Collisionless Stellar Dynamics + Vlasov / Jeans Equation

February 26

Layout of the Course

Lectures

- Feb 2: Course Introduction, Overview, and Galaxy Formation Basics
- Feb 9: Disk Galaxies (I)
- Feb 12: Disk Galaxies (II)
- Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics
- Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations
- Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)
- Mar 9: Elliptical Galaxies (II)
- Mar 23: Elliptical Galaxies (III)
- Mar 30: Dark Matter Halos
- Apr 13: Large Scale Structure
- Apr 20: Galaxy Stellar Populations
- Apr 23: Lessons from Large Galaxy Samples at z<0.2
- May 4: Evolution of Galaxies with Redshift
- May 11: Galaxy Evolution at z>1.5 / Review for Final Exam

No lecture or practical classes the following week!!!

Problem Set 2

(Distributed earlier this week, due March 16)

Galaxies: Structure, Dynamics, and Evolution
Problem Set 2
Instructor: Dr. Bouwens

Here is Problem Set 2. The entire problem set will be due before class on Monday, March 16 (email them to Wout). Be sure to pay extra attention to problem 4, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. How does the epicyclic (radial) frequency κ for both an isothermal potential (v_c = constant) and a potential with the form $\phi(r) = r^{-3/4}$ compare with the azimuthal frequency Ω? How many radial/epicyclic oscillations will a star undergo for each orbit around a galaxy?

2. Calculate the epicyclic frequency for both an isothermal potential (v_c = constant) and a potential with the form $\phi = -1/(1+r^2)$. Draw the behaviour of $\Omega - n\kappa/m$ as in figure 6.11 in the supplementary reading.

3. In class, we calculated the relaxation time for a star in some dynamical system (e.g., a galaxy) by considering the effect of its interactions with other stars in a galaxy. In calculating the relaxation time, we only considered impact parameters b from b_{min} to the size of the galaxy R (where b_{min} was the minimum impact parameter where our simple formula for the velocity kick was approximately valid). What about the effect of impact parameters $b = 0$ to b_{min}? How does this impact the relaxation time?
(a) Calculate the probability that a star will pass by another star with impact parameter b less than or equal to b_{min}? (Ignore the curvature of the orbit.) Adopt the standard variables N, v, m, and G used in the derivation during lecture.

(b) If any star passes by another star with impact parameter $b < b_{min}$, its velocity will be so perturbed that it "will lose all memory of its initial orbit." Let us say it relaxes with just one encounter. Calculate the relaxation time assuming that $b < b_{min}$ encounters are the only meaningful relaxation process. Calculate the relaxation time for the same choice of N, m, G, and v considered in class for a galaxy (i.e., $N = 10^{10}$, $v = 100$ km s^{-1}, $r = 10$ kpc, $t_{cross} = 10^8$ yr). How does this compare with the relaxation time derived in class considering only $b > b_{min}$ encounters?

This will be the graded problem ←

4. To help yourself visualize the Inner and Outer Lindblad Resonances work, as well as the corotational radius, I will ask you to sketch out a number of snapshots of the movement of a spiral arm around a galaxy. Assume that a galaxy has two spiral arms and that it is an isothermal sphere.
(a) Suppose that circular velocity of the galaxy is 200 km/s and that the inner Lindblad resonance for a galaxy is at a radius of 2.5 kpc. What is the pattern speed (or frequency) Ω_p?
(b) What is the period of epicyclic motion at the Inner Lindblad resonance?
(c) What is the corotation radius and radius of the Outer Lindblad resonance for the pattern speed you computed in part (a)?
(d) In multiples of the epicyclic period of stars at the inner Lindblad resonance (consider multiples out to 10), please sketch out the motion of stars in a spiral galaxy (similar to what I do in lecture, but now moving indicative stars at the Inner and Outer Lindblad resonances and corotation radius self consistently). Please indicate the position of the spiral arms, a star at the radius of the Inner Lindblad resonance, a star at the corotation radius, and a star at the radius of the outer Lindblad resonance. Assume that the position angle of the spiral arms and the stars at all three radii are all 0 at time t = 0.

5. How many integrals of motion are there for a particle with the following force law $F(r) = -\hat{r}/r^2$ where r is the radius?

6. Show that the distribution function $f(\epsilon, L)$

$$f(\epsilon, L) = \begin{cases} F\delta(L^2)(\epsilon - \epsilon_0)^{-1/2} & \text{for } \epsilon > \epsilon_0, \\ 0 & \text{otherwise.} \end{cases}$$

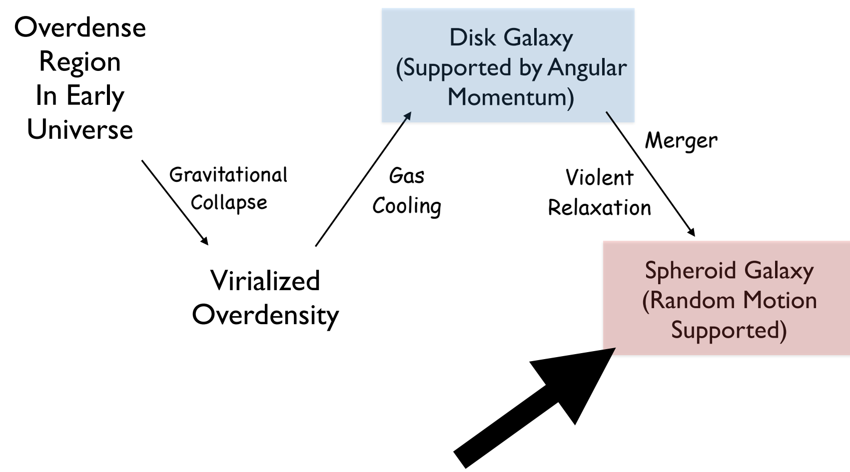
where F and ϵ_0 are constants and δ is the familiar delta function. Show that this distribution function self-consistently generates a model with density

$$\rho(r) = \begin{cases} Cr^{-2} & \text{for } r < r_0 \\ 0 & \text{otherwise.} \end{cases}$$

where C is a constant and the relative potential at r_0 satisfies $\Psi(r_0) = \epsilon_0$. This is the only analytic stellar system known to us in which all stars are on perfectly radial orbits.

First, let's review the important material from last week

Galaxy Formation: Major Steps



Next topic is elliptical galaxies...

Elliptical galaxies consist of large numbers of stars on diverse orbits.

While spiral galaxies are rotation supported, elliptical galaxies are supported by the random motions of stars they contain

Their behavior can largely be described using collisionless dynamics.

REVIEW Point from Bachelor Course: The time scale for the relaxation time of individual stars to collisions will other stars is very high, i.e., 10^{16} years, and thus can be ignored in modeling the dynamics of stars in a galaxy. Consequently, it is possible to model the potential and phase space as smoothly varying.

Earlier this week, we showed that the stars in a galaxy will have their transverse component of velocity scattered as follows:

$\langle v_\perp^2 \rangle / v^2 = 8 (\ln N) / N$ (per crossing time)

where N is the # of stars in a system

For a star to lose all memory of its initial orbit, this star must cross the galaxy enough times so that the scatter in the transverse component equals the overall velocity itself.

This results in the following expression:

$$n_{cross,relax} \langle v_\perp^2 \rangle = v^2$$

REVIEW

What is the relaxation time then?

$$t_{relax} = t_{cross}\, n_{cross,relax} = t_{cross}(N / (8 \ln N))$$

t_{cross}: 10^8 years

$n_{cross,relax}$: 10^{10} stars / $(8 \ln 10^{10})$

$$t_{relax} = 10^{16} \text{ years}$$

Example of Time Scales

System	Mass $M_\odot$	Radius kpc	Velocity $\mathrm{km\,s^{-1}}$	N	$t_{\rm cross}$ yr	$t_{\rm relax}$ yr
Galaxy	10^{10}	10	100	10^{10}	10^{8}	$> 10^{15}$
DM Halo	10^{12}	200	200	$> 10^{50}$	10^{9}	$> 10^{60}$
Cluster	10^{14}	1000	1000	10^{3}	10^{9}	$\sim 10^{10}$
Globular	10^{4}	0.01	2	10^{4}	5×10^{6}	5×10^{8}

- **Dark Matter Haloes and Galaxies are collisionless**
- **Collisions may or may not be important in clusters of galaxies**
- **Relaxation is expected to have occured in (some) globular clusters**

Credit: van den Bosch

REVIEW point from Bachelor course: The time evolution of the distribution function is defined by the distribution function at that time, spatial derivatives, and the gradients of the potential (Vlasov-Equation). This follows directly from a conservation equation on the stars.

At any time t, one can describe the collective positions and velocities for stars in a dynamical system by a distribution function f(**x**,**v**,t)

For the distribution function, one very useful equation is the Collisionless Boltzmann equaton which guarantees stars are conserved in moving through phase space.

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

or

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f - \vec{\nabla}\Phi.\frac{\partial f}{\partial \vec{v}} = 0$$

also can write as $\frac{df}{dt} = 0$

REVIEW Point from Bachelor Course: Integrals of motion are functions of x and v which are constant along an orbit. They are not explicit functions of time. Examples: energy, angular momentum. Most 3D densities allow for 3 integrals of motion, 2 of which are non-classical.

Integrals of motion can be a very useful concept for characterizing the orbits of stars in a galaxy.

They are useful in the case that they are isolating integrals of motion since the reduce the dimensionality of the phase space in which a star travels during its orbit.

There can be no more than 6 integrals of motion. Typically there is at least one integral of motion (energy).

What are some examples of isolating integrals of motion?

- Energy is always an integral of motion for a star in a static potential.

The energy per unit mass for a star remains constant throughout its orbit: E(**x**,**v**) = (1/2) $\mathbf{v}^2$ + Φ(**x**)

- L_z: angular momentum in the z direction (for an axisymmetric potential)

- **L**: all three components of the angular momentum in spherically symmetric potential

Integrals of motion tend to arise from some symmetry in the system.

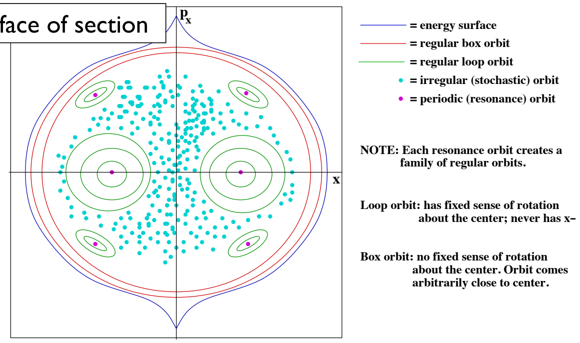
However, dynamical systems can also have other isolating integrals of motion outside of the classical ones (i.e., energy, angular momentum)

One alternate way of determining how restricted the orbital manifold of galaxies are is to construct poincare surfaces of section:

To investigate whether the orbits admit any additional (hidden) isolating integrals of motion, Poincaré introduced the surface-of-section (SOS)

Consider the intersection of $\mathcal{M}_3$ with the surface $y = 0$. Integrate the orbit, and everytime it crosses the surface $y = 0$ with $\dot{y} > 0$, record the position in the (x, p_x)-plane. After many orbital periods. the accumulated points begin to show some topology that allows one to discriminate between regular, irregular and resonance orbits.

Example surface of section



Credit: van den Bosch

Setting up equilibrium models for a collisionless system.

It is not necessarily an easy thing to do

We must set up a self-consistent system whereby each of the following steps imply the next:

(1) given density distribution ρ(r), calculate the potential Φ(r) the density distribution would imply

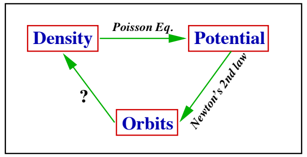
(2) given some potential Φ, determine the set of orbits that stars would undergo

(3) calculate the density distribution that would result from the collective orbits of all the stars in a system

The Density Distribution derived in step #3 must be the same as assumed in step #1

The relevant equations are:

$$\rho(\vec{x}) = \int f(\vec{x}, \vec{v})\, d^3\vec{v}$$
$$\nabla^2\Phi(\vec{x}) = 4\pi G\rho(\vec{x})$$
$$\frac{df}{dt} = 0$$



New Material

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

There is a huge amount of freedom in finding a distribution function that satisfies the collisionless boltzmann equation for some equilibrium model.

How can we find a solution?

One approach is to make the distribution function a function of the integrals of motion themselves.

By definition, the first time derivative of an integral of motion is zero. Since this is the same requirement the collisionless boltzmann equation makes on the distribution function, we know that any arbitrary function of the integrals of motion satisfies the collisionless boltzmann equation.

Jean's theorem

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

By constructing the distribution function f as such, we ensure that we already solve one of the 3 equations needed to construct a self-consistent equilibrium system:

The equations we need to satisfy:

$$\begin{aligned} \rho(\vec{x}) &= \int f(\vec{x},\vec{v})\, d^3\vec{v} \\ \nabla^2\Phi(\vec{x}) &= 4\pi G\rho(\vec{x}) \\ \frac{df}{dt} &= 0 \end{aligned}$$

Using Jeans theorem to construct a self-consistent equilibrium, we automatically satisfy this equation

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

Now we move onto solving this one:

$$\begin{aligned} \rho(\vec{x}) &= \int f(\vec{x},\vec{v})\, d^3\vec{v} \\ \nabla^2\Phi(\vec{x}) &= 4\pi G\rho(\vec{x}) \\ \frac{df}{dt} &= 0 \end{aligned}$$

Since E = energy is an integral of motion, let's use f(E) for the distribution function.

Assume:

$$\psi = -\Phi + \Phi_0$$

$$\varepsilon = -E + \Phi_0 = \psi - \tfrac{1}{2}v^2$$

choose Φ_0 such that
$f > 0$ for $\varepsilon > 0$
$f = 0$ for $\varepsilon \leq 0$

What is the density of stars for such a distribution function?

Review Point from Bachelor course. Equilibrium models can easily be made by requiring that the distribution function be a function of integrals of motion. By definition, the distribution function is an integral of motion.

The density is integral of distribution function over all velocities:

$$\rho(r) = \int f(\varepsilon) d\vec{v}$$

$$= \int_0^{v_{max}} f(\varepsilon) 4\pi v^2 dv$$

We show in class that

$$\rho(r) = 4\pi \int_0^{\psi} f(\varepsilon)\sqrt{2(\psi-\varepsilon)}\, d\varepsilon$$

where ψ is the maximum potential energy of a star (i.e., = E).

Finally, all that remains to be done to set an equilibrium system satisfying Poisson's equation is the following:

$$\nabla^2\Phi(\vec{x}) = 4\pi G\rho(\vec{x})$$

REVIEW (from Bachelor course)

JEANS EQUATIONS

Jeans Equations build on the material just presented and provide us with another means to estimate the masses and mass profiles in galaxies...

The Jeans equations are derived by taking various velocity moments of the Collision Boltzmann Equation.

This is a useful approach because velocity moments of the stars are straightforward to derive from the observations.

REVIEW point from Bachelor course. An important method for deriving the total mass of individual galaxy involves the Jeans equations and moments of the stellar velocity.

First define several velocity moments of the distribution function:

0. Spatial density of stars (0th moment):

$$\nu(\vec{x}) = \int f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v}$$

1. Mean velocity of stars (1st moment):

$$\overline{v_i}\,(\vec{x}) \equiv \frac{1}{\nu}\int v_i f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v}, \quad i=1,2,3$$

2. Second moment of stars (2nd moment):

$$\overline{v_i v_j}\,(\vec{x}) \equiv \frac{1}{\nu}\int v_i v_j f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v}, \quad j=1,2,3$$

Velocity Dispersion Tensor:

$$\sigma_{ij}^2 \equiv \overline{(v_i - \overline{v_i})(v_j - \overline{v_j})} = \overline{v_i v_j} - \overline{v_i}\;\overline{v_j}$$

Jeans Equation 1 (Continuity equation):

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\nu\,\overline{v_i} = 0$$

Jeans Equation 2 (The Force Equation):

$$\frac{\partial(\nu\,\overline{v_j})}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\nu\,\overline{v_i v_j}) + \nu\frac{\partial \Phi}{\partial x_j} = 0$$

Jeans Equation 3 (Rewrite of Equation 2)

$$\nu\frac{\partial\,\overline{v_j}}{\partial t} + \sum_{i=1}^{3} \nu\,\overline{v_i}\,\frac{\partial\,\overline{v_j}}{\partial x_i} = -\nu\frac{\partial \Phi}{\partial x_j} - \sum_{i=1}^{3}\frac{\partial}{\partial x_i}\nu\sigma_{ij}^2$$

Let us start out by deriving the first Jeans Equation from the collisionless Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

↓ Derive ↓

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\nu\,\overline{v_i} = 0$$

This is the continuity equation and is anologous to the following expression from fluid mechanics:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

Let us start out by deriving the second Jeans Equation from the collisionless Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

↓ Derive ↓

$$\frac{\partial(\nu\,\overline{v_j})}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\nu\,\overline{v_i v_j}) + \nu\frac{\partial \Phi}{\partial x_j} = 0$$

Finally let us derive the third Jeans Equation from the first two Jeans Equations:

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu \, \overline{v_i} = 0 \qquad \frac{\partial (\nu \, \overline{v_j})}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (\nu \, \overline{v_i v_j}) + \nu \frac{\partial \Phi}{\partial x_j} = 0$$

↓ For a future problem set ↓

$$\nu \frac{\partial \, \overline{v_j}}{\partial t} + \sum_{i=1}^{3} \nu \, \overline{v_i} \, \frac{\partial \, \overline{v_j}}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu \sigma_{ij}^2$$

This is the "force" equation and is analogous to the Euler equation from fluid mechanics:

$$\frac{\rho \partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \vec{\nabla})\vec{v} = -\rho \vec{\nabla} \Phi - \vec{\nabla} p = 0$$

$$\frac{\partial \nu}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu \, \overline{v_i} = 0$$

$$\frac{\partial (\nu \, \overline{v_j})}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (\nu \, \overline{v_i v_j}) + \nu \frac{\partial \Phi}{\partial x_j} = 0$$

$$\nu \frac{\partial \, \overline{v_j}}{\partial t} + \sum_{i=1}^{3} \nu \, \overline{v_i} \, \frac{\partial \, \overline{v_j}}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \nu \sigma_{ij}^2$$

- These are the Jeans Equations. There are three equations and six unknowns. Possible to find many solutions to these equations.

- For a stationary problem (where the time derivatives are zero and net streaming motion is zero, i.e., $\overline{v_i}$ = 0), the left terms disappear completely. We then have the stellar velocity dispersion counteracting the force of gravity, in the same way that gas pressure counteracts the force of gravity in a star. Note however that there is no equation of state.

- Velocity dispersion tensor σ_{ij}^2 is symmetric and so an orthogonal coordinate system can be found where it is diagonal, i.e., σ_{ij}^2 = 0 if i does not equal j.

Jeans Equations for Spherically Symmetric Models

Adopt a spherical coordinate system (r, θ, ϕ). Assume invariant under rotations about center and there are no streaming motions. Hence,

$$\overline{v_r} = \overline{v_\theta} = \overline{v_\phi} = 0$$

$$\overline{v_r v_\theta} = \overline{v_r v_\phi} = \overline{v_\theta v_\phi} = 0$$

$$\overline{v_\theta^2} = \overline{v_\phi^2}$$

The second Jeans equation (in spherical coordinates) reduces to the following:

$$\frac{d(\nu \, \overline{v_r^2})}{dr} + \frac{\nu}{r}\left[2\,\overline{v_r^2} - 2\,\overline{v_\theta^2}\right] = -\nu \frac{d\Phi}{dr}$$

Define anisotropy function

$$\beta(r) = 1 - \overline{v_\theta^2} \, / \, \overline{v_r^2} \, .$$

Jeans Equations for Spherically Symmetric Models

Clearly, β <= 1. Manipulating the previous expression, we find

$$\frac{1}{\nu}\frac{d}{dr}\nu \, \overline{v_r^2} + 2\frac{\beta}{r}\,\overline{v_r^2} = -\frac{d\Phi}{dr}$$

Total Enclosed Mass and Rotation Curve

Assuming a particle is on a circular orbit at radius r, there is a relationship between the circular velocity v_c and $\frac{d\Phi}{dr}$

$$\frac{d\Phi}{dr} = \frac{GM(<r)}{r^2} = \frac{v_c^2}{r}$$

Using this relation, the Jeans Equation can be written as

$$v_c^2 = \frac{GM(<r)}{r} = -\overline{v_r^2}\left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\overline{v_r^2}}{d\ln r} + 2\beta\right)$$

From this expression, we see that if we can measure the density of stars ν, the velocity dispersion in the radial direction, and anisotropy function, we can determine the enclosed mass inside some radius

What other applications do the Jeans Equations have?

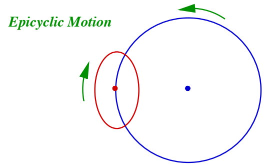
A few examples:

1. Estimating the surface mass density of our own Milky Way Galaxy based on the velocities of stars perpendicular to the plane.

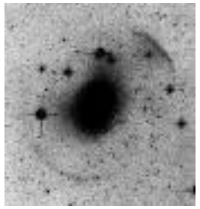
2. Understanding quantitatively why stars in spiral galaxies have lower tangential velocities than gas (i.e., why their tangential velocities are less than the circular velocity)

   => due to random motions, stars effectively move slower than gas due to asymmetric drift (and due to small amount of pressure support)

3. Understanding quantitatively the small epicyclical motion that stars rotating around a spiral galaxy experience.



What is the nature of elliptical galaxies?



End state of galaxy formation!

Dominated by the random motions of its component parts (stars, hot gas, dark matter)

While progenitors to elliptical galaxies experienced lots of star formation, elliptical galaxies themselves experience almost no star formation

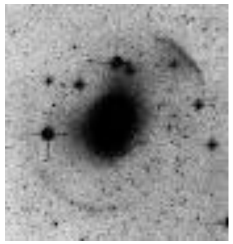
Only way for an elliptical galaxy to transform into another type of galaxy (e.g., spiral) is if lots of cold gas cools onto it (but this may not happen!)

What is the nature of elliptical galaxies?

What can we learn about elliptical galaxies from their structure?

(from Photometry) We begin by looking at the two dimensional surface brightness profiles of elliptical galaxies.

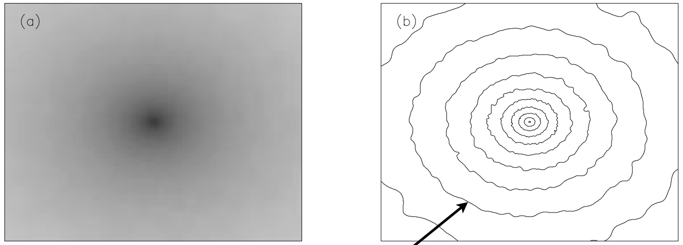
Not even necessary to use a telescope now to investigate -- as there is a lot of data from surveys like the Sloan Digital Sky Survey



Structure of Elliptical Galaxies

What does the situation look like for elliptical galaxies?

Light is almost constant on elliptical "isophotes"



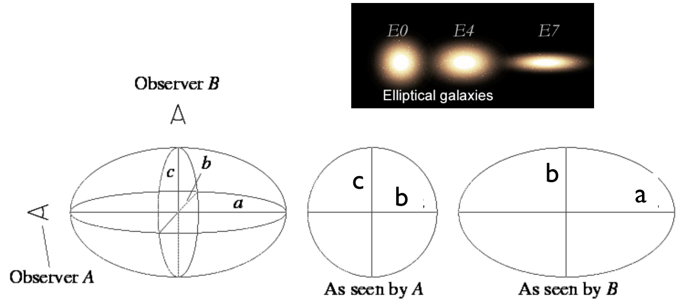
Well described by ellipses!

Note that we can derive an ellipticity and position angle for each ellipse

Structure of Elliptical Galaxies

Ellipticity is defined in the following way:

As En where n = 10ε and ε = 1−b/a is the ellipticity

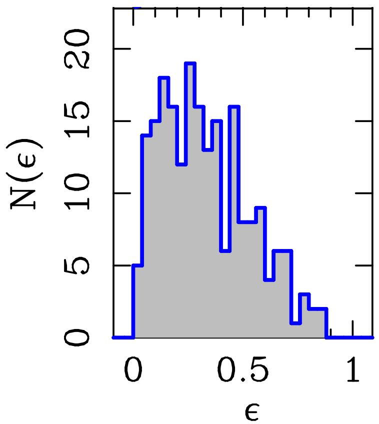


Note that defined ellipticity depends on the vantage point from which we view a galaxy. The defined ellipticity is not intrinsic.

Ellipticity Distribution (Nearby Galaxies)

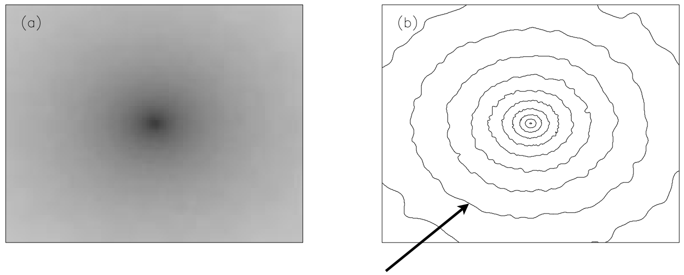
Here is a determination of ellipticity distribution for nearby elliptical galaxies:

Ellipticities generally less than 0.5, median about 0.3



Elliptical Isophote Structure

Light is almost constant on elliptical "isophotes"



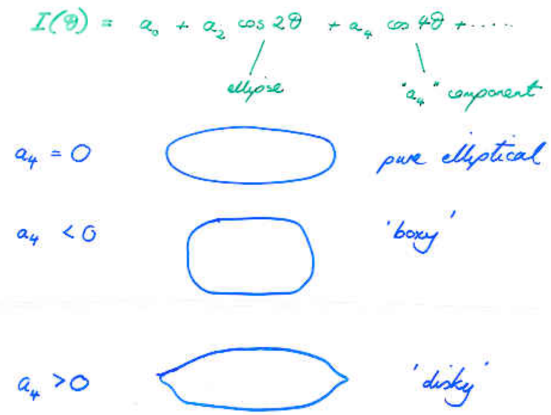
Well described by ellipses!

There are however very small deviations from pure elliptical isophotes, on the order of a few percent

Deviations from pure ellipses classified as "disky" or "boxy"

"Disky" vs. "Boxy" Isophotes

One can quantify deviations from an elliptical shape using this boxyness parameter a_4:



Isophotes for Boxy Galaxy

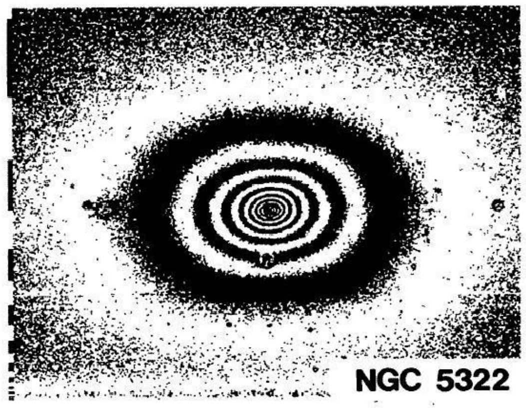


FIGURE 7. — R-image of NGC 5322, an elliptical galaxy with box-shaped isophotes $(a(4)/a \sim -0.01)$.

Isophotes for Disky Galaxy

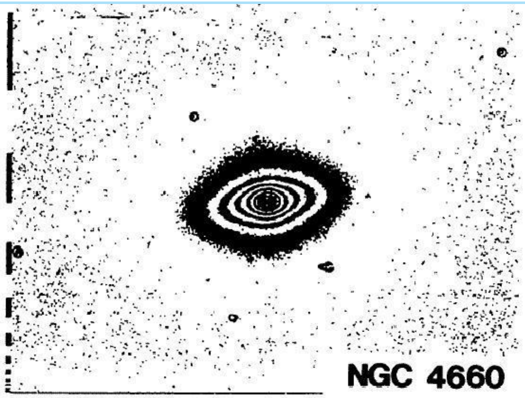
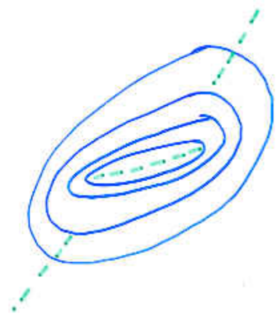


FIGURE 6. — R-image of NGC 4660, an elliptical galaxy with a disk-component in the isophotes $(a(4)/a \sim +0.03)$.

Structure of Elliptical Galaxies

Light is almost constant on elliptical "isophotes"



Position Angles and Ellipticities vary slightly (or twist) with radius

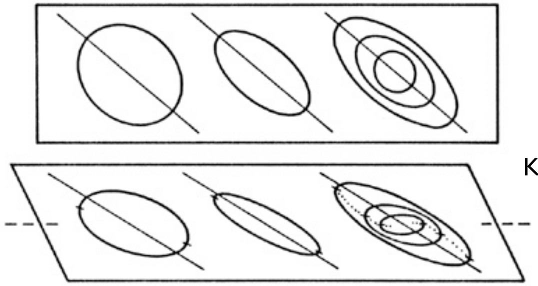
Such twisting of the position angles of the isophotes can easily result from projection effects if an elliptical galaxy has a triaxial shape.

Structure of Elliptical Galaxies

Let's see if we can't understand how this "isophote twisting" works:

"Original"

"After Projection"



Credit: Kormendy

Notice that the position angle of major axis of the ellipses change, after projection. Depending on the initial ellipticity of the original ellipse, the change in position angle can be quite large.

Such twists in the position angle of the major axis is expected, if elliptical galaxies have triaxial profiles.

Twisting isophotes in real galaxies

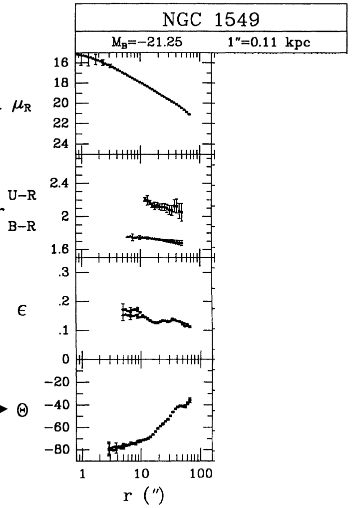
Example galaxy!

Surface Brightness →

Color

Note best-fit ellipticity varies with radius slightly →

Also position angle! →



Twisting isophotes in real galaxies

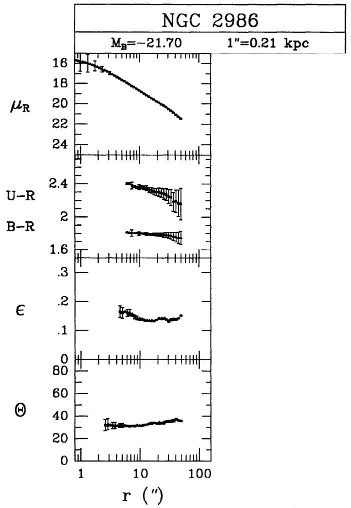
Another galaxy!

Surface Brightness →

Color

Note best-fit ellipticity varies with radius slightly →

Also position angle! →



Surface Brightness vs. Radius

Intensity Profiles in Galaxies Well Described by Sersic Profile:

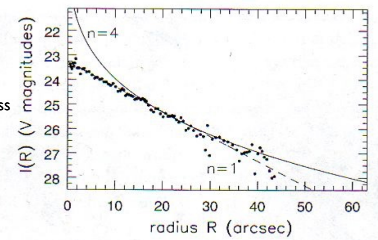
$$I(R) = I_e 10^{b_n[(R/R_e)^{1/n}-1]}$$

where R_e is the half-light Radius and n is the Sersic Index (from 2 to 5 for most Elliptical Galaxies)

For large n, there is a lot of light at very large radii and very small radii -- i.e., extended wings to light profile and bright center

For small n, the light is less concentrated in center. Also the light profile cuts off at large radii.

Surface Brightness



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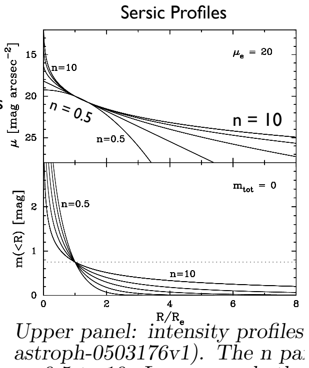
Surface Brightness

How measured magnitude depends on radius of aperture in which measurement is made



Structure of Elliptical Galaxies

Typical value is n=4

"de-Vaucouleur Profile"

Surface Brightness

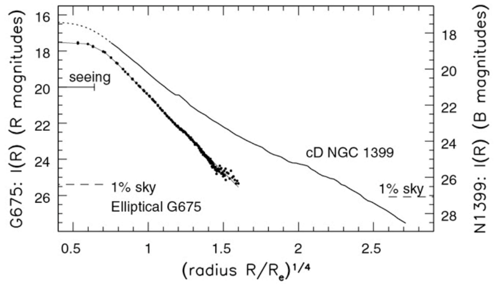


Fig 6.3 (Saglia, Caon) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Note that it is very challenging to trace the surface brightness profile of ellipticals to large radii, due to their low surface brightness there and high sky brightness

Seeing by the atmosphere can also affect the measurement of the profile at small radii

Center of Ellipticals: Cusp vs. Core

Some deviation from a Sersic profile can be seen in some galaxies.

In their centers, some elliptical galaxies show cusps and some show cores.

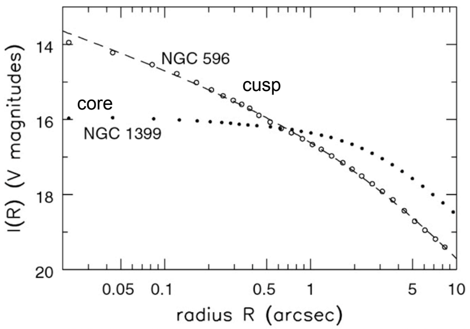


Fig 6.7 (T. Lauer) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

elliptical galaxies with cusps are also called "power-law" galaxies

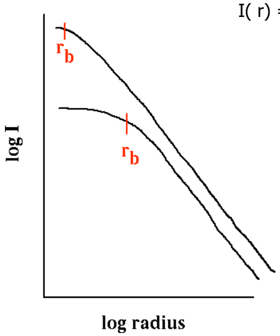
elliptical galaxies with core are also called "break" galaxies

Definition of Break Radius

The radius where galaxies show this break from the power law (or cuspy) profile is called the "break" radius.

$I(r) = I_b 2^{(\beta-\gamma)/\alpha} (r_b/r)^{\gamma} [1+(r/r_b)^{\alpha}]^{(\gamma-\beta)/\alpha}$ $r_b)^{\alpha}]^{(\gamma-\beta)/\alpha}$

Surface Brightness



r_b = radius where power-law changes shape
I_b = surface brightness where power-law changes shape
β = power-law at large radii
γ = power-law at small radii
α = sharpness of transition between power-law slopes

can be a five parameter fit!

deficit of stars at the center of ellipticals is thought to be due to scouring by the central super massive black hole

Shells and Ripples

Elliptical galaxies also show a significant number of shells and ripples
(presumably indicative of mergers between galaxies in the past)

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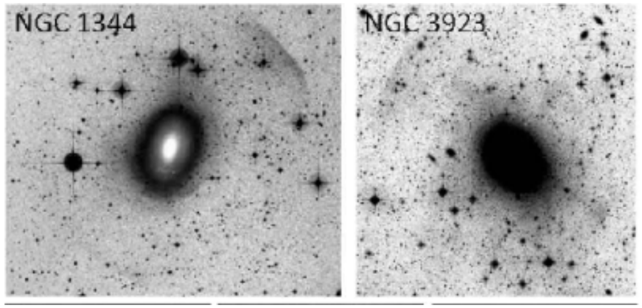
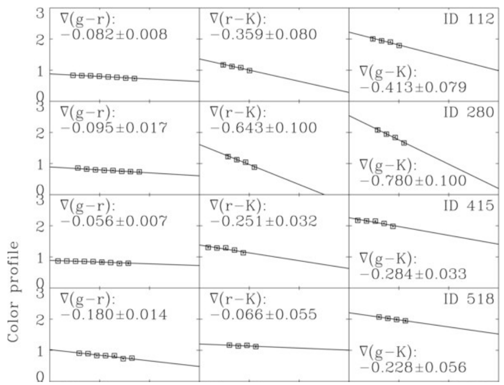


Color Gradients in Ellipticals



Kim & Im (2013)

Weak Colors Gradients -- Inner Parts Redder

Likely due to systematic differences in the metallicities of the stars

Kinematics of Elliptical Galaxies

We can also learn things from the kinematic information
we can derive from elliptical galaxies.

While there are far too many stars to observe the signatures of stars individually, one can consider the velocity distribution of stars along the line of sight:

$F(v_{los})$

Typically this function is assumed to be very close to a Gaussian:

$$F(v_{los}) = exp\left[\frac{-(v_{los} - \overline{v_{los}})^2}{2\sigma_{los}^2}\right]$$

In reality, one cannot describe the velocity distribution as a perfect Gaussian, so one allows for a third or fourth moment in the distribution.

Make use a complete orthogonal polynomial series ("Gauss Hermite") multiplied by an exponential:

$$e^{-k} [1 + \Sigma_{k=3,n} h_k H_k (w)]$$

h_3: Gauss-Hermite information $(2w^3-3w)/3^{1/2}$

measures "skewness" or deviation from symmetry. Large h_3 represents a secondary bump at v > <v>, so the peak of the line is at v < <v>

h_4: Gauss-Hermite information $(4w^4-12w^2+1)/24^{1/2}$

measures "kurtosis" or symmetric departures from a Gaussian. Large h_4 indicates a boxy profile centered on <v>.

Deriving Kinematic Information

We can determine the mean line of sight velocity and the dispersion of a galaxy (and other higher order velocity components) by taking a high signal to noise spectrum of this galaxy and then comparing its spectrum with a star.

Model Spectral Energy Distribution:

$$\mathrm{Model(L)} = \text{convolved star spectrum}(L) =$$

$$= \int S(L - v_{los}/c) F(v_{los}) dv_{los}$$

where S is the spectrum of the stars that dominate the light in a galaxy.

The integral is over the many different velocities v_{los} that stars in a galaxy could have.

Fitting the Velocity Dispersion

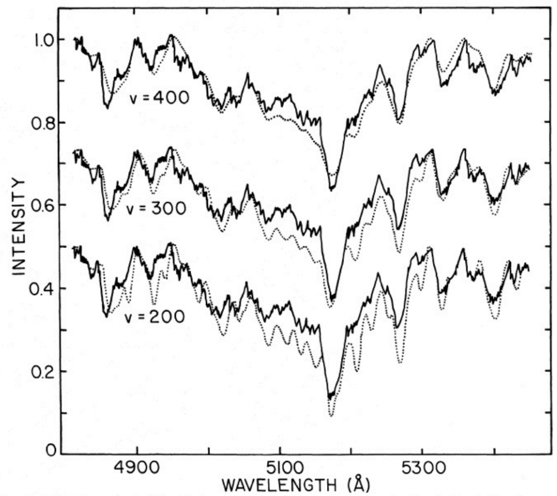


FIG. 3.—NGC 4472 compared with standard star HR 1805 (K3 III), broadened by various line-of-sight velocities (*dotted line*)

Deriving Kinematic Information

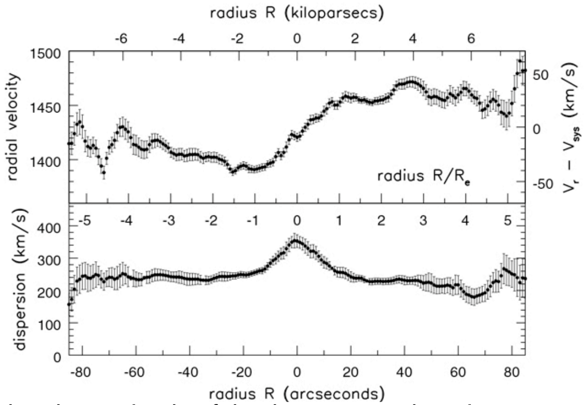
Find the model that gives the minimum X^2 with respect to the observed spectrum:

$$\chi^2 = \Sigma \left(\frac{(G(L) - Model(L))^2}{error(L)^2} \right)$$

In practice, the model spectrum never fits the observed spectral energy distribution exactly (due to the fact that the stars in the model never have exactly the same distribution of metallicities or temperatures as in the observations), so one needs to include some freedom in the models to overcome this effect.

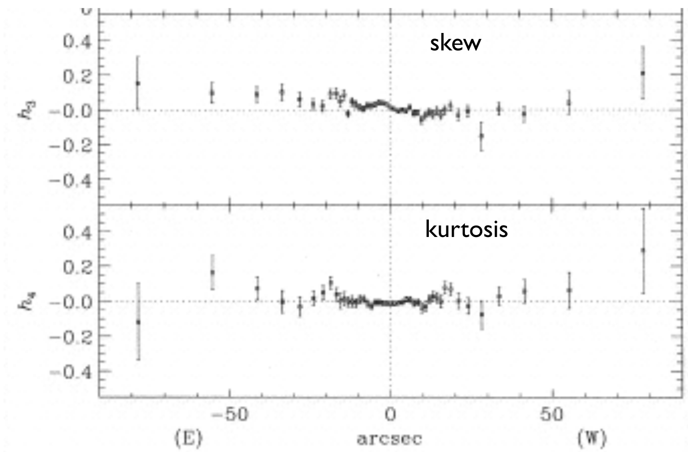
How does the mean line of sight velocity and velocity dispersion depend on the radius of an elliptical galaxy?

One Example:

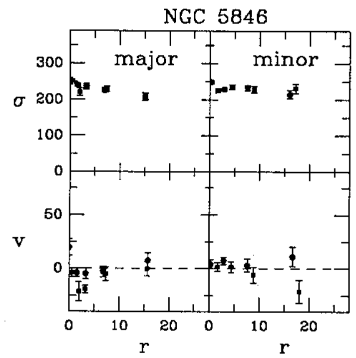


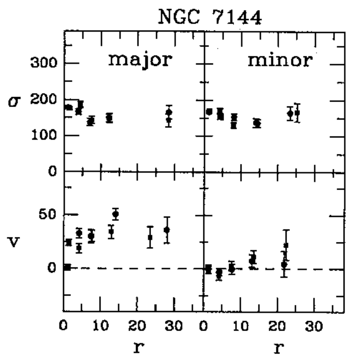
Notice that the amplitude of the dispersion in the velocity is much higher in general than the rotational velocities in these galaxies.

Can also attempt to quantify to the skew and the kurtosis of the velocity distribution as a function of radius in galaxies



Other examples of results for individual galaxies:





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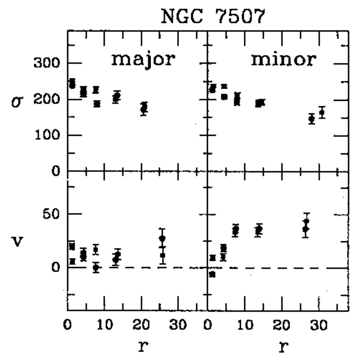

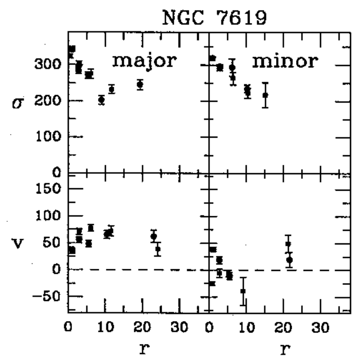

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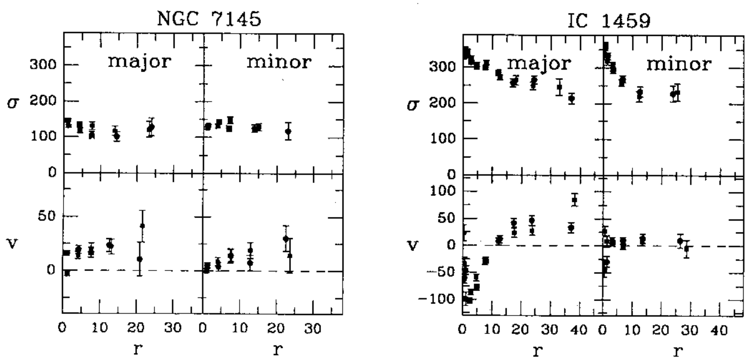
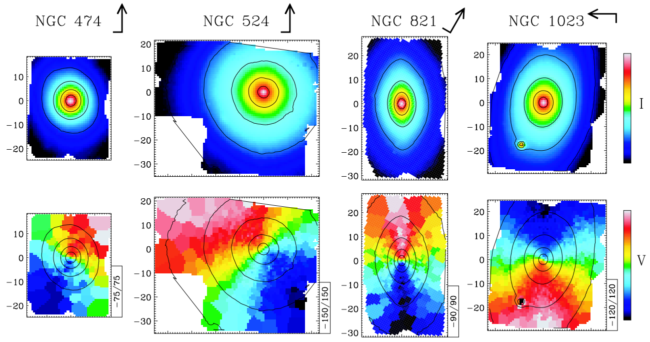


Can also measure the velocity distribution of galaxies as a function of both coordinates on the sky:

(this work done with an IFU: Integrated Field Unit)

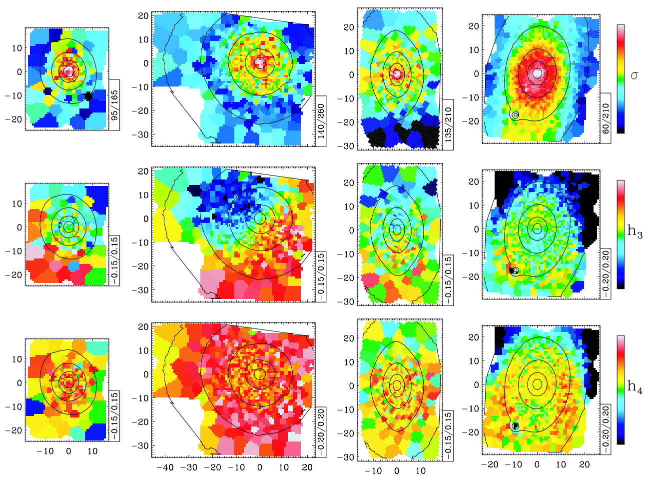


Surface Brightness

Average velocity (line of sight)

Dipole-like structure indicates a net rotation about the center

Can also measure the velocity distribution of galaxies as a function of both coordinates on the sky:



Velocity Dispersion

“Skew” in Velocity Distribution

“Kurtosis” in Velocity Distribution

General Conclusions from Kinematic Studies

Elliptical Galaxies Have Dispersions between 100 and 300 km/s

Rotation Velocity of galaxies is smaller ~50 km/s

Ratio of Rotation Velocity v to velocity dispersion σ, i.e., v/σ, is generally between 0 and 1

Most of Rotation on Major Axis, but also on Minor Axis

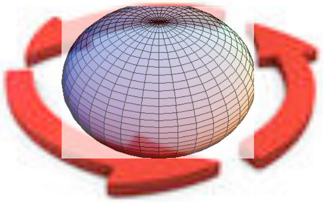
From examples we have just given -- NGC7144, NGC7145, IC1459, NGC7619, NGC821, NGC1023 show more rotation along major axis

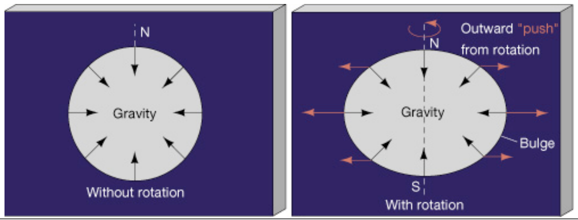
From examples we have just given -- NGC7507 shows more rotation along minor axis

Most of the Stellar Motion is Random, not Systematic

Why do Ellipticals Have an Elliptical Morphology?

One possibility = rotational flattening





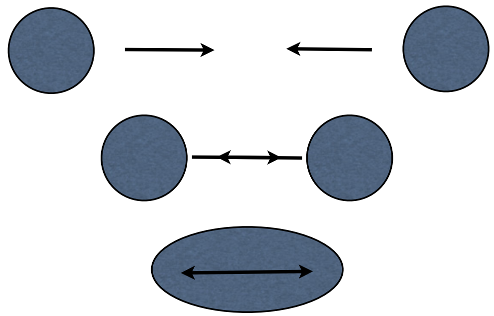
rotating objects tend to be elongated away from axis of rotation

this mechanism has an impact on the shape of the planets in the solar system, especially Jupiter and Saturn

Why do Ellipticals Have an Elliptical Morphology?

Second possibility = anisotropy in the velocity dispersion

consider the formation of elliptical galaxies from the merger of two smaller galaxies



Is the Flattening of Elliptical Galaxies Due to Rotation?

Make use of 3D virial theorem:

moment of inertia tensor ⟶

$$1/2\frac{d^2 I_{j,k}}{dt^2} = 2T_{jk} + \Pi_{jk} + Wjk$$

where

$$T_{jk} = 1/2\int \rho\,\overline{v_j}\;\overline{v_k}\,d^3x$$

$$\Pi_{jk} = 1/2\int \rho\sigma_{jk}^2 d^3x$$

$$\overline{v_j} = \int v_j f(x,v) d^3v/\rho$$

$$\sigma_{jk} = \int (v_j - \overline{v_j})(v_k - \overline{v_k}) f(x,v) d^3v/\rho$$

$$W_{jk} = -\int \rho(x) x_j \frac{\delta\Psi}{\delta x_k} d^3x$$

(potential energy tensor)

NOTE:

moment of inertia tensor ⟶

$$1/2\frac{d^2 I_{j,k}}{dt^2} = 2T_{jk} + \Pi_{jk} + Wjk$$

This expression is very similar to the expression for the virial theorem:

$$d^2I/dt^2 = 2K + W$$

if we make the following association

$$2K_{jk} = 2T_{jk} + \Pi_{jk}$$

where K_{jk} is defined to equal

$$K_{jk} \equiv \frac{1}{2}\int \rho\overline{v_j v_k} d^3\mathbf{x}.$$

Why are Elliptical Galaxies Flattened?

For a stationary system, the second time derivative of the moment of inertia tensor is 0.

Also, let assume that any rotation in the elliptical galaxy is along the z axis and that the galaxy is axisymmetric (i.e., assume elliptical galaxy is oblate)

By definition then, $W_{xx} = W_{yy}$, and $W_{ij} = 0$ for $i \neq j$

Also, $\Pi_{xx} = \Pi_{yy}$, and $\Pi_{ij} = 0$ for $i \neq j$

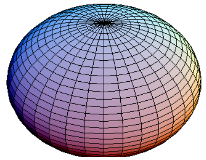
Also, $T_{xx} = T_{yy}$, and $T_{ij} = 0$ for $i \neq j$

$T_{zz} = 0$, by definition

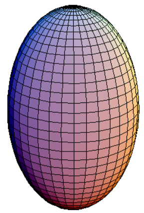
This results in the two equations

$$2T_{xx} + \Pi_{xx} + W_{xx} = 0; 2T_{zz} + \Pi_{zz} + W_{zz} = 0$$

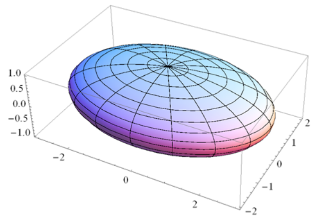
Oblate vs. Prolate vs. Triaxial



oblate



prolate



triaxial spheroid

Why are Elliptical Galaxies Flattened?

Expressing the T_{xx} and Π_{xx} in terms of some rotational velocity and velocity dispersion

$$2T_{xx} = 1/2 \int \rho \, \overline{v_\phi}^{\,2} d^3x = 1/2 M v_0^2$$

$$\Pi_{xx} = M\sigma_0^2,$$

the following can be shown:

$$\frac{v_0^2}{\sigma_0^2} = 2(1-\delta)\frac{W_{xx}}{W_{zz}} - 2$$

where δ is implicitly defined from the following relation:

$$\Pi_{zz} = (1-\delta)\Pi_{xx} = (1-\delta)M\sigma_0^2.$$

δ indicates the anisotropy in the velocity-dispersion tensor

From the following relation $\frac{v_0^2}{\sigma_0^2} = 2(1-\delta)\frac{W_{xx}}{W_{zz}} - 2$

we can predict how the ratio of the rotation velocity to velocity dispersion would depend on the ellipticity of the galaxy for an oblate isotropic rotator...

It can be shown that

$W_{xx}/W_{zz} \sim (1-\varepsilon)^{-0.9}$

Does not depend on the density profile

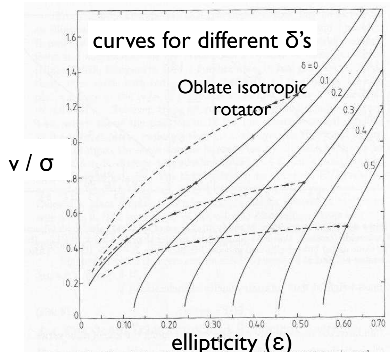
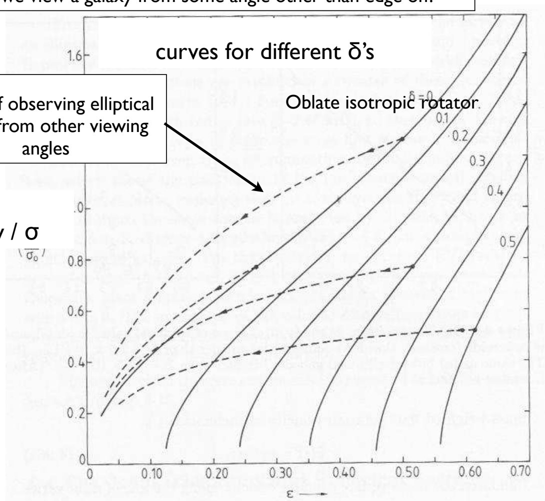


Figure 4-5. The relationship between the rotation parameter v/σ and ellipticity ϵ predicted by (4-95) for elliptical galaxies whose isodensity surfaces are similar coaxial oblate spheroids. The dashed curves show the movement of the point corresponding to the observable quantities $\tilde{v}/\tilde{\sigma}$ and $\tilde{\epsilon}_a$ when the galaxy's inclination angle i is decreased from $i = 90°$.

What if we view a galaxy from some angle other than edge on?



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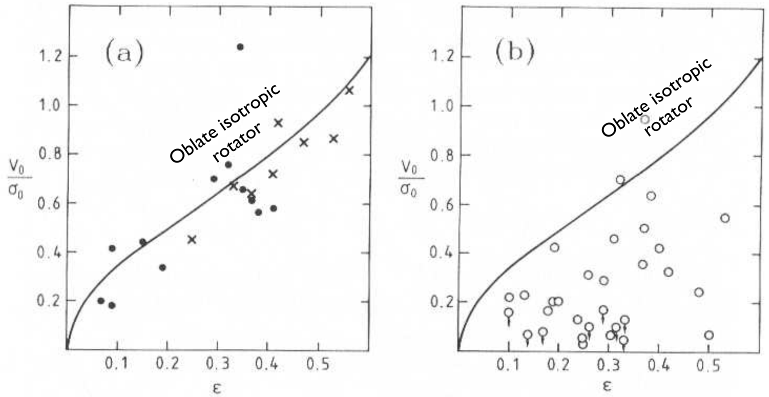
How does the predicted relationship between v/σ and ellipticity (ε) compare to that found for elliptical galaxies observed in the real universe?

It depends on the luminosity of the elliptical galaxy.

Lower Luminosity Spheroids | More Luminous Spheroids



From the observed v/σ versus ellipticity (ε) relationship, it seems clear that there is a qualitative difference between the most luminous elliptical galaxies and lower luminosity elliptical galaxies:

This dichotomy is seen in many of the other properties of elliptical galaxies as well:

Luminosity	High	Low
Physical Mechanism for Flattening	Anisotropy	Rotation
Isophotes	Boxy	Disky
Shape	Triaxial	Oblate
Profile	Core/ Break	Cuspy/ Power-Law
X-ray/radio	Loud	Quiet

This suggests that higher luminosity and lower luminosity elliptical galaxies may form in different ways!