Formation of Disk Galaxies (Part II)

February 12

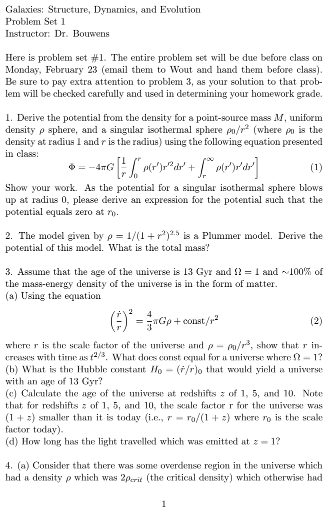
Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics
Feb 9: Disk Galaxies (I)
Feb 12: Disk Galaxies (II) ←
Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics
Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations
Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)
Mar 9: Elliptical Galaxies (II)
Mar 23: Elliptical Galaxies (III)
Mar 30: Dark Matter Halos
Apr 13: Large Scale Structure
Apr 20: Galaxy Stellar Populations
Apr 23: Lessons from Large Galaxy Samples at z<0.2
May 4: Evolution of Galaxies with Redshift
May 11: Galaxy Evolution at z>1.5 / Review for Final Exam

Problem Set 1

(Distributed last week, due on Feb 23)



Galaxies: Structure, Dynamics, and Evolution
Problem Set 1
Instructor: Dr. Bouwens

Here is problem set #1. The entire problem set will be due before class on Monday, February 23 (email them to Wout and hand them before class). Be sure to pay extra attention to problem 3, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. Derive the potential from the density for a point-source mass M, uniform density ρ sphere, and a singular isothermal sphere ρ_0/r^2 (where ρ_0 is the density at radius 1 and r is the radius) using the following equation presented in class:

$$\Phi = -4\pi G \left[\frac{1}{r}\int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r' dr'\right] \qquad (1)$$

Show your work. As the potential for a singular isothermal sphere blows up at radius 0, please derive an expression for the potential such that the potential equals zero at r_0.

2. The model given by $\rho = 1/(1+r^2)^{2.5}$ is a Plummer model. Derive the potential of this model. What is the total mass?

3. Assume that the age of the universe is 13 Gyr and $\Omega = 1$ and $\sim$100% of the mass-energy density of the universe is in the form of matter.
(a) Using the equation

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{4}{3}\pi G\rho + \text{const}/r^2 \qquad (2)$$

where r is the scale factor of the universe and $\rho = \rho_0/r^3$, show that r increases with time as $t^{2/3}$. What does const equal for a universe where $\Omega = 1$?
(b) What is the Hubble constant $H_0 = (\dot{r}/r)_0$ that would yield a universe with an age of 13 Gyr?
(c) Calculate the age of the universe at redshifts z of 1, 5, and 10. Note that for redshifts z of 1, 5, and 10, the scale factor r for the universe was $(1+z)$ smaller than it is today (i.e., $r = r_0/(1+z)$ where r_0 is the scale factor today).
(d) How long has the light travelled which was emitted at $z = 1$?

4. (a) Consider that there was some overdense region in the universe which had a density ρ which was $2\rho_{crit}$ (the critical density) which otherwise had

1

← This will be the graded problem

February 19 Practical Session

(In 7 days)

Problems 5 and 6 (to be discussed)

Eugenia Rodendo Gonzalez	Andrea Gibilaro
Noah Kaijser	Susana Carneiro

5. In lecture, we examined an arbitrary dynamical system and determined how that dynamical system can be scaled in position, mass, and velocity and still maintain the same qualitative form.
(a) Show explicitly that the virial theorem produces the same result for the scaling relations.
(b) Derive Kepler's Third Law using the scaling relations found in class.
(c) Do the same sort of scaling relations exist for stars? Is it possible to scale the position, velocity, and mass for particles in a star in the same way – and have a system with the same qualitative form? Which equilibrium is retained and which is lost?

6. Prove that $M \propto T^{3/2}/n^{1/2}$. Use the fact that $\sigma^2 \propto T$ and $n \propto M/R^3$. Comment on the importance of this scaling relative to the T vs. n diagram used to understand for which mass sources $T_{cool} < T_{dyn}$ (i.e., where galaxy formation is efficient).

February 19 Practical Session

(In 7 days)

Problems 5 and 6 (to be discussed)

Eugenia Rodendo Gonzalez	Andrea Gibilaro
Noah Kaijser	Susana Carneiro

Why attend?

To prepare for final exam!   Exam will include ~1-2 homework problems!

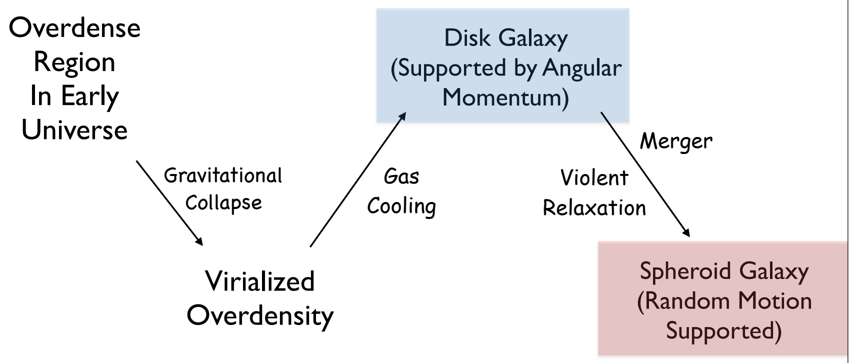
20% of Homework Grade is from Attendance in Practical Classes
(5% of your final grade)

Helpful for learning the material! Learn from your peers!

Note that there will be 6 more practical sessions.

Review of Material from Last Week

Galaxy Formation: Major Steps



Overdense Region In Early Universe

Gravitational Collapse

Virialized Overdensity

Gas Cooling

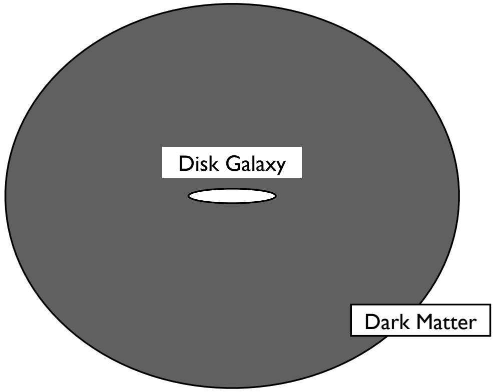
Disk Galaxy (Supported by Angular Momentum)

Merger

Violent Relaxation

Spheroid Galaxy (Random Motion Supported)

Let's consider a collapsed object with both dark matter (does not cool) + Baryons (can cool)



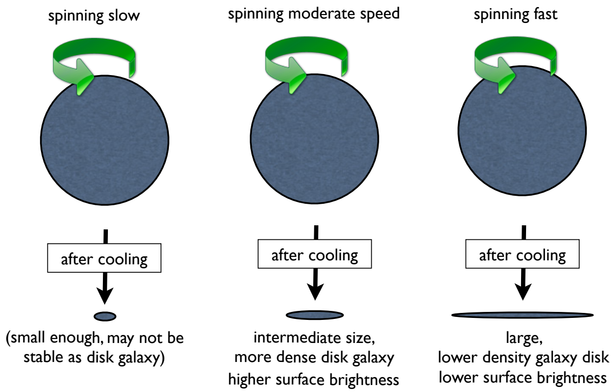
How extended is the baryonic mass at the center of collapsed sources?

Global Properties of Disk Galaxies

What is the reason for their disk-like, flat geometry?

A rotating disk is minimum energy configuration which preserves angular momentum

The Size of Disk Galaxies is likely determined by the angular momentum of the halo



spinning slow

spinning moderate speed

spinning fast

after cooling

after cooling

after cooling

(small enough, may not be stable as disk galaxy)

intermediate size, more dense disk galaxy higher surface brightness

large, lower density galaxy disk lower surface brightness

The other important variable is their mass which sets their luminosity and rotation

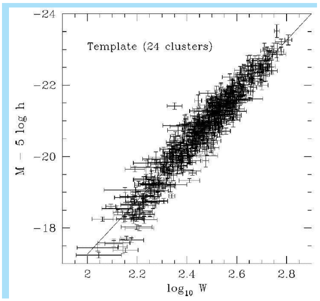
---

Global Properties of Disk Galaxies

Two variables (mass + angular momentum) of collapsed halos appear to determine most of the physical properties of a spiral galaxy.

And so other variables like their circular velocity appear to closely trace what we observe based on their luminosity and mass

Tully Fisher Relationship



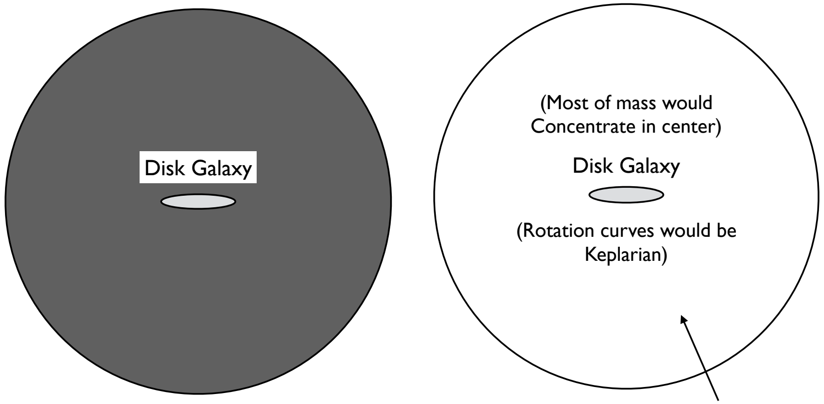
luminosity

~2 x circular velocity

Luminosity ∝ v_c^4

Disk Galaxy with Dark Matter

Disk Galaxy (if no Dark Matter)



Disk Galaxy

Dark Matter Halo

(Most of mass would Concentrate in center)

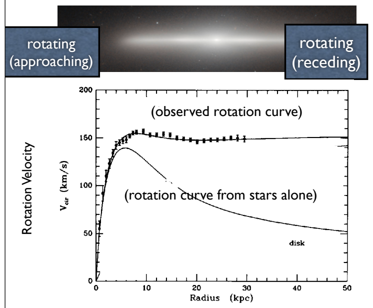
Disk Galaxy

(Rotation curves would be Keplarian)

The halo out here would likely contain much less mass

What is the evidence for significant mass in galaxies from dark matter?

Mass of Galaxies:
Inferences from the Rotation Curve



(observed rotation curve)

(rotation curve from stars alone)

V_c^2 (observed) = $G(M_{stars}(R)+M_{halo}(R))/R = GM_{stars}(R)/R + GM_{halo}(R)/R$

Mass of Galaxy Clusters:

Inferences from Velocity Dispersion of Cluster Galaxies

$M \sim v^2 R / G$

Inferences assuming Hydrostatic Equilibrium of X-ray gas

$$\frac{dp}{dr} = -\rho\frac{GM(r)}{r^2}$$

Pressure Gradient Gravitational Force

Determine temperature, density of gas to derive pressure

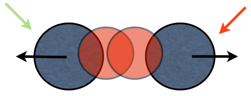
Inferences from Gravitational Lensing

Use inferred deflection to infer mass of cluster

Inferences from Cluster Collisions

-- dark matter from the colliding clusters pass right through each other

-- ionized gas from the colliding clusters "run into each other" forming a shock

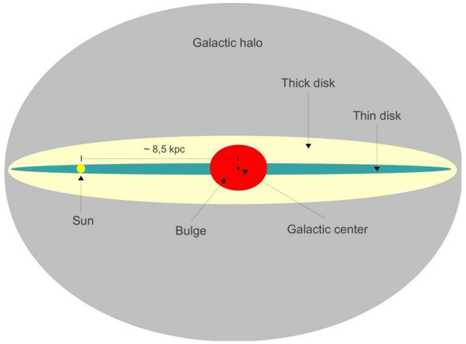


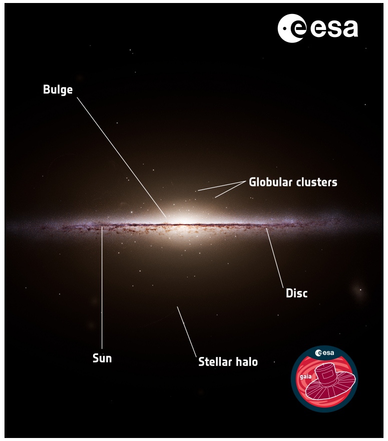
this presents us with a situation where the light (from baryons) and mass (from dark matter) are in different places

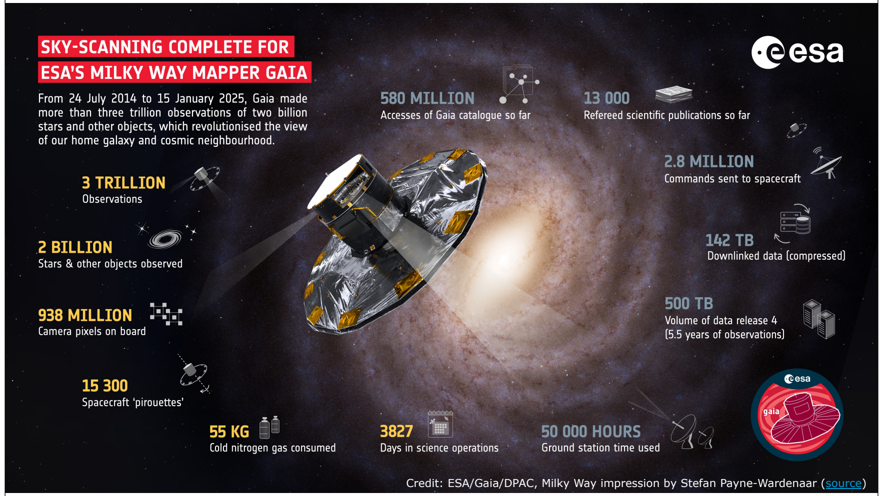
Brief Context: Structure of Disk Galaxy

Four Basic Components:

1. Thin Disk
2. Thick Disk
3. Halo
4. Bulge







SKY-SCANNING COMPLETE FOR ESA'S MILKY WAY MAPPER GAIA

From 24 July 2014 to 15 January 2025, Gaia made more than three trillion observations of two billion stars and other objects, which revolutionised the view of our home galaxy and cosmic neighbourhood.

3 TRILLION
Observations

2 BILLION
Stars & other objects observed

938 MILLION
Camera pixels on board

15 300
Spacecraft 'pirouettes'

55 KG
Cold nitrogen gas consumed

580 MILLION
Accesses of Gaia catalogue so far

3827
Days in science operations

13 000
Refereed scientific publications so far

2.8 MILLION
Commands sent to spacecraft

142 TB
Downlinked data (compressed)

500 TB
Volume of data release 4
(5.5 years of observations)

50 000 HOURS
Ground station time used

Credit: ESA/Gaia/DPAC, Milky Way impression by Stefan Payne-Wardenaar (source)

Existence of Both Thin and Thick Disk

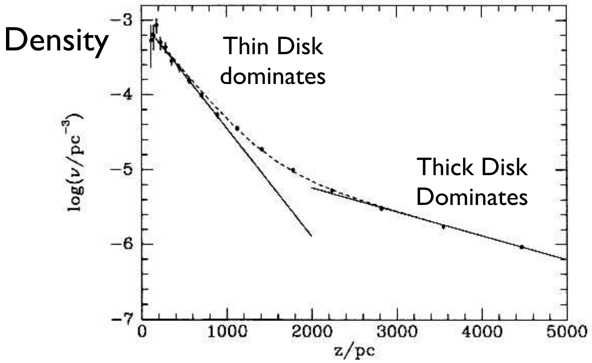
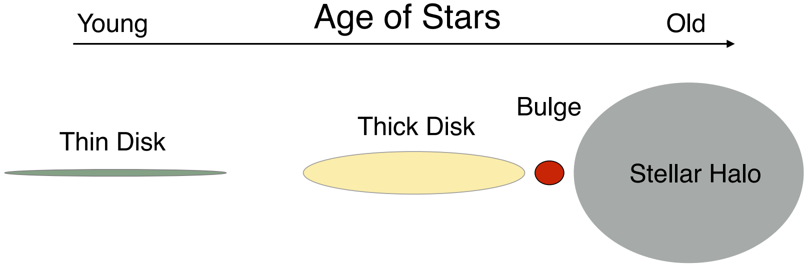


Figure 10.25 The space density as a function of distance z from the plane of MS stars with absolute magnitudes $4 \leq M_V \leq 5$. The full lines are exponentials with scale heights $z_0 = 300\,\mathrm{pc}$ (at left) and $z_0 = 1350\,\mathrm{pc}$ (at right). The dashed curve shows the sum of these two exponentials. [From data published in Gilmore & Reid (1983)]

Of course, with recent Gaia data, it is clear that even these distinctions are too simple.

Properties of Components



Young — Age of Stars — Old

Thin Disk

Thick Disk

Bulge

Stellar Halo

Higher Metallicity

Lower Metallicity

Higher Rotation Speed

Low Rotation Speed

Ages & Metal Abundances of Different Components

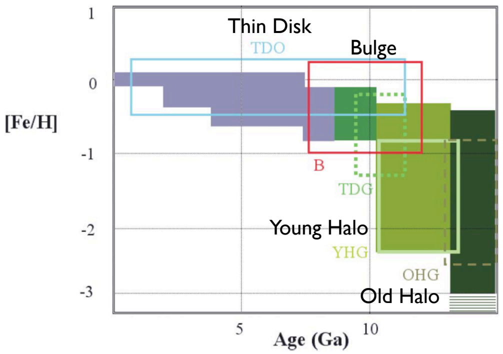


Fig. 1.16 The age-metallicity relation of the Galaxy for the different components (see text): TDS – thin disk stars; TDO – thin disk open clusters; ThDS – thick disk stars; ThDG – thick disk globulars; B – bulge; YHG – young halo globulars; OHG – old halo globulars.

Characteristics of Stars in the Stellar Halo of Milky Way very different than the Disk

Stellar Halo	Thin Disk
Low Metallicity	High Metallicity
Old Ages	Young Ages
Little Rotation	High Rotation
Older, lower metallicity star clusters	Younger, higher metallicity star clusters

This motivated deriving a model to explain the halo.

Two Competing Models for Formation of Stellar Halo in Milky Way

ELS Monolithic Collapse

Halo formed in first Gyr

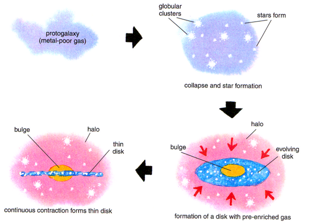


Figure 6. The "ELS" model holds that the Milky Way formed from the rapid collapse of a single cloud of gas. Stars formed early in the collapse maintained the dynamics of the metal-poor gas and so now travel around the Galaxy in elliptical orbits within the halo. As the cloud collapsed *(red arrows)* preferentially along its rotational axis, it formed a disk that had been enriched with the metals produced by the early generations of halo stars.

Searle & Zinn Hierarchical Model

Halo built up from mergers

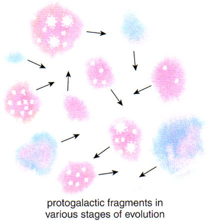
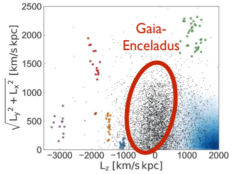


Figure 7. The Searle and Zinn model proposes that the Milky Way formed from an aggregation of several cloud fragments. This model helps to explain the observed differences in the metallicity of globular clusters in the galactic halo. Since each of the cloud fragments had independent histories, some may have evolved more than others, and so have produced objects of greater metallicity.

Angular Momenta of Halo Stars from GAIA



Appears to Be Largely Correct (Using new GAIA data) ✔

Properties of Stars in Disk Galaxy:

1. the velocity dispersion of a population of stars depends on the age. The older the population of stars, the higher the velocity dispersion.

   Heating is thought to be driven by mergers (important) and also the impact of the spiral arms and molecular clouds.

2. There is a relation between the metallicity of stars and their age

   Old stars have a lower metallicity than your stars. This suggests that the metallicity of the gas (from which the stars formed) increased gradually with time.

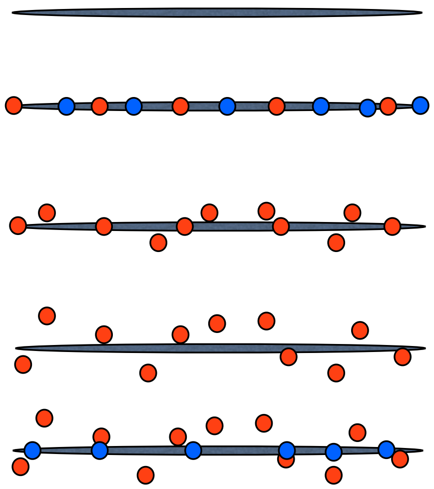
3. Abundance ratio of elements are also a function of the metallicity.

   This can be due to the fact that different enrichment mechanisms (i.e., supernovae) produce metals in different ratios.

SN II (fast time scale) → SN Ia (slow time scale)

produce alpha elements — time — produce Fe

How the Scale Height of Stars in Disk Galaxies Changes with Time



cooling causes gas to settle in a very thin disk (i.e., the minimum energy configuration that preserves angular momentum)

stars form from the cool gas and begin their lives in the plane of the disk

as time goes on, due to some heating mechanism (e.g., mergers with small galaxies), stars are given small kicks out of the plane (and oscillate in and out of plane)

the scale height of the stars increases as time goes on (due to more collisions)

while old stars have large scale heights, newly formed stars have small scale heights (formed in the gas disk)

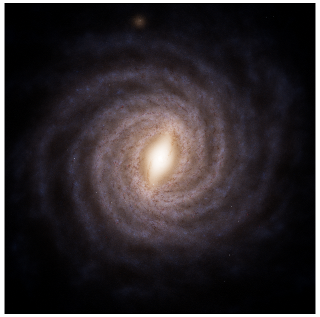
Gas Composition of Spiral Galaxies

Obviously, spiral galaxies have gas (neutral hydrogen, molecular hydrogen, ionized hydrogen). Most of the gas resides in the disk. Most of the molecular gas content is in the center of the galaxy, while most of the neutral gas content is on the outer parts of galaxies.

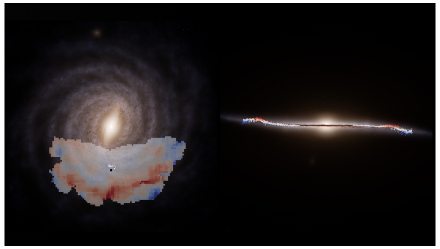
New Material

What else has been learned about the Milky Way from GAIA?

Milky Way is a Barred Spiral



The Disk of the Milky Way is Significantly Warped



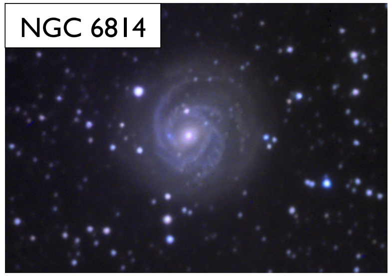
What else has been learned about the Milky Way from GAIA?

How can we understand spiral structure
in disk galaxies?

Well-defined spiral structure is present in many galaxies.

Whirlpool Galaxy
Messier 51

NGC 6814



In many cases, the spiral structure is so well organized that
the galaxies are called "grand-design" spirals

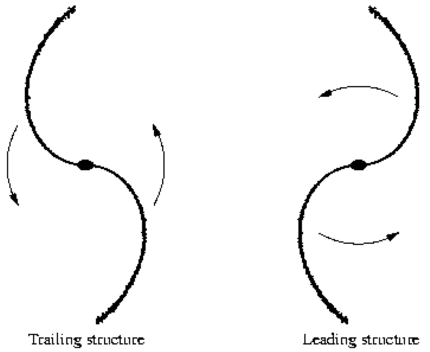
Other times the spiral structure is less well organized

Flocculent Spiral Galaxy: NGC 2841

How is such spiral structure put in place?

How does it evolve?

As disk galaxies rotate, do spiral arms lead or trail the rotation?

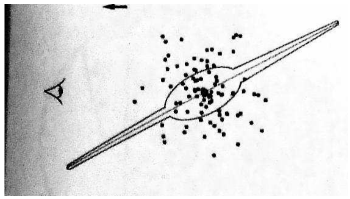


As disk galaxies rotate, do spiral arms lead or trail the rotation?

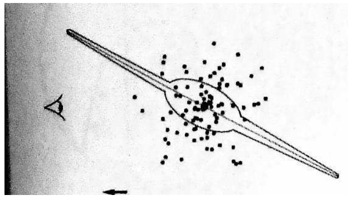
How can we settle this observationally?

Impossible to tell for face-on spiral galaxies or edge on galaxies

Use galaxies that are mildly inclined



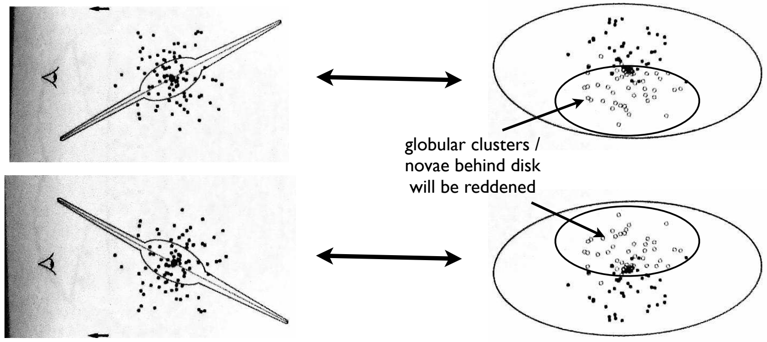
How can we distinguish the above from this?



As disk galaxies rotate, do spiral arms lead or trail the rotation?

How can we settle this observationally?

Look at globular clusters / novae in spiral galaxies

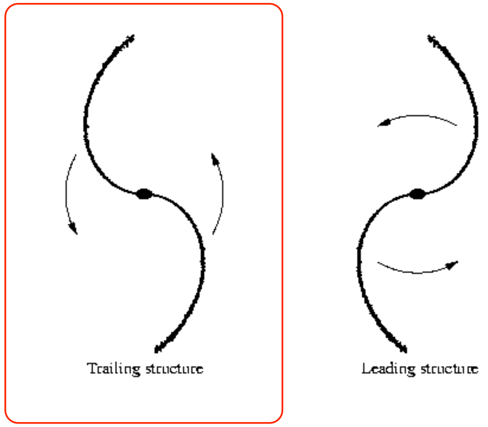


globular clusters / novae behind disk will be reddened

Globular clusters / novae behind disk will be highly reddened

As disk galaxies rotate, do spiral arms lead or trail the rotation?

Most spiral arms are found to be trailing.

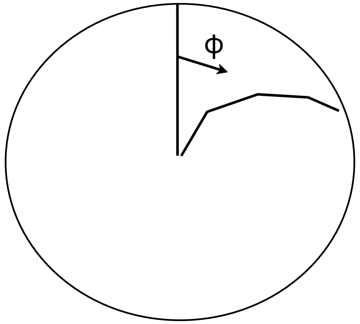


How do the arms in spiral galaxies evolve with time?

Now let us consider the time evolution of azimuthal position of each spiral arm:

$$\phi(R,t) = \phi_0 + \Omega(R)t.$$

which is also a function of radius R (because of differential rotation)



$\Omega(R)$ = angular rotation speed

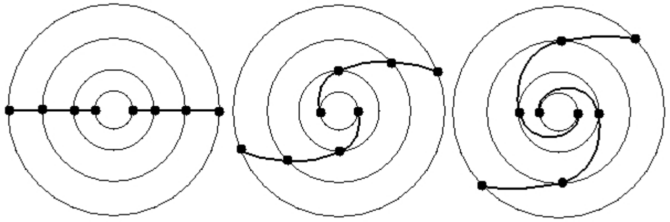
$\Omega(R) = v_{circular} / R$

~ constant

Implies angular rotation speed is smaller at large radii

Winding Problem

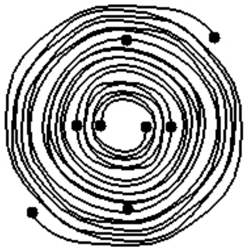
The revolution time for stars is smaller for stars on smaller radial orbits.



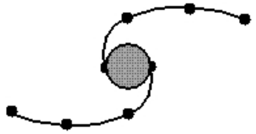
Differential rotation: stars near the center take less time to orbit the center than those farther from the center. Differential rotation can create a spiral pattern in the disk in a short time.

If the spiral arms rotate in the same way as the particles located in the spiral arms, differential rotation would cause the spiral arms to wind up.

Winding Problem



Prediction: 500 million years



Observation: 15,000 million years

Assuming that the spiral arms rotate in the same way as the particles in these arms, one would predict that the spiral arms in a galaxy would wind up very quickly.

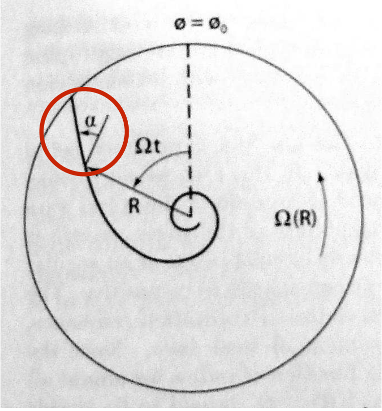
This is in contrast to what is observed!

Winding Problem: How big is the discrepancy?

Consider the pitch angle.

We define the pitch angle α for spiral arms as follows:

$$\cot \alpha = \left| R \frac{\partial \phi}{\partial R} \right| ,$$

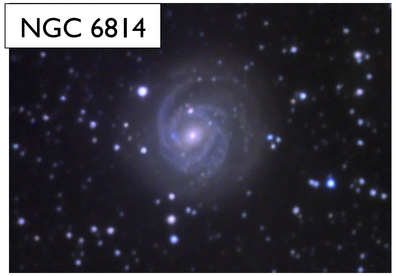


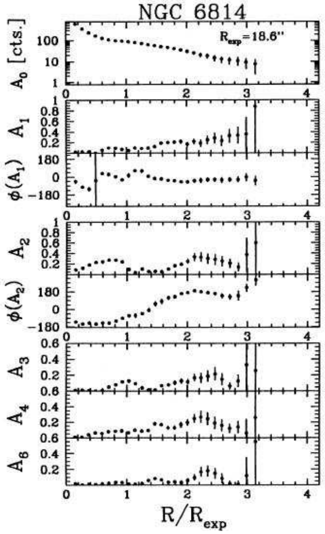
Fitting 2D light profiles of Spiral Galaxies

We can try to fit the two dimension surface brightness profile of spiral galaxies with the function:

$$\frac{I(R,\phi)}{\bar{I}(R)} = 1 + \sum_{m=1}^{\infty} A_m(R) \cos m[\phi - \phi_m(R)]$$

NGC 6814



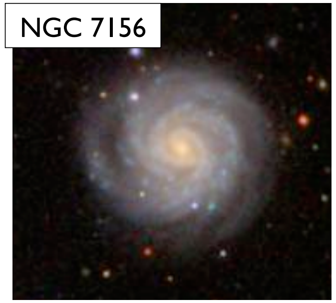


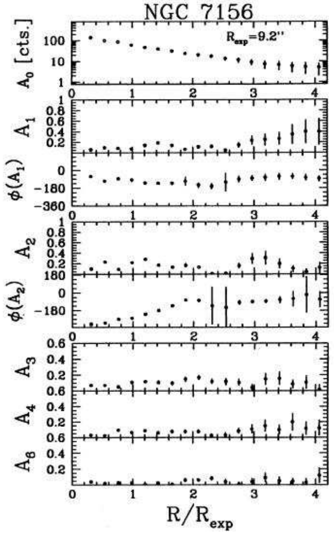
Fitting 2D light profiles of Spiral Galaxies

We can try to fit the two dimension surface brightness profile of spiral galaxies with the function:

$$\frac{I(R,\phi)}{\bar{I}(R)} = 1 + \sum_{m=1}^{\infty} A_m(R) \cos m[\phi - \phi_m(R)]$$

NGC 7156





What range of pitch angles are observed?

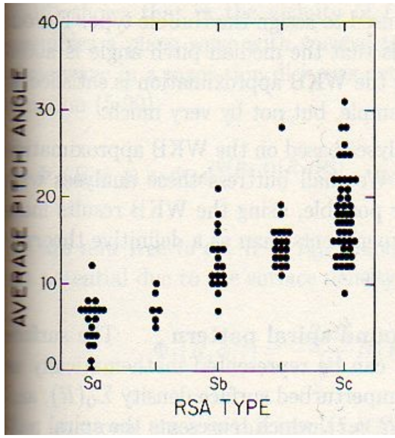


Figure 6-12. Measured pitch angle as a function of Hubble type for 113 galaxies (Kennicutt 1981). Reprinted by permission of *The Astronomical Journal.*

How do we predict the pitch angle will change?

$$\cot\alpha = \left|R\frac{\partial\phi}{\partial R}\right|, \quad \longrightarrow \quad \cot\alpha = Rt\left|\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right|$$

$$\phi(R,t) = \phi_0 + \Omega(R)t.$$

Note $\Omega(R) = v_{circular} / R$

$\longrightarrow$ $\cot \alpha = Rt\,(v_c / R^2) = v_c t/R$

For galaxies with a flat rotational curve $v_c = R\Omega = 200$ km/s, R = 5 kpc, and t = 10 Gyr, then $\alpha \sim 0.15$ degrees (much smaller than observed)

Observed pitch angles of ~10-20 degrees differs dramatically from expectation of 0.15 degrees from this simple baseline model.

How can we solve the winding problem?

Density Wave Theory

Lin & Shu (1964-1966)

The spiral arms in disk galaxies are not fixed structures that rotate around the center of disk galaxies, but rather density waves.

These density waves can move at a different speed than the stars within the galaxy itself.

The speed at which the spiral density waves propagate around the disk of a spiral galaxy is called the pattern speed Ω_p.

We will investigate this in more detail, but first let us look at epicyclic motion by stars in galaxies!

REVIEW POINT from Bachelor Course — The dynamical time scale of galaxies is much shorter than the age of the universe - implying that galaxies are largely in a state of equilibrium.

Assume for galaxy:

radius ~ 5 kpc / mass ~ 3×10^{10} solar masses

$v^2 = \phi(R) = GM/R$ (virial relation)

$v = (GM/R)^{1/2}$

1 pc ~ 3.09×10^{18} cm
1 solar mass ~ 2×10^{33} g
G ~ 6.67×10^{-8} cm^3 g^{-1} s^{-2}

=> velocities ~ 164 km/s,

=> t_{dyn} = 2 (radius) / velocity
t_{dyn} = 2 (5 kpc) / (164 km/s) ~ 6×10^7 years

$t_{dyn} \sim 6 \times 10^7$ years $<< t_{univ} = 1.3 \times 10^{10}$ years

---

How can we solve the winding problem?

Density Wave Theory

Lin & Shu (1964-1966)

The spiral arms in disk galaxies are not fixed structures that rotate around the center of disk galaxies, but rather density waves.

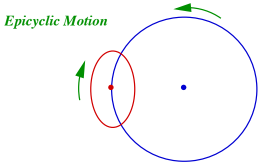
These density waves can move at a different speed than the stars within the galaxy itself.

The speed at which the spiral density waves propagate around the disk of a spiral galaxy is called the pattern speed Ω_p.

We will investigate this in more detail, but first let us look at epicyclic motion by stars in galaxies!

Epicyclic orbits

Stars that rotate around the center of disk galaxies are on epicyclic orbits:



This may not seem intuitive to you, but it is actually expected and you encountered this concept already in your study of the rotation of planets around the sun in the solar system.

Epicyclic orbits

Let us analyze the orbit of a star in some axisymmetric potential $\Phi(R)$

Assume that the star has angular momentum L_z

The energy of a star in this potential is as follows:

$$E = \frac{1}{2}\left[\dot{R}^2 + (R\dot{\theta})^2 + \dot{z}^2\right] + \Phi$$

where

$$\Phi_{\text{eff}}(R,z) = \Phi(R,z) + \frac{L_z^2}{2R^2}$$

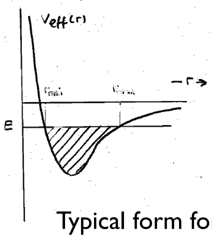
Gravitational Potential

Centrifugal barrier

Epicyclic orbits

How does $\Phi_{\text{eff}}(R)$ behave?

$$\Phi_{\text{eff}}(R,z) = \Phi(R,z) + \frac{L_z^2}{2R^2}$$



Typical form for $\Phi_{\text{eff}}(R)$

Different Cases:

Point Mass: $\Phi(R) \sim 1/R$

Isothermal Sphere: $\Phi(R) \sim \log R$

Homogeneous Density $\Phi(R) \sim R^2$

What happens to $\Phi_{\text{eff}}(R)$ at large and small radii?

As $R \to 0$, $L_z^2/2R^2$ centrifugal term always dominates.

As $R \to \infty$, $\Phi(R)$ term dominates.

$\Phi_{\text{eff}}(R)$ has a minimum at some radius R_g. Stars orbiting around a galaxy at that radius will be on a circular orbit.

Epicyclic orbits

Expand the potential $\Phi_{\text{eff}}(R)$ about the radial position R_g and the vertical position z=0 as a Taylor series:

$$\Phi_{\text{eff}} = \Phi_{\text{eff}}(R_g,0) + \frac{1}{2}\left(\frac{\partial^2\Phi_{\text{eff}}}{\partial R^2}\right)_{(R_g,0)} x^2 + \frac{1}{2}\left(\frac{\partial^2\Phi_{\text{eff}}}{\partial z^2}\right)_{(R_g,0)} z^2 + \mathrm{O}(xz^2). \quad (3.76)$$

where $x = R - R_g$.

The first order terms in this expansion $d\Phi_{\text{eff}}(R)/dx$, $d\Phi_{\text{eff}}(R)/dz$ and the second order term $d^2\Phi_{\text{eff}}(R)/dxdz$ are zero given that we are expanding the potential about a local minimum.

Represent the second derivatives of $\Phi_{\text{eff}}(R)$ with respect to R and z as κ and ν:

$$\kappa^2(R_g) \equiv \left(\frac{\partial^2\Phi_{\text{eff}}}{\partial R^2}\right)_{(R_g,0)} \quad ; \quad \nu^2(R_g) \equiv \left(\frac{\partial^2\Phi_{\text{eff}}}{\partial z^2}\right)_{(R_g,0)}$$

Epicyclic orbits

Then the time evolution of x and z are as follows:

$$\ddot{x} = -\kappa^2 x,$$

$$\ddot{z} = -\nu^2 z.$$

Since $d\Phi_{eff}/dR = 0$ at $R = R_g$

$$\frac{\partial \Phi_{\mathrm{eff}}}{\partial R} = \frac{\partial \Phi}{\partial R} - \frac{L_z^2}{R^3} = 0$$

and since $\Phi_{eff} = \Phi + L_z^2 / 2R^2$, we can also rewrite κ as

$$\kappa^2(R_g) = \left(\frac{\partial^2 \Phi}{\partial R^2}\right)_{(R_g,0)} + \frac{3L_z^2}{R_g^4} = \left(\frac{\partial^2 \Phi}{\partial R^2}\right)_{(R_g,0)} + \frac{3}{R_g}\left(\frac{\partial \Phi}{\partial R}\right)_{(R_g,0)},$$

Epicyclic orbits

Since we can write the orbital frequency Ω(R) as follows:

$$\Omega^2(R) = \frac{1}{R}\left(\frac{\partial \Phi}{\partial R}\right)_{(R,0)} = \frac{L_z^2}{R^4},$$

We then rewrite κ as follows:

$$\kappa^2(R_g) = \left(R\frac{\mathrm{d}\Omega^2}{\mathrm{d}R} + 4\Omega^2\right)_{R_g}$$

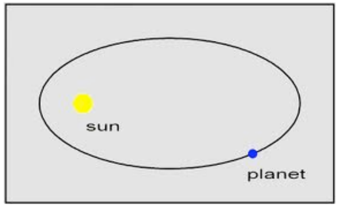
For a point mass ($\Omega \propto R^{-3/2}$),	κ = Ω
For an isothermal sphere ($\Omega \propto R^{-1}$),	$\kappa = \Omega\ (2)^{1/2}$
For solid body rotation (Ω = constant),	κ = 2Ω

In general, Ω < κ < 2 Ω

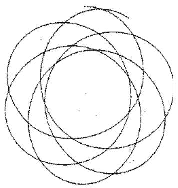
Therefore, a star can only undergo 2 revolutions in its epicyclic orbit in the time it finishes an entire orbit around the center of the galaxy.

Epicyclic orbits

For the case of a point mass ($\Omega \propto R^{-3/2}$), e.g., solar system, the epicyclical time perfectly matches the rotation time around the central body so that orbits close on each other.



In general, this is not true, however. Orbits regress and one finds a planar rosette.



Epicyclic orbits

Using the measured values for κ and Ω at the radial position of the sun in our galaxy is as follows:

κ = 1.3 Ω

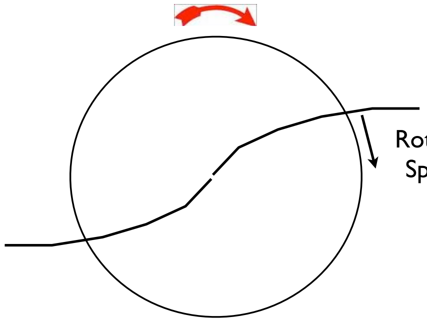
similar to the case for an isothermal sphere...

Period for orbit around galaxy = 2π/Ω

Period for epicyclic orbit = 2π/κ

Which resonances drive spiral density wave growth?

Now let us now consider a possible spiral density wave in the disk of a galaxy:

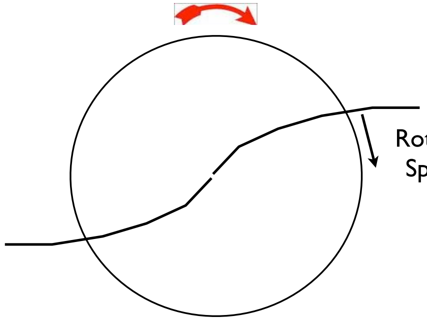


Rotational Frequency of Spiral Density Wave = Ω_p

In these illustrations, let's adopt the most common type of "grand design" spiral galaxy where we just have 2 arms (rotational symmetry = 180 degrees)

Which resonances drive spiral density wave growth?

When might we expect growth of a spiral density wave?



Rotational Frequency of Spiral Density Wave = Ω_p

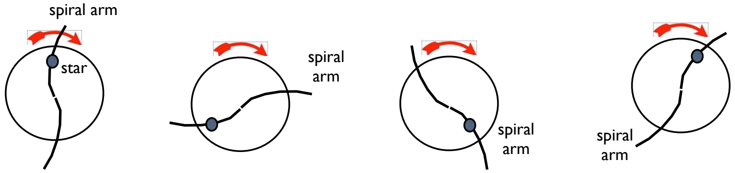
We might expect such if a star completes one period of epicyclic motion every time it encounters the spiral density wave in its orbit around the galaxy.

Which resonances drive spiral density wave growth?

Let us consider a few examples of the orbit of stars that would finish a complete epicyclic orbit in the spiral density wave itself:

Example #1: The star is moving at the same speed as the spiral density in orbiting around the center of a galaxy.

Let's consider snapshots in time where the star completes an entire epicyclic orbit. Typically a star must complete 70% of a revolution around a galaxy before this happens.



one epicyclic orbit
time = $2\pi/\kappa$
spiral pattern ~ 250 deg
star orbitted ~ 250 deg

two epicyclic orbits
time = $4\pi/\kappa$
spiral pattern ~ 500 deg
star orbitted ~ 500 deg

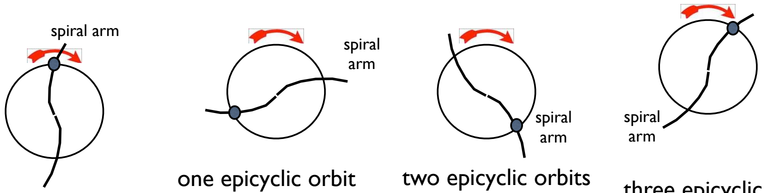
three epicyclic orbits
time = $6\pi/\kappa$
spiral pattern ~ 750 deg
star orbitted ~ 750 deg

Which resonances drive spiral density wave growth?

Let us consider a few examples of the orbit of stars that would finish a complete epicyclic orbit in the spiral density wave itself:

Example #2: The star is traveling much faster than the speed of the spiral density wave.

Let's consider snapshots in time where the star completes an entire epicyclic orbit. In this case, the star again completes 70% of an orbit, but the spiral arm orbits 0.2 times



time = 0

one epicyclic orbit
time = $2\pi/\kappa$
spiral pattern ~ 70 deg
star orbitted ~ 250 deg

two epicyclic orbits
time = $4\pi/\kappa$
spiral pattern ~ 140 deg
star orbitted ~ 500 deg

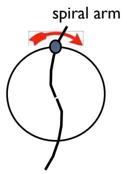
three epicyclic orbits
time = $6\pi/\kappa$
spiral pattern ~ 210 deg
star orbitted ~ 750 deg

Which resonances drive spiral density wave growth?

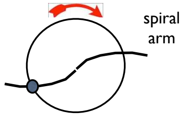
Let us consider a few examples of the orbit of stars that would finish a complete epicyclic orbit in the spiral density wave itself:

Example #3: The star is traveling much slower than the speed of the spiral density wave.

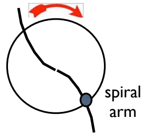
Let's consider snapshots in time where the star completes an entire epicyclic orbit. In this case, the star again completes 70% of an orbit, but the spiral arm orbits 1.2 times (instead of just 0.2 times)



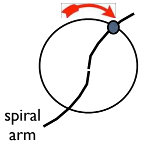
time = 0



one epicyclic orbit
time = 2π/κ
spiral pattern ~ 430 deg
star orbitted ~ 250 deg



two epicyclic orbits
time = 4π/κ
spiral pattern ~ 860 deg
star orbitted ~ 500 deg

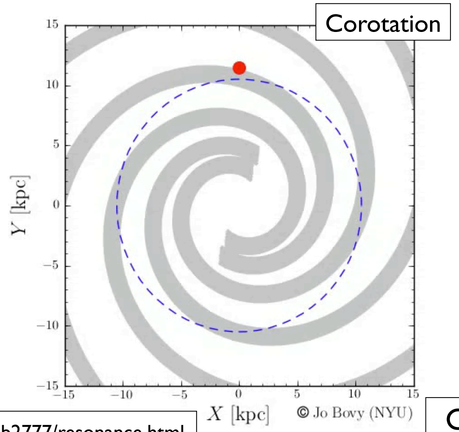


three epicyclic orbits
time = 6π/κ
spiral pattern ~ 1290 deg
star orbitted ~ 750 deg

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:

Corotation



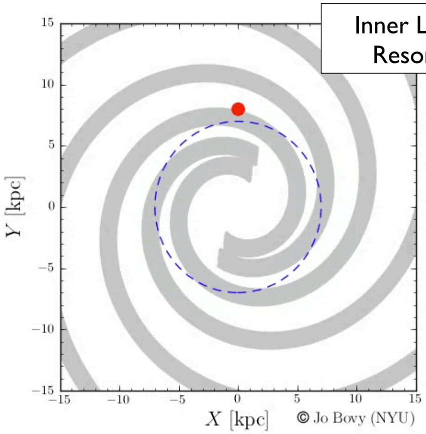
http://cosmo.nyu.edu/~jb2777/resonance.html

Credit: Jo Bovy

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:

Inner Lindblad Resonance

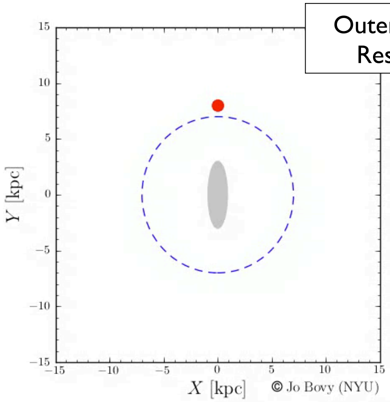


Credit: Jo Bovy

Which resonances drive spiral density wave growth?

Let us look at a few movies that illustrate these concepts rather directly:

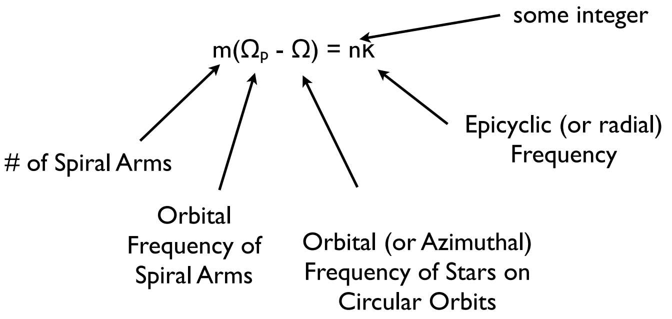
Outer Lindblad Resonance



Credit: Jo Bovy

Which resonances drive spiral density wave growth?

To ensure that some arbitrary star can complete an epicyclic orbit in the same time it takes to move from one region in the spiral arm to another, the following condition must be satisfied:



some integer

$m(\Omega_p - \Omega) = n\kappa$

Epicyclic (or radial) Frequency

# of Spiral Arms

Orbital Frequency of Spiral Arms

Orbital (or Azimuthal) Frequency of Stars on Circular Orbits

The only integers n for this relation that are interesting are 0, +1, -1.

Which resonances drive spiral density wave growth?

This results in a number of well known resonances:

Inner Lindblad resonance: $\Omega_p = \Omega - \kappa/m$

Most relevant cases: $\Omega_p = \Omega - \kappa/2$

Outer Lindblad resonance: $\Omega_p = \Omega + \kappa/m$ $\quad \Omega_p = \Omega + \kappa/2$

Corotational radius: $\Omega_p = \Omega$ $\quad \Omega_p = \Omega$

In most cases, the only relevant case is that of two spiral arms, i.e., m = 2

And note that physics behind bar-like features in spiral galaxies is similar

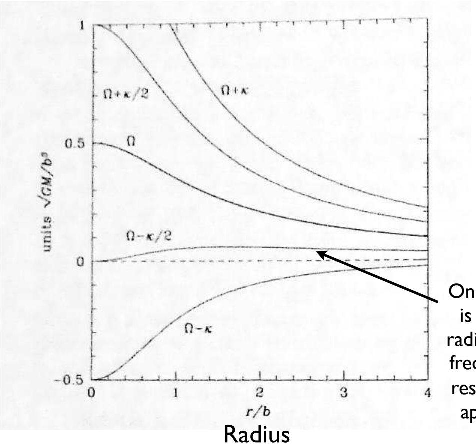
At what orbital frequencies for the spiral arms are these resonances relevant?

Compute $\Omega_p = \Omega - \kappa/2, \Omega, \Omega + \kappa/2$

How does the resonant frequencies vary by radius?

Ω_p

Orbital frequency for spiral arms at which these resonances become important



Radius

isochrone potential

One thing you should note is the extended range in radius where the rotational frequency for one of these resonances, i.e., $\Omega - \kappa/2$ is approximately constant.

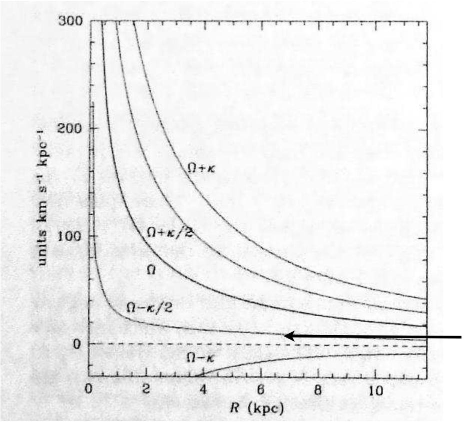
At what orbital frequencies for the spiral arms are these resonances relevant?

Compute $\Omega_p = \Omega - \kappa/2, \Omega, \Omega + \kappa/2$

How does the resonant frequencies vary by radius?

Ω_p

Rotational frequency for spiral arms at which these resonances become important



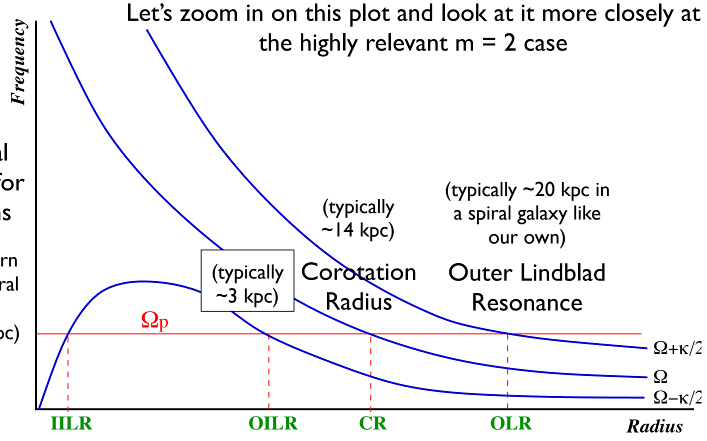
Radius

model 1 for our Galaxy from BT 2.7

Again $\Omega - \kappa/2$ is almost independent of radius!

What are the typical physical radii where these resonances apply?

Let's zoom in on this plot and look at it more closely at the highly relevant m = 2 case



Rotational frequency for spiral arms

(typical pattern speed for spiral arms Ω_p ~15 km/s / kpc)

(typically ~3 kpc)

(typically ~14 kpc)

Corotation Radius

(typically ~20 kpc in a spiral galaxy like our own)

Outer Lindblad Resonance

Inner Lindblad Resonances

(Can be more than one)

Additional Properties of Spiral Density Waves

Spiral density waves can only survive and grow between the inner Lindblad resonance and outer Lindblad resonance.

These waves cannot pass through the inner Lindblad resonance (they are damped inside this radius)

What conditions are important for gas in spiral galaxies for material to collapse gravitationally, feeding spiral density waves?

Jeans Instability

Consider homogeneous fluid that is in equilibrium, with density ρ, pressure p, with no internal motion.

Assume that the fluid is spherically symmetric. We shall consider the fluid is the matter inside some sphere with radius r.

Suppose that we compress the fluid element so that it now has a radius $r(1-\alpha/3)$ where α is much smaller than 1.

What will be the force acting on the surface of the sphere after this small compression?

To first order, the density perturbation is $\rho_1 = \alpha\rho$

To first order, the pressure perturbation is $p_1 = (dp/d\rho)\alpha\rho = \alpha\rho v_s^2$ where v_s is the sound speed.

Jeans Instability

The pressure force per unit mass is

$F_p = \nabla p / \rho$

The additional pressure force per unit mass is

$dF_p = \nabla(\alpha \rho v_s^2)/\rho$

$dF_p = \alpha v_s^2/r$

The gravitational force per unit mass is

$F_g = \rho GM/r^2/\rho$

The additional gravitational force per unit mass is

$dF_g = \alpha GM/r^2$

If the additional force on the surface of the sphere from the pressure of the fluid, i.e., dF_p, is greater than the additional force on the surface of the sphere from gravity, i.e., dF_g, then the pressure force resists the radial perturbation.

However, if the additional force from the gas pressure dF_p is less than the additional force from gravity dF_g, then the force of gravity will only accelerate the collapse.

Jeans Instability

In summary, for $dF_p > dF_g$ ==> fluid pressure resists gravitational collapse

However, for $dF_p < dFg$ ==> system undergoes gravitational collapse

$dF_p = dFg$ represents a specific physical scale.

$dF_p = \alpha v_s^2/r = \alpha GM/r^2 = dF_g$

Using $M = \rho\ 4/3\ \pi\ r^3$,

$v_s^2/r = G\rho\ 4/3\ \pi\ r$

We find:

$r_J \sim (3\ v_s^2 / (4\pi\ G\rho))^{1/2}$

Perturbations on a larger scale than the Jeans scale r_J will result in a gravitational collapse.

Toomre Instability Criterion

In spiral/disk galaxies, the stability criterion is more complex, due to the shearing type motion

Similarly for a disk galaxies, there is also a stability criterion. Any perturbations with wavelength larger than λ_{crit} are unstable.

$$\lambda_{crit} = 2\pi/k_{crit} = 4\pi^2 G\Sigma/\kappa^2$$

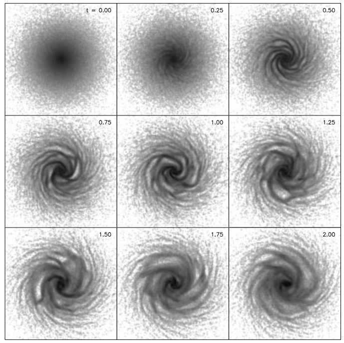
Random motions by stars tend to stabilize disks. Disks with stellar motions are unstable if

$$Q = \frac{\sigma_R \kappa}{3.36\ G\ \Sigma} < 1$$

This is the Toomre criterion. It assume that the disk is thin and unstable modes are much smaller than the size of the sky. Again, significant dispersion in the velocities of stars will stabilize the disk.

What happens if Q < 1?

This is an illustration of how the disk becomes clumpy because of a Toomre-like instability.



Credit: Barnes

What is Q for our galaxy?

For our disk where κ = 37±3 km/s/kpc, σ_R = 38±2 km/s, Σ = 36±5 $M_\odot$ / pc², Q* = 2.7±0.4.

Including interstellar gas 13 $M_\odot$ / pc², Q = 1.5.

What impact does the measured Q value have on the growth of spiral density waves?

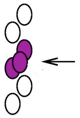
For lower values of the Q parameter — as high as Q <~ 2 — gravitational instabilities can feed the growth of spiral structure.

This is particularly relevant at the coronation radius, given the similar speed of the material (stars, gas) and spiral density waves.

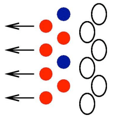
Now let’s return to spiral density waves in spiral galaxies

What astrophysical processes drive these spiral density waves as they rotate around a spiral disk?

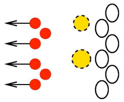
When the gas in the spiral density wave is compressed, it results in the formation of stars (due to the high gas densities induced by these compression waves)



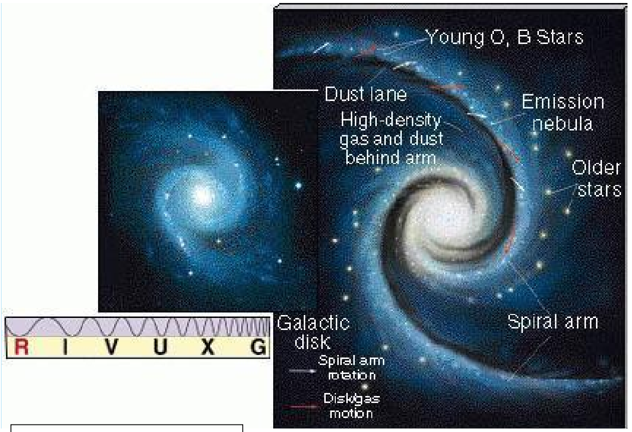
After the stars form, they will approximately move at the circular velocity of the spiral galaxy -- which is often faster than the pattern speed of the spiral arm



The high mass stars formed in the spiral density compression waves die (SNe explosions or otherwise) shortly after leaving the spiral arm compression wave, but the lower mass (redder) stars continue to rotate around the disk.



What astrophysical processes drive these spiral density waves as they rotate around a spiral disk?



Credit: van der Kruit

at inner radii in spiral galaxies, stars travel faster than the spiral density wave.

gas and dust lanes (formed from the metal output of the supernovae explosions) indicate the position of the high density spiral density wave

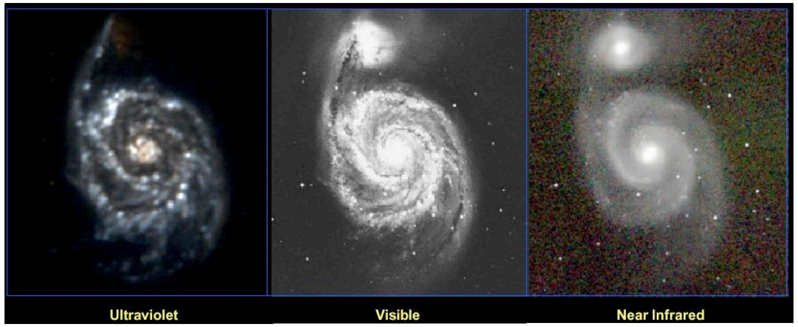
hot (massive) stars do not travel much beyond the spiral density wave in which they are formed

it is only the old (low mass) stars that can travel far enough to get ahead of the spiral wave

the hot stars are somewhat ahead of the gas/dust lane, since there is some time lag between when gravitational collapse begins and when the stars finally form (i.e., are on the main sequence)

Where is the spiral structure most evident?

Because of the hot blue stars being predominantly formed in the spiral density waves in disk galaxies and living for a very short time, we would expect the spiral structure to be much clearer at bluer or ultraviolet wavelengths where we just see the hot blue stars.

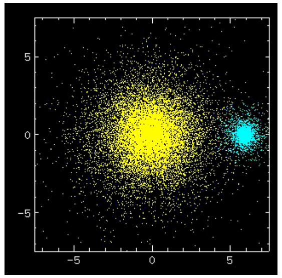


Besides high gas densities, what else can drive spiral density waves?

What else can drive spiral density waves in disk galaxies?

Asymmetries in the dark and/or halo (galaxy formation processes)

Or from interactions with a nearby neighbor (as in the case of spiral galaxy M51)



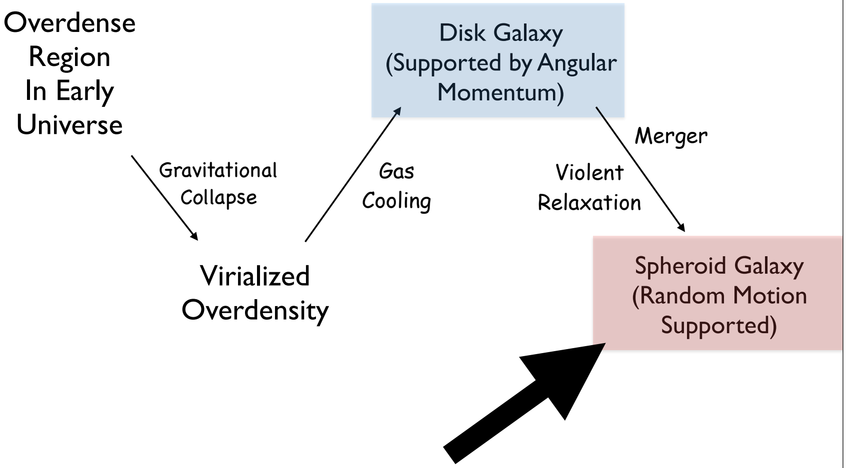
Next topic is elliptical galaxies...

Elliptical galaxies consist of large numbers of stars on diverse orbits.

While spiral galaxies are rotation supported, elliptical galaxies are supported by the random motions of stars they contain

Their behavior can largely be described using collisionless dynamics.

Galaxy Formation: Major Steps



Overdense Region In Early Universe

Gravitational Collapse

Virialized Overdensity

Gas Cooling

Disk Galaxy (Supported by Angular Momentum)

Merger

Violent Relaxation

Spheroid Galaxy (Random Motion Supported)

We will therefore be reviewing some concepts from collisionless dynamics from the Leiden Bachelor course

We will discuss how to model the dynamics of $>10^{10}$ stars that form a self-gravitating system.

(this will require ~1 to ~1.5 lectures)

REVIEW Point from Bachelor Course: Collisions between individual stars are a non-issue in modeling galaxies - given the typical density, velocity, and cross section of stars. The main challenge is modeling their collective gravitational potential.

Let's quantify this by estimating the time scale for collisions:

Let's assume:

radius (galaxy) ~ 5 kpc
# of stars (galaxy) ~ 1×10^{10} stars
star diameter ~ 1.4×10^6 km
all stars have a mass equal to the sun

In 6×10^7 years, a typical star crosses pathes with N stars:

N = (Distance Covered)(# stars / size^3)

Collisions per crossing time = $N \pi r^2$

$= (10 \text{ kpc})(1 \times 10^{10} \text{ stars}/(5 \text{ kpc})^3)\pi(1.4 \times 10^6 \text{ km})^2 = 5 \times 10^{-12}$ per crossing time

$= 5 \times 10^{-12} / 6 \times 10^7 \sim 8 \times 10^{-20}$ / year

Hence stars collide with each other very rarely!

REVIEW Point from Bachelor Course: In contrast to situations with fluids, the force on individual stars does not come primarily from its immediate neighbors, but from stars at all distances in the galaxy

Force from region of galaxy on a star

$= Gm(\rho r^2 dr d\Omega)/r^2$
$= Gm\rho dr d\Omega$

Force is independent of r! This means that a given star in a galaxy feels essentially the same force from stars at 1 kpc and stars at 4 kpc.

This is very different from hydrodynamics where short range pressure forces dominate!

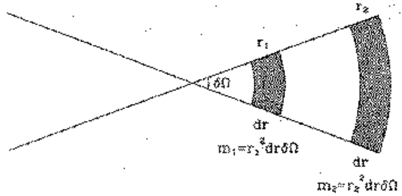


Figure 4-1. If the density of stars were everywhere the same, the stars in each of the shaded segments would make equal contributions to the net force on a star at the cone's apex. Thus the acceleration of a star at the apex is determined by the large-scale gradient in the density of stars within the galaxy.

BT4: pages 187

REVIEW Point from Bachelor Course: The time scale for the relaxation time of individual stars to collisions will other stars is very high, i.e., 10^{16} years, and thus can be ignored in modeling the dynamics of stars in a galaxy. Consequently, it is possible to model the potential and phase space as smoothly varying.

First let's look at velocity perturbation created by one star passing by another.

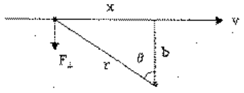


Figure 4-2. A field star approaches the test star at speed v and impact parameter b. We estimate the resulting impulse to the test star by approximating the field star's trajectory as a straight line.

BT4: pages 187-190

Relevant variables:

m = mass of "stationary" star
b = impact parameter
v = velocity of moving star
$v_\perp$ = velocity perturbation

We begin by showing that the velocity perturbation is the following:

$$v_\perp = 2Gm / bv$$

In this problem, we make the assumption that the impact parameter undergoes no meaningful change during the encounter.

However, this will not be true if the velocity kick is on order of the original velocity of the star.

So, we can show that our derivation breaks down if $b_{min} = Gm / v^2$

But, each star is perturbed by not just one star, but many stars along the line of sight. Each perturbation is in a random direction.

While on average the perturbations cancel each other out, the many perturbations introduces a spread in the overall distribution

---

In calculating the dispersion in the velocity distribution caused by perturbations by other stars, we add velocity kicks in quadrature

Let's first considering the effect from an interaction with some random star in a star and then let's consider all N stars in a galaxy.

Fraction of Stars with impact parameter b = $2\pi b db/\pi R^2$

Radius of galaxy

During derivation, we make use of the virial theorem

Total mass $v^2 = GM/R = (GNm)/R$

N = # of stars per galaxy

Here is the result, i.e., dispersion in velocity kicks divided by typical velocity in system is just a function of the number of particles in a dynamical system....

$$\langle v_\perp^2 \rangle / v^2 = 8 (\ln N) / N \qquad \text{(per crossing time)}$$

Number of times star must cross galaxies such that the dispersion in its velocity kick equals the typical velocity gives us the relaxation time.

$$n_{cross,relax} \langle v_\perp^2 \rangle = v^2$$

$$t_{relax} = t_{cross}\, n_{cross,relax} = t_{cross}(N / (8 \ln N))$$

Example of Time Scales

System	Mass $M_\odot$	Radius kpc	Velocity $km\,s^{-1}$	N	t_{cross} yr	t_{relax} yr
Galaxy	10^{10}	10	100	10^{10}	10^8	$> 10^{15}$
DM Halo	10^{12}	200	200	$> 10^{50}$	10^9	$> 10^{60}$
Cluster	10^{14}	1000	1000	10^3	10^9	$\sim 10^{10}$
Globular	10^4	0.01	2	10^4	5×10^6	5×10^8

- **Dark Matter Haloes and Galaxies are collisionless**
- **Collisions may or may not be important in clusters of galaxies**
- **Relaxation is expected to have occured in (some) globular clusters**

Credit: van den Bosch

MOTIVATION:

We have this very complicated situation:
how can we model the orbits of all the stars in a galaxy simultaneously

Seems difficult!

Before even thinking about how to solve it, how shall we even try to model it?

Ignore the fact that stars are discrete sources and assume that we treat them as a fluid with each star individually having an infinitesimal mass.

We will use a 7-dimension distribution function to describe where they are in 6-dimensional phase space at some time t.

REVIEW point from Bachelor course: The time evolution of the distribution function is defined by the distribution function at that time, spatial derivatives, and the gradients of the potential (Vlasov-Equation). This follows directly from a conservation equation on the stars.

At any time t, one can describe the collective positions and velocities for stars in a dynamical system by a distribution function f(**x**,**v**,t)

To describe the time evolution, we define a six dimensional vector
w = (**x**,**v**)

The flow of stars in the six dimensional phase can be described as d**w**/dt = (**v**,-**∇**Φ)

The flow d**w**/dt conserves stars...

BT4.1:page 190-195

At any time t, one can describe the collective positions and velocities for stars in a dynamical system by a distribution function f(**x**,**v**,t)

$$\int_S (f\dot{\vec{w}})\cdot d^2S = -\int_V \frac{\partial f}{\partial t}$$

"stars moving in and out of volume through some surface"

"change in the total number of stars in volume"

equal to each other from divergence theorem from vector calculus

$$\int_V \vec{\nabla}\cdot(f\dot{\vec{w}}) = -\int_V \frac{\partial f}{\partial t}$$

"divergence of flow inside volume"

"change in the total number of stars in volume"

$$\frac{\partial f}{\partial t} + \vec{\nabla}\cdot(f\dot{\vec{w}}) = 0$$

At any time t, one can describe the collective positions and velocities for stars in a dynamical system by a distribution function f(**x**,**v**,t)

$$\frac{\partial f}{\partial t} + \vec{\nabla}\cdot(f\dot{\vec{w}}) = 0$$

Collisionless Boltzmann Equation
"Vlasov Equation"

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

or

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f - \vec{\nabla}\Phi.\frac{\partial f}{\partial \vec{v}} = 0$$

also can write as $\frac{df}{dt} = 0$

REVIEW Point from Bachelor Course: Integrals of motion are functions of x and v which are constant along an orbit. They are not explicit functions of time. Examples: energy, angular momentum. Most 3D densities allow for 3 integrals of motion, 2 of which are non-classical.

Integrals of motion can be a very useful concept for characterizing the orbits of stars in a galaxy.

They are useful in the case that they are isolating integrals of motion since the reduce the dimensionality of the phase space in which a star travels during its orbit.

There can be no more than 6 integrals of motion. Typically there is at least one integral of motion (energy).

What are some examples of isolating integrals of motion?

- Energy is always an integral of motion for a star in a static potential.

The energy per unit mass for a star remains constant throughout its orbit: E(**x**,**v**) = (1/2) $\mathbf{v}^2 + \Phi(\mathbf{x})$

- L_z: angular momentum in the z direction (for an axisymmetric potential)

- **L**: all three components of the angular momentum in spherically symmetric potential

Integrals of motion tend to arise from some symmetry in the system.

However, dynamical systems can also have other isolating integrals of motion outside of the classical ones (i.e., energy, angular momentum)

How can integrals of motion reduce the phase space explored by an orbit?

Consider a spherically symmetric potential:

- 1. Spherical potentials:

E, L_x, L_y, L_z are integrals of motion, but also $E, |L|$ and the direction of $\vec{L}$ (given by the unit vector $\vec{n}$, which is defined by two independent numbers). $\vec{n}$ defines the plane in which $\vec{x}$ and $\vec{v}$ must lie. Define coordinate system with z axis along $\vec{n}$

$x_3 = 0$

$v_3 = 0$

$$\vec{x} = (x_1, x_2, 0)$$

$$\vec{v} = (v_1, v_2, 0)$$

$\rightarrow$ $\vec{x}$ and $\vec{v}$ constrained to 4D region of the 6D phase space. In this 4 dimensional space, $|L|$ and E are conserved. This constrains the orbit to a 2 dimensional space. Hence the velocity is uniquely defined for a given $\vec{x}$

Energy

because of L_z conservation

$$v_r = \pm\sqrt{2(E-\Phi) - L^2/r^2}$$

$$v_\psi = \pm L/r$$

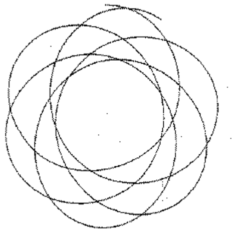


Figure 3-1. A typical orbit in a spherical potential forms a rosette.

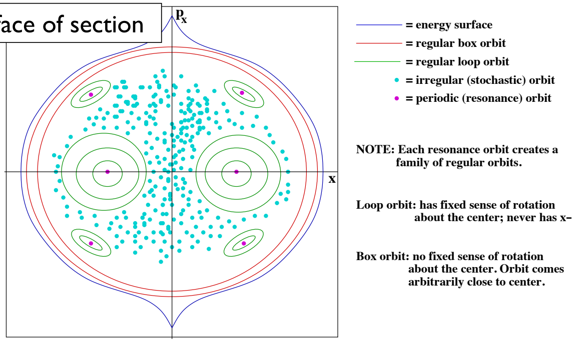
>=4 integrals of motion:
Energy
L_x
L_y
L_z

One alternate way of determining how restricted the orbital manifold of galaxies are is to construct poincare surfaces of section:

To investigate whether the orbits admit any additional (hidden) isolating integrals of motion, Poincaré introduced the surface-of-section (SOS)

Consider the intersection of $\mathcal{M}_3$ with the surface $y = 0$. Integrate the orbit, and everytime it crosses the surface $y = 0$ with $\dot{y} > 0$, record the position in the (x, p_x)-plane. After many orbital periods. the accumulated points begin to show some topology that allows one to discriminate between regular, irregular and resonance orbits.

Example surface of section



NOTE: Each resonance orbit creates a family of regular orbits.

Loop orbit: has fixed sense of rotation about the center; never has x–

Box orbit: no fixed sense of rotation about the center. Orbit comes arbitrarily close to center.

Credit: van den Bosch

Setting up equilibrium models for a collisionless system.

It is not necessarily an easy thing to do

We must set up a self-consistent system whereby each of the following steps imply the next:

(1) given density distribution ρ(r), calculate the potential Φ(r) the density distribution would imply

(2) given some potential Φ, determine the set of orbits that stars would undergo

(3) calculate the density distribution that would result from the collective orbits of all the stars in a system

The Density Distribution derived in step #3 must be the same as assumed in step #1

The relevant equations are:

$$\begin{aligned} \rho(\vec{x}) &= \int f(\vec{x},\vec{v})\,\mathrm{d}^3\vec{v} \\ \nabla^2\Phi(\vec{x}) &= 4\pi G\rho(\vec{x}) \\ \frac{\mathrm{d}f}{\mathrm{d}t} &= 0 \end{aligned}$$