

Galaxies: Structure, Dynamics, and Evolution

Galaxies: Structure, Dynamics, and Evolution

(from website)

“Galaxies are the basic building blocks of the universe, and we use them to trace the evolution of the universe. Fundamental processes such as star formation, recycling and enrichment of gas, formation of planets etc. all take place in galaxies.

The course describes the structure of the galaxies, including dark matter, stars, and gas as well as the large scale structure in which galaxies are embedded. It discusses ongoing surveys of the nearby and distant universe. A special focus will be on the evolution of galaxies. The course builds on the bachelor lecture course “Galaxies and Cosmology” (Sterrenstelsels en Kosmologie), and assumes that the material in this course is known to the student. A very brief recapitulation will be given of the most important material.”

6 EC course

Galaxies: Structure, Dynamics, and Evolution

(from website)

The approach we take in this course will be very empirical / observational.

We find these interesting physical objects called galaxies nearby with observations. In this course, we will outline their properties and try to explain them using various physical ideas.

This contrasts with the approach taken in the galaxy formation course originally taught by Joop Schaye -- where one tries to form galaxies ab initio just from theory.

Lectures

Rychard Bouwens

BW.3.44

bouwens@strw.leidenuniv.nl

Lecture Hours:

BW.0.05

Monday 9:00-10:45

Course Website:

<http://www.strw.leidenuniv.nl/~bouwens/galstrdyn/>

Textbook?

Useful Textbooks for the course will be
“Galaxy Formation and Evolution” by Houjun Mo, Frank van
den Bosch, and Simon White

“Galaxy Dynamics” by James Binney & Scott Tremaine

“Galactic Astronomy” by James Binney & Michael Merrifield

The textbook includes a useful discussion of the material, but the course will not be organized to follow the presentation in the book.

However, I will advise you as to where you can find the relevant material in the textbook.

Teaching Assistant

Wout Goesart

BW.3.2I

goesaert@strw.leidenuniv.nl

Wout will also be available by appointment to answer your questions, and he also may hold office hours

Wout is pursuing a PhD thesis, with some guidance from Mariska Kriek, so he also has considerable expertise in the subject matter of this course.

Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics

Feb 9: Disk Galaxies (I)

Feb 12: Disk Galaxies (II)

Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics

Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations

Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)

Mar 9: Elliptical Galaxies (II)

Mar 23: Elliptical Galaxies (III)

Mar 30: Dark Matter Halos

Apr 13: Large Scale Structure

Apr 20: Galaxy Stellar Populations

Apr 23: Lessons from Large Galaxy Samples at $z < 0.2$

May 4: Evolution of Galaxies with Redshift

May 11: Galaxy Evolution at $z > 1.5$ / Review for Final Exam

Layout of the Course

Practical Sessions

Feb 19: Board Work + Problem Set 1

Mar 12: Board Work + Problem Set 2

Mar 26: Problem Set 3 / Paper Presentations (4 slots)

Apr 2: Paper Presentations (7 slots)

Apr 16: Problem Set 4 / Paper Presentations (4 slots)

Apr 30: Problem Set 5 / Paper Presentations (4 slots)

May 7: Problem Set 6 / Paper Presentations (4 slots)

Exams

June 4: Written Exam

June 26: Retake Exam

How will this class be taught?

70% Powerpoint....

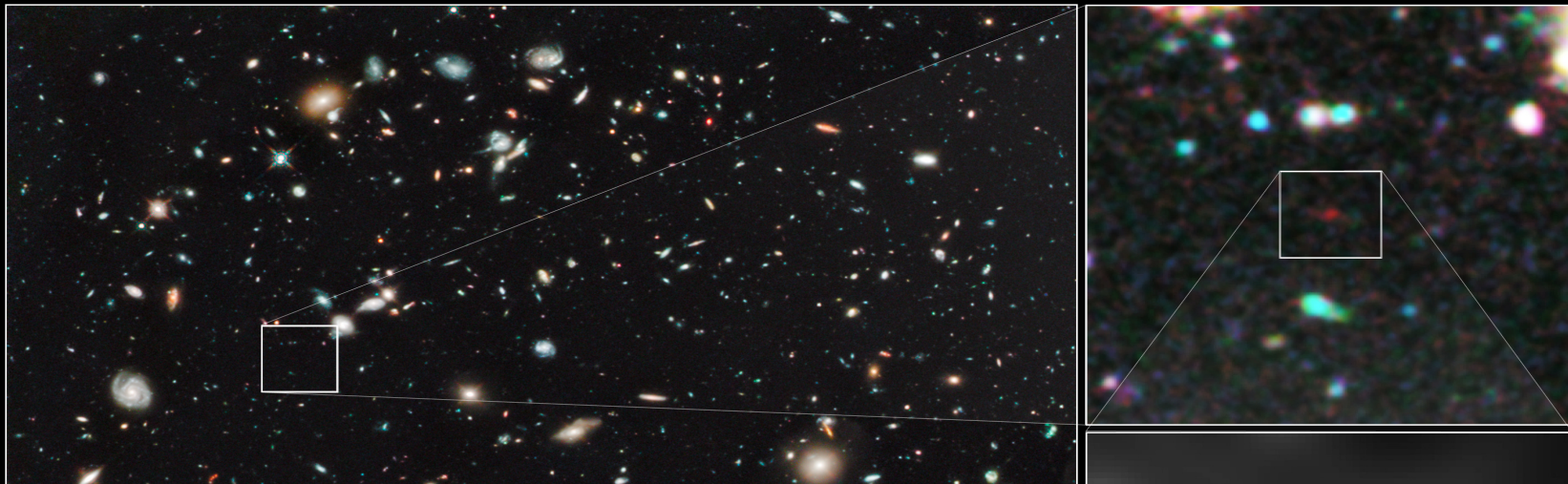
30% Derivations on the blackboard...

Who am I?

My name is Rychard Bouwens
(studied in the United States: University of
California, Berkeley & Santa Cruz)

A strong understanding of galaxies is very
important for my research, especially leveraging
the observations

Discovery of Plausible Galaxy just ~400-450 Myr after Big Bang



Candidate for the most distant galaxy ever discovered by 2011!



UDFj-39546284

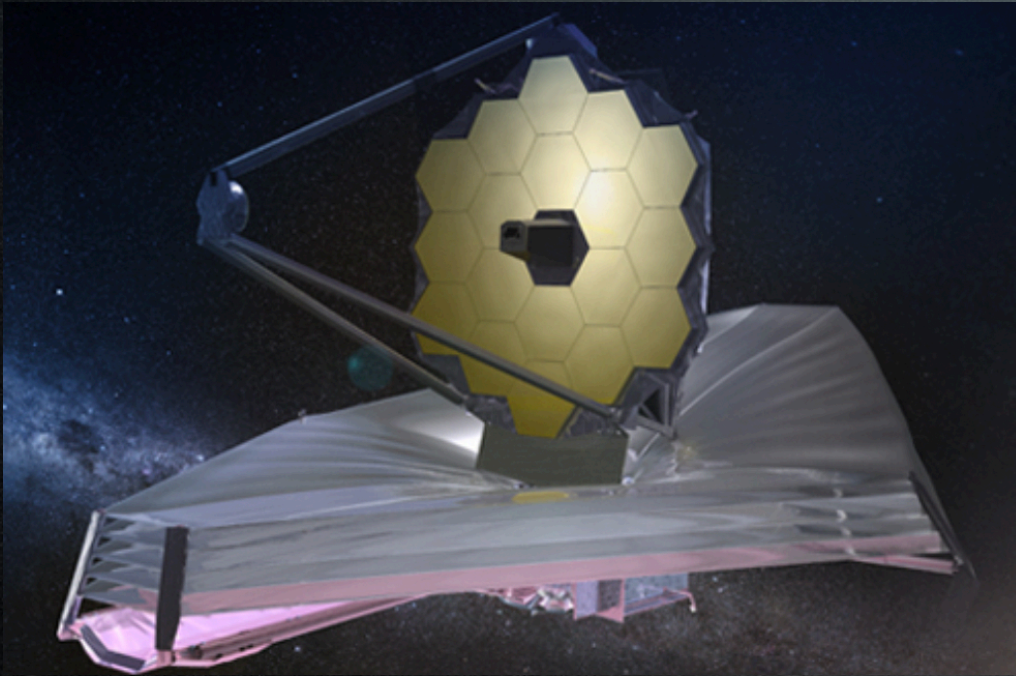
Hubble Ultra Deep Field 2009–2010
Hubble Space Telescope • WFC3/IR

NASA, ESA, G. Illingworth (University of California, Santa Cruz),
R. Bouwens (University of California, Santa Cruz and Leiden University), and the HUDF09 Team

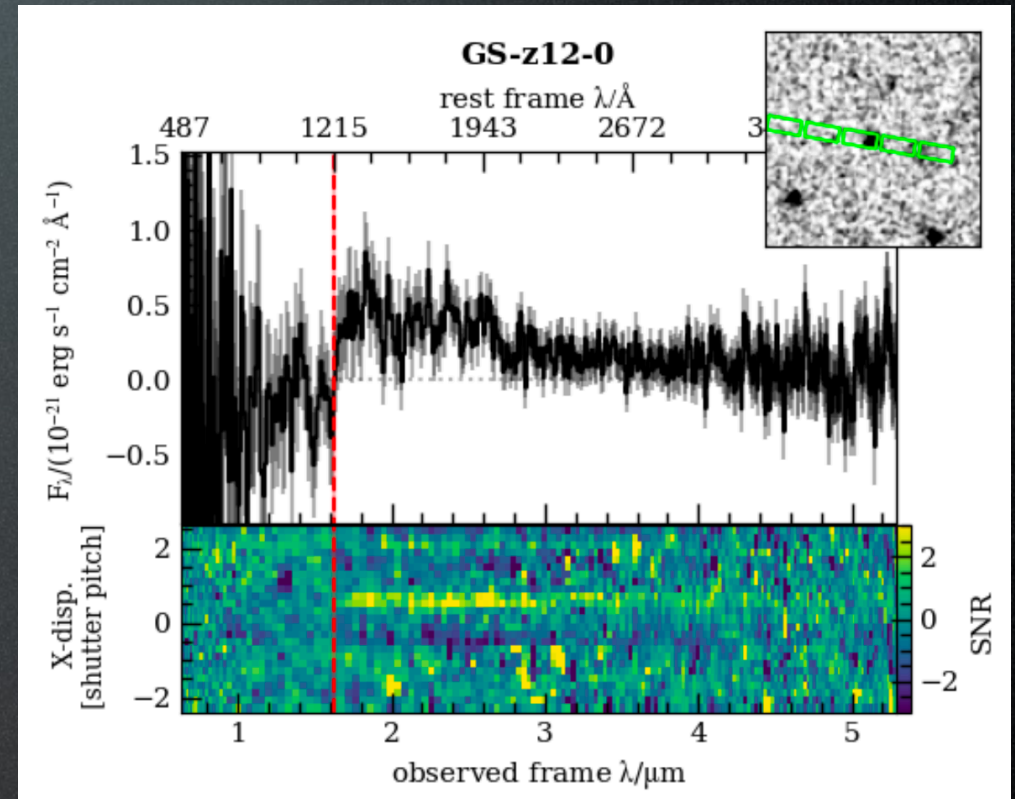
STScI-PRC11-05

What Happened with That Source?

According to a spectrum
taken by JWST



$z = 11.122$
(390 Myr after the Big Bang)



Curtis-Lake+2023

Most Galaxy Ever Seen with HST !!!

How will you be evaluated?

Final Exam (Written) — 60%

Problem Sets — 25%

Paper Presentation — 15%

Problem Sets?

You will have ~6 problem sets.

During the practical sessions, there will be an opportunity to further discuss the most challenging problems from the sets.

Teams of ~3-4 students will be assigned to present a solution to these problems in the practical sessions, with hints from the TA and me.

Will ask you to vote on which problems you want to discuss in advance.

What about the Paper Presentations?

To connect with the material from the course with a more contemporary, in-depth analysis, part of your grade will be based on a 10-minute presentation on manuscript from the literature.

I will provide a list of 40 papers from which to choose to present — which will be extensions of the material covered in this course.

Depending on the paper you choose, it will be suggested that you give a presentations towards the mid or end point of the course.

It will be first come, first serve, for papers, so by choosing early, you will have more choice regarding the paper/topic to cover.

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Practical Sessions — What will we do?

- * Derivations on the Blackboard *(by me)*
- * Solutions Presented to Hardest Homework Problems
(mostly by teams of 3-4 students chosen a week in advance)
- * Paper Presentations
(During the last 2 months of the course, there will be paper presentations given by a few students in each session)

The purpose of these presentations will be to show observational studies of galaxies has evolved over the past 10-15 years.

Your performance in presenting both papers and the solution to ~1-2 homework problems will impact your grade.

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Who are you?

Could I see a show of hands?

Physics or Astronomy?

Master's Student?

First or Second Year?

Please ask questions

This is **your** course. It is your opportunity to learn.

By asking questions, you allow me to clarify issues

It is assumed that you all are familiar with the following material from the Bachelor course here.

Background (assumed known)

Brief content of the course Galaxies and Cosmology (Bachelors)

1) Introduction

What is a galaxy ?
Classifications
Photometry, exponentials, $r^{1/4}$ profiles, luminosity function

2) Keeping a galaxy together: Gravity Potentials for spherical systems

3) Galactic Dynamics Equilibrium collisions, Virial Theorem

4) Galactic Dynamics continued Timescales Orbits

5) Collisionless Boltzmann Equation equilibrium, phase mixing derivation of distribution function

6) Velocity Moments Jeans equations comparison to observations

7) Mass distribution and dark matter Evidence for dark matter from rotation curves Solar neighborhood, Oort limit Elliptical galaxies and hot gas

Clusters of galaxies, the universe
Candidate dark matter particles

- 8) Galaxy formation
Universe expansion
Growth of galaxies by gravity
Galaxy scaling relations
- 9) Galaxy formation - forming the stars
Gas cooling and star formation
formation of disks
dynamical friction and mergers
tidal tails in mergers
- 10) Observing galaxy formation
High redshift galaxies from HST
Fair samples of galaxies at high redshift

It is assumed that you all are familiar with the following material from the Bachelor course here.

The gravitational force caused by a point mass M at \vec{x}_0 on a unit mass at position \vec{x} is

$$\vec{F}(\vec{x}) = GM \frac{\vec{x}_0 - \vec{x}}{|\vec{x}_0 - \vec{x}|^3}$$

In general, the gravitational force is related to the potential Φ by

$$\vec{F}(\vec{x}) = -\vec{\nabla}\Phi(\vec{x})$$

so that

$$\Phi(\vec{x}) = -\frac{GM}{|\vec{x}_0 - \vec{x}|}$$

The density follows from the potential by Poisson's equation:

$$4\pi G\rho(\vec{x}) = \vec{\nabla}^2\Phi(\vec{x})$$

It is assumed that you all are familiar with the following material from the Bachelor course here.

Generalizing $\Phi(\vec{x}) = -\frac{GM}{|\vec{x}_0 - \vec{x}|}$

The gravitational potential for **extended** density distribution $\rho(\vec{x})$ can be obtained by integrating over the density distribution

$$\Phi(\vec{x}) = -G \iiint \frac{\rho(\vec{x}_0) d^3 \vec{x}_0}{|\vec{x}_0 - \vec{x}|}$$

It is assumed that you all are familiar with the following material from the Bachelor course here.

For a spherically symmetric distribution, the potential can be calculated much more simply.

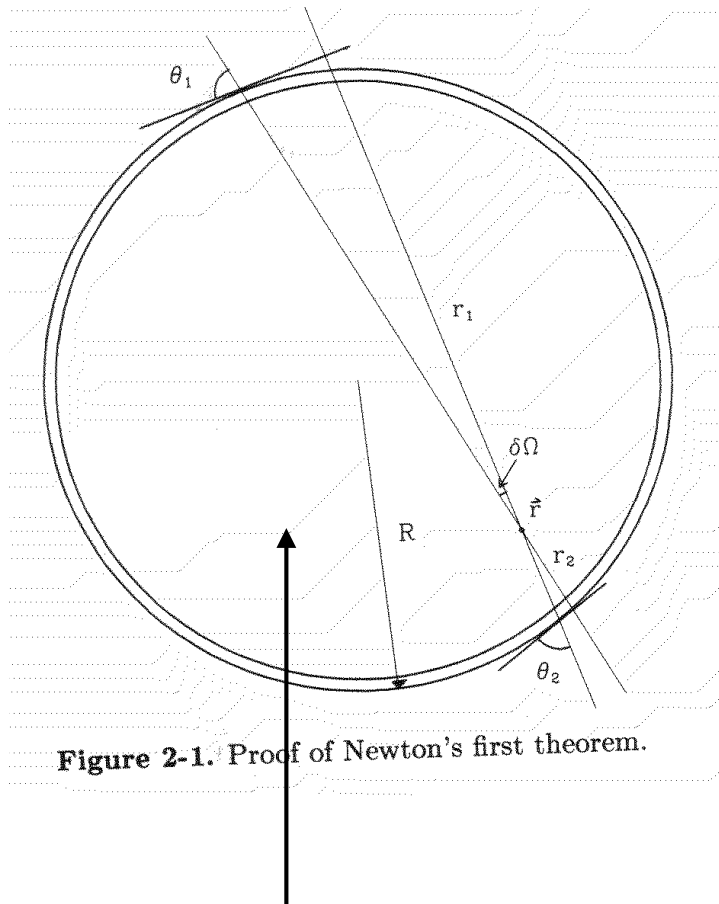


Figure 2-1. Proof of Newton's first theorem.

No net gravitational forces inside sphere

Newton's First Theorem

A body inside an infinitesimally thin spherical shell of matter experiences no net gravitational force from that shell

Why? Looking at the forces on a particle inside the sphere from the mass on the shell over the solid angle $d\Omega$ in either direction.

$$F_1 = Gm\sigma r_1^2 / r_1^2$$

$$F_2 = Gm\sigma r_2^2 / r_2^2$$

We see that $F_1 = F_2$, so the force from the mass in the shell at either end of the sphere offset each other perfectly.

Given the lack of force, the gravitational potential is constant and for the center, $\Phi = -GM/R$,

It is assumed that you all are familiar with the following material from the Bachelor course here.

For a spherically symmetric distribution, the potential can be calculated much more simply.

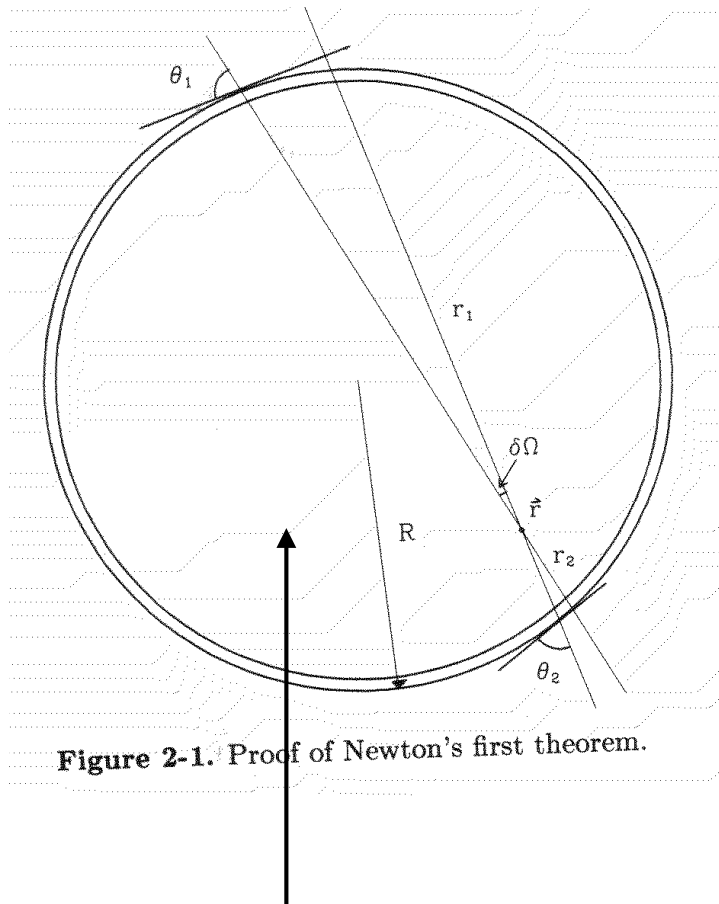


Figure 2-1. Proof of Newton's first theorem.

No net gravitational
forces inside sphere

Newton's Second Theorem

The gravitational force on a body outside a closed spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.

It is assumed that you all are familiar with the following material from the Bachelor course here.

Given Newton's theorems, it is very easy to calculate the potential for a mass with spherical symmetry.

In calculating the potential at a given point in space from a spherical mass, one must distinguish the mass in shells at radii greater than the given point in space.

- 1) For mass in shells at smaller radius r (than radius r' where the potential is being evaluated), treat the mass as if it were concentrated at the center of the sphere
- 2) For mass in shells at larger radius (than radius r' where the potential is being evaluated), treat the mass as if it were concentrated at the center of the sphere but viewed from a distance r' the radius of the shell.

Hence total potential:

$$\Phi = -4\pi G \left[\frac{1}{r} \int_0^r \rho(r') r'^2 dr' + \int_r^\infty \rho(r') r' dr' \right].$$

Galaxies have many different morphologies

The Hubble Tuning Fork

Ellipticals



E0



E5



Sa



Sb



Sc

Unbarred spirals



Lenticular
S0

SBa



SBb



SBc

Barred spirals



Think for a moment about what
complex entities galaxies are:

Whirlpool Galaxy
Messier 51



Elliptical Galaxy
ESO 325-G004



Sombrero Galaxy
Messier 104



Note:

- (1) the remarkable spiral structure.
- (2) prominent nucleus

Think for a moment about what
complex entities galaxies are:

Whirlpool Galaxy
Messier 51



Elliptical Galaxy
ESO 325-G004



Sombrero Galaxy
Messier 104



Note:
(1) how homogeneous this galaxy
looks

Think for a moment about what
complex entities galaxies are:

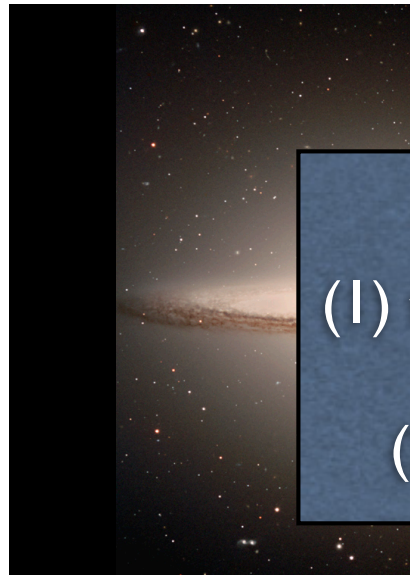
Whirlpool Galaxy
Messier 51



Elliptical Galaxy
ESO 325-G004



Sombrero Galaxy
Messier 104



Note:

- (1) the faint disk for this mostly spherical galaxy
- (2) notice the dark band (dust lane)

To illustrate how differently a galaxy can look on different scales, note these are different views of the same galaxy!

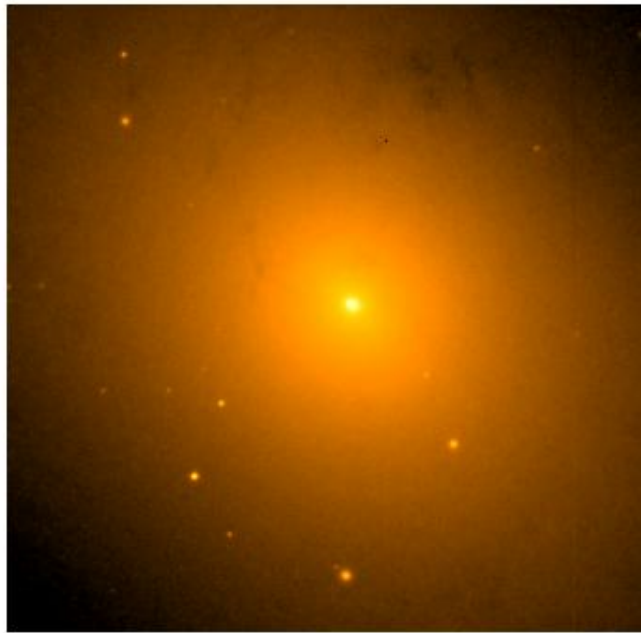
M 31

The Andromeda Galaxy



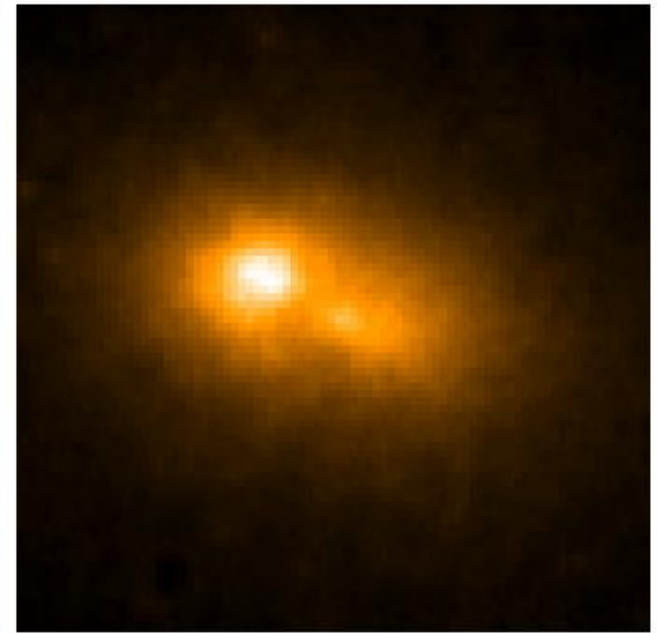
40,000 LY

Ground View of Galaxy



2,000 LY

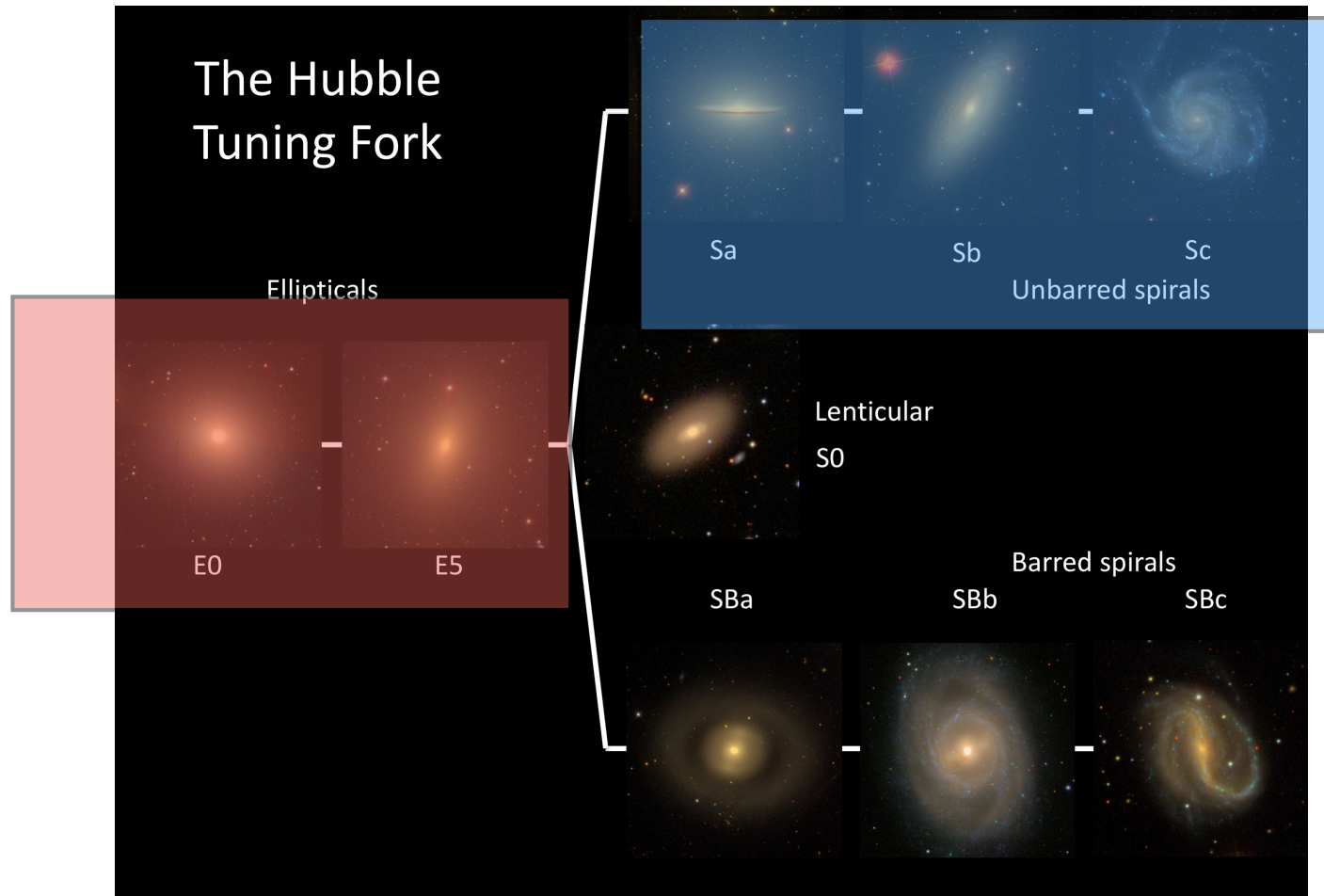
Ground View of Galaxy Core



40 LIGHT-YEARS

HST View of Galaxy Nucleus

Galaxies have many different morphologies



Not surprisingly, these different morphologies reflect different formation mechanisms.

In this course, we will focus on many of these mechanisms, as we aim to understand galaxy formation and evolution.

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Feb 23: Collisionless Stellar Dynamics

Feb 26: Vlasov/Jeans Equations

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Apr 13: Large Scale Structure

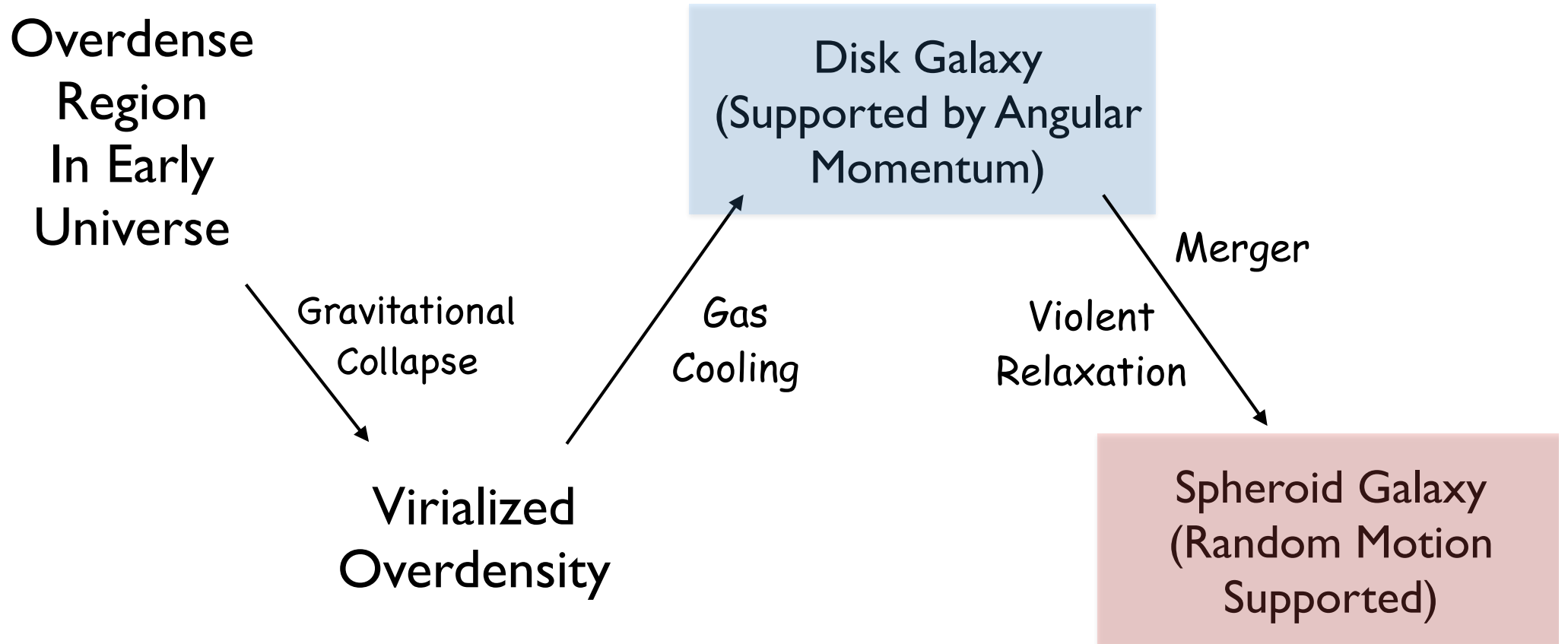
Apr 20: Galaxy Stellar Populations

Apr 27: Lessons from Large Galaxy Samples at $z < 0.2$

May 4: Evolution of Galaxies with Redshift

May 11: Galaxy Evolution at $z > 1.5$ / Review for Final Exam

Galaxy Formation: Major Steps



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FIRST STEP: Gravitational Collapse

Galaxy formation is driven by the impact of gravity in forming collapsed, virtualized halos, consistently primarily of dark matter.

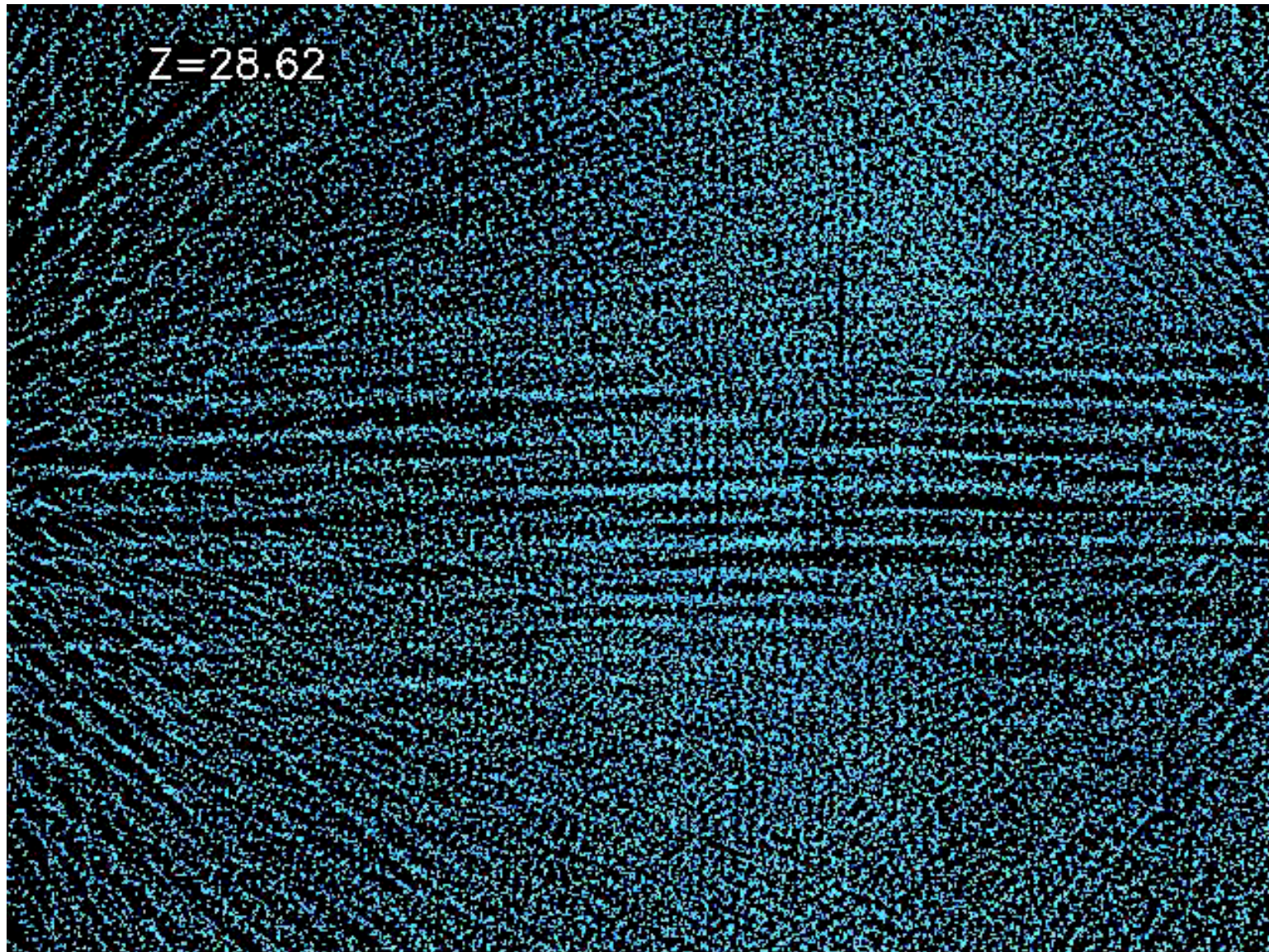
From the cosmic microwave background, we can see that the universe is exceedingly uniform at early times (fluctuations $\sim 10^{-5}$)

The universe is not so smooth today (galaxies, galaxy clusters)...

Galaxies form by gravitational forces through the growth of small overdensities in early universe...

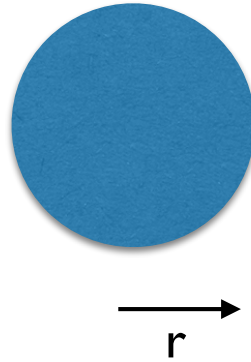
FIRST STEP: Gravitational Collapse

Galaxy formation is driven by the impact of gravity in forming collapsed, virtualized halos, consistently primarily of dark matter.



FIRST STEP: Gravitational Collapse

Let us first consider the expansion of the universe as a whole... Let us consider a small region of the universe which we will assume to be spherically symmetric. Let us assume this region has mass M and radius r .



FIRST STEP: Gravitational Collapse

Let us first consider the expansion of the universe as a whole... Let us consider a small region of the universe which we will assume to be spherically symmetric. Let us assume this region has mass M and radius r .

The acceleration the outer shell of this spherical region will feel can be described by the following equation:

$$\ddot{r} = -\frac{GM(< r)}{r^2}$$

Multiplying both sides of the equation by dr/dt and then integrating both sides of the equation:

$$\frac{1}{2}(\dot{r})^2 = \frac{GM}{r} + c$$

FIRST STEP: Gravitational Collapse

We can manipulate this to derive the energy E of the sphere which will determine its long term evolution:

$$E = \frac{1}{2}(\dot{r})^2 - \frac{GM}{r} = c$$

If $E < 0$, the sphere will eventually collapse.

If $E \geq 0$, the sphere will expand forever.

For energy $E = 0$, the density of the sphere can be said to have a critical density, since any lower density would result in an eventual collapse.

FIRST STEP: Gravitational Collapse

What is this critical density ρ_c ?

Inserting $M = \rho(4\pi/3)r^3$ into this equation, one can easily show that

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{8}{3}\pi G\rho + \frac{c}{r^2}$$

H^2
(Hubble constant squared)

$c = 0$ is the dividing line
between collapse and
expanding forever

Then...

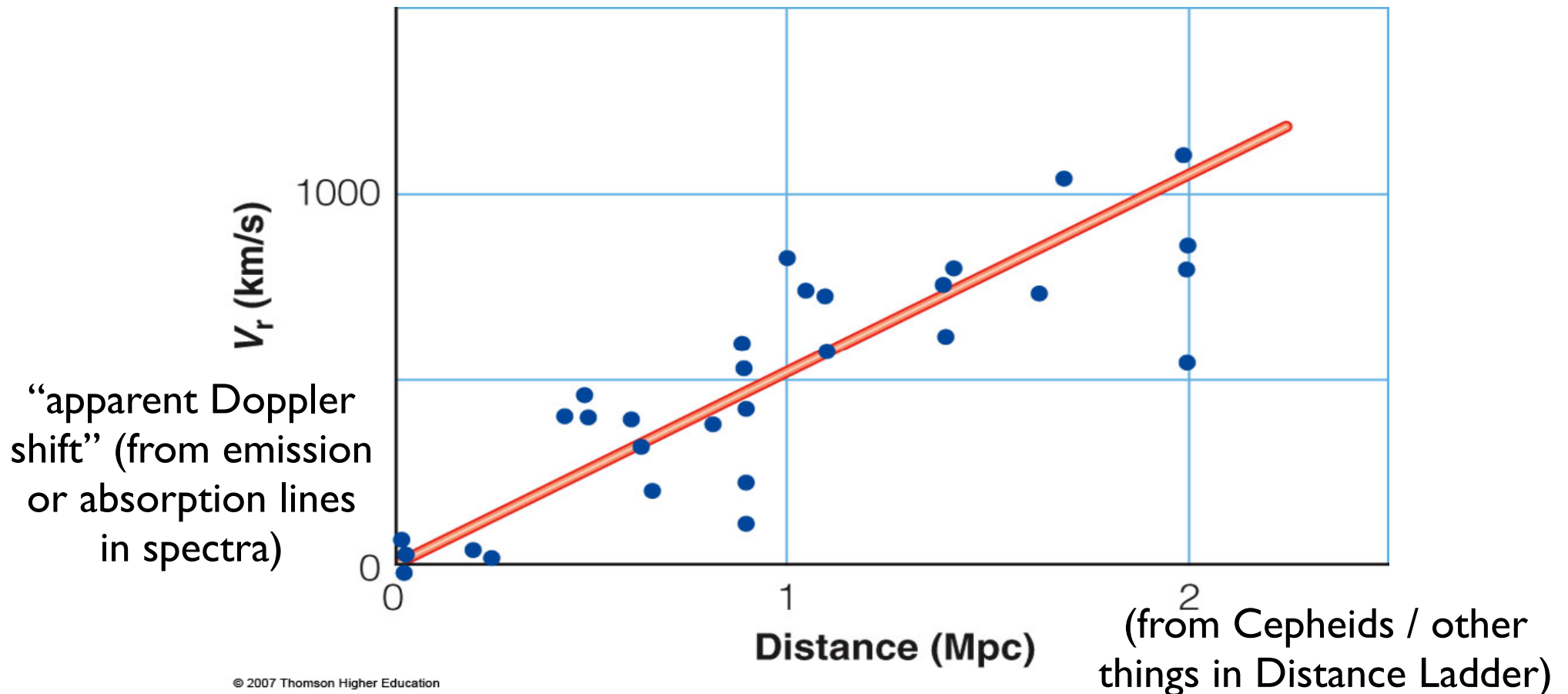
$$\rho_c = \frac{H^2}{\frac{8}{3}\pi G} = \frac{3H^2}{8\pi G}$$

The expansion rate of the universe

In 1929, Hubble showed that the velocities and distances are linearly correlated, and satisfy

$$v = H_0 d$$

where v is the recessional velocity (km/s) and d is the distance (Mpc). H_0 is a constant, “Hubble’s Constant” and has units of $\text{km s}^{-1} \text{Mpc}^{-1}$.



FIRST STEP: Gravitational Collapse

Actual density of the universe is often expressed in relation to this critical density as Ω :

$$\Omega = \rho / \rho_c = \frac{8\pi G \rho}{3H^2}$$

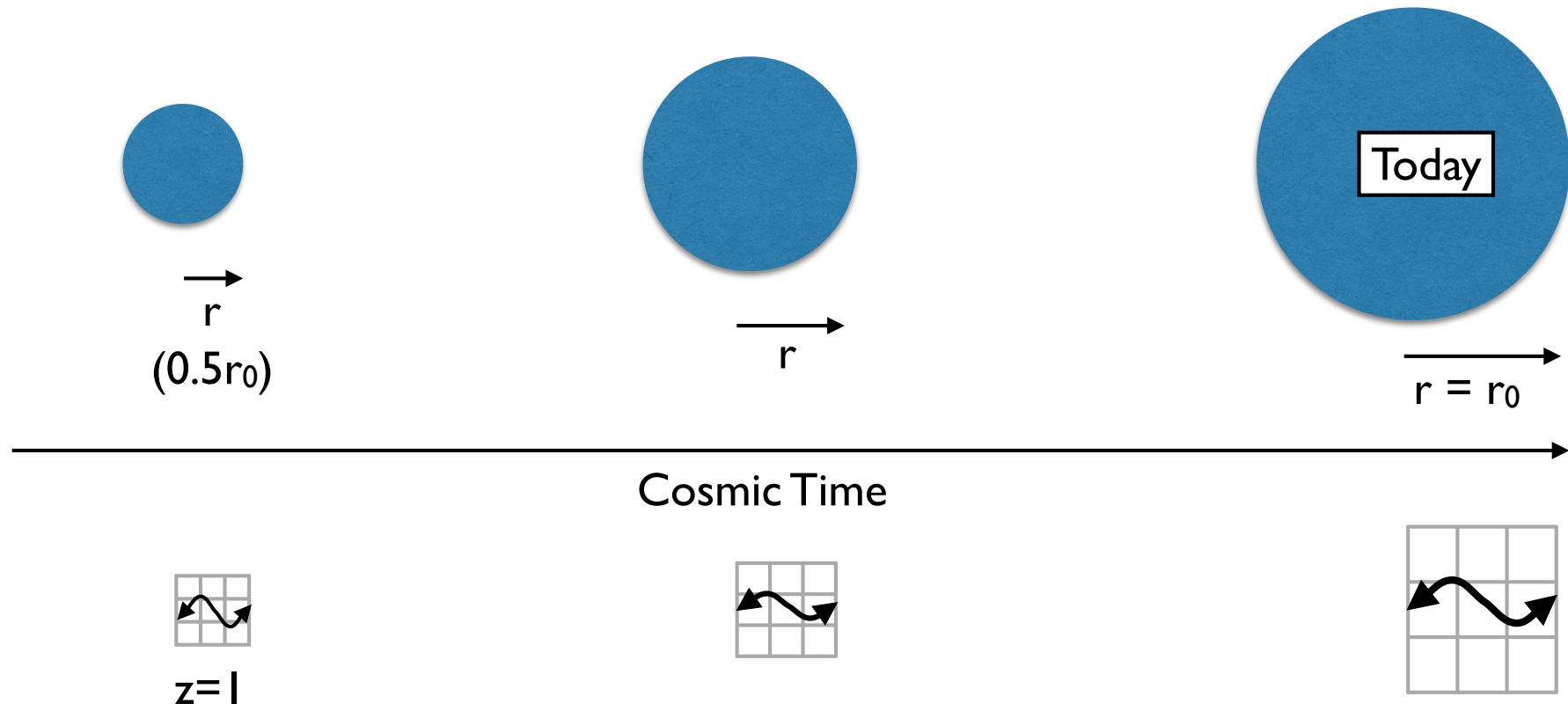
How does Ω evolve with redshift?

We can manipulate previous expressions such that

$$\frac{3E}{4\pi G \rho r^2} = \frac{3H^2}{8\pi G \rho} - 1 \qquad \frac{3E}{4\pi G} \frac{1}{\rho r^2} = \frac{1}{\Omega} - 1$$

FIRST STEP: Gravitational Collapse

Let us first consider the expansion of the universe as a whole... Let us consider a small region of the universe which we will assume to be spherically symmetric. Let us assume this region has mass M and radius r .



λ_{obs} proportional to r (scale factor of universe)

$$\lambda_{\text{obs}} = \lambda_{\text{rest}} (1+z)$$

$$r/r_0 = 1/(1+z)$$

FIRST STEP: Gravitational Collapse

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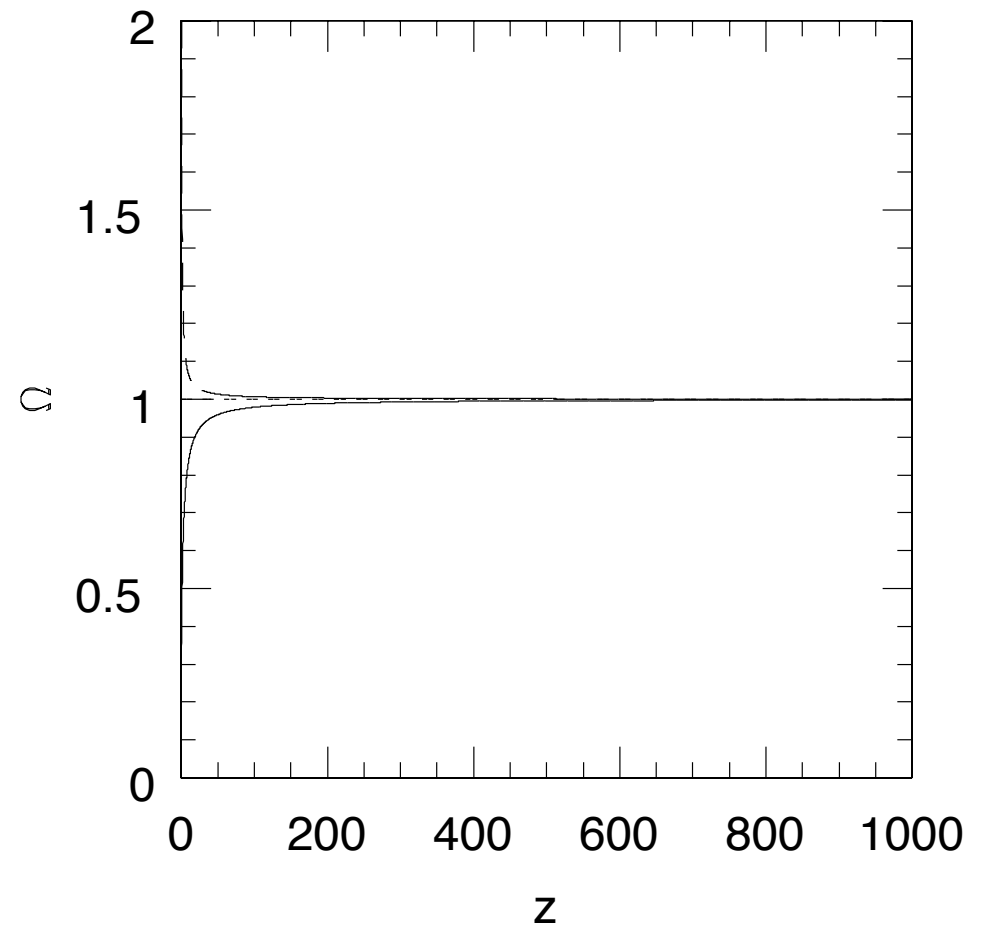
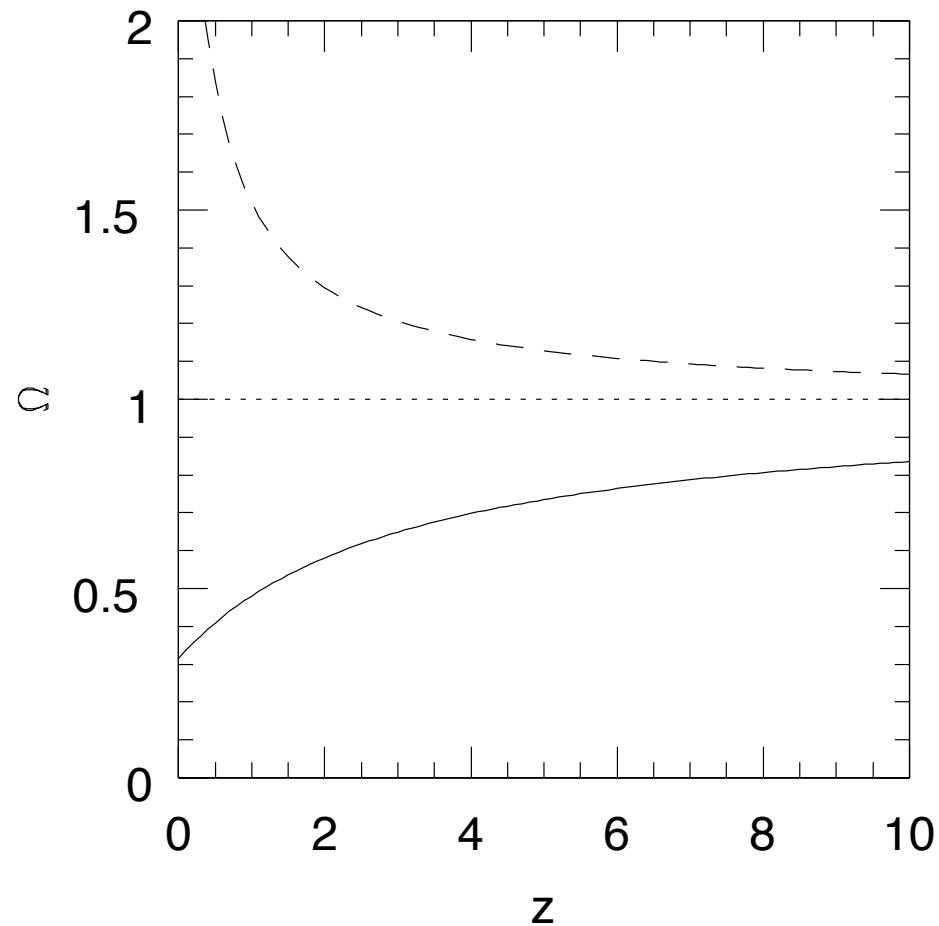
$$\frac{3E}{4\pi G \rho r^2} = \frac{3H^2}{8\pi G \rho} - 1 \qquad \frac{3E}{4\pi G} \frac{1}{\rho r^2} = \frac{1}{\Omega} - 1$$

Note that $1/\rho r^2 = r^3/(r^2 \rho_0) = r/\rho_0 = 1/(1+z)/\rho_0$

$$\frac{1}{\Omega} - 1 = \text{const}/(1+z)$$

FIRST STEP: Gravitational Collapse

The above equation implies that Ω reverts to 1 at early times....

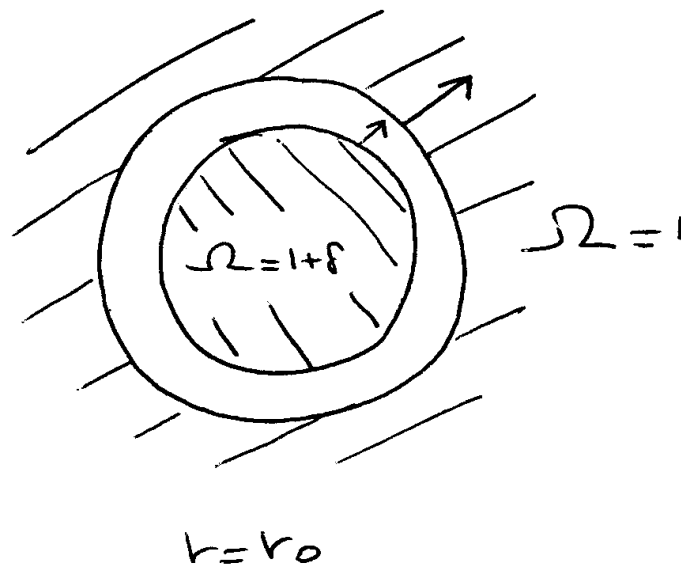


Independent of its current value, Ω is very close to 1 at early times!

FIRST STEP: Gravitational Collapse

Due to the proximity of Ω to 1 at early times, a small amount of extra matter could cause various regions of the universe to have a density $\rho > \rho_c$

How would spherically symmetric regions of the universe at supercritical density $\rho > \rho_c$ evolve?



$$\delta = \rho / \rho_c - 1$$

δ = Overdensity
relative to critical...

Because of spherical symmetries, can analyze force on outside of spherically symmetric region using only mass internal to it.

Can analyze the evolution of this region of the universe as if it were a “separate universe” with its own Ω . If Ω for this region is > 1 , then the region will eventually collapse.

FIRST STEP: Gravitational Collapse

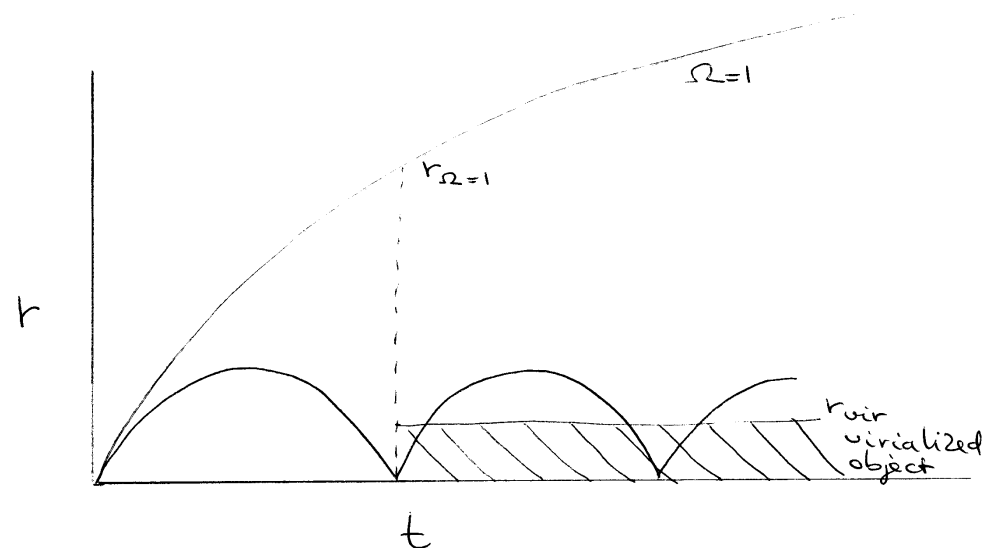
The time evolution of the effective Ω in the overdense sphere should evolve as we previously found:

$$\frac{1}{\Omega} - 1 = \text{const}/(1+z)$$

Substituting $\Omega = 1 + \delta$, we find that

$$\delta = \text{const} / (1+z) = \text{const} (t / t_0)^{2/3}$$

The expected time evolution of the collapsed sphere is as follows:



This is known as the spherical collapse model!

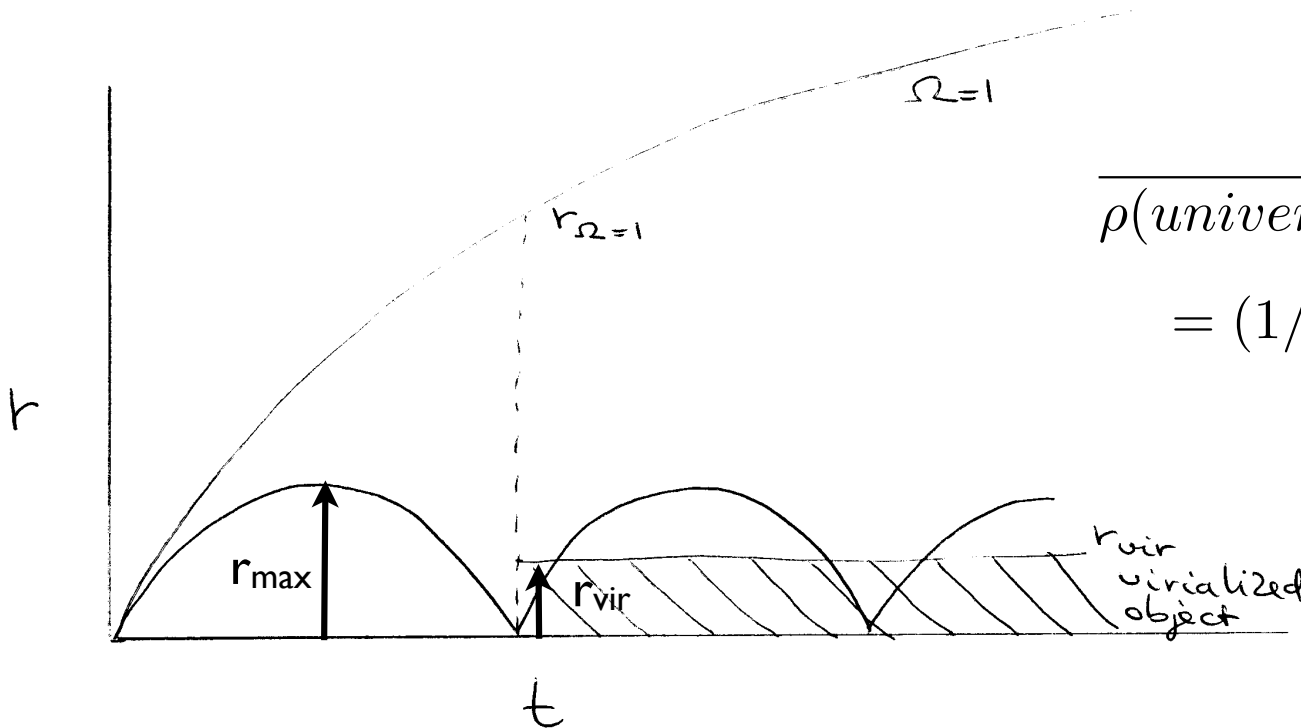
FIRST STEP: Gravitational Collapse

Substituting $r = A(1 - \cos \theta)$, $t = B(\theta - \sin \theta)$ in the differential equation for the outer spherical shell, i.e.,

$$\ddot{r} = -\frac{GM(< r)}{r^2}$$

$$r_{max} = \frac{1}{\frac{1}{4}(12\pi)^{2/3}} r_{\Omega=1}(t_{collapse}) \quad (\approx 0.36 r_{\Omega=1}(t_{collapse}))$$

$$r_{vir} = \frac{1}{\frac{1}{2}(12\pi)^{2/3}} r_{\Omega=1}$$



$$\frac{\rho_{vir}}{\rho(universe)(z = z_{vir})} = (r_{\Omega=1}/r_{vir})^3$$

$$= (1/2(12\pi)^{2/3})^3 = 18\pi^2 = 178$$

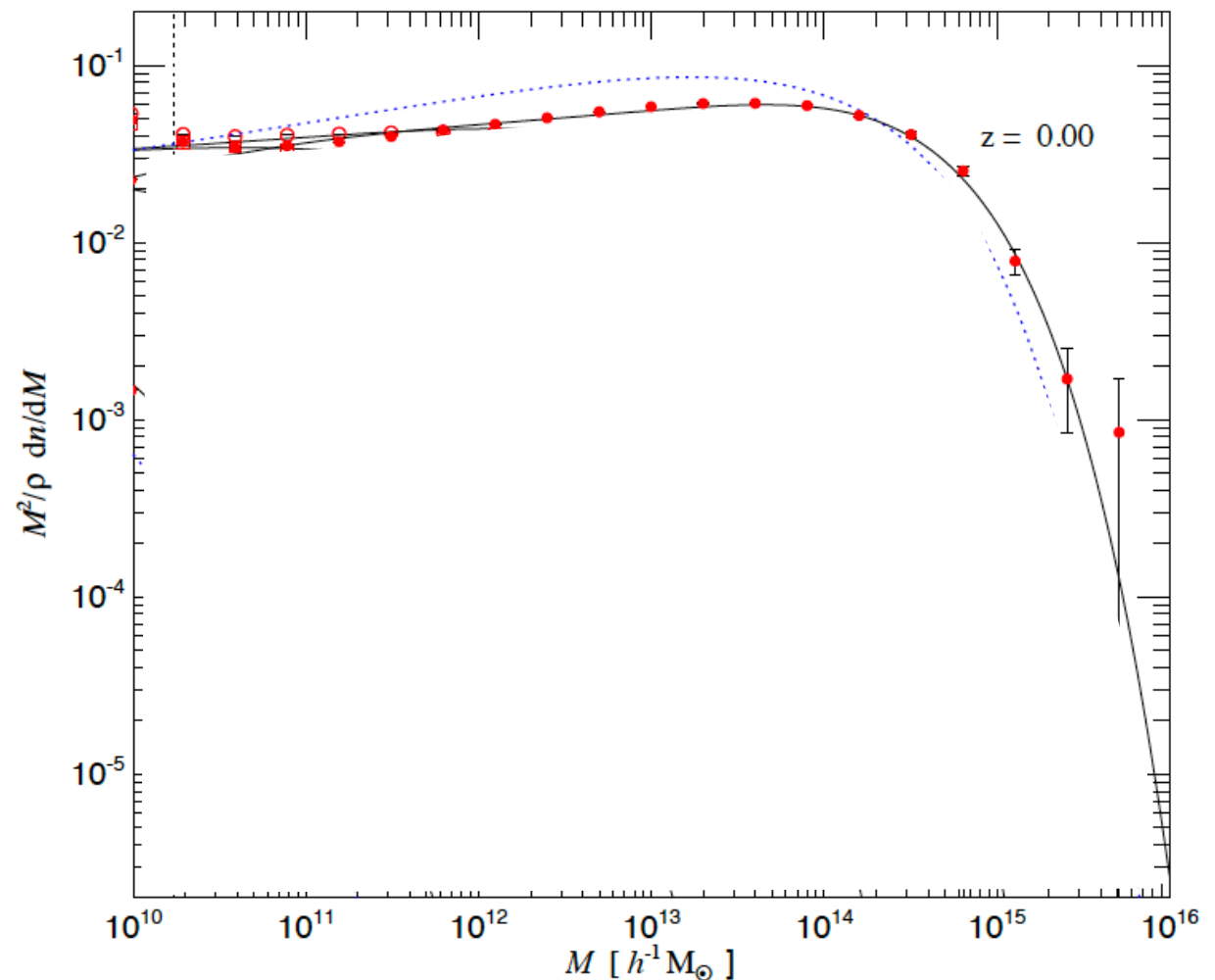
FIRST STEP: Gravitational Collapse

Of course, there exist overdensities on a wide variety of scales and masses, each of which can collapse and form virtualized objects.

One can quantify the density of these collapsed halos on different mass scales with the halo mass function

But on which scales do these halos turn into galaxies?

Halo Mass Function



Given that the equations of motions for self-gravitating systems are simple and can be scaled by an arbitrary factor, gravity itself cannot set a scale for galaxy formation.

In a system where gravity is the only important force:

$$F = m \, d^2x/dt^2 = \sum_i G m m_i / (x - x_i)^2$$

We will consider the substitutions:

“new” “existing”

$$m^* = a_m m$$

$$v^* = a_v v$$

$$x^* (t^*) = a_x x(a_t t)$$

$$F = m \, d^2x/dt^2 = \sum_i G m m_i / (x - x_i)^2$$

$$a_m \, a_x \, a_t^2 (m \, d^2x/dt^2) = a_m^2 \, a_x^{-2} (\sum_i G m m_i / (x - x_i)^2)$$

$$a_m \, a_x \, a_t^2 (\sum_i G m m_i / (x - x_i)^2) = a_m^2 \, a_x^{-2} (\sum_i G m m_i / (x - x_i)^2)$$

$$a_m \, a_x \, a_t^2 = a_m^2 \, a_x^{-2}$$

$$a_x^3 \, a_t^2 = a_m$$

$$v^* = dx^*/dt = d(a_x x(a_t \, t))/dt = a_x a_t \, dx(a_t \, t)/dt$$

$$a_v v = a_x a_t \, dx(a_t \, t)/dt$$

$$a_v v = a_x a_t \, v$$

$$a_v = a_x a_t$$

$$a_v/a_x = a_t$$

$$a_x^3 \, a_t^2 = a_m \rightarrow$$

$$a_x^3 \, (a_v/a_x)^2 = a_m \rightarrow$$

$$a_x \, a_v^2 = a_m$$

This means it is always possible to scale the masses, spatial positions, and velocities of particles in a system and have it behave in the same manner

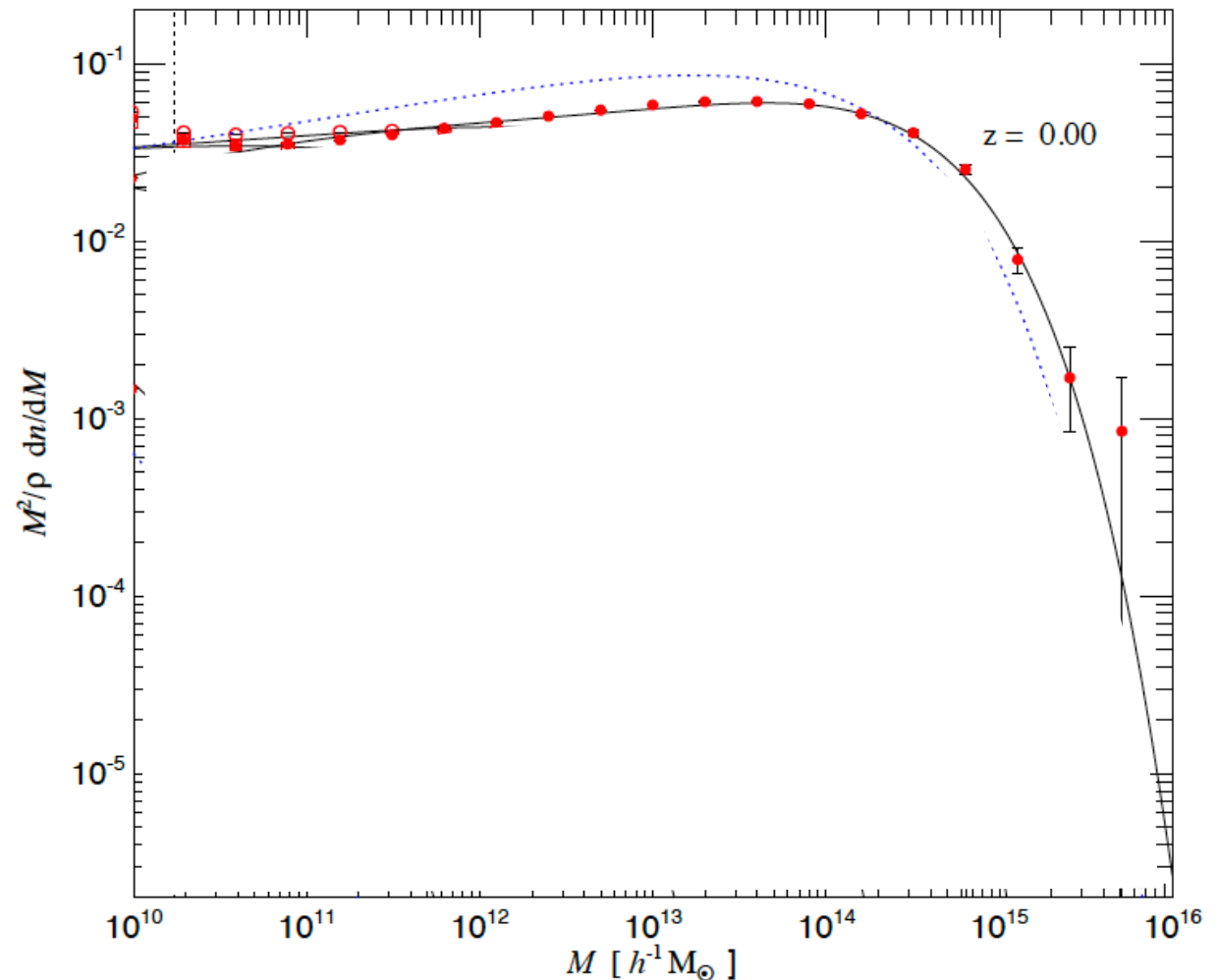
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Halo Mass Function



SECOND STEP: Gas cooling

The second essential step in the formation of galaxies is the need for baryons to be able to cool to the center of the collapsed halo.

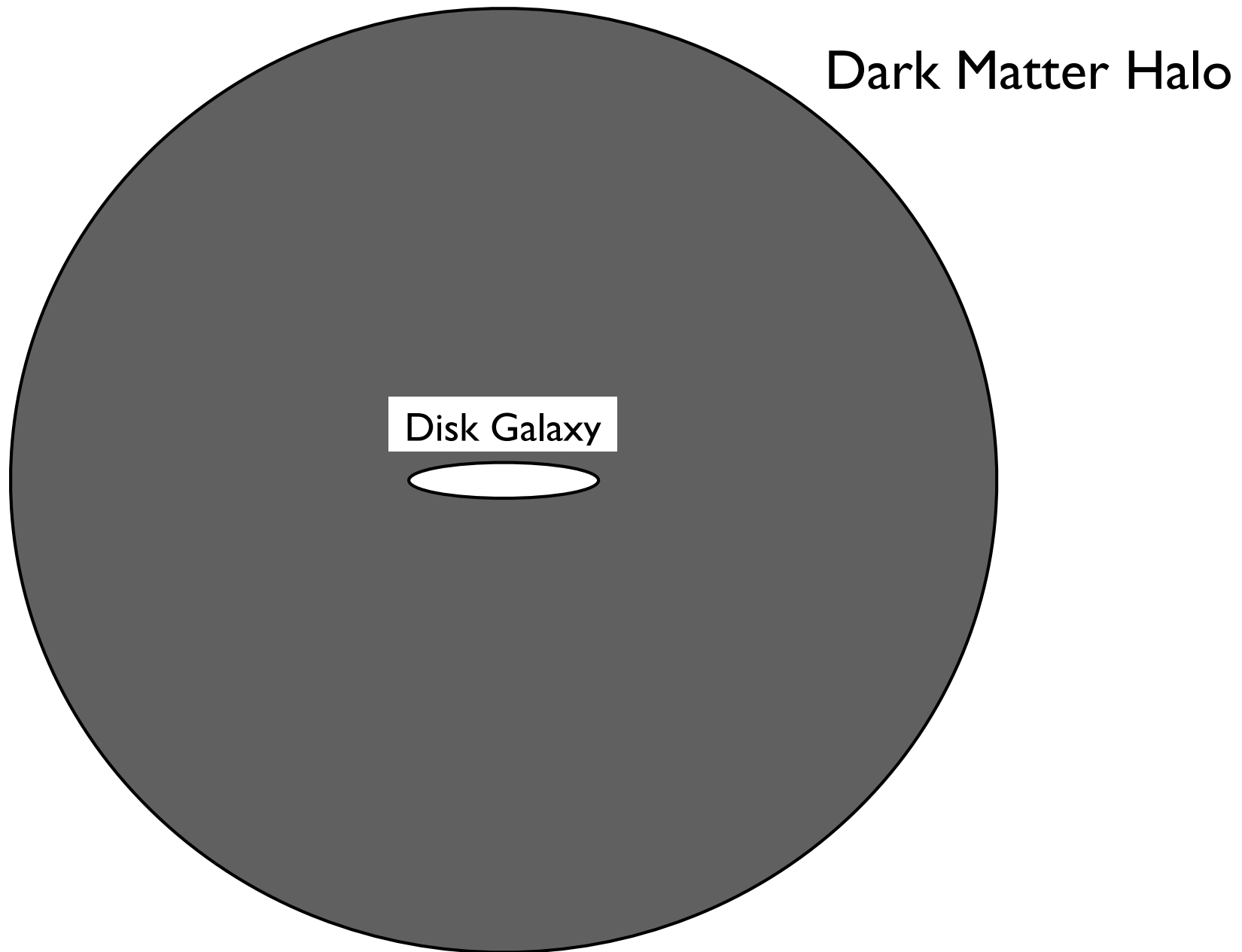
During the initial gravitational collapse of an overdensity, we would expect the baryons to be distributed in a very similar way to the dark matter.

Both matter distributions are supported by the random motions of the particles.

However, in favorable conditions, baryons sink to the center of the gravitational potential, but dark matter remains where it is.

Baryonic gas particles can lose energy through radiative processes, but dark matter cannot.

Schematic Picture of a Galaxy



SECOND STEP: Gas cooling

When are conditions favorable to the baryons cooling to the center of a collapsed halo?

SECOND STEP: Gas cooling

The cooling rate can be expressed as

$$\text{cooling rate} = n^2 \Lambda(T)$$

where n is the gas volume density and $\Lambda(T)$ is the temperature dependent cooling rate.

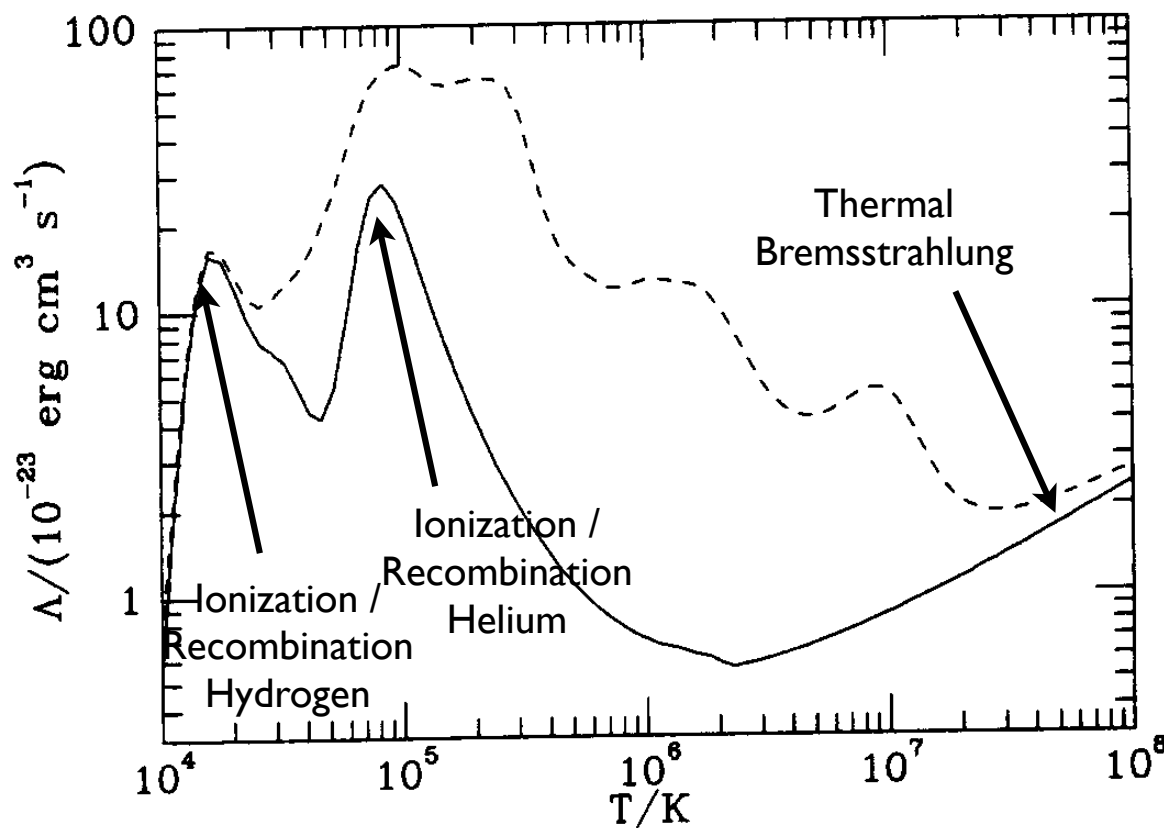


Figure 24.19 The cooling function $\Lambda(T)$. The solid line corresponds to a gas mixture of 90% hydrogen and 10% helium, by number. The dashed line is for solar abundances. (Figure from Binney and Tremaine, *Galactic Dynamics*, Princeton University Press, Princeton, NJ, 1987.)

SECOND STEP: Gas cooling

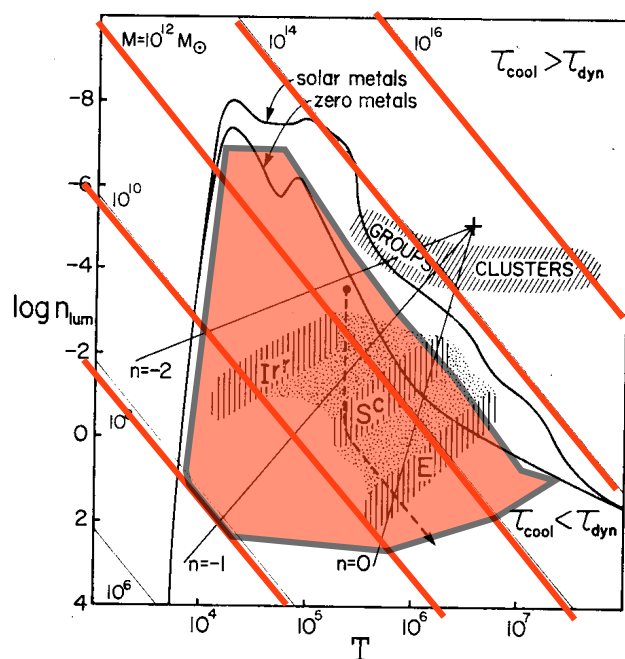
There are two time scales of interest in thinking about galaxy formation:

(1) dynamical time scale $t_{\text{dyn}} \propto n^{-1/2}$

(2) cooling time scale t_{cool}

For cooling to have any impact, $t_{\text{cool}} \ll t_{\text{dyn}}$

How do these time scales compare for various mass halos?



For sources with galaxy-sized masses ($10^{10} M_{\text{sol}}$), cooling is efficient...

However for cluster-scale masses ($10^{14} M_{\text{sol}}$), cooling is not efficient... and have lots of hot gas.

constant mass lines

FIG. 1. Ostriker-Rees cooling diagram showing the actual locations of present-day galaxies and clusters. Data on galaxies came from the following sources - E's: σ 's from Terlevich *et al.* (1981), M_{lum}/L from Faber and Gallagher (1979); Sc's: v_{rot} 's and radii from Burstein *et al.* (1981), M_{lum}/L_B assumed to be 1.6; Irr's: Thuan and Seitzer (1979). Data on groups and clusters from Rood and Dickel (1978). Cross is mean mass turning around today (White and Rees 1978). Heavy straight lines are clustering loci for various values of n . Light lines are the mass of a self-gravitating body composed purely of ordinary matter. Dashed line is