459, NGC7619, NGC821, NGC1023 show more rotation along major axis

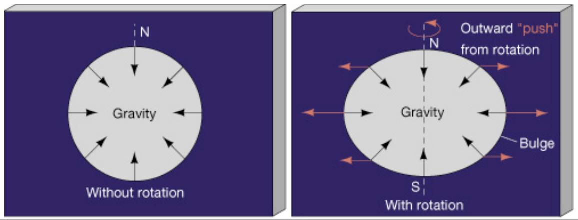
From examples we have just given -- NGC7507 shows more rotation along minor axis

Most of the Stellar Motion is Random, not Systematic

Why do Ellipticals Have an Elliptical Morphology?

One possibility = rotational flattening





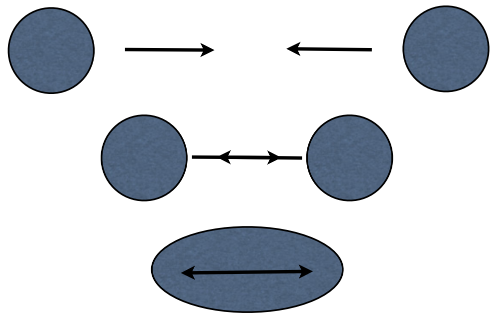
rotating objects tend to be elongated away from axis of rotation

this mechanism has an impact on the shape of the planets in the solar system, especially Jupiter and Saturn

Why do Ellipticals Have an Elliptical Morphology?

Second possibility = anisotropy in the velocity dispersion

consider the formation of elliptical galaxies from the merger of two smaller galaxies



Is the Flattening of Elliptical Galaxies Due to Rotation?

Make use of 3D virial theorem:

moment of inertia tensor →

$$1/2\frac{d^2 I_{j,k}}{dt^2} = 2T_{jk} + \Pi_{jk} + Wjk$$

where

$$T_{jk} = 1/2 \int \rho \, \overline{v_j} \; \overline{v_k} \, d^3x$$

$$\Pi_{jk} = 1/2 \int \rho \sigma_{jk}^2 d^3x$$

$$\overline{v_j} = \int v_j f(x,v) d^3v/\rho$$

$$\sigma_{jk} = \int (v_j - \overline{v_j})(v_k - \overline{v_k}) f(x,v) d^3v/\rho$$

$$W_{jk} = -\int \rho(x) x_j \frac{\delta\Psi}{\delta x_k} d^3x$$

(potential energy tensor)

NOTE:

moment of inertia tensor →

$$1/2\frac{d^2 I_{j,k}}{dt^2} = 2T_{jk} + \Pi_{jk} + Wjk$$

This expression is very similar to the expression for the virial theorem:

$$d^2I/dt^2 = 2K + W$$

if we make the following association

$$2K_{jk} = 2T_{jk} + \Pi_{jk}$$

where K_{jk} is defined to equal

$$K_{jk} \equiv \tfrac{1}{2} \int \rho \overline{v_j v_k} d^3\mathbf{x}.$$

Why are Elliptical Galaxies Flattened?

For a stationary system, the second time derivative of the moment of inertia tensor is 0.

Also, let assume that any rotation in the elliptical galaxy is along the z axis and that the galaxy is axisymmetric (i.e., assume elliptical galaxy is oblate)

By definition then, $W_{xx} = W_{yy}$, and $W_{ij} = 0$ for $i \neq j$

Also, $\Pi_{xx} = \Pi_{yy}$, and $\Pi_{ij} = 0$ for $i \neq j$

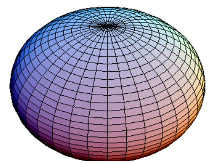
Also, $T_{xx} = T_{yy}$, and $T_{ij} = 0$ for $i \neq j$

$T_{zz} = 0$, by definition

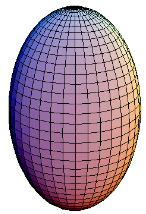
This results in the two equations

$$2T_{xx} + \Pi_{xx} + W_{xx} = 0; 2T_{zz} + \Pi_{zz} + W_{zz} = 0$$

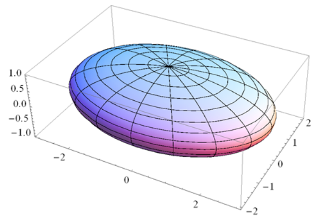
Oblate vs. Prolate vs. Triaxial



oblate



prolate



triaxial spheroid

Why are Elliptical Galaxies Flattened?

Expressing the T_{xx} and Π_{xx} in terms of some rotational velocity and velocity dispersion

$$2T_{xx} = 1/2 \int \rho \, \overline{v_\phi}^{\,2} d^3x = 1/2 M v_0^2$$

$$\Pi_{xx} = M\sigma_0^2,$$

the following can be shown:

$$\frac{v_0^2}{\sigma_0^2} = 2(1-\delta)\frac{W_{xx}}{W_{zz}} - 2$$

where δ is implicitly defined from the following relation:

$$\Pi_{zz} = (1-\delta)\Pi_{xx} = (1-\delta)M\sigma_0^2.$$

δ indicates the anisotropy in the velocity-dispersion tensor

From the following relation $\frac{v_0^2}{\sigma_0^2} = 2(1-\delta)\frac{W_{xx}}{W_{zz}} - 2$

we can predict how the ratio of the rotation velocity to velocity dispersion would depend on the ellipticity of the galaxy for an oblate isotropic rotator...

It can be shown that

$W_{xx}/W_{zz} \sim (1\text{-}\varepsilon)^{-0.9}$

Does not depend on the density profile

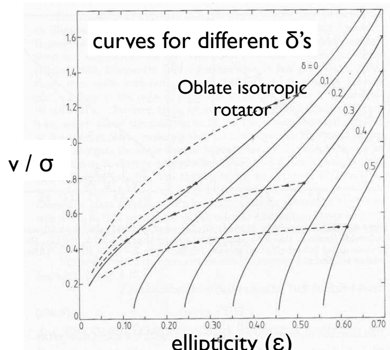


Figure 4-5. The relationship between the rotation parameter v/σ and ellipticity ϵ predicted by (4-95) for elliptical galaxies whose isodensity surfaces are similar coaxial oblate spheroids. The dashed curves show the movement of the point corresponding to the observable quantities $\bar{v}/\bar{\sigma}$ and $\bar{\epsilon}$ when the galaxy's inclination angle i is decreased from $i = 90°$.

What if we view a galaxy from some angle other than edge on?

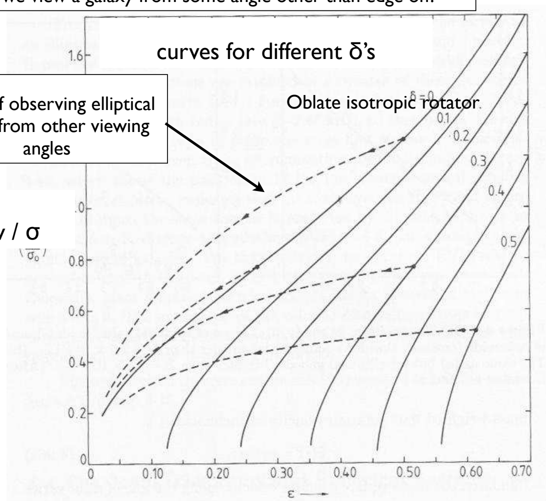
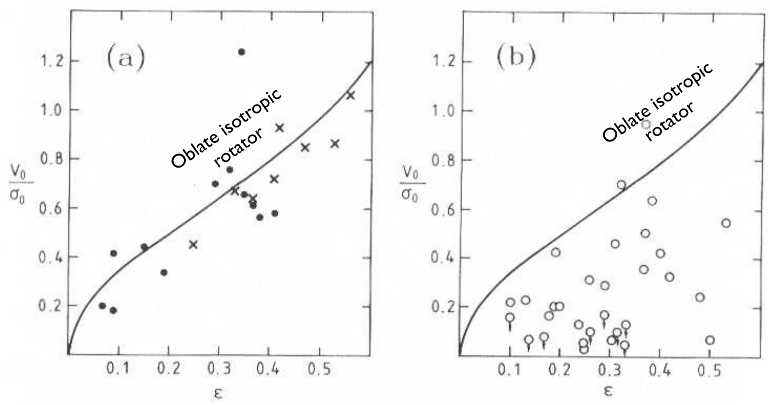


Figure 4-5. The relationship between the rotation parameter v/σ and ellipticity ϵ predicted by (4-95) for elliptical galaxies whose isodensity surfaces are similar coaxial oblate spheroids. The dashed curves show the movement of the point corresponding to the observable quantities $\tilde{v}/\tilde{\sigma}$ and $\tilde{\epsilon}_a$ when the galaxy's inclination angle i is decreased from $i = 90°$.

How does the predicted relationship between v/σ and ellipticity (ε) compare to that found for elliptical galaxies observed in the real universe?

It depends on the luminosity of the elliptical galaxy.

Lower Luminosity Spheroids | More Luminous Spheroids



From the observed v/σ versus ellipticity (ε) relationship, it seems clear that there is a qualitative difference between the most luminous elliptical galaxies and lower luminosity elliptical galaxies:

This dichotomy is seen in many of the other properties of elliptical galaxies as well:

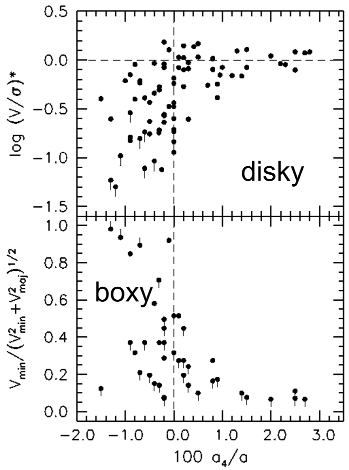
Luminosity	High	Low
Physical Mechanism for Flattening	Anisotropy	Rotation
Isophotes	Boxy	Disky
Shape	Triaxial	Oblate
Profile	Core/ Break	Cuspy/ Power-Law
X-ray/radio	Loud	Quiet

This suggests that higher luminosity and lower luminosity elliptical galaxies may form in different ways!

How the apparent rotational flattening of elliptical galaxies depend upon its isophotal structure (boxy vs. disky)

(v / σ)*

relative to that required for rotational support



a4 parameter defining "boxiness" or "diskiness"

How the radio power and x-ray luminosity of elliptical galaxies depend upon its isophotal structure (boxy vs. disky)

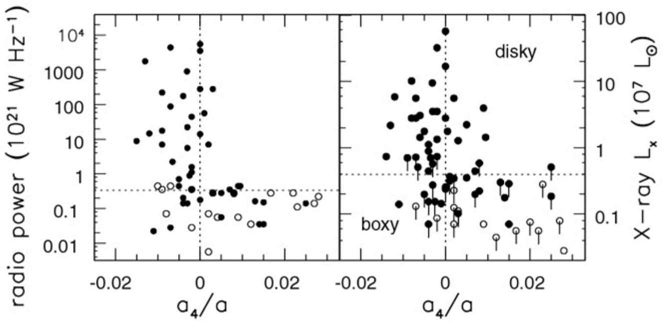
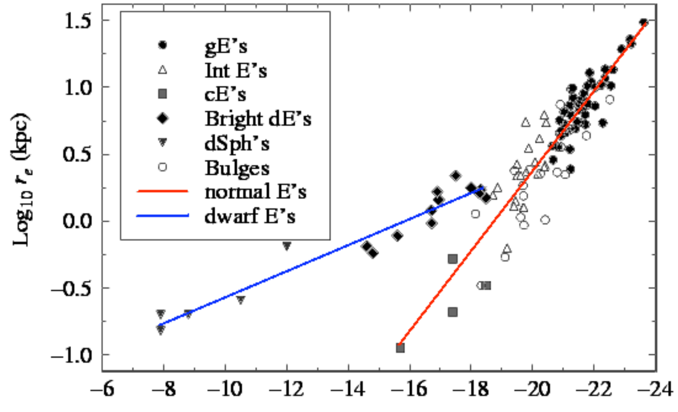


Fig 6.11 (R. Bender) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Part of the above trend may be due to the fact that boxy galaxies are more luminous overall -- and larger galaxies are also expected to brighter at x-ray and radio wavelengths

We can also see this dichotomy between the properties luminous ellipticals and lower luminosity ellipticals in the radius vs. luminosity relation:

Half-Light Radius



Faint
Bright
Optical Luminosity

How might these two classes of elliptical galaxies arise?

Case #1: "Wet" Mergers

(e.g., between two spiral galaxies)

(tends to occur more frequently for lower mass galaxies, when galaxy evolution less advanced)

Galaxy (with gas) →  ← Galaxy (with gas)

Resulting in Disky Ellipticals?

Case #2: "Dry" Mergers

(e.g., between two elliptical galaxies)

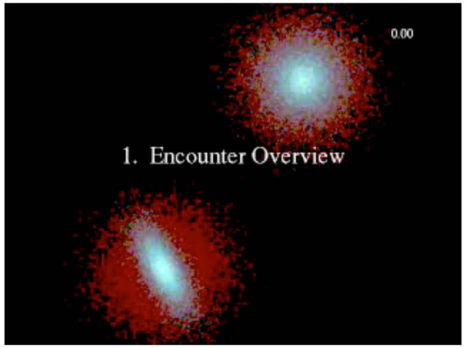
(frequently occurs after many previous mergers, when mass is higher)

Galaxy (without gas) →  ← Galaxy (without gas)

Resulting in Boxy Ellipticals?

REVIEW point from Bachelor course. Dark matter halos grow in mass as a result of merging with smaller halos. Merging is very efficient in the universe, due to the fact that halos are extended.

Due to the impact of dynamical friction, halos of galaxies efficiently merge with one another to build larger systems.



Barnes & Hernquist

REVIEW point from Bachelor course. Dark matter halos grow in mass as a result of merging with smaller halos. Merging is very efficient in the universe, due to the fact that halos are extended.

Due to the impact of dynamical friction, halos of galaxies efficiently merge with one another to build larger systems.

Even galaxies on hyperbolic orbits, i.e., formally unbound, can merge!

What is dynamical friction?

It is drag force that collisionless gravitationally bound systems experience as they pass by each other.

What is dynamical friction?

Consider a system with high mass passing through a sea of particles with lower mass m.

The motion of the high mass system creates a wake of smaller mass particles behind it.

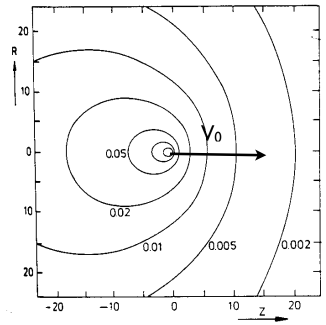


Figure 7-3. A mass travels from left to right at speed v through a homogeneous Maxwellian distribution of stars with one-dimensional dispersion $\sigma = v$. Deflection of the stars by the mass enhances the stellar density downstream more than upstream. Contours of equal stellar density are labeled with the corresponding fractional density enhancement. (From Mulder 1983.)

The wake of lower mass particles is an overdensity and will impose a gravitational pull on high mass system, slowing it down.

The amplitude of the wake is proportional to GM, which means the gravitational force on high mass system is proportional to $(GM)^2$

The impact of dynamical friction can be quantified by considering a similar two body "collision" as we considered earlier:

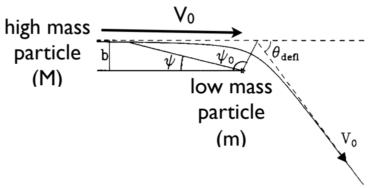


Figure 7-2. The motion of the reduced particle during a hyperbolic encounter.

By explicitly considering a Keplerian orbit and conserving angular momentum, one can show that the perturbations to the velocity of the high mass particle are as follows:

$$|\Delta v_{M,\perp}| = \frac{2mbV_0^3}{G(M+m)^2}\left[1+\frac{b^2V_0^4}{G^2(M+m)^2}\right]^{-1}$$

$$|\Delta v_{M,||}| = \frac{2mV_0}{(M+m)}\left[1+\frac{b^2V_0^4}{G^2(M+m)^2}\right]^{-1}$$

On average, perturbations in the perpendicular direction will cancel. However, perturbations in the horizontal direction will add.

As before, we can integrate over all impact parameters b and considering the phase-space density of small mass particles f(v) that the high mass system M is encountering.

$$\frac{d\vec{v}_M}{dt} = 2\pi\ln(1+\Lambda^2)G^2m(M+m)f(\vec{v}_m)d\vec{v}_m\frac{(\vec{v}_m-\vec{v}_M)}{|\vec{v}_m-\vec{v}_M|^3}$$

where

$$\Lambda \equiv \frac{b_{max}V_0^2}{G(M+m)}$$

For relative velocities between the systems v_M, we can show that

$$\frac{dv_M}{dt} = -\frac{4\pi\ln(\Lambda)G^2(M+m)\rho_m}{v_M^2}$$

where ρ_m is the mass density in the cloud of particles against which the high mass system is colliding.

Note that the impact of dynamical friction becomes more important as the mass of the systems increase

Also note that dynamical friction depends on the density of particles in the systems under collision, not the mass of the individual particles.

Due to the impact of dynamical friction, halos of galaxies efficiently merge with one another to build larger systems.

What happens with the baryonic matter inside halos as their host halos merge? It depends.

For sufficiently low mass halos of similar masses, mergers between the baryonic matter at their centers can occur quite quickly.

However, for very high mass halos (galaxy clusters) or halos where one galaxy is substantially lower mass than the other (e.g., a dwarf galaxy merging with the Milky Way halo), the eventual merger of two systems can take a very long time.

Why do Elliptical Galaxies Look So Similar?

Why do Elliptical Galaxies All Look So Much Alike?

i.e., why do their surface brightness profiles approximately satisfy a de-Vaucouleur law?

As we have seen from the first few lectures, there is a huge freedom in constructing collisionless systems that are in equilibrium.

So, these profiles must arise in some other way!

Violent Relaxation

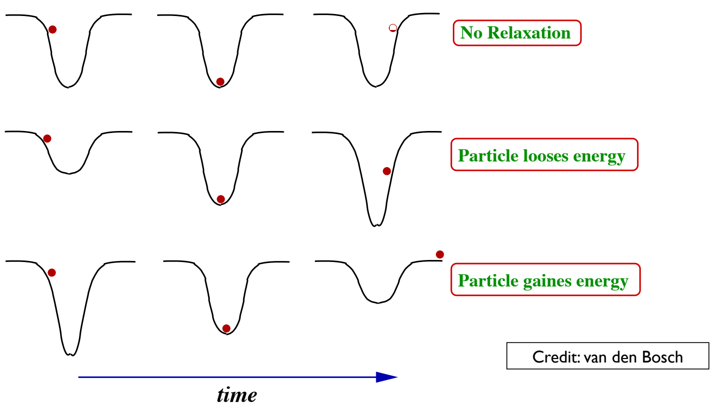
One way to set up the equilibrium phase-space structure in elliptical galaxies is through violent relaxation!

Under normal conditions, for equilibrium systems, we would expect minimal changes in phase space structure of galaxies

However, what would happen if there was a collision between two galaxies?

In this case, individual stars would experience a rapidly changing gravitational potential.

How would the time-varying gravitational potential affect individual particles?



Individual particles can gain or lose energy (due to the changing potential)

As a result, while the total energy of the colliding galaxies does not change, there is a considerable energy exchange between individual stars in the colliding galaxies.

The redistribution of energy between particles rapidly moves the phase space distribution to being more Maxwellian in form (and its density profile more like an isothermal sphere)

The time scale for violent relaxation is equal to

$$t_{\rm vr} = \left\langle \frac{(\mathrm{d}E/\mathrm{d}t)^2}{E^2} \right\rangle^{-1/2} = \left\langle \frac{(\partial \Phi/\partial t)^2}{E^2} \right\rangle^{-1/2} = \frac{3}{4}\langle \dot{\Phi}^2/\Phi^2 \rangle^{-1/2}$$

where the last equation comes from the time-dependent virial theorem (Lynden-Bell 1967)

As such, the time scale for violent relaxation is basically the time scale for the merger itself -- which is a few dynamical times. It is fast!

The redistribution of energy between particles proceeds in such a way as to be independent of the mass of stars in galaxies (since it only occurs due to time variations in the potential).

It therefore very different from what happens during collisional relaxation where there is a dependence on the mass of the colliding stars (e.g., as in globular clusters where the most massive stars tend to collect in the center).

Let us consider one concrete example of a system undergoing violent relaxation

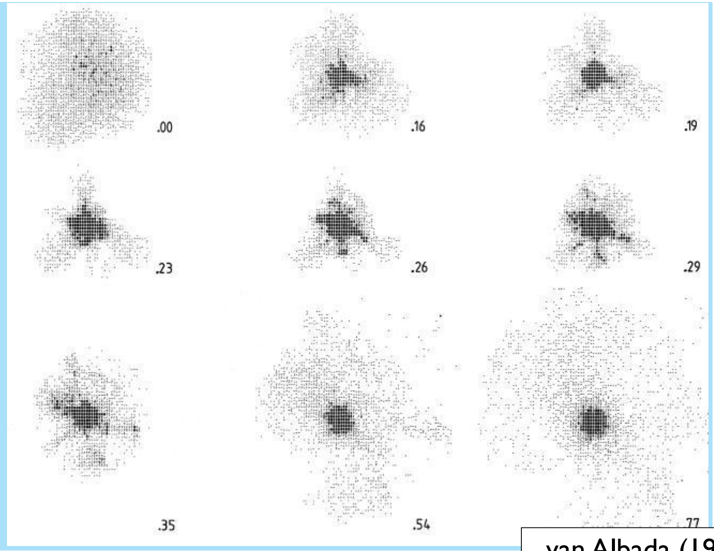
One such system is a cloud of particles that start out locally at rest.

Gravity will cause this cloud of particles to collapse onto the center of the cloud...

How does this system evolve with time?

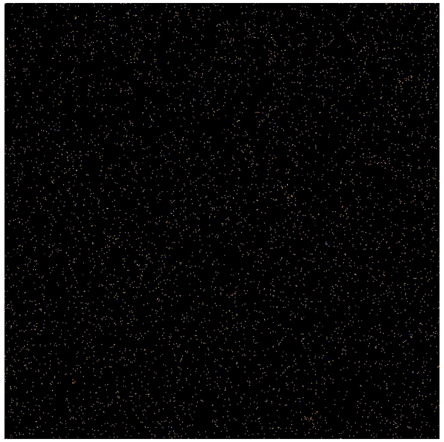
This problem was initially considered by van Albada while working in Groningen (1982)

How does such a system evolve with time?



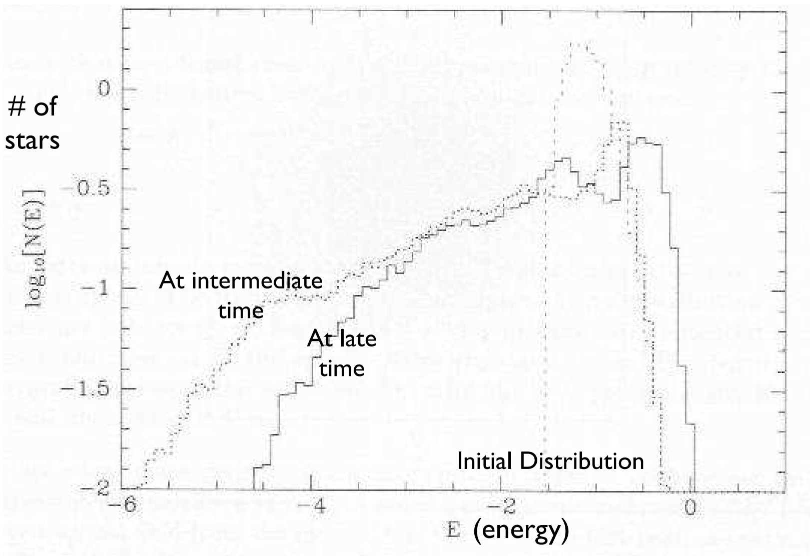
van Albada (1982)

Here is a short movie illustrating such a collapsing cloud:

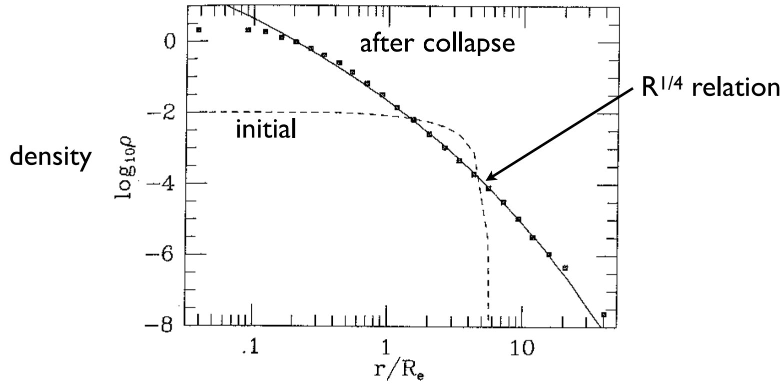


Note that the flow is initially very uniform and ordered! After collapse, individual stars have a wide range of velocities and energies!

How does this re-distribution of energy look for this collapsing cloud?



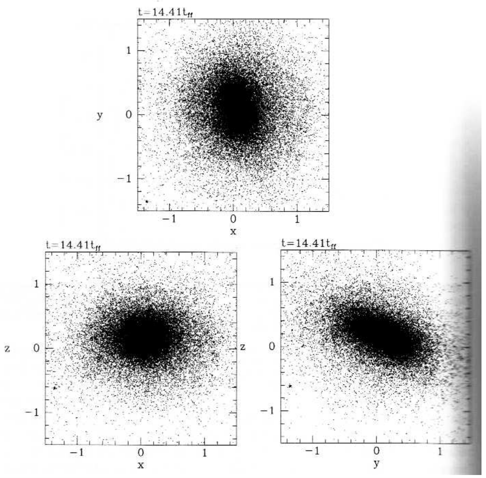
What is the final density profile for this collapsing cloud?



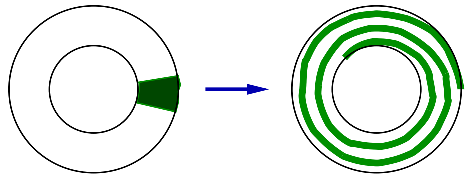
Remarkably, it was found that the final density profile of the equilibrium system that formed from the collapsing cloud followed a $R^{1/4}$ law.

Such density distributions that result from such a collapse are also naturally triaxial in form.

Here are projections of the density distribution of stars in the xy, xz, and yz planes.



Additional smoothing out of the phase space distribution is provided by the process of phase mixing:

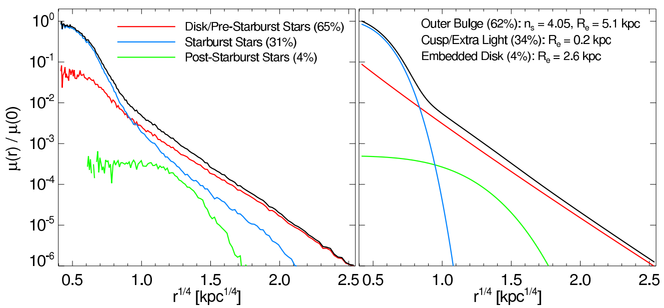


where due to differences in the oscillation frequencies for different stars in a collisionless system, stars spread out to more completely fill phase space.

What about the merger between two galaxies?

Amazingly, one finds very similar density profiles to what one finds for the collapsing clouds. Here are some results from Hopkins et al. (2008):

surface brightness



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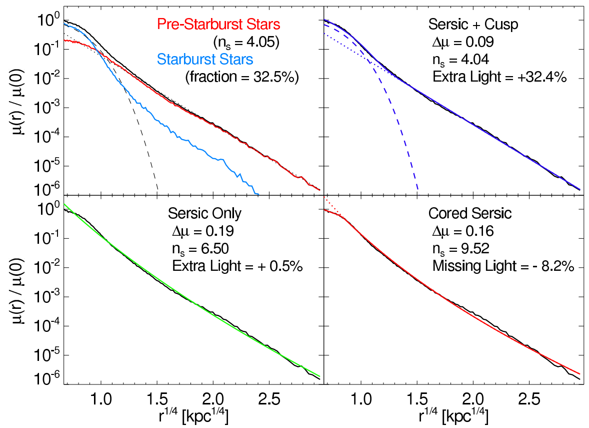


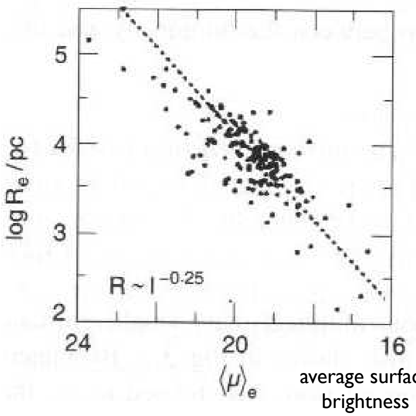
These merging galaxies also largely follow $R^{1/4}$ density profiles.

Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

half-light radius

i.e., radius containing half of the light



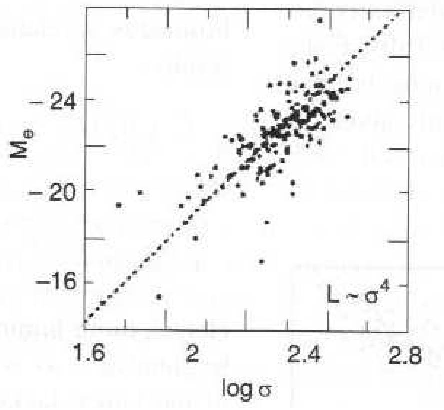
average surface brightness

As we noted earlier, such correlations would not need to be present for a generic collisionless system and tell us something fundamental about the formation of galaxies themselves.

Intrinsic Correlations between Galaxy Properties

Strong correlations are observed between the masses, sizes, and velocity dispersions of elliptical galaxies.

absolute magnitude inside the half-light radius



velocity dispersion

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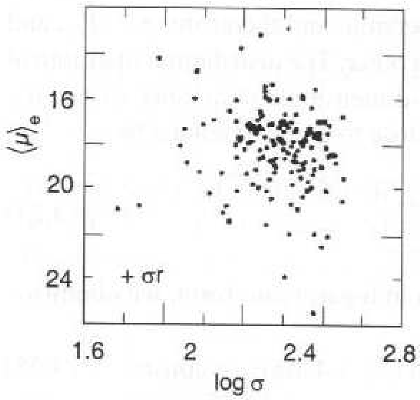

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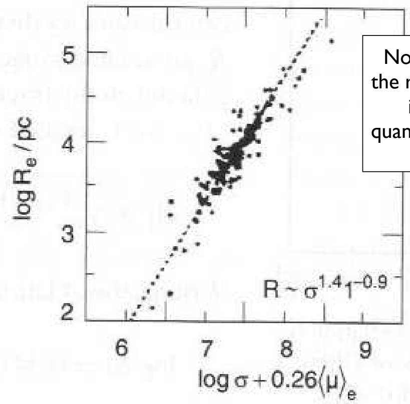


Notice how much tighter the relationship can become if one combines the quantities in the appropriate fashion!

velocity dispersion

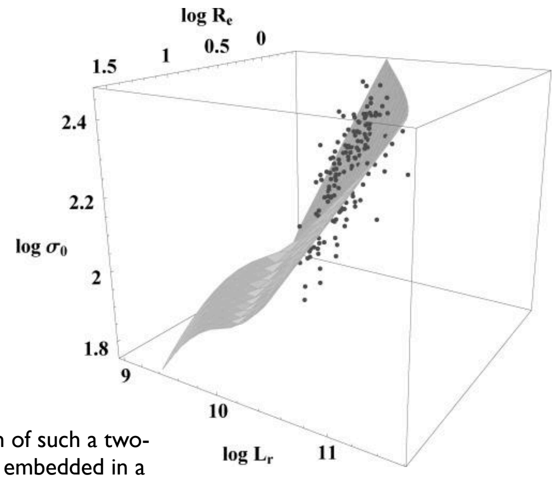
Intrinsic Correlations between Galaxy Properties

These correlations are such that galaxies populate a fundamental plane in these parameters, so that if you know two of the following three variables for a galaxy, you can determine the third.

$$R \propto \sigma^{1.4} \mu_e^{-0.9}$$

where R is the size (radius), σ is the velocity dispersion, and μ is the galaxy surface brightness.

The properties of galaxies occupy a two dimensional plane in the three dimensional parameter space.



Here is an illustration of such a two-dimensional surface embedded in a three-dimensional space.

How shall we interpret this fundamental plane relation?

Let us interpret in terms of the mass to light ratio of galaxies:

$$\frac{M}{L} = \frac{M}{\mu_e R_e^2} = \frac{M}{\sigma^{1.5} R_e^{-1.1} R_e^2} = \frac{M}{M^{0.75} R_e^{-0.75} R_e^{-1.1} R_e^2} = M^{0.25} R_e^{-0.15}$$

$\mu_e \sim \sigma^{1.5} R_e^{-1.1}$

From virial relation:

$M \sim \sigma^2 R_e$

$M^{0.75} \sim \sigma^{1.5} R_e^{0.75}$

The fundamental plane relation implies that the mass-to-light ratio scales as $M^{0.25} R_e^{-0.15}$.

It is unclear what causes this mass dependence. It could be due to the stellar populations (i.e., that the baryonic component in massive galaxies have higher mass-to-light ratios) or due to dark matter playing a larger dynamical role in more massive galaxies

What about Baryonic Gas in Ellipticals? Do they have any?

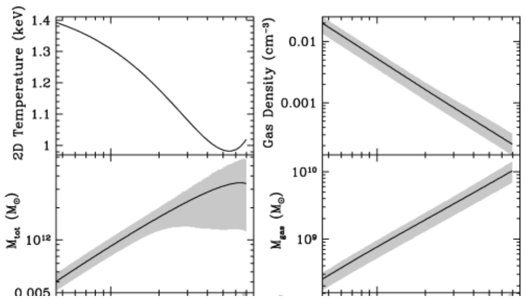
Yes -- lots of hot ionized gas!

Luminosity of the hot ionized gas scales approximately as the optical luminosity squared!

Total gas mass ranges from 10^9 to 10^{11} solar masses

Hot ionized gas appears to originate partially from mass loss from AGB stars.

Heating is from SNe and movement of stars through gas



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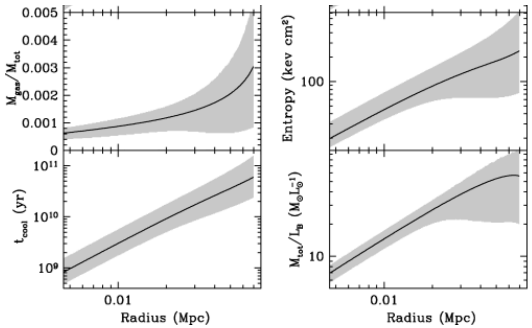

What about Baryonic Gas in Ellipticals? Do they have any?

Occasionally there is cool gas

However, most of the time it is outside the center of a gas.

It is expected to be rare, since elliptical galaxies do not undergo much star formation.