Formation of Disk Galaxies (Part III) + Collisionless Stellar Dynamics

February 16

Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics
Feb 9: Disk Galaxies (I)
Feb 12: Disk Galaxies (II)
Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics
Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations
Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)
Mar 9: Elliptical Galaxies (II)
Mar 23: Elliptical Galaxies (III)
Mar 30: Dark Matter Halos
Apr 13: Large Scale Structure
Apr 20: Galaxy Stellar Populations
Apr 23: Lessons from Large Galaxy Samples at $z<0.2$
May 4: Evolution of Galaxies with Redshift
May 11: Galaxy Evolution at $z>1.5$ / Review for Final Exam

Problem Set 1

(Distributed last week, due on Feb 23)

Galaxies: Structure, Dynamics, and Evolution
Problem Set 1
Instructor: Dr. Bouwens

Here is problem set #1. The entire problem set will be due before class on Monday, February 23 (email them to Wout and hand them before class). Be sure to pay extra attention to problem 3, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. Derive the potential from the density for a point-source mass M, uniform density ρ sphere, and a singular isothermal sphere ρ_0/r^2 (where ρ_0 is the density at radius 1 and r is the radius) using the following equation presented in class:

$$\Phi = -4\pi G \left[\frac{1}{r} \int_0^r \rho(r') r'^2 dr' + \int_r^\infty \rho(r') r' dr' \right] \tag{1}$$

Show your work. As the potential for a singular isothermal sphere blows up at radius 0, please derive an expression for the potential such that the potential equals zero at r_0.

2. The model given by $\rho = 1/(1+r^2)^{2.5}$ is a Plummer model. Derive the potential of this model. What is the total mass?

3. Assume that the age of the universe is 13 Gyr and $\Omega = 1$ and $\sim$100% of the mass-energy density of the universe is in the form of matter. ← This will be the graded problem
(a) Using the equation

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{8}{3}\pi G\rho + \mathrm{const}/r^2 \tag{2}$$

where r is the scale factor of the universe and $\rho = \rho_0/r^3$, show that r increases with time as $t^{2/3}$. What does const equal for a universe where $\Omega = 1$?
(b) What is the Hubble constant $H_0 = (\dot{r}/r)_0$ that would yield a universe with an age of 13 Gyr?
(c) Calculate the age of the universe at redshifts z of 1, 5, and 10. Note that for redshifts z of 1, 5, and 10, the scale factor r for the universe was $(1+z)$ smaller than it is today (i.e., $r = r_0/(1+z)$ where r_0 is the scale factor today).
(d) How long has the light travelled which was emitted at $z = 1$?

4. (a) Consider that there was some overdense region in the universe which had a density ρ which was $2\rho_{crit}$ (the critical density) which otherwise had

February 19 Practical Session

(In 3 days)

Problems 5 and 6 (to be discussed)

Eugenia Rodendo Gonzalez
Noah Kaijser
Andrea Gibilaro
Susana Carneiro

5. In lecture, we examined an arbitrary dynamical system and determined how that dynamical system can be scaled in position, mass, and velocity and still maintain the same qualitative form.
(a) Show explicitly that the virial theorem produces the same result for the scaling relations.
(b) Derive Kepler's Third Law using the scaling relations found in class.
(c) Do the same sort of scaling relations exist for stars? Is it possible to scale the position, velocity, and mass for particles in a star in the same way – and have a system with the same qualitative form? Which equilibrium is retained and which is lost?

6. Prove that $M \propto T^{3/2}/n^{1/2}$. Use the fact that $\sigma^2 \propto T$ and $n \propto M/R^3$. Comment on the importance of this scaling relative to the T vs. n diagram used to understand for which mass sources $T_{cool} < T_{dyn}$ (i.e., where galaxy formation is efficient).

February 19 Practical Session

(In 3 days)

Problems 5 and 6 (to be discussed)

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Why attend?

To prepare for final exam! Exam will include ~1-2 homework problems!

20% of Homework Grade is from Attendance in Practical Classes
(5% of your final grade)

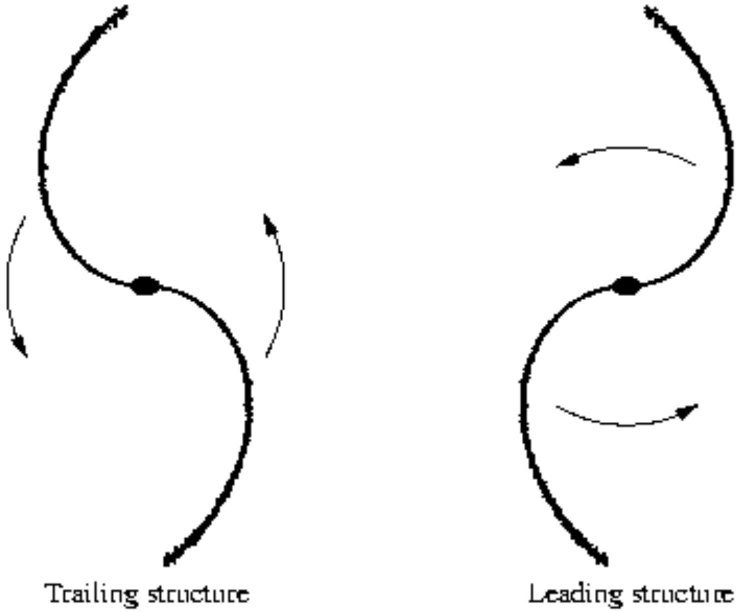
Helpful for learning the material! Learn from your peers!

Note that there will be 6 more practical sessions.

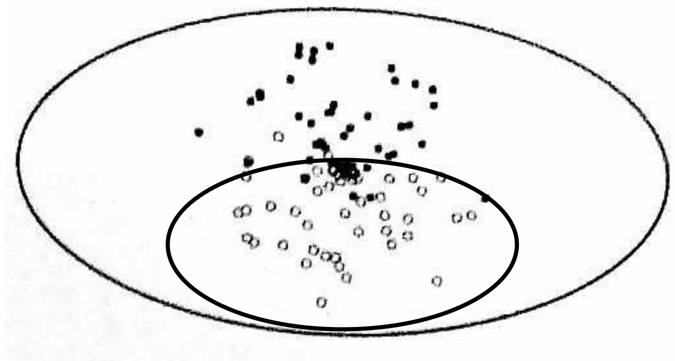
First, let's review the important material from last week

How do the arms in spiral galaxies evolve with time?

Most spiral arms are found to be trailing.

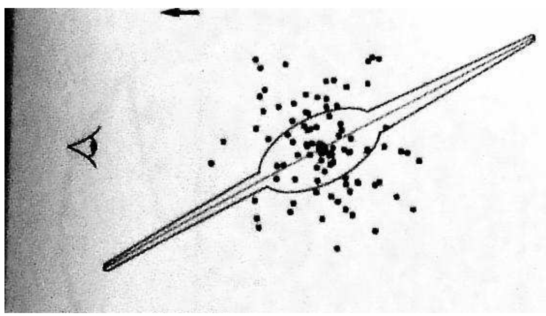


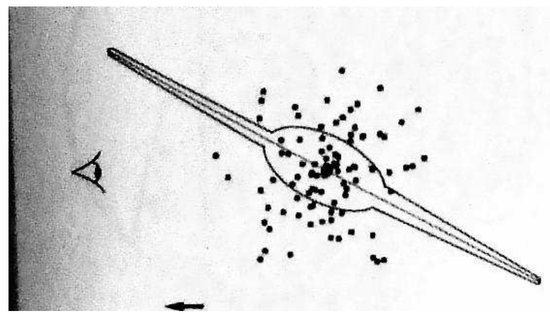
This could be determined by looking at reddening in globular clusters / novae



globular clusters seen around disk galaxy. amount of reddening indicated by whether circles are solid or open

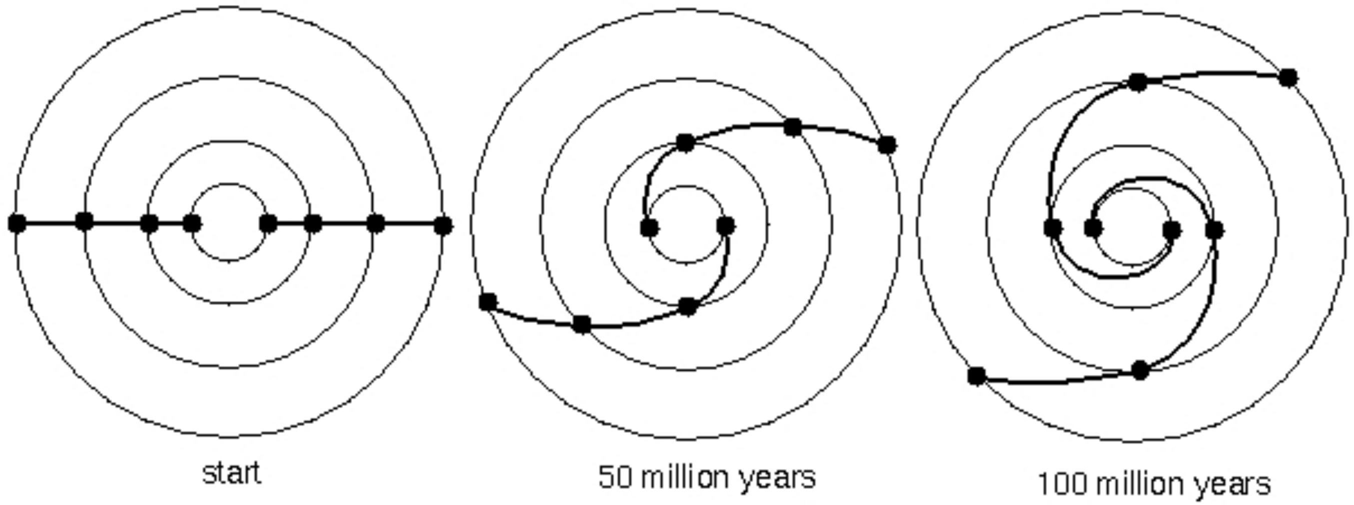
allows us to determine which way a spiral galaxy is tilted.



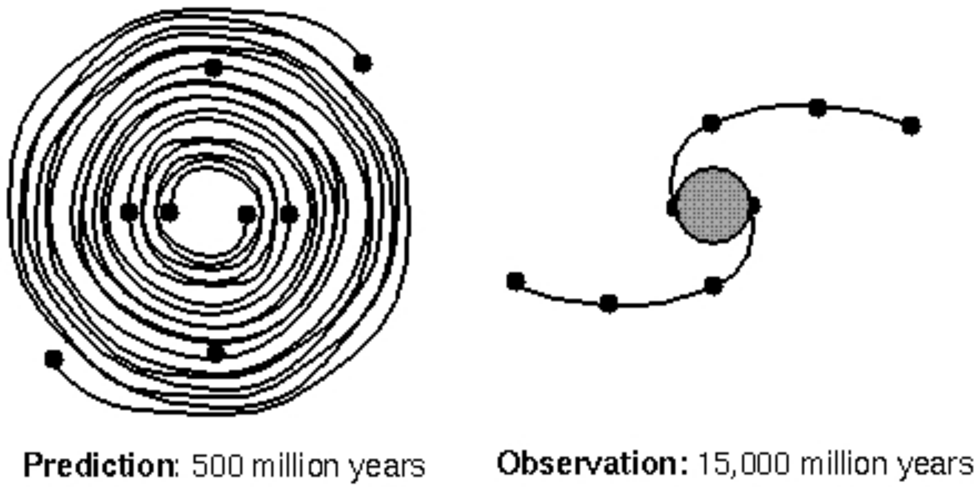


What is nature of the arms in spiral arms?

Do the spiral arms travel at the same speed as the stars? If spiral arms did,one would predict that the spiral arms in a galaxy would wind up very quickly.



Differential rotation: stars near the center take less time to orbit the center than those farther from the center. Differential rotation can create a spiral pattern in the disk in a short time.



The predicted outcome is in contrast to what is observed!

How can we solve the winding problem?

Density Wave Theory

Lin & Shu (1964-1966)

The spiral arms in disk galaxies are not fixed structures that rotate around the center of disk galaxies, but rather density waves.

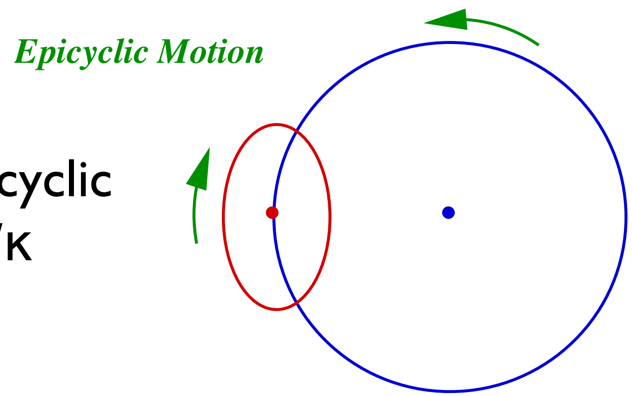
These density waves can move at a different speed than the stars within the galaxy itself.

The speed at which the spiral density waves propagate around the disk of a spiral galaxy is called the pattern speed Ω_P.

Stars in Spiral Galaxies are on Epicyclic orbits

The motion can be approximately described as the combination of orbital motion around a disk galaxy and an epicyclic motion in radius:

Frequency of epicyclic motion = 2π/κ



Orbital Frequency around Disk Galaxy = 2π/Ω

The orbital frequency of a star Ω(R) can be written as follows

$$\Omega^2(R) = \frac{1}{R}\left(\frac{\partial \Phi}{\partial R}\right)_{(R,0)} = \frac{L_z^2}{R^4},$$

Meanwhile, the frequency of epicyclic motion κ(R) can be written as follows:

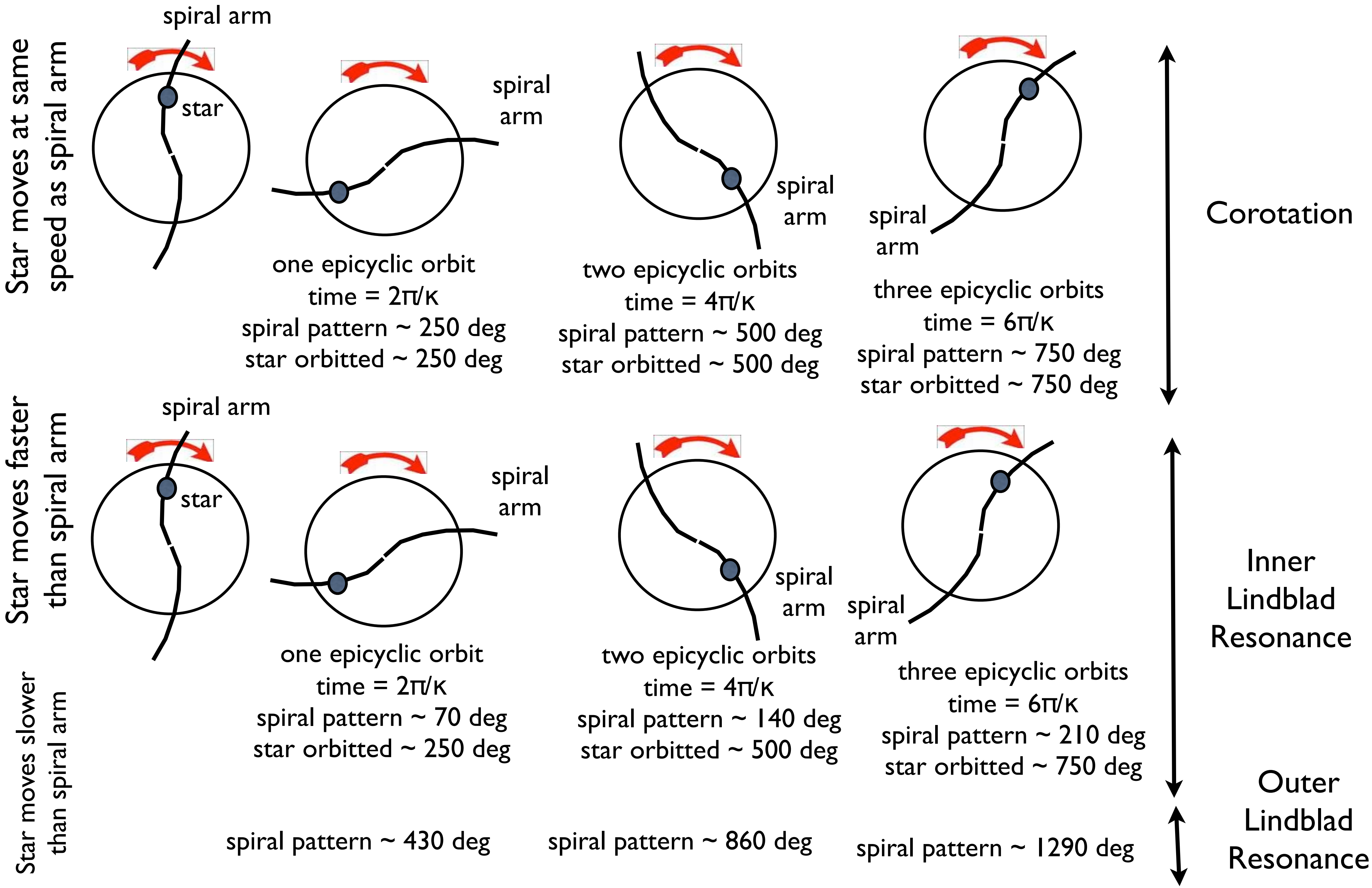
$$\kappa^2(R_g) = \left(R\frac{d\Omega^2}{dR} + 4\Omega^2\right)_{R_g}.$$

The frequency of epicycle motion is very similar to the orbital frequency:

In general, $\Omega < \kappa < 2\,\Omega$

Near the solar system, the epicycle frequency $\kappa \sim 1.3\,\Omega$

Let's consider snapshots in time where the star completes an entire epicyclic orbit. Typically a star must complete 70% of a revolution around a galaxy before this happens.



New Material

Galaxy Formation: Major Steps

Overdense Region In Early Universe

→ Gravitational Collapse →

Virialized Overdensity

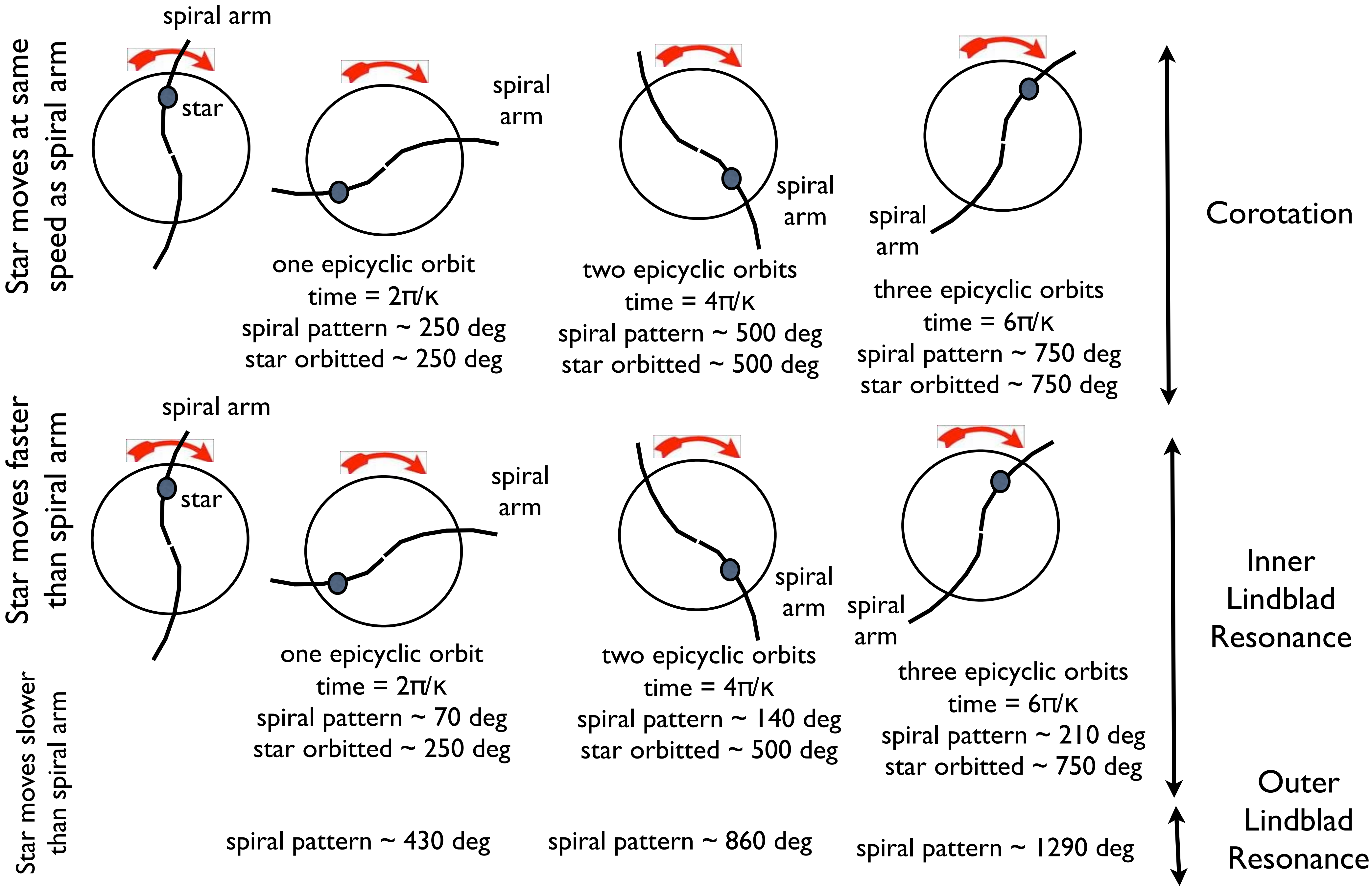
→ Gas Cooling →

Disk Galaxy (Supported by Angular Momentum)

→ Merger, Violent Relaxation →

Spheroid Galaxy (Random Motion Supported)

Let's consider snapshots in time where the star completes an entire epicyclic orbit. Typically a star must complete 70% of a revolution around a galaxy before this happens.



Which resonances drive spiral density wave growth?

To ensure that some arbitrary star can complete an epicyclic orbit in the same time it takes to move from one region in the spiral arm to another, the following condition must be satisfied:

some integer

$$m(\Omega_p - \Omega) = n\kappa$$

# of Spiral Arms

Orbital Frequency of Spiral Arms

Orbital (or Azimuthal) Frequency of Stars on Circular Orbits

Epicyclic (or radial) Frequency

The only integers n for this relation that are interesting are 0, +1, -1.

Which resonances drive spiral density wave growth?

This results in a number of well known resonances:

Inner Lindblad resonance:

$$\Omega_p = \Omega - \kappa/m$$

Outer Lindblad resonance:

$$\Omega_p = \Omega + \kappa/m$$

Corotational radius:

$$\Omega_p = \Omega$$

Most relevant cases:

$$\Omega_p = \Omega - \kappa/2$$

$$\Omega_p = \Omega + \kappa/2$$

$$\Omega_p = \Omega$$

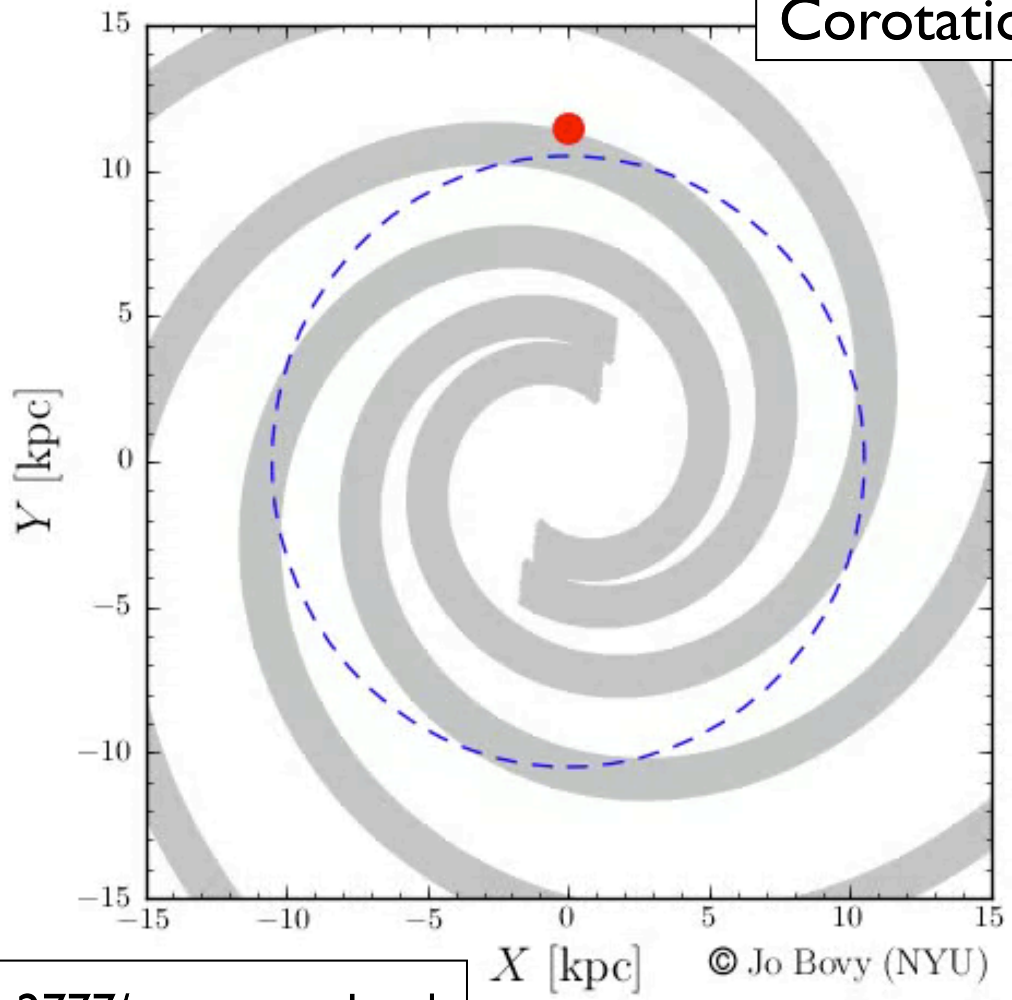
In most cases, the only relevant case is that of two spiral arms, i.e.,
$m = 2$

And note that physics behind bar-like features in spiral galaxies is similar

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:

Corotation

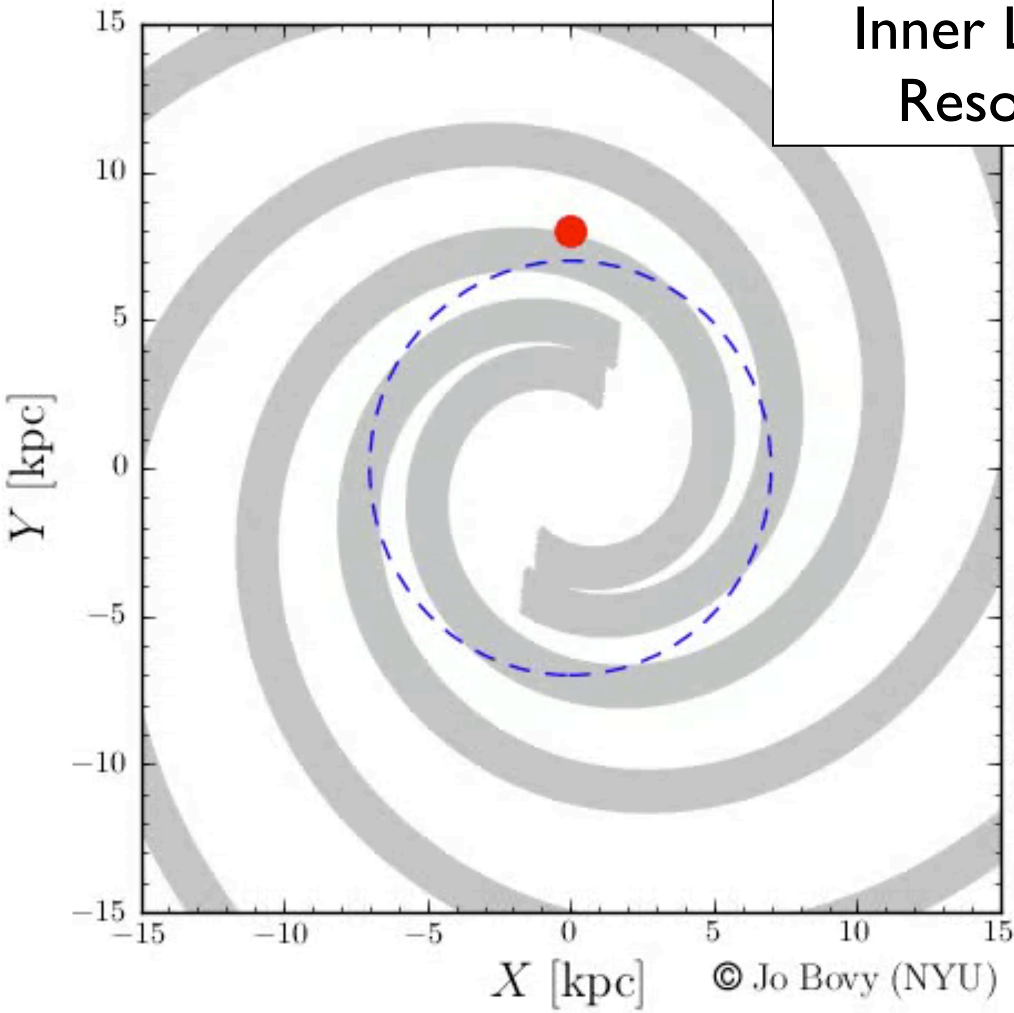


http://cosmo.nyu.edu/~jb2777/resonance.html

Credit: Jo Bovy

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:



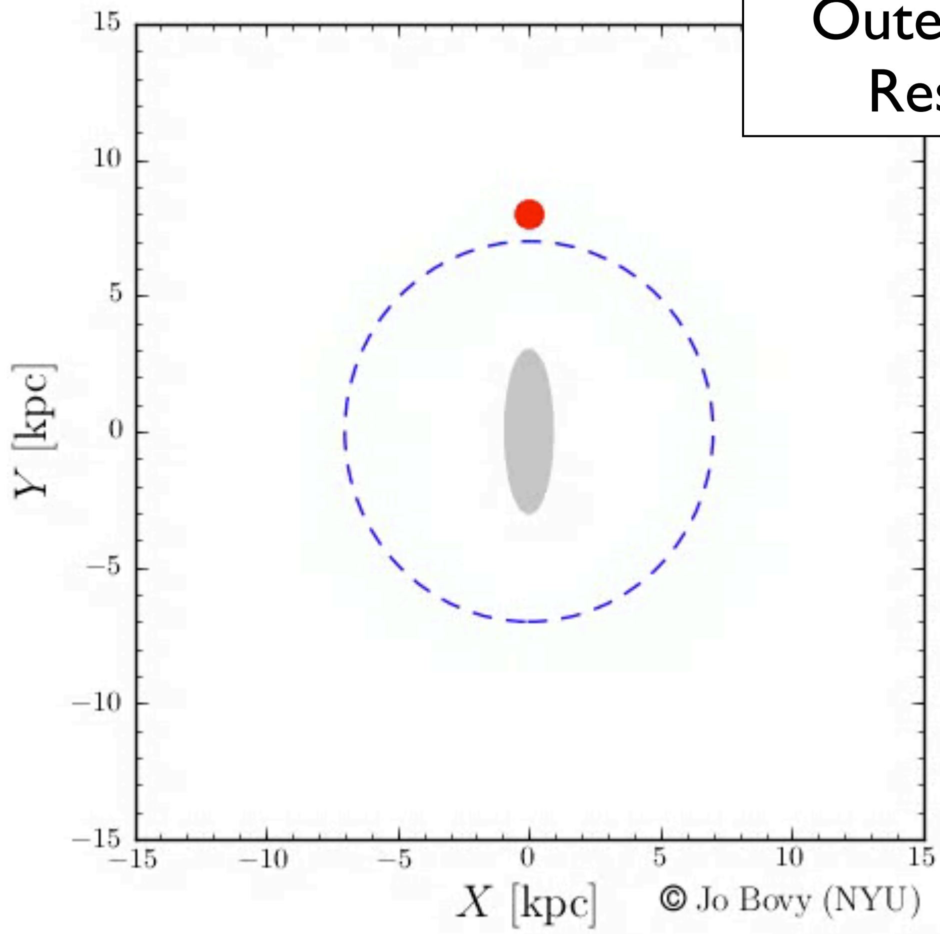
Inner Lindblad Resonance

Credit: Jo Bovy

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:

Outer Lindblad Resonance



Credit: Jo Bovy

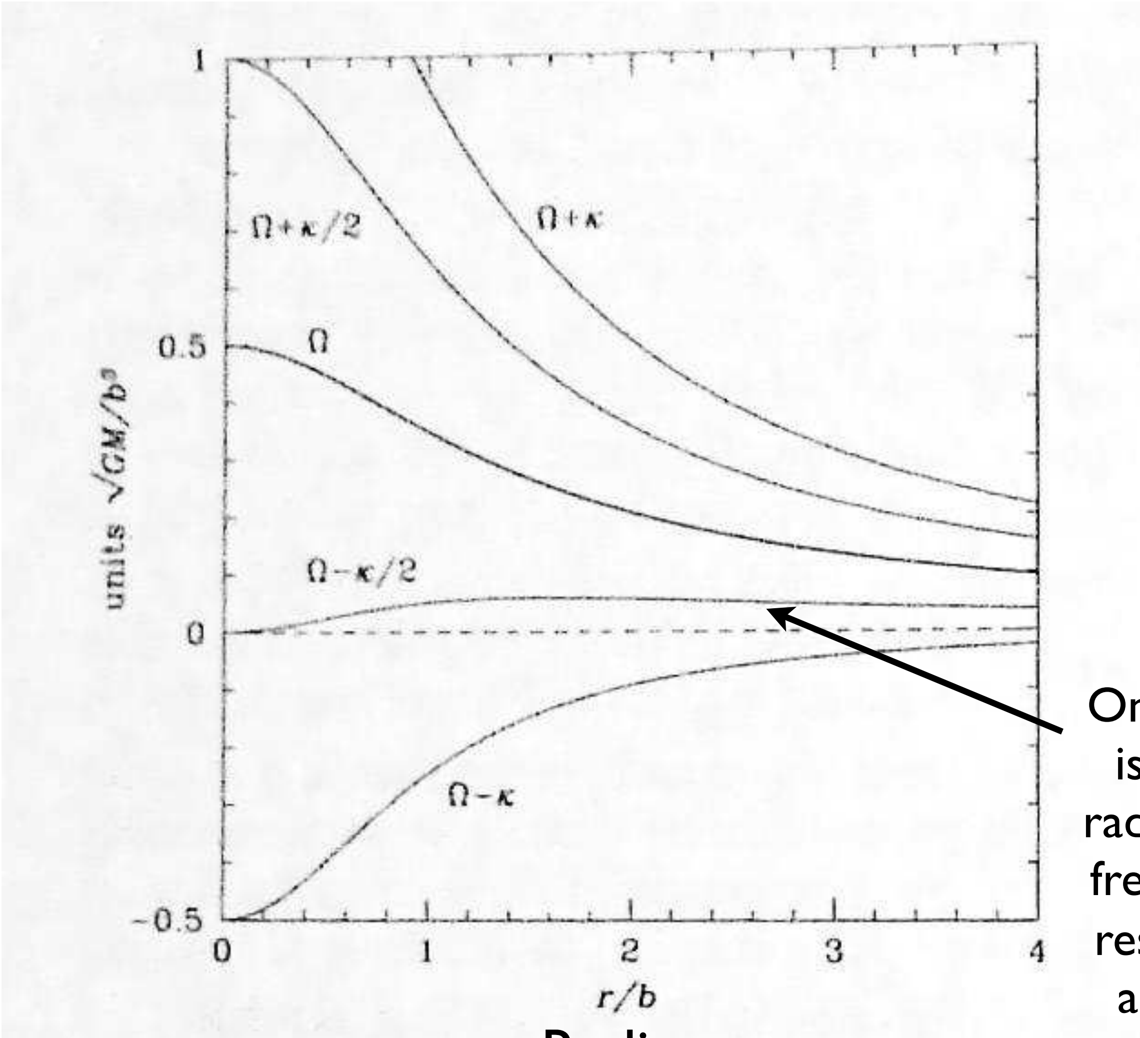
At what orbital frequencies for the spiral arms are these resonances relevant?

Compute $\Omega_p = \Omega - \kappa/2, \Omega, \Omega + \kappa/2$

How does the resonant frequencies vary by radius?

Ω_p

Orbital frequency for spiral arms at which these resonances become important



Radius

isochrone potential

One thing you should note is the extended range in radius where the rotational frequency for one of these resonances, i.e., $\Omega - \kappa/2$ is approximately constant.

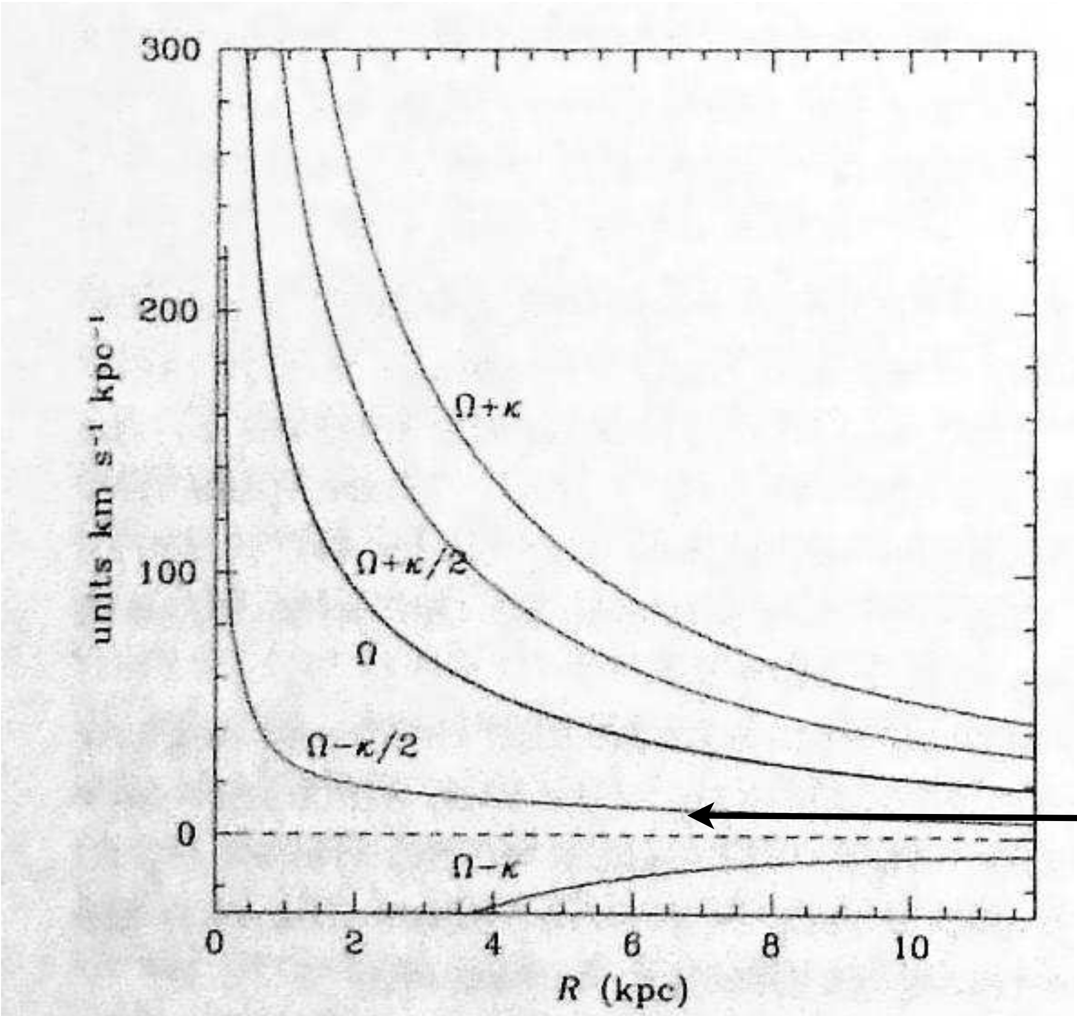
At what orbital frequencies for the spiral arms are these resonances relevant?

Compute $\Omega_p = \Omega - \kappa/2, \Omega, \Omega + \kappa/2$

How does the resonant frequencies vary by radius?

Ω_p

Rotational frequency for spiral arms at which these resonances become important



Radius

model 1 for our Galaxy from BT 2.7

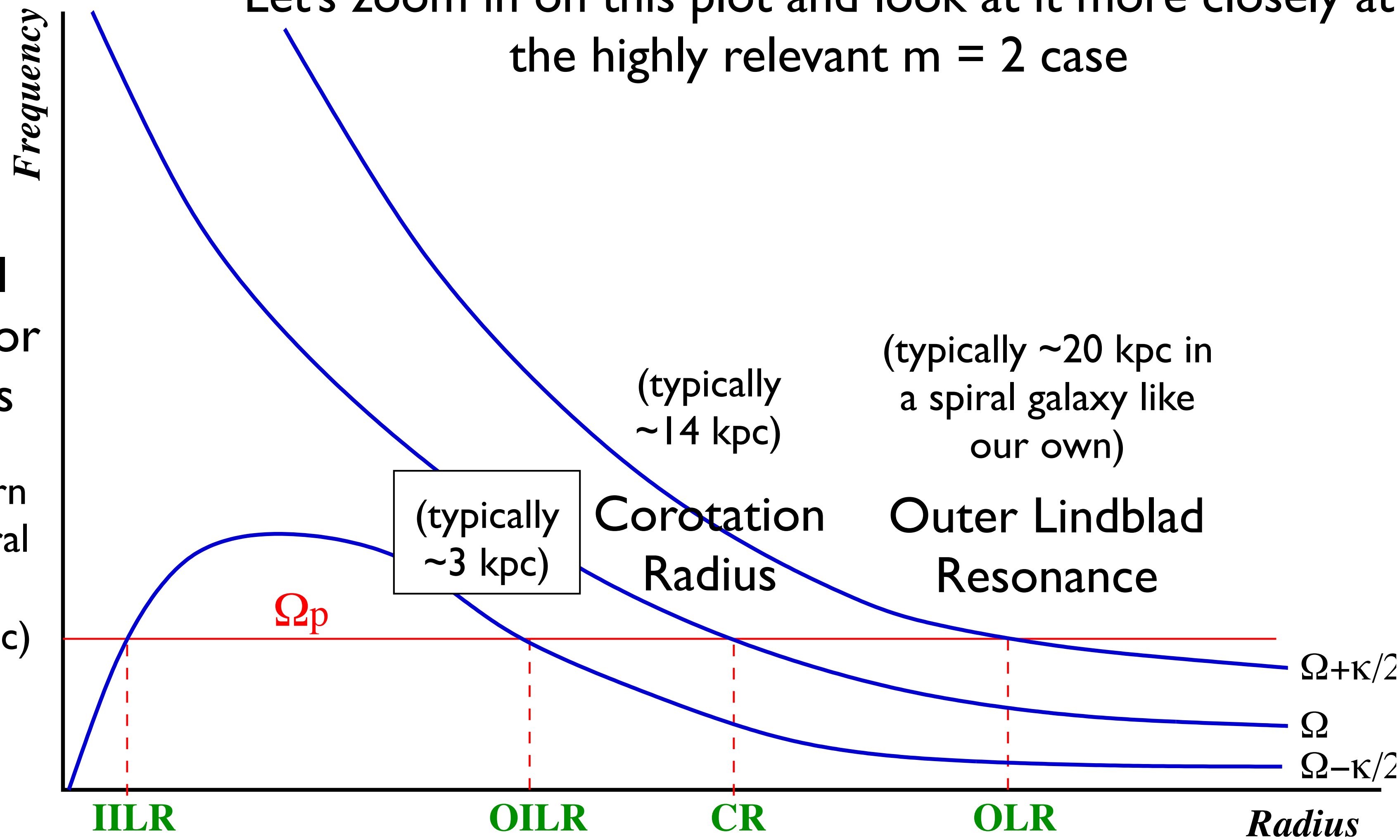
Again $\Omega - \kappa/2$ is almost independent of radius!

What are the typical physical radii where these resonances apply?

Let's zoom in on this plot and look at it more closely at the highly relevant m = 2 case

Rotational frequency for spiral arms

(typical pattern speed for spiral arms Ω_p ~15 km/s / kpc)



Inner Lindblad Resonances (Can be more than one)

Additional Properties of Spiral Density Waves

Spiral density waves generally begin their growth at the corotation radius where overdensities can readily grow and collapse

Spiral density waves can only survive and grow between the inner Lindblad resonance and outer Lindblad resonance.

These waves cannot pass through the inner Lindblad resonance (they are damped inside this radius)

In general the waves follow the following WKB (Wentzel–Kramers–Brillouin) relation:

$$(\omega - m\Omega)^2 = \kappa^2 + \sigma_R^2 k^2 - 2\pi G\Sigma|k|\mathcal{F}.$$

for perturbations of the form

$$\propto e^{i(m\phi + kR - \omega t)},$$

where ω is the angular frequency of a perturbation, σ_R is the radial velocity dispersion, k gives the radial wavenumber, and F is the reduction factor.

What conditions are important for gas in spiral galaxies for material to collapse gravitationally, feeding spiral density waves?

Jeans Instability

Consider homogeneous fluid that is in equilibrium, with density ρ, pressure p, with no internal motion.

Assume that the fluid is spherically symmetric. We shall consider the fluid is the matter inside some sphere with radius r.

Suppose that we compress the fluid element so that it now has a radius $r(1-\alpha/3)$ where α is much smaller than 1.

What will be the force acting on the surface of the sphere after this small compression?

To first order, the density perturbation is $\rho_1 = \alpha\rho$

To first order, the pressure perturbation is $p_1 = (dp/d\rho)\alpha\rho = \alpha\rho v_s^2$ where v_s is the sound speed.

Jeans Instability

The pressure force per unit mass is

$$F_p = \nabla p \,/\, \rho$$

The gravitational force per unit mass is

$$F_g = \rho GM/r^2/\rho$$

The additional pressure force per unit mass is

$$dF_p = \nabla(\alpha\rho v_s^2)/\rho$$

$$dF_p = \alpha v_s^2/r$$

The additional gravitational force per unit mass is

$$dF_g = \alpha GM/r^2$$

If the additional force on the surface of the sphere from the pressure of the fluid, i.e., dF_p, is greater than the additional force on the surface of the sphere from gravity, i.e., dF_g, then the pressure force resists the radial perturbation.

However, if the additional force from the gas pressure dF_p is less than the additional force from gravity dF_g, then the force of gravity will only accelerate the collapse.

Jeans Instability

In summary, for $dF_p > dF_g$ ==> fluid pressure resists gravitational collapse

However, for $dF_p < dFg$ ==> system undergoes gravitational collapse

$dF_p = dFg$ represents a specific physical scale.

$$dF_p = \alpha v_s^2/r = \alpha GM/r^2 = dF_g$$

Using $M = \rho\ 4/3\ \pi\ r^3$,

$$v_s^2/r = G\rho\ 4/3\ \pi\ r$$

We find:

$$r_J \sim (3\ v_s^2 / (4\pi\ G\rho))^{1/2}$$

Perturbations on a larger scale than the Jeans scale r_J will result in a gravitational collapse.

Toomre Instability Criterion

In spiral/disk galaxies, the stability criterion is more complex, due to the shearing type motion

Similarly for a disk galaxies, there is also a stability criterion. Any perturbations with wavelength larger than λ_{crit} are unstable.

$$\lambda_{crit} = 2\pi/k_{crit} = 4\pi^2 G\Sigma/\kappa^2$$

where κ is again the epicyclic frequency and Σ is the surface density of stars

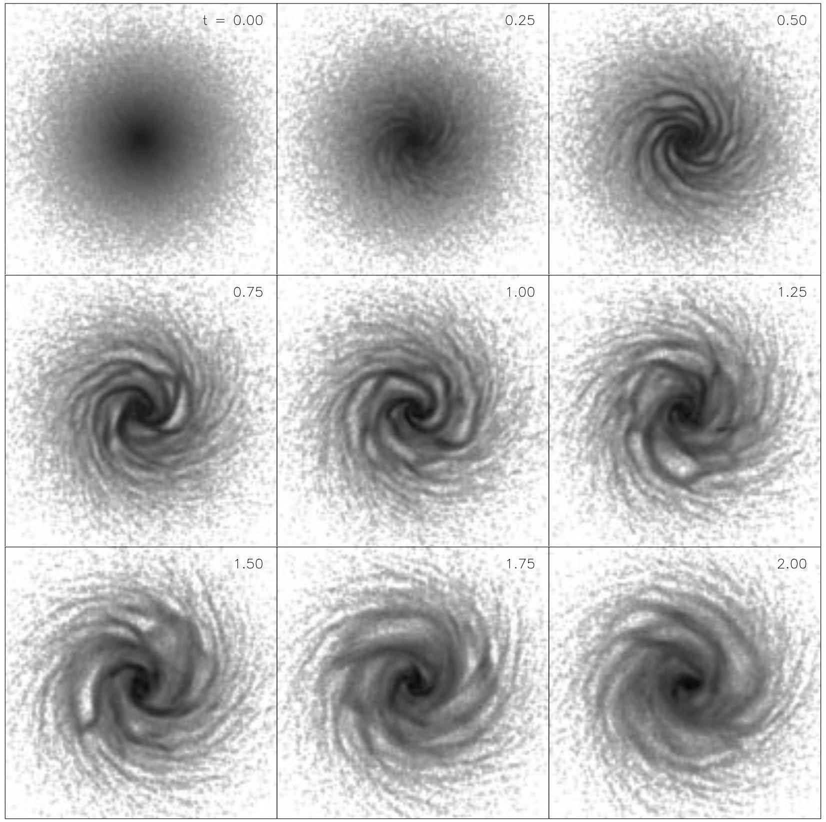
Random motions by stars tend to stabilize disks. Disks with stellar motions are unstable if

$$Q = \frac{\sigma_R \kappa}{3.36\ G\ \Sigma} < 1$$

This is the Toomre criterion. It assume that the disk is thin and unstable modes are much smaller than the size of the galaxy. Again, significant dispersion in the velocities of stars will stabilize the disk.

What happens if Q < 1?

This is an illustration of how the disk becomes clumpy because of a Toomre-like instability.



Credit: Barnes

What is Q for our galaxy?

For our disk where κ = 37±3 km/s/kpc, σ_R = 38±2 km/s, Σ = 36±5 $M_\odot$ / pc^2, Q* = 2.7±0.4.

Including interstellar gas 13 $M_\odot$ / pc^2, Q = 1.5.

WKB Dispersion relation

The value of Q determines the relative size of the terms in the WKB dispersion relation (governing propaganda and stability of wave-like perturbations in a disk):

$$(\omega - m\Omega)^2 = \kappa^2 + \sigma_R^2 k^2 - 2\pi G \Sigma |k| \mathcal{F}.$$

if Q large, this term large

if Q small, this term large

If Q is small, gravitational collapse results in star formation and injects energy and Q increases

If Q is large, eventually gas cools and increases in surface density, resulting in a decrease of Q.

This self regulation drives the Q parameter towards ~1-2

What impact does the measured Q value have on the growth of spiral density waves?

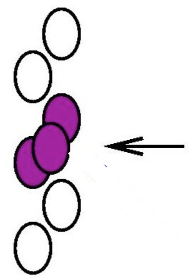
For lower values of the Q parameter — as high as Q <~ 2 — gravitational instabilities can feed the growth of spiral structure.

This is particularly relevant at the corotation radius, given the similar speed of the material (stars, gas) and spiral density waves.

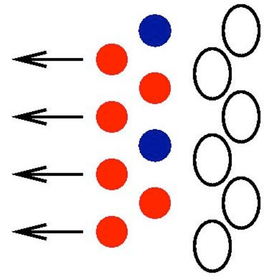
Now let's return to spiral density waves in spiral galaxies

What astrophysical processes drive these spiral density waves as they rotate around a spiral disk?

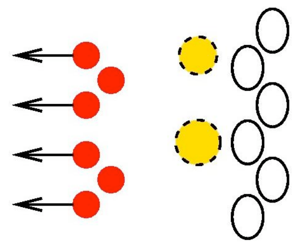
When the gas in the spiral density wave is compressed, it results in the formation of stars (due to the high gas densities induced by these compression waves)



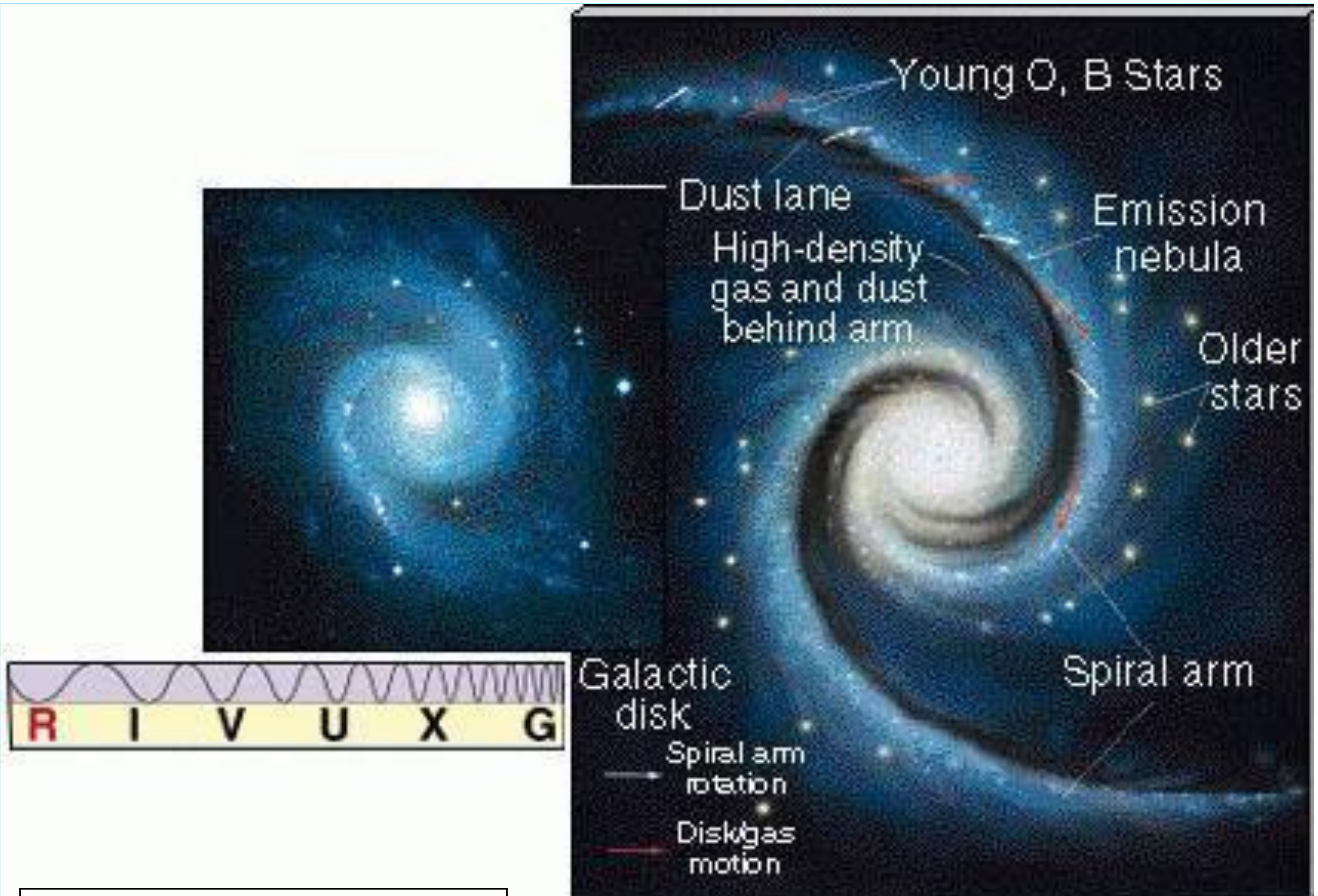
After the stars form, they will approximately move at the circular velocity of the spiral galaxy -- which is often faster than the pattern speed of the spiral arm



The high mass stars formed in the spiral density compression waves die (SNe explosions or otherwise) shortly after leaving the spiral arm compression wave, but the lower mass (redder) stars continue to rotate around the disk.



What astrophysical processes drive these spiral density waves as they rotate around a spiral disk?



Credit: van der Kruit

at inner radii in spiral galaxies, stars travel faster than the spiral density wave.

gas and dust lanes (formed from the metal output of the supernovae explosions) indicate the position of the high density spiral density wave

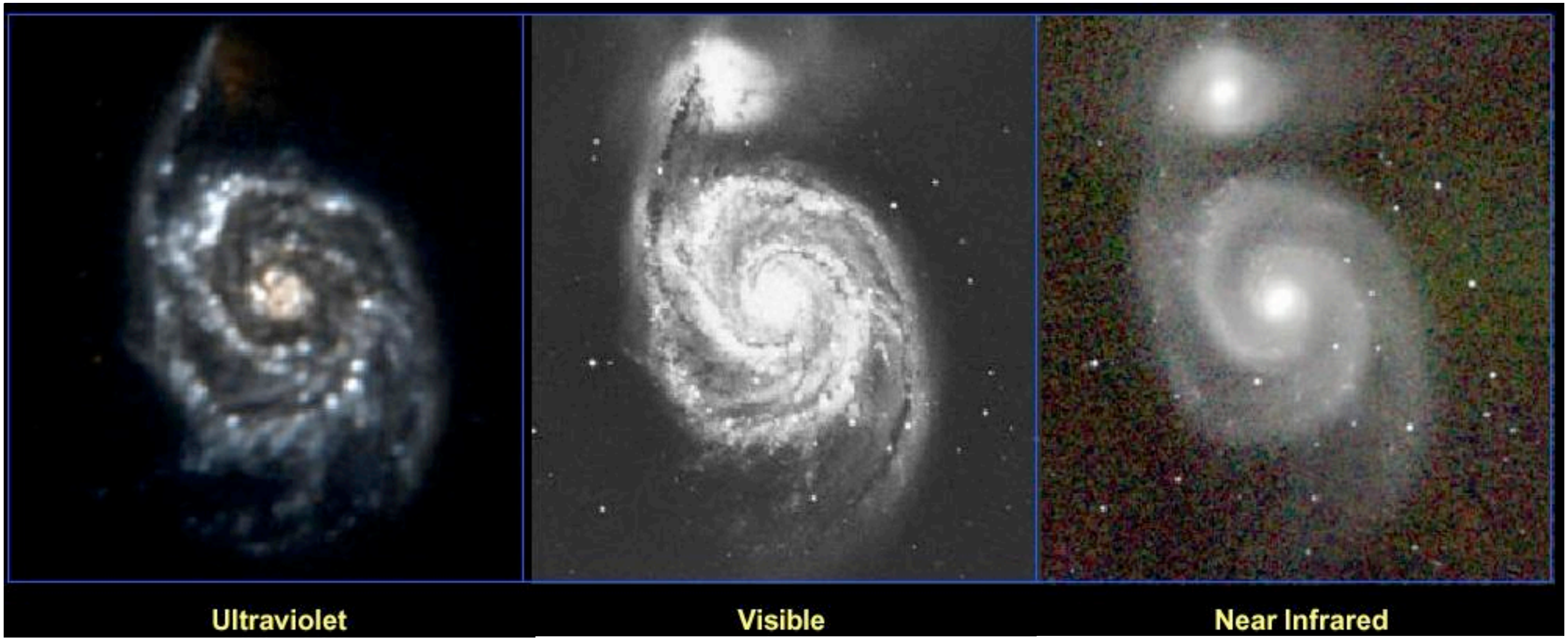
hot (massive) stars do not travel much beyond the spiral density wave in which they are formed

it is only the old (low mass) stars that can travel far enough to get ahead of the spiral wave

the hot stars are somewhat ahead of the gas/dust lane, since there is some time lag between when gravitational collapse begins and when the stars finally form (i.e., are on the main sequence)

Where is the spiral structure most evident?

Because of the hot blue stars being predominantly formed in the spiral density waves in disk galaxies and living for a very short time, we would expect the spiral structure to be much clearer at bluer or ultraviolet wavelengths where we just see the hot blue stars.

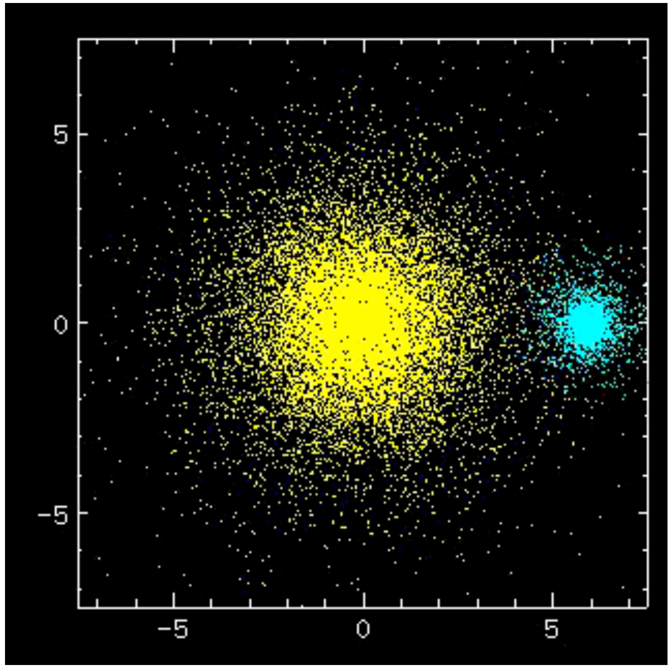


Besides high gas densities, what else can drive spiral density waves?

What else can drive spiral density waves in disk galaxies?

Asymmetries in the dark and/or halo (galaxy formation processes)

Or from interactions with a nearby neighbor (as in the case of spiral galaxy M51)



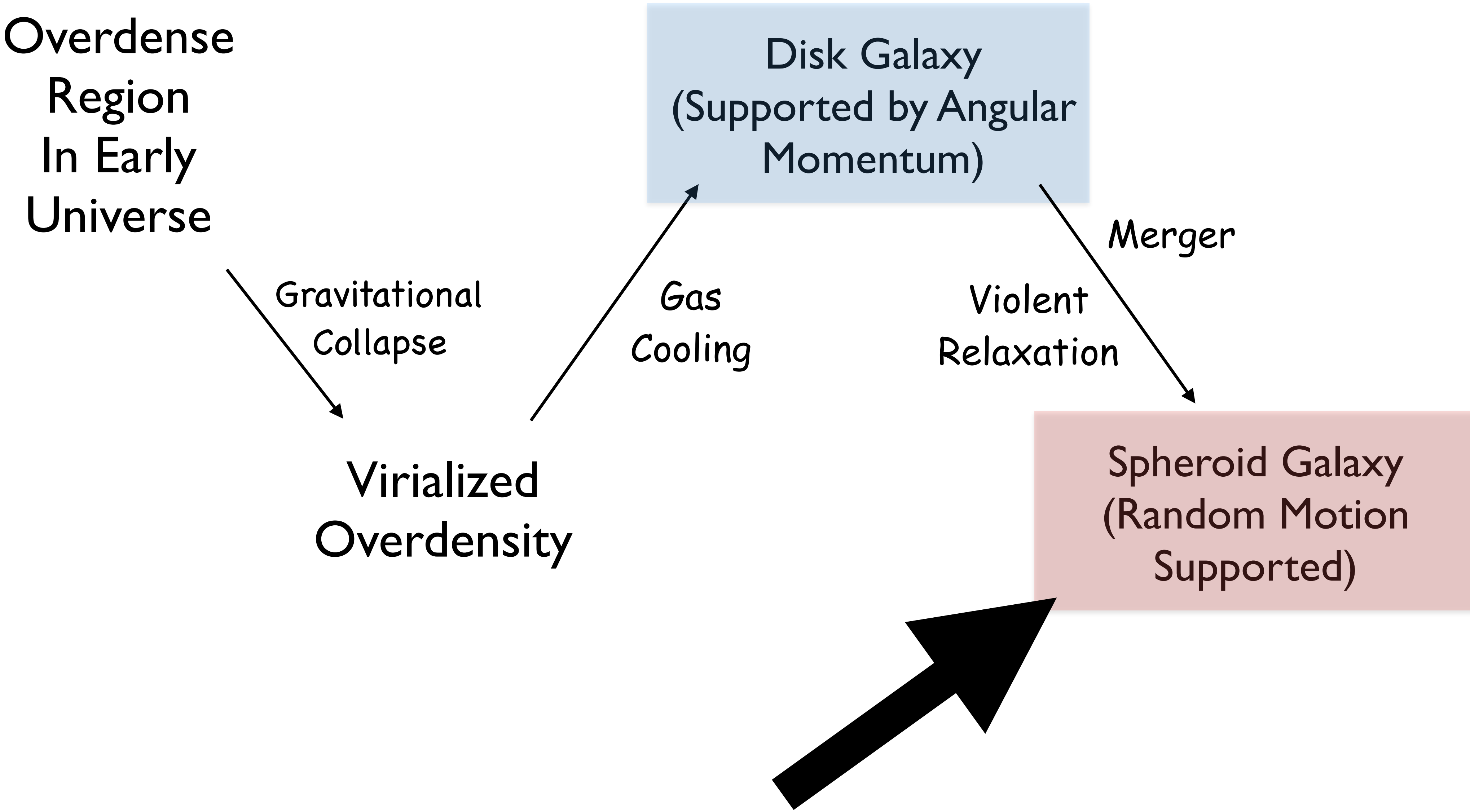
Next topic is elliptical galaxies...

Elliptical galaxies consist of large numbers of stars on diverse orbits.

While spiral galaxies are rotation supported, elliptical galaxies are supported by the random motions of stars they contain

Their behavior can largely be described using collisionless dynamics.

Galaxy Formation: Major Steps



We will therefore be reviewing some concepts from collisionless dynamics from the Leiden Bachelor course

We will discuss how to model the dynamics of $>10^{10}$ stars that form a self-gravitating system.

(this will require ~1 to ~1.5 lectures)

REVIEW Point from Bachelor Course: Collisions between individual stars are a non-issue in modeling galaxies - given the typical density, velocity, and cross section of stars. The main challenge is modeling their collective gravitational potential.

Let's quantify this by estimating the time scale for collisions:

Let's assume:

radius (galaxy) ~ 5 kpc
# of stars (galaxy) ~ 1×10^{10} stars
star diameter ~ 1.4×10^6 km
all stars have a mass equal to the sun

In 6×10^7 years, a typical star crosses pathes with N stars:

$$N = (\text{Distance Covered})(\#\ \text{stars} / \text{size}^3)$$

$$\text{Collisions per crossing time} = N \pi r^2$$

$$= (10\ \text{kpc})(1 \times 10^{10}\ \text{stars}/(5\ \text{kpc})^3)\pi(1.4 \times 10^6\ \text{km})^2 = 5 \times 10^{-12}\ \text{per crossing time}$$

$$= 5 \times 10^{-12} / 6 \times 10^7 \sim 8 \times 10^{-20} / \text{year}$$

Hence stars collide with each other very rarely!

REVIEW Point from Bachelor Course: In contrast to situations with fluids, the force on individual stars does not come primarily from its immediate neighbors, but from stars at all distances in the galaxy

Force from region of galaxy on a star

$$= Gm(\rho r^2 dr d\Omega)/r^2$$
$$= Gm\rho dr d\Omega$$

Force is independent of r! This means that a given star in a galaxy feels essentially the same force from stars at 1 kpc and stars at 4 kpc.

This is very different from hydrodynamics where short range pressure forces dominate!

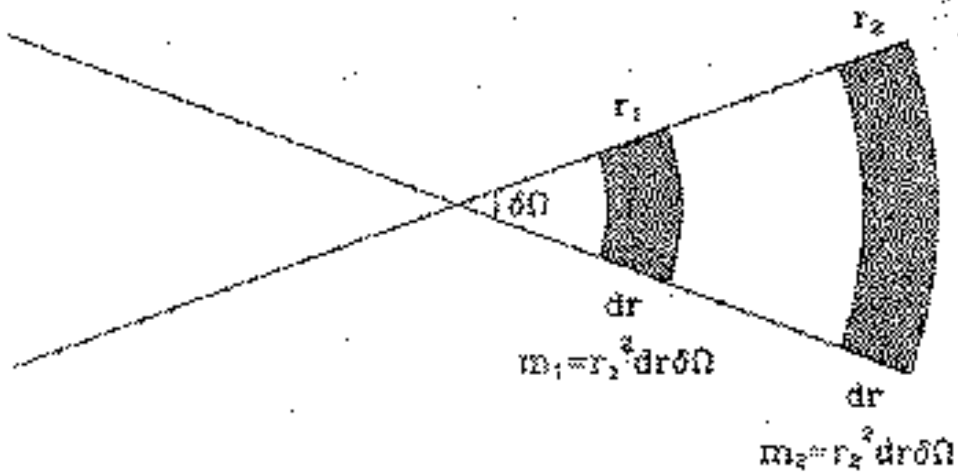


Figure 4-1. If the density of stars were everywhere the same, the stars in each of the shaded segments would make equal contributions to the net force on a star at the cone's apex. Thus the acceleration of a star at the apex is determined by the large-scale gradient in the density of stars within the galaxy.

BT4: pages 187

REVIEW Point from Bachelor Course: The time scale for the relaxation time of individual stars to collisions will other stars is very high, i.e., 10^{16} years, and thus can be ignored in modeling the dynamics of stars in a galaxy. Consequently, it is possible to model the potential and phase space as smoothly varying.

First let's look at velocity perturbation created by one star passing by another.

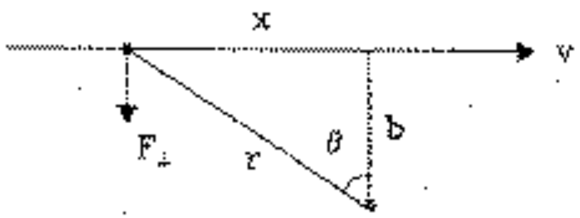


Figure 4-2. A field star approaches the test star at speed v and impact parameter b. We estimate the resulting impulse to the test star by approximating the field star's trajectory as a straight line.

BT4: pages 187-190