

Galaxies: Structure, Dynamics, and Evolution

Analysis of Galaxy Stellar Populations

“What can we learn about galaxies by putting together a large survey of galaxies in the nearby universe”

Layout of the Course

Lectures

Feb 2: Course Introduction, Overview, and Galaxy Formation Basics

Feb 9: Disk Galaxies (I)

Feb 12: Disk Galaxies (II)

Feb 16: Disk Galaxies (III) / Collisionless Stellar Dynamics

Feb 23: Collisionless Stellar Dynamics + Vlasov/Jeans Equations

Feb 26: Vlasov/Jeans Equations / Elliptical Galaxies (I)

Mar 9: Elliptical Galaxies (II)

Mar 23: Dark Matter Halos

Mar 30: Connecting Galaxies to Dark Matter Halos

Apr 13: Galaxy Stellar Populations + Lessons from Galaxies at $z < 0.2$

Apr 20: Lessons from Galaxy Samples at $z < 0.2$ + Gas Cycle

Apr 23: Gas Cycle + Evolution of Galaxies with Redshift

May 4: Galaxy Evolution at $z > 1.5$

May 11: Galaxy Evolution at $z > 6$. / Review for Final Exam



Practical Sessions

Feb 19: Board Work + Problem Set 1

Mar 12: Board Work + Problem Set 2

Mar 26: Problem Set 3 / Paper Presentations (3 slots)

Apr 2: Problem Set 3 (cont'd) / Paper Presentations (6 slots)

Apr 16: Problem Set 4 / Paper Presentations (3 slots)

Apr 30: Problem Set 5 / Paper Presentations (3 slots)

May 7: Problem Set 6 / Paper Presentations (3 slots)



April 16 Practical Session

(In 3 days)

Paper Presentations (12 + 3 minutes)

Thin & Thick Disk as Seen By Gaia: Eilers+2019, Nitschai+2021

Zuzanna Ryduchowska, Erin Corcoran

Core/Cusp in Elliptical Galaxies and SMBHs: Kormendy+2013, Savorgnan+2016

Ottavia Zanello & Arianna Zirotti

Missing Satellite Problem: Boylin-Kolchin+2011, Simon 2019

Bente Zandbergen & Frederique van Holk

April 16 Practical Session

(Today or In 3 days)

Problem Set 4 - Problems 2, 7 (to be discussed)

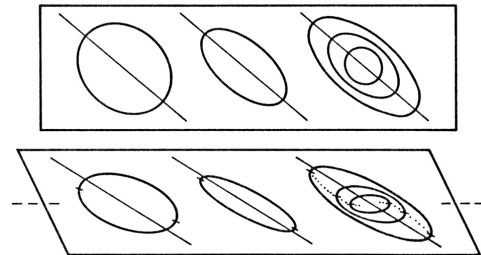
Ines Bercuk
George-Luca Iconaru

Yifan Li

Galaxies: Structure, Dynamics, and Evolution
Problem Set 4
Instructor: Dr. Bouwens

Here is Problem Set 4. The entire problem set will be due before class on Monday, April 27 (email them to Wout and include GSD in the subject line). Be sure to pay extra attention to problem 6, as your solution to that problem will be checked carefully and used in determining your homework grade.

1. Determine the impact of projection effects on the apparent isophotal twist (for elliptical galaxies). Consider two ellipses with their major axis oriented 45 degrees away from some line (that line would be horizontal on the following diagram):



Suppose that the axial ratio is 1.15 for the one ellipse (similar to the leftmost ellipse shown in the above figure) and 2.8 for the other ellipse (similar to the center ellipse shown in the above figure). Suppose that we are viewing the ellipses face on and then we rotate the ellipses by 60 degrees about an axis (parallel to the aforementioned line) so that the ellipses are viewed almost edge on. What ellipticity would we measure for each of our two ellipses? What would be the apparent position angle of the major axis of each ellipse relative to aforementioned line?

2. (a) Derive the enclosed mass $M(< r)$ for the NFW profile $\rho(r) = \rho_s / [(r/r_s)(1+r/r_s)^2]$. Use $r/(1+r)^2 = 1/(r+1) - 1/(1+r)^2$
- (b) Use this to show $\rho_s = \frac{200}{3} \rho_{cr}(z) \frac{e^3}{\ln(1+c) - c/(1+c)}$ given our parameterization $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$
- (c) Derive the circular velocity as a function of radius for an NFW profile.

3. Consider the collapse of a uniform cloud of stars initially at rest. Assume the cloud has a total mass of $5 \times 10^{10} M_\odot$, is entirely composed of stars with $1 M_\odot$, and has approximate dimensions of $2 \text{ kpc} \times 2 \text{ kpc} \times 2 \text{ kpc}$. Assume that the collapse finishes in one free fall time, $1/\sqrt{G\rho}$. What is the time scale for violent relaxation? [Approximate order-of-magnitude estimates are fine for this first step.] If the system were instead in equilibrium (i.e., not undergoing collapse), what relaxation time scale would we estimate for stars in this system using the equations we derived in Lecture #5? How do these time scales compare?

4. Determine what the b_n normalization factor in the Sersic law must be such that the integral of the surface brightness profile $10^{b_n [(R/R_e)^{1/n} - 1]}$ over all radii is equal to one. What is this normalization factor in the case $n = 1$ and $n = 4$?

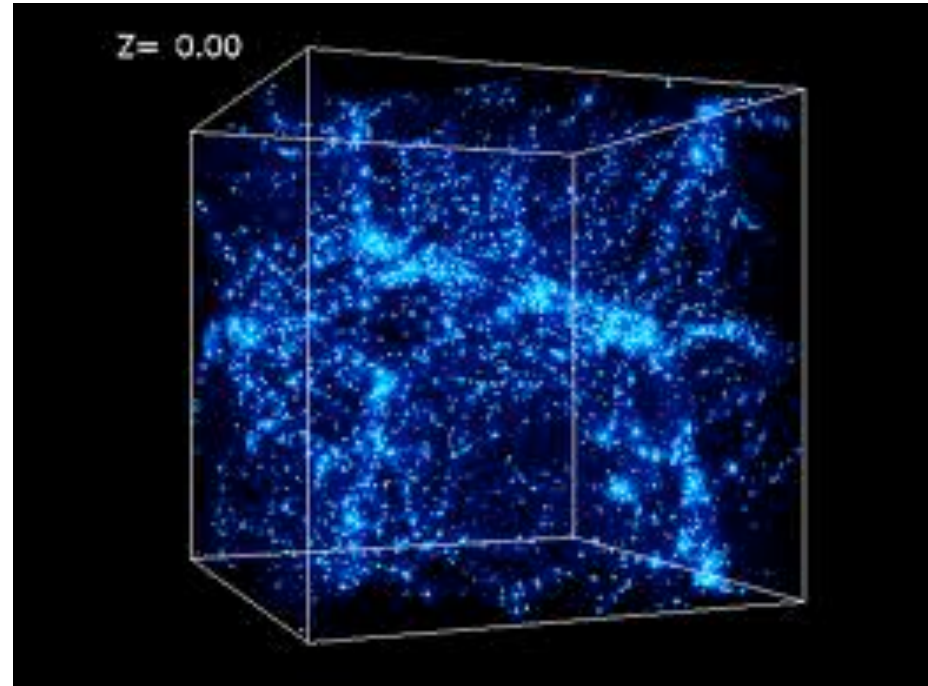
5. Look at the angular correlation functions for luminous galaxies $-22 < M_{UV,AB} < -21$ and lower luminosity galaxies $-19 < M_{UV,AB} < -18$ (shown in the last lecture). What is the ratio of bias factors for these galaxies at a scale of $1.5 h^{-1} \text{ Mpc}$? [Make your best guess for the bias factors based on the figure shown in lecture.]

6. Derive the Fundamental Plane that one would find if the mass-to-light ratio is a function of mass only $M/L = M^{0.25}$ and more generally $M/L = M^\gamma$. (The Fundamental Plane is the relation of the form $R_e \propto \sigma^\alpha \mu_e^\beta$ where R_e is the half-light radius.) Assume that the galaxies are homologous, i.e., they have similar density profiles, but scaled up or down with respect to each other. Note that the assumption of homology results in the following relation: $\sigma^2 \propto M/R_e$.

7. The number density of galaxies is about $0.01 h^3 \text{ Mpc}^{-3}$. The correlation length r_0 is $5h^{-1} \text{ Mpc}$.
 - a) Why does the density and the correlation length depend on h ($= H_0 / (100 \text{ km/s/Mpc})$)
 - b) The correlation function gives the relative excess of galaxies at a given radius. Calculate the integrated correlation function, i.e., the excess from within a radius smaller than r .
 - c) Now combine this with the average number density to estimate the radius r within which each galaxy has on average 1 neighbor.
 - d) What would this radius be if the galaxies are not correlated?

**First, let's review the important
material from last week**

“How Do Galaxies Distribute Themselves in Space?”

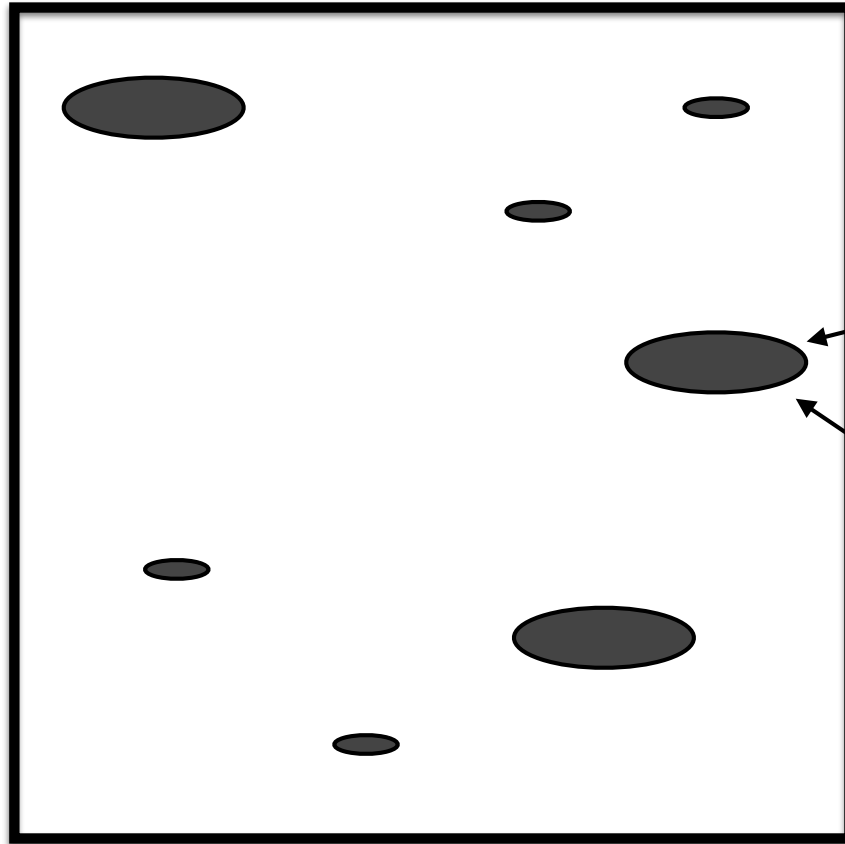


What does this teach us?”

It provides insight into the masses or properties of the collapsed dark matter halos in galaxies form and evolve.

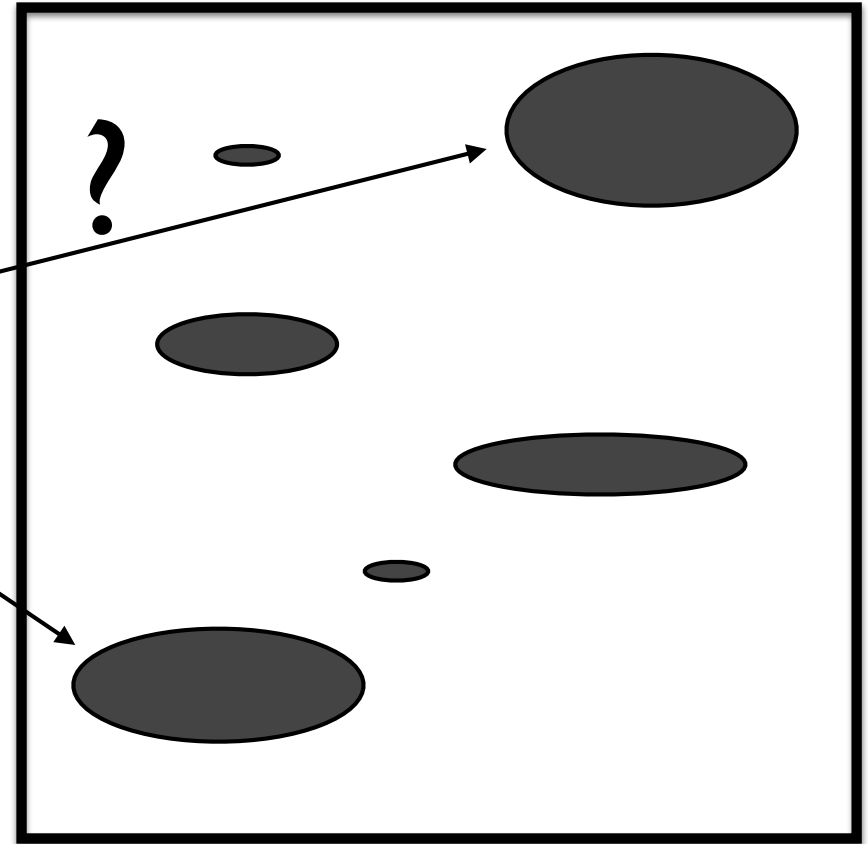
Key Question: How do we determine what a given galaxy at $z \sim 1$ looked like at both earlier and later times?

Survey of a volume of the universe at $z \sim 1$



By constraining the halo mass of galaxies here

Survey of a volume of the universe at $z \sim 0$



By constraining the halo mass of galaxies here

\Rightarrow Since we can accurately model how the mass in halos grow with time due to gravity, we can associate galaxies in both time slices.

How can we gain insight into the collapsed structures in which galaxies live?

We assume that galaxies are found in collapsed dark matter halos and we find the halos that match power spectrum seen in observations:

$$P(k)_{\text{galaxies}} = b^2 P(k)_{\text{DM}} = P(k)_{\text{DMHalo}}$$

$P(k)_{\text{galaxies}}$ can be computed from the observed correlation function $\xi(r)$ for galaxies

$P(k)_{\text{DM}}$ is the matter power spectrum

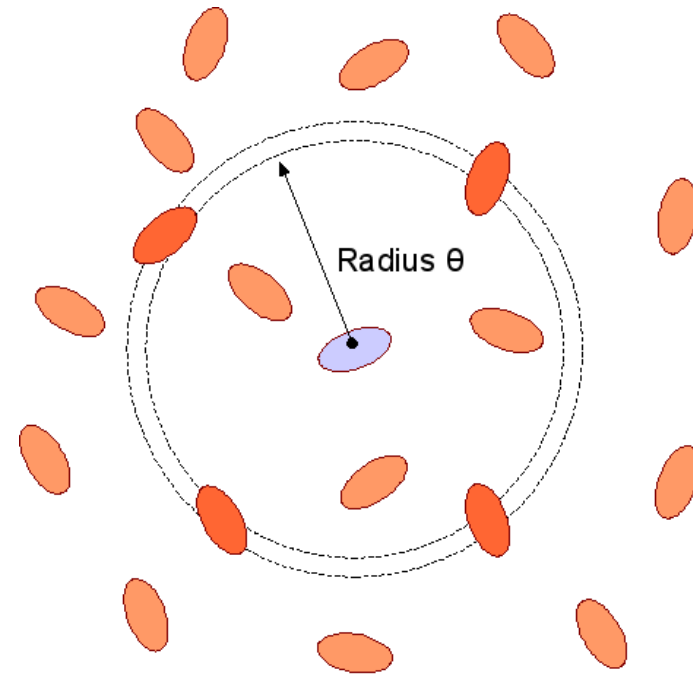
$P(k)_{\text{DMHalo}}$ is the power spectrum in dark matter halo distribution

b is the bias factor
(≥ 1 in general)

Calculating $P(k)_{\text{galaxies}}$

The correlation function $\xi(r)$ describes the excess probability above random that one finds a galaxy at a given distance r .

It is calculated by examining the distances between every pair of galaxies in a survey and comparing it to a random distribution



Correlations between points can be determined by counting pairs.

The power spectrum is the Fourier transform of the correlation function ξ

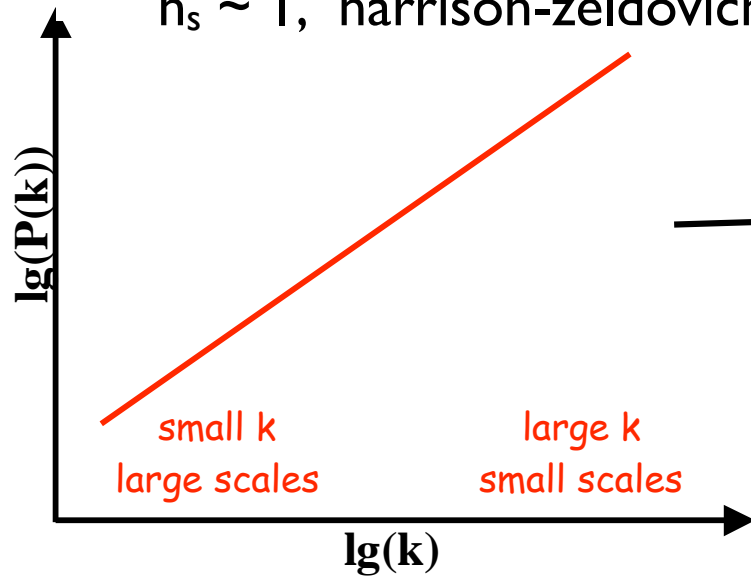
$$P(k) = \int \xi(r) e^{ik \cdot r} d^3 r \equiv \int \xi(r) \frac{\sin(kr)}{kr} r^2 dr$$

Calculating $P(k)_{DM}$

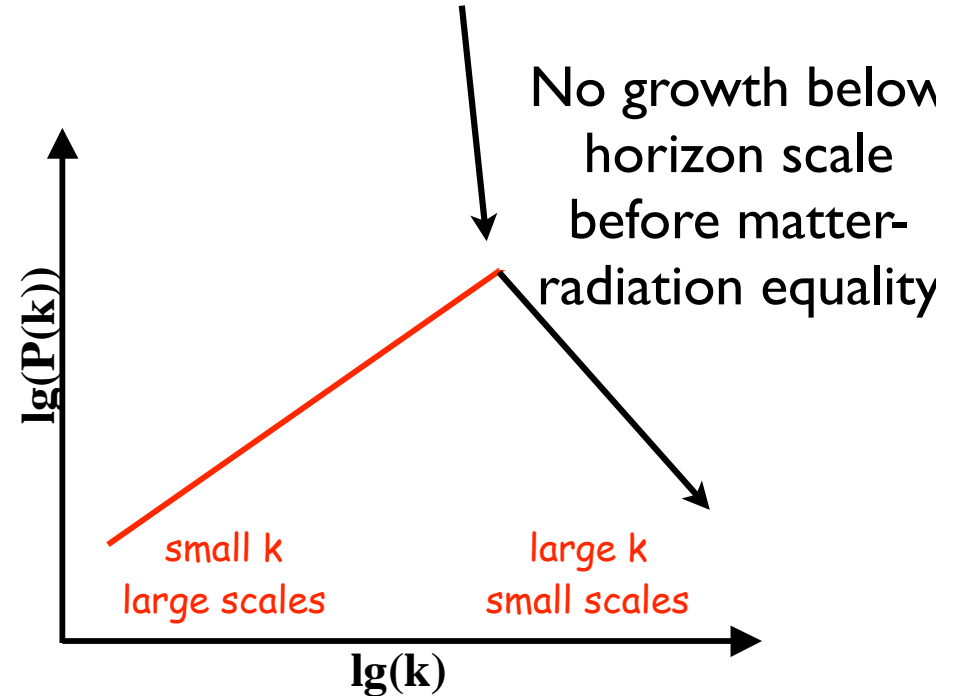
The initial power spectrum of fluctuations is the following:

$$P_0(k) = A k^{n_s}$$

$n_s \sim 1$, "harrison-zeldovich"



Position of turn-over determined by horizon size @ matter-radiation equality



Formal Equation:

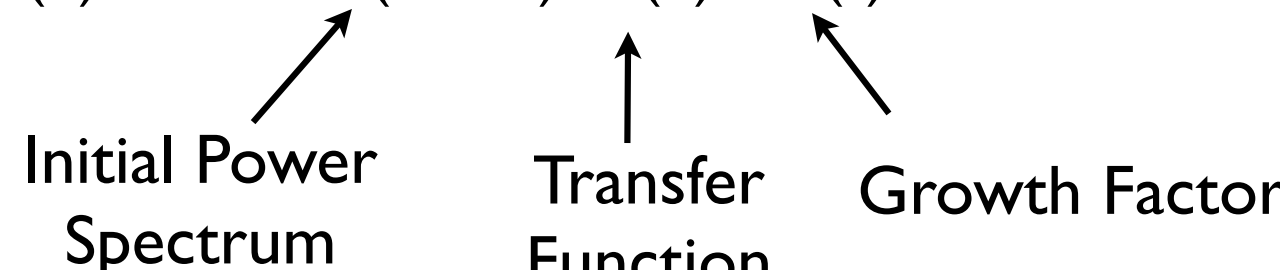
$$P(k)_{DM} = P(k, t=0) T^2(k)$$

Calculating $P(k)_{DM}$

As you learn in Joop's galaxy course and Henk's "origin and evolution of the universe" course, small perturbations are amplified at the same rate as the size of the universe (in the distant past when the universe has $\Omega = 1$)

So, the $P(k)_{DM}$ in the current day universe is as follows:

Formal Equation:

$$P(k)_{DM} = P(k, t=0) T^2(k) D_+^2(t)$$


Initial Power Spectrum Transfer Function Growth Factor

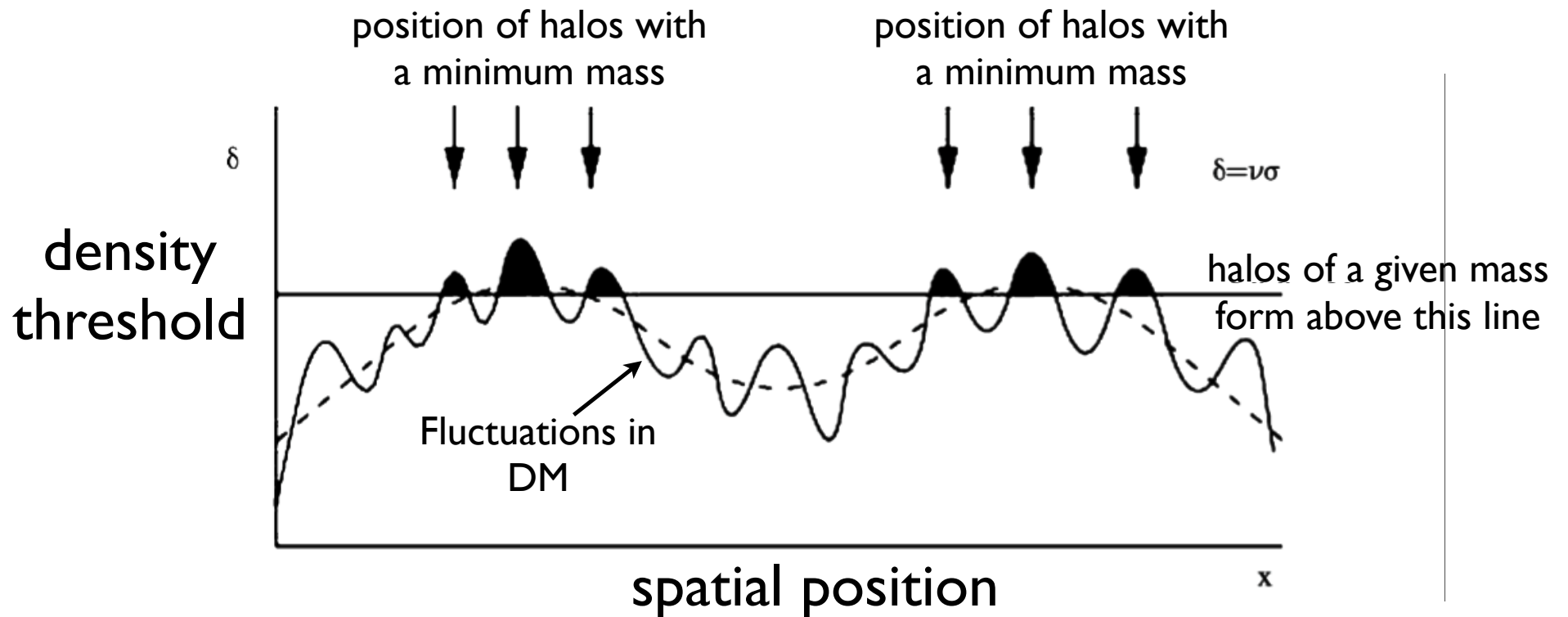
Typically the growth factor scales as the size of the universe (if $\Omega = 1$)

Calculating $P(k)_{\text{DMhalo}}$

$P(k)_{\text{DMhalo}}$ is calculated by running a big N-body simulation with billions of particles and then extracting the power spectrum of halos from the result.

$P(k)_{\text{DMhalo}}$ will be higher than the $P(k)_{\text{DM}}$, particularly if one consider the power in the spatial distribution of the most massive

This is due to massive halos forming on top of large-scale overdensities and small fluctuations can push one over the density threshold (needed to produce the massive halo)

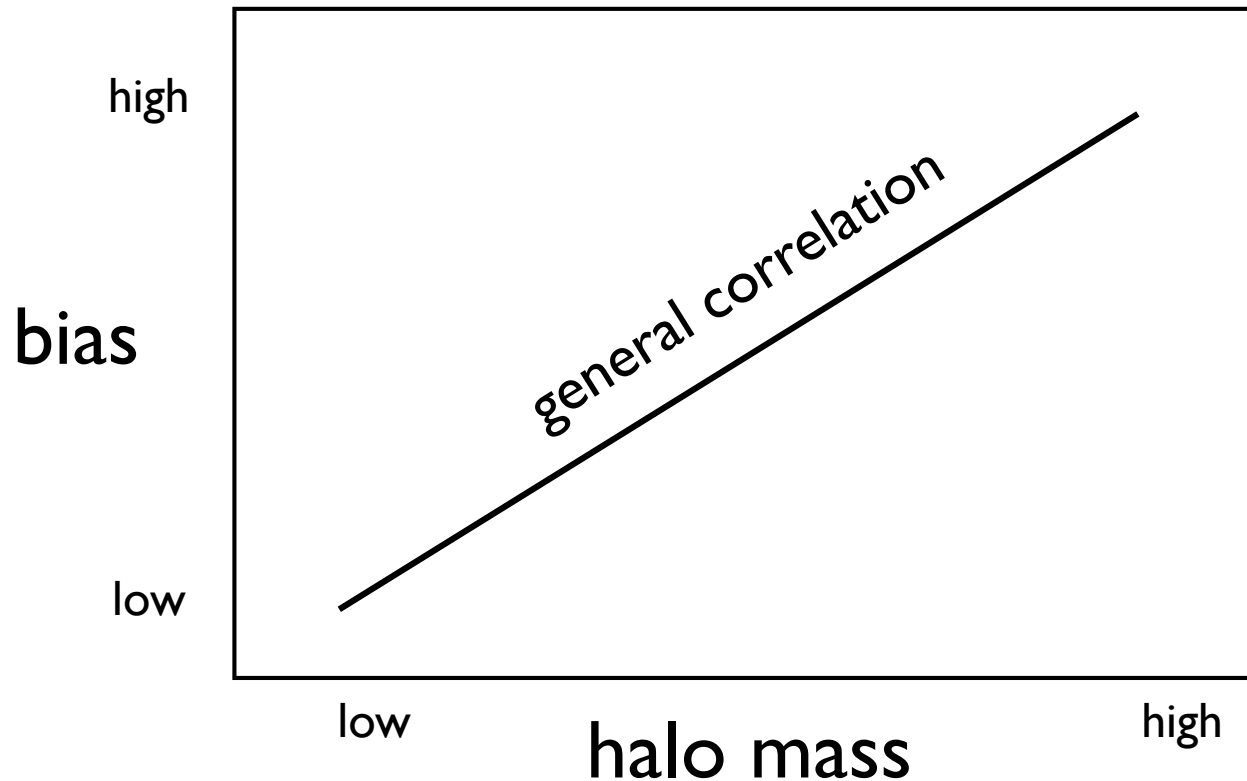


The bias factor b relates the following two power spectra:

$$P(k)_{\text{DMHalos}} = b^2 P(k)_{\text{DM}}$$

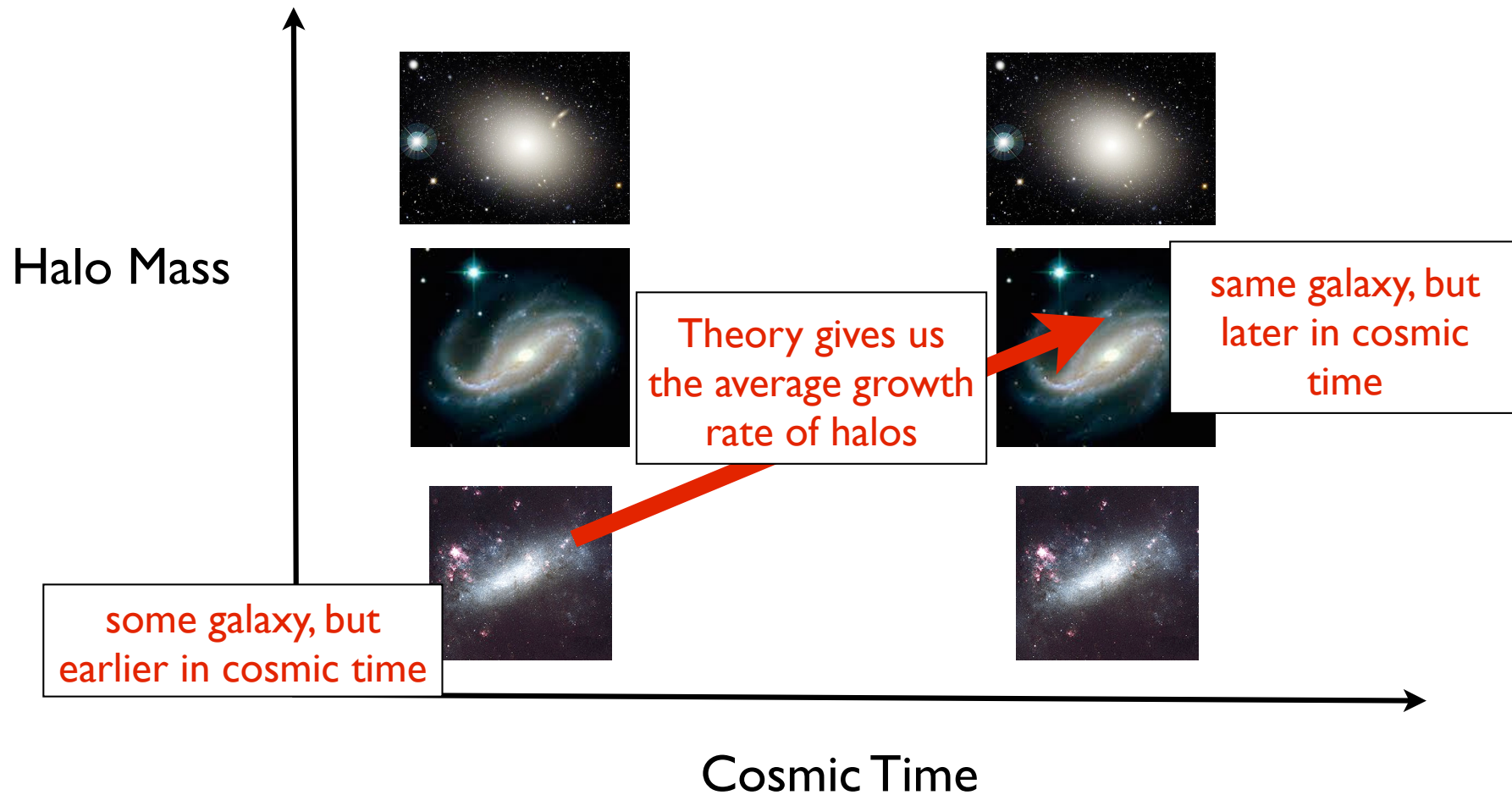
How does it depend on mass?

Less massive galaxies, on the other hand, are less biased tracers of the underlying dark matter distribution.



Why is it useful to learn about the dark matter halos in which galaxies live?

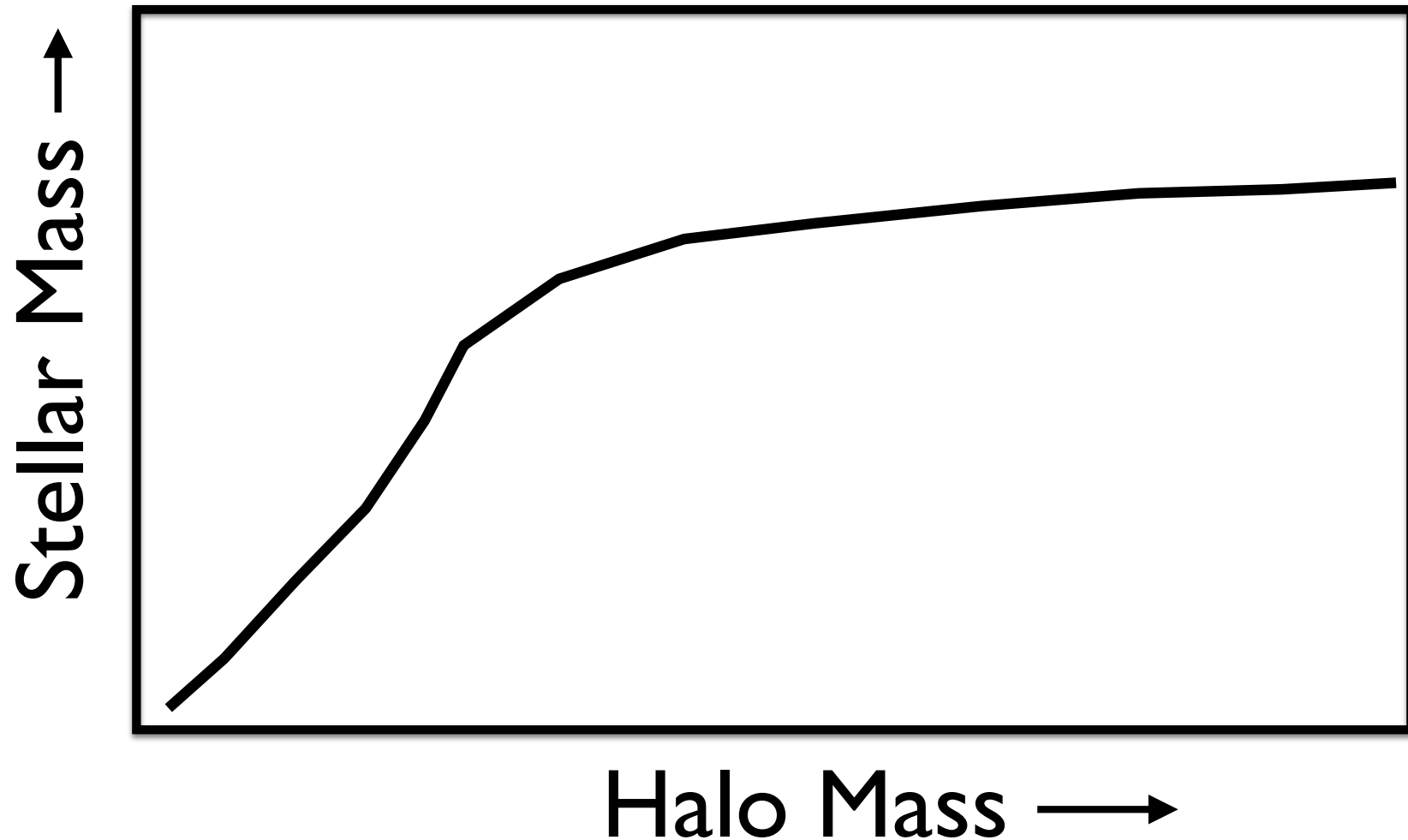
It provides us a powerful tool for tracing the same population of galaxies through cosmic time.



Besides clustering, is there another commonly used method to estimate halo mass?

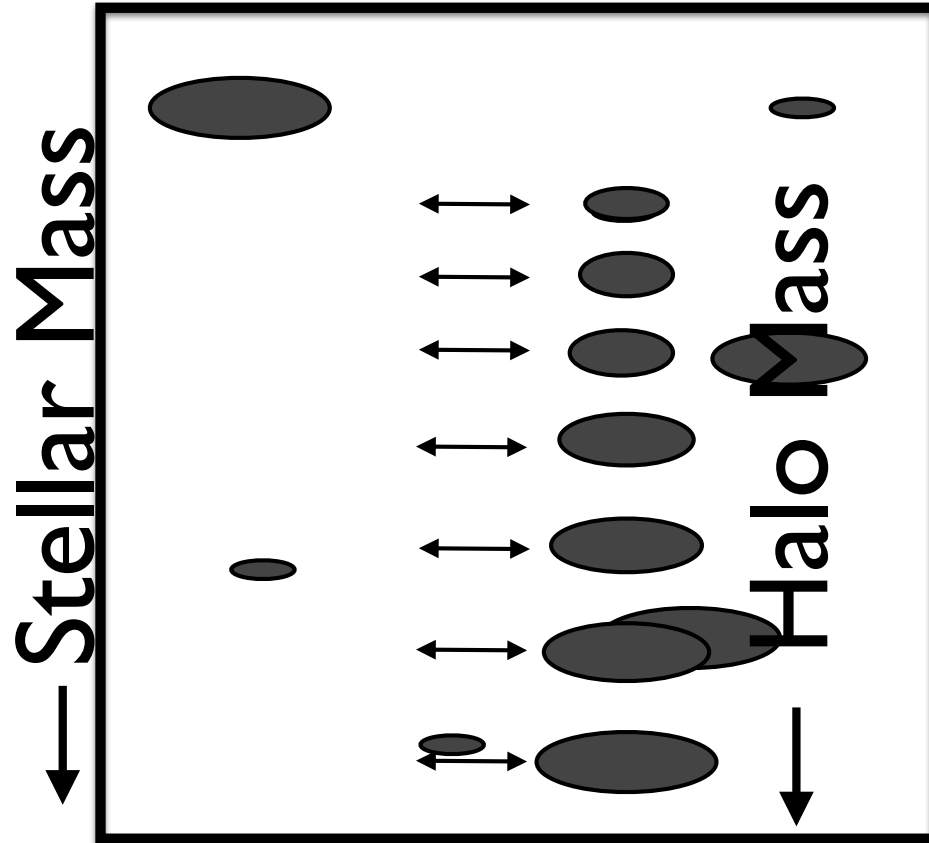
Yes — and it is called abundance matching

Basic idea leverages stellar mass in galaxies likely being a strictly increasing function of halo mass



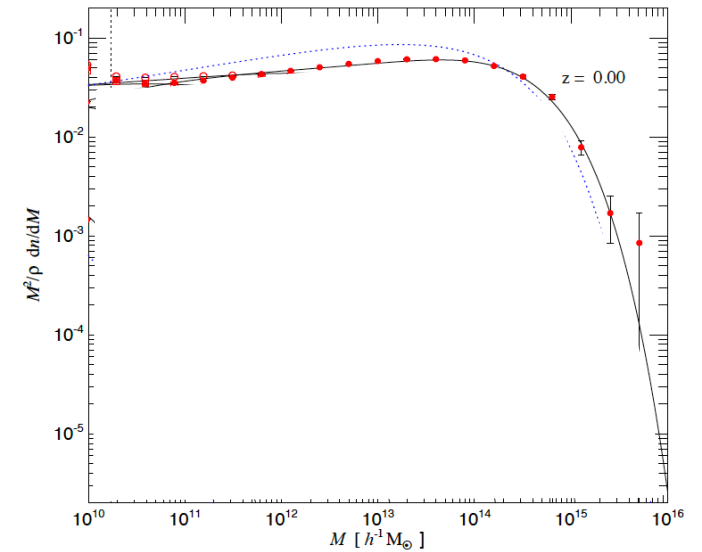
What are the halo masses of galaxies in this volume?

Survey of a volume of the universe at $z \sim 1$



Volume
Density of
Dark
Matter
Halos of a
Given
Mass

Halo Mass Function



Dark Matter Halo Mass

Assuming a monotonic 1-1 correspondence between stellar and halo mass, abundance matching allows for an estimate of the halo masses for individual galaxies

What can learn about the formation and evolution of galaxies from the stars we find in these galaxies?

What can learn about the formation and evolution of galaxies from the stars we find in these galaxies?

Ideally, we would use the observed stars to reconstruct the history of star formation in a galaxy.

We would like to determine the function:

SFR(t)

where t is time.

Generally, this area of study is divided into two subfields:

1) Resolved Stellar Population Analyses

Can measure the luminosity and color of individual stars in the nearby object

Useful for studies of Nearby Galaxies and Star Clusters

2) Integrated (Unresolved) Stellar Population Analyses

One cannot resolve the light from individual stars and they all blend together.

Used in Studying Distant Galaxies (galaxies greater than a few Mpc away)

I) Resolved Stellar Population Analyses

For especially nearby galaxies, we can resolve out the individual stars and place them on a color-magnitude diagram.

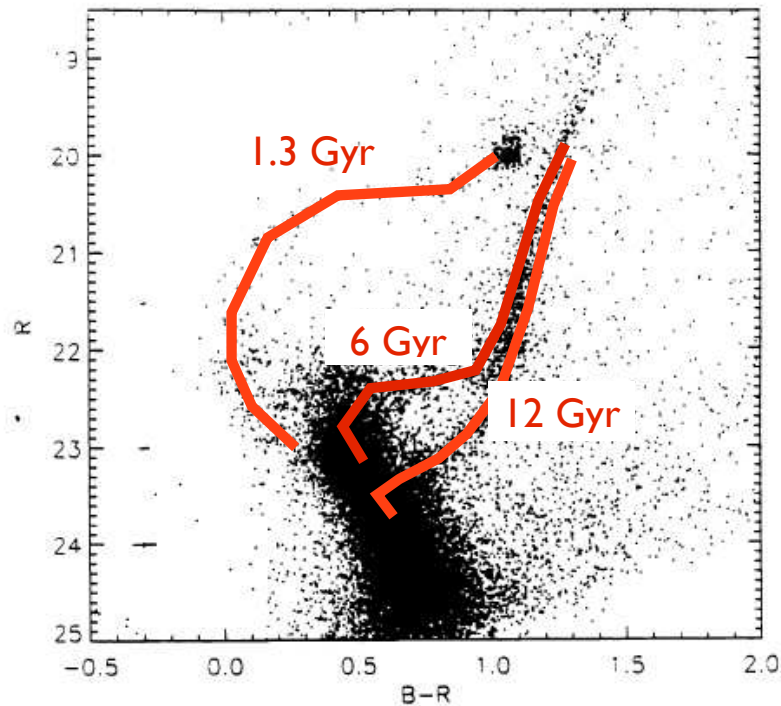


Figure 1. The CMD of the Carina dSph. The median internal photometric error at magnitudes $R = 21.5$, 23, and 24 are shown as on the left-hand side.

The above galaxy shows evidence for forming stars at three distinct points in time.

By comparing the position of the stars on the color magnitude diagram with that expected for specific isochrones, we can reconstruct the SF history

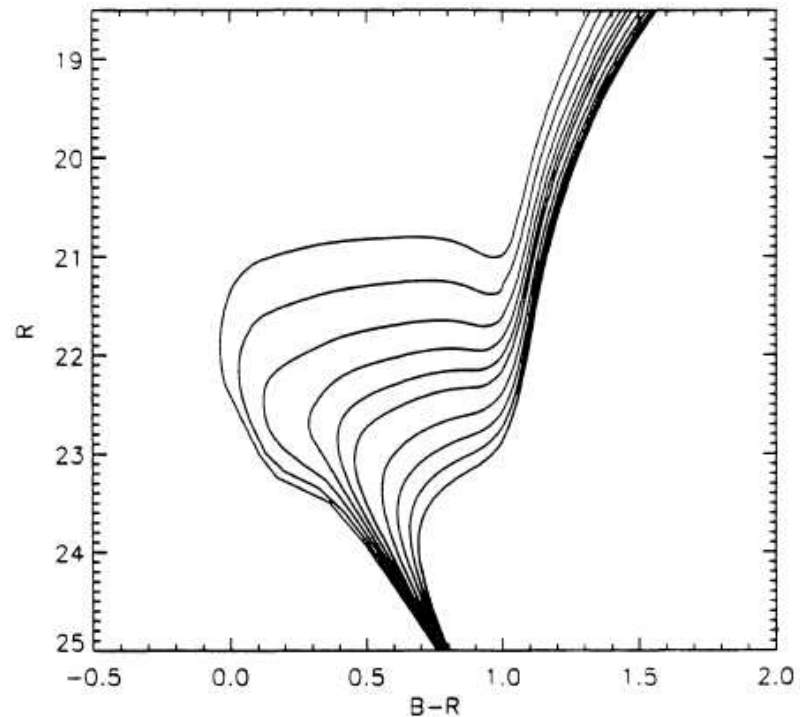


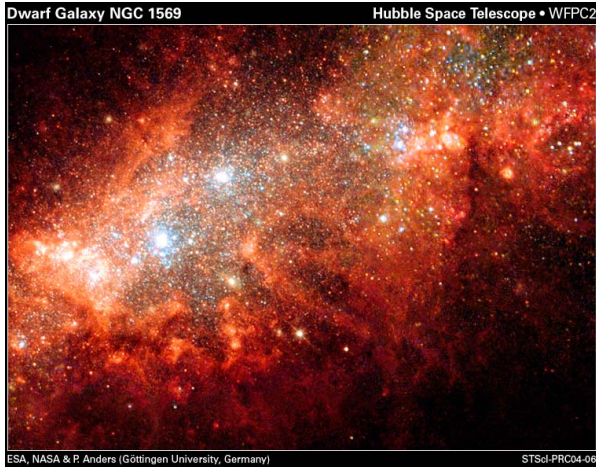
Figure 2. New theoretical isochrones from Vandenberg *et al.* (1996) for $[\text{Fe}/\text{H}] = -1.84$ and ages of 1.3, 2, 3, 4, 5, 6, 8, 10, 12, and 14 Gyr at the distance and reddening of the Carina dSph.

NOW new material for this
week

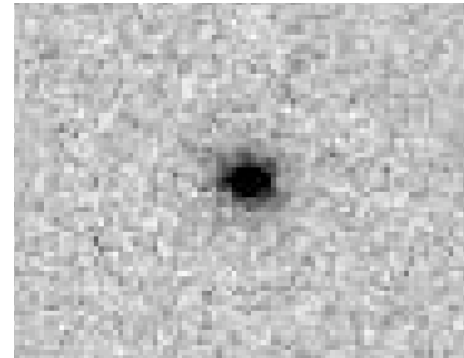
2) Integrated Stellar Population Analyses

If one cannot resolve out the individual stars, one must look at the total spectrum coming from all stars in a galaxy.

The Problem



“What you’d like to get”



“What you get”

(George Hau)

WHAT CAN WE MEASURE?

Broad-band colours (B-V, J-K, etc).

Surface brightness fluctuations (sometimes)

Spectroscopic features (absorption lines)

Credit: Russell Smith

2) Integrated Stellar Population Analyses

One attempts to model the observed spectrum using the following inputs:

- I) Stellar Initial Mass Function $\phi(m)$: Defines the Fraction of Forming Stars as a function of the stellar mass m . Appears to be well described by the Salpeter power-law function $m^{-2.35}$ at intermediate to high masses.

At low-to-intermediate stellar masses, the number of stars formed are lower than one would predict based on the Salpeter power law.

One IMF that takes this into account is the Chabrier IMF:

$$m\phi(m) = \exp\left[-\frac{(\log(m) - \log(m_c))^2}{2\sigma^2}\right] \text{ for } m \leq 1 M_{\odot}$$
$$= m^{-1.3} \text{ for } m \geq 1 M_{\odot}$$

with $m_c = 0.08M_{\odot}$ and $\sigma = 0.69$

Tracks & Isochrones

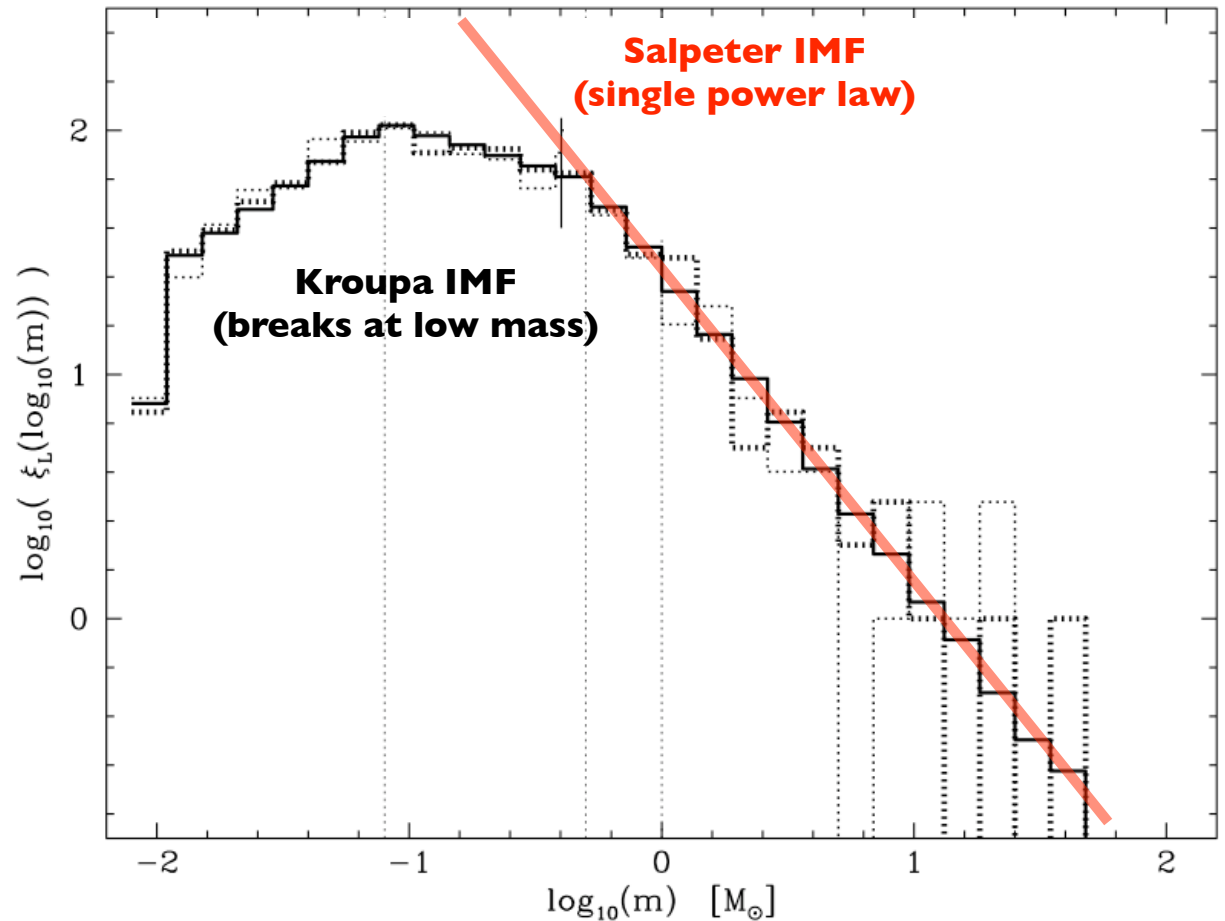
INITIAL MASS FUNCTION

How many stars formed per unit mass?

Determines number of stars expected at each point along isochrone.

Constrained from detailed observations of star-forming regions in the MW.

Applied to wide range of environments beyond MW!



Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

2) Metallicity and abundance ratios of the stars which are forming:

Metallicity can have a significant impact on the light emitted by stars, making them appear redder. It can also slow down the evolution of stars somewhat.

Aside: What Exactly is Metallicity?

“METAL” CONTENT OF THE GAS CLOUD FROM WHICH THE STARS FORMED

Usually assume the surface composition reflects the original composition.

REMEMBER: ASTRONOMERS THINK CARBON IS A METAL....

Stellar modellers express chemical mixture of material as mass fractions

X or H = mass fraction of H , Y = mass fraction of He

Z = mass fraction of everything else = “metals”

EMPIRICAL NOTATION

For measurements of metallicity in stellar atmospheres, we usually express abundances in terms of number density (not mass fractions). Total metallicity is often expressed as $[Z/H] = \log(N_Z/N_H) - \log(N_Z/N_H)_{\text{sun}}$. Then:

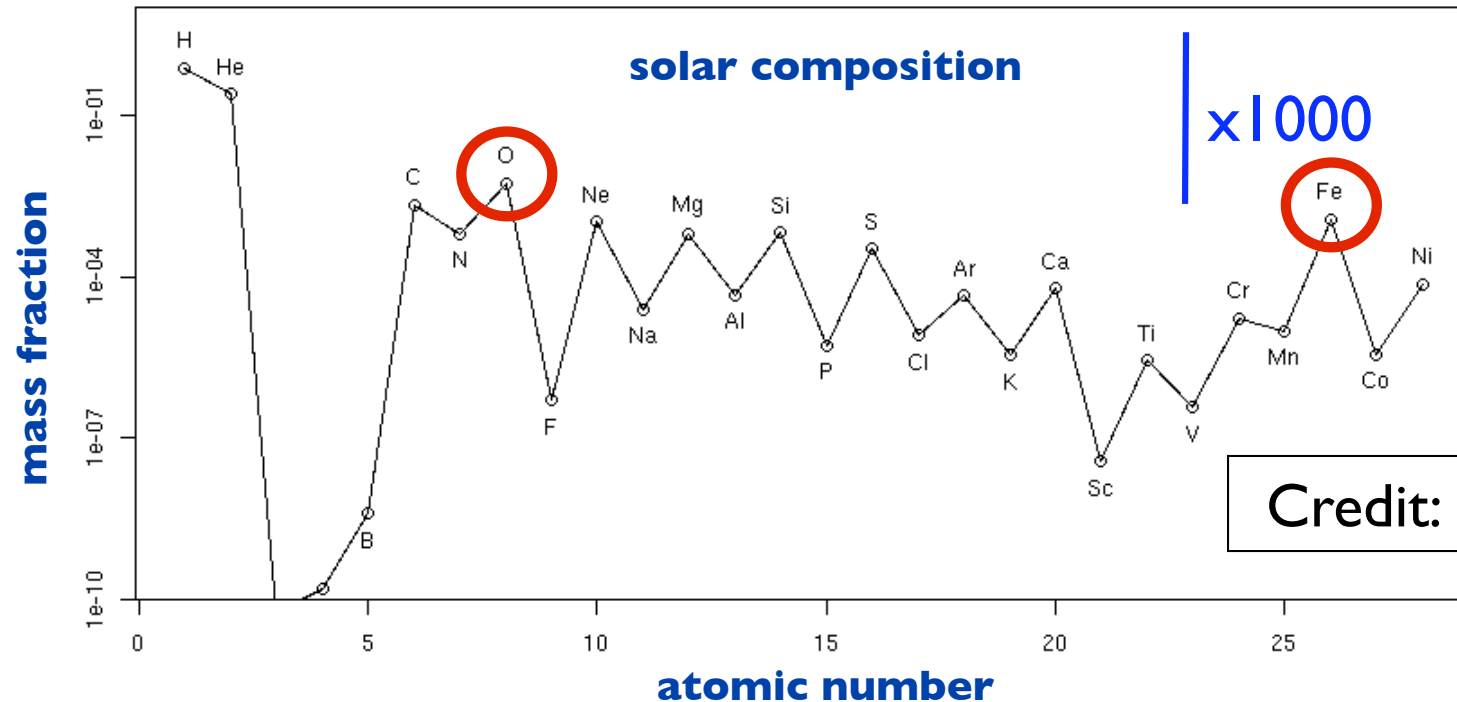
$[Z/H] = 0$ is “solar metallicity”,

$[Z/H] = +0.3$ is “twice-solar”,

$[Z/H] = -1$ is “one tenth solar”, etc.

Credit: Russell Smith

More on Metallicity



Credit: Russell Smith

BUT WHAT IS COMPOSITION OF “Z” ?

Note that O is the most important element for stellar evolution: it is abundant and a big contributor to the opacities.

But unfortunately it is very hard actually to measure O from stellar spectra!

Much easier to measure Fe which has lots of absorption lines in the optical.

So we often talk about $[Fe/H]$ instead. These are equivalent if $[O/Fe]$ is solar, i.e. mixture between metals always the same. (We'll come back to this!)

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

3) Detailed Stellar Evolution Models:

While most phases of stellar evolution seem to be well understood, other rarer phases of stellar evolution like the horizontal branch evolution or the asymptotic giant branch evolution are less well understood. This can make the predictions of the models uncertain.

Tracks & Isochrones

TRACKS

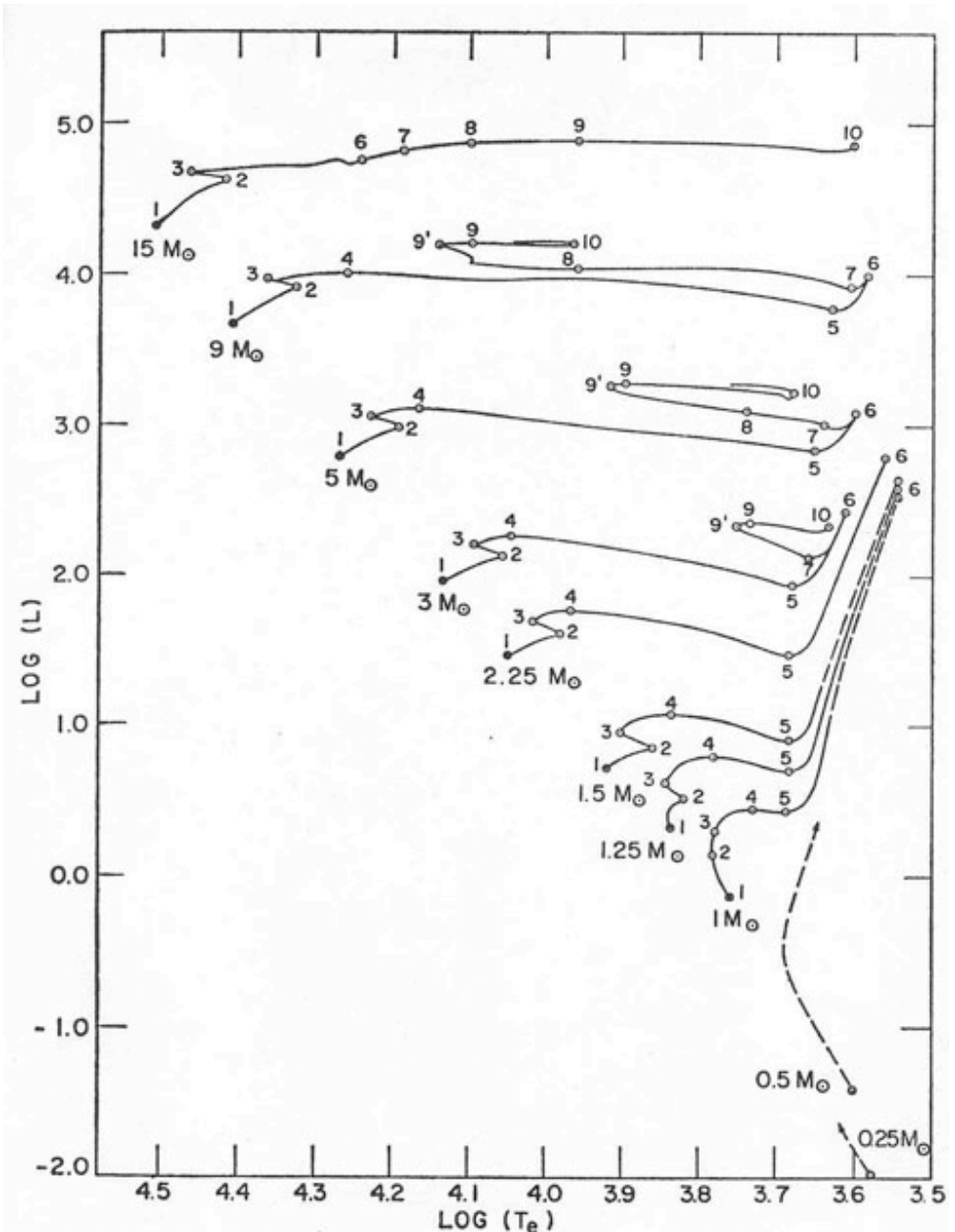
“Tracks” are trajectories of individual stars in the HRD.

Stellar evolutionary tracks are tables describing the evolving properties:

- luminosity,
- temperature,
- evolving mass, etc

as a function of initial mass (and initial chemical composition).

In detail, the tracks are computed from stellar evolution models (Padova, Geneva, BaSTI etc).



2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

4) Spectra of Stars at a given temperature, metallicity, surface gravity.

One can calculate the spectra of stars theoretically from stellar atmosphere modelling. However, the predictions from theory often differ from observations -- suggesting that one may want to use real spectra of actual stars. The challenge with using real

Empirical Spectral Libraries

OBSERVED SPECTRA OF STARS

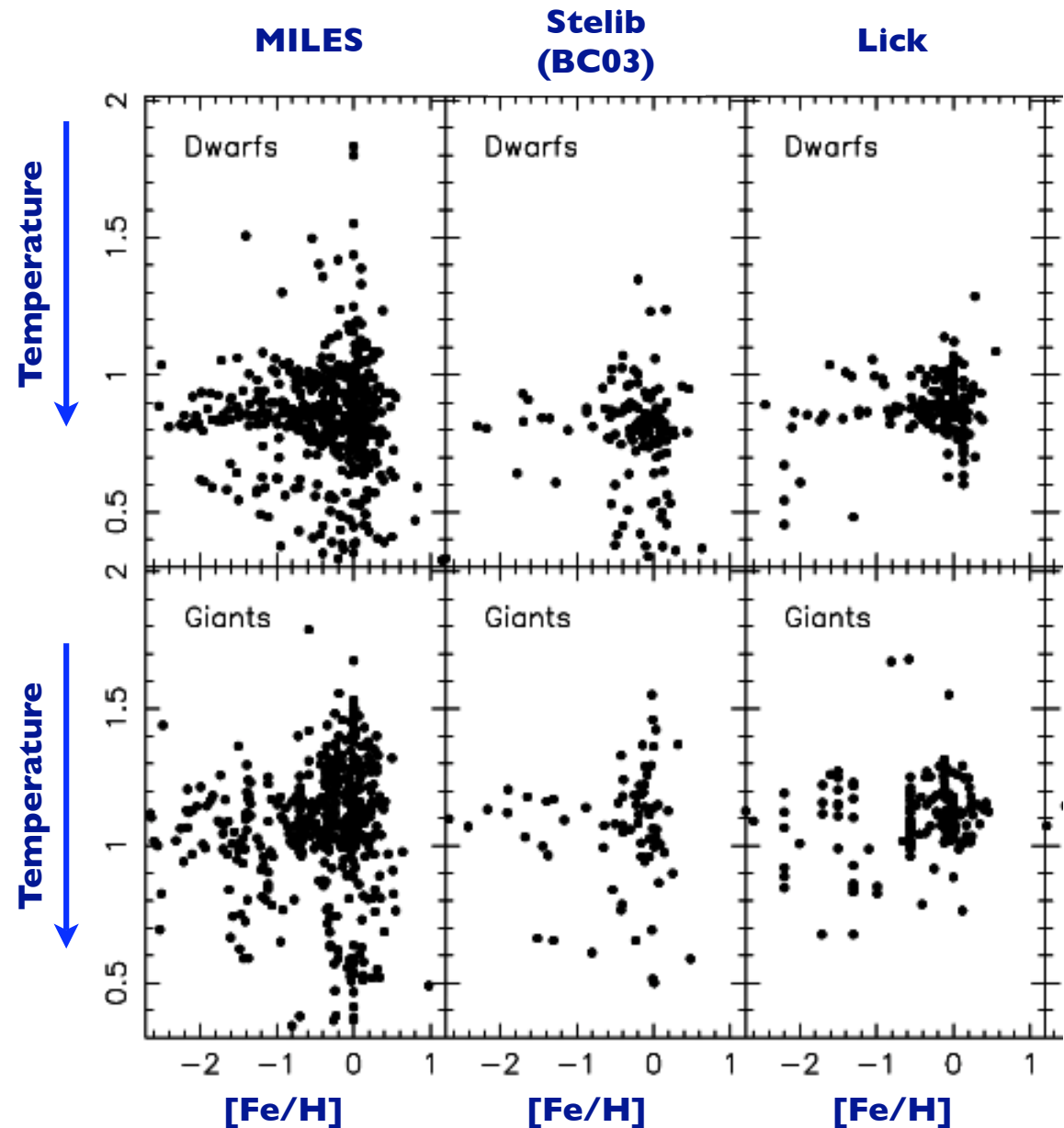
Desirable to cover large range in T_{eff} , $\log g$ (=“gravity” i.e. dwarf vs giant) and Fe/H .

And to know the atmospheric parameters of the stars (difficult for the coolest stars.)

PROBLEMS

All the stars in empirical libraries are in our galaxy, which limits parameter coverage.

Valid application of the models implicitly restricted to systems with stars “like” those in our galaxy.



Sanchez-Blazquez et al. (2006)

Credit: Russell Smith

Theoretical Spectral Libraries

THEORETICAL LIBRARIES

Based on stellar atmosphere models, e.g. ATLAS9 (Castelli & Kurucz 2003) MARCS (Gustafsson et al. 2003)

ADVANTAGES

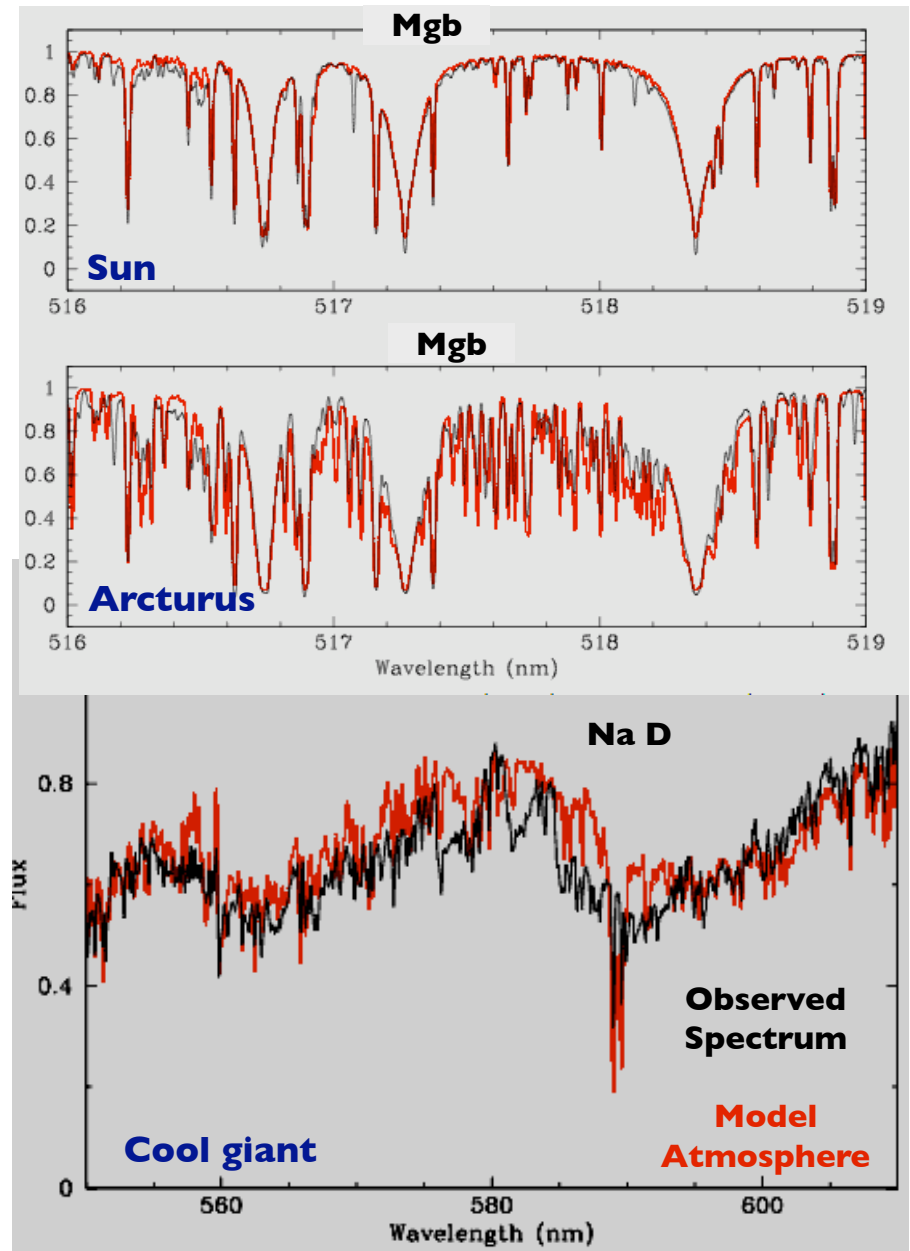
Much more flexible than empirical libraries: In principle can obtain spectra for any value of T_{eff} , $\log g$ and Fe/H (and any chemical mixture).

CHALLENGES

Complex atmosphere physics (especially for hottest and coolest stars) may not be adequately modelled.

Empirical atomic and (especially) molecular line-lists may not be complete enough.

Include QM-predicted lines not verified in lab? These can be badly wrong in detail, but needed for accurate colours (Coelho et al.)



Coelho et al. (2007)

Credit: Russell Smith

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

5) Star Formation History of Galaxies

This is the typical input that people assume changes from galaxy to galaxy.

The simplest model is single burst stellar population models... where one assumes all the stars in a galaxy formed at a single point in the past.

Such models can work well for describing the stellar populations of elliptical galaxies and globular clusters, where most of the stars were first formed long ago in the past.

2) Integrated Stellar Population Analyses

Other inputs to stellar population analyses:

5) Star Formation History of Galaxies

Another simple model is to assume that stars in a galaxy formed at a fixed constant rate with time.

Such models can work well for describing the stellar populations of late spiral and irregular galaxies.

One can try to parameterize all star formation histories between a constant star formation model and a fixed burst in the past adopting an exponentially declining star formation rate:

$$e^{-t/\tau}$$

where τ is the time scale on which the star-formation rate of some galaxy declines with time.

Note that $\tau = 0$ corresponds to a simple stellar population (all stars formed at some time in the past)

while $\tau = \infty$ corresponds to a constant star formation model.

Interpolating between $\tau = 0$ and $\tau = \infty$ gives SFR history for Hubble sequence galaxies in between ellipticals and irregular galaxies

2) Integrated Stellar Population Analyses

From the models, we can calculate the integrated spectrum of a galaxy, if all the stars formed at certain times in the past.

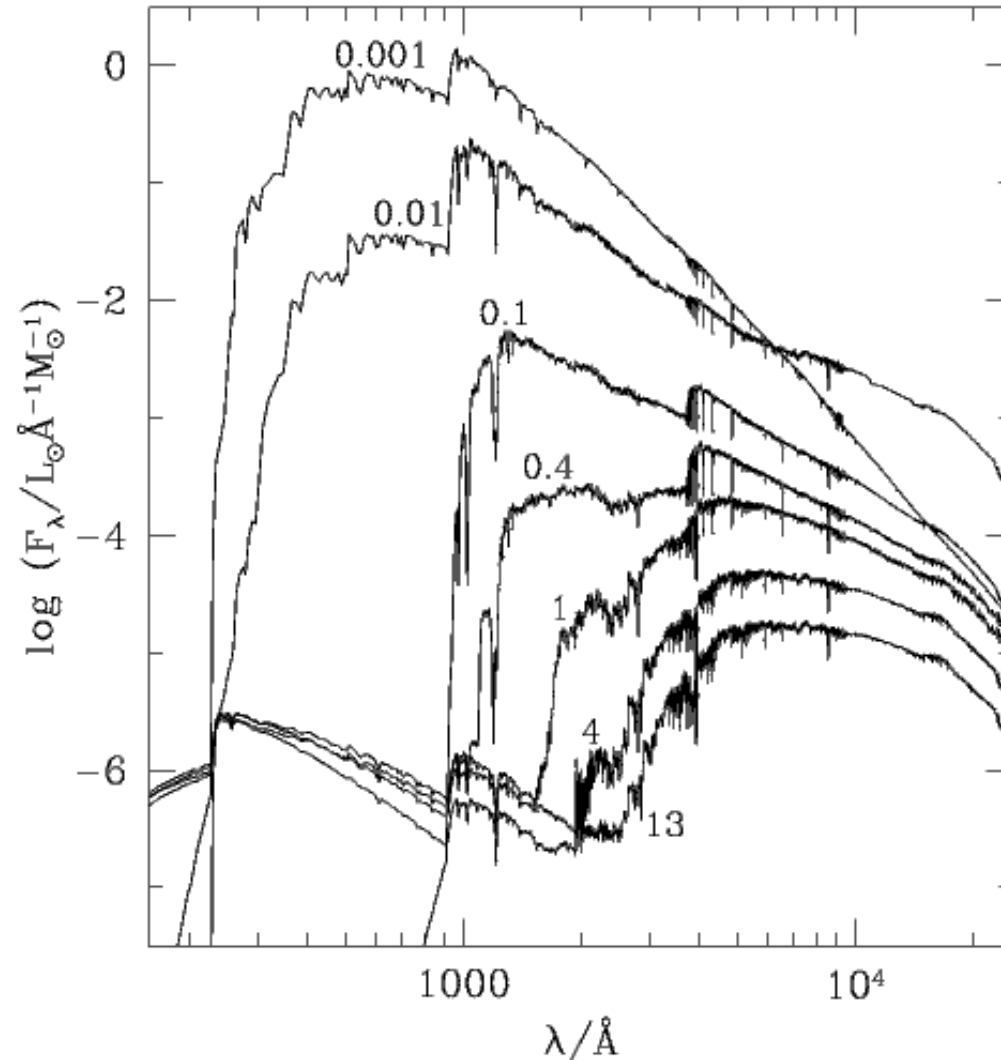


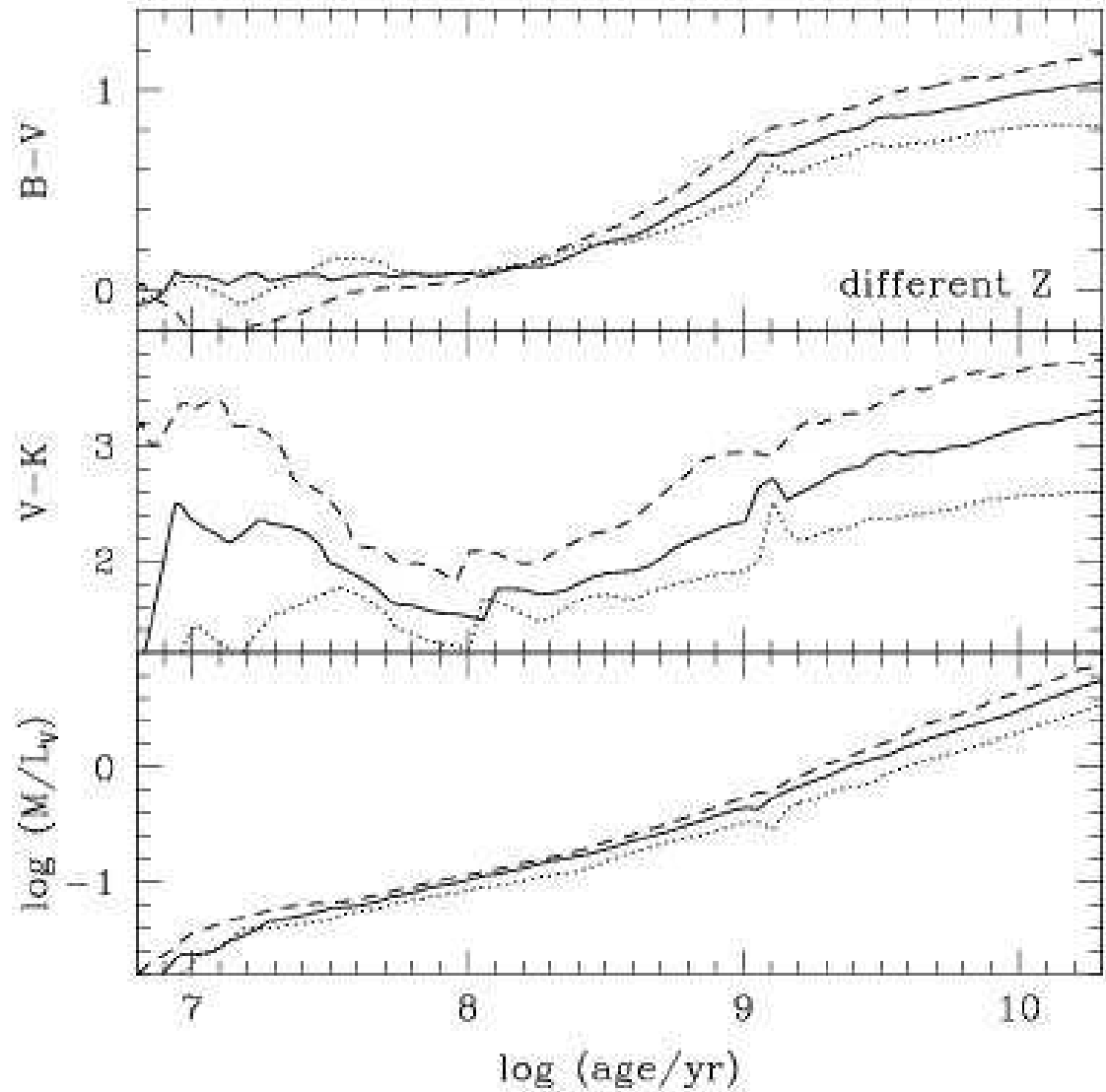
Figure 9. Spectral evolution of the standard SSP model of Section 3 for the solar metallicity. The STELIB/BaSeL 3.1 spectra have been extended blueward of 3200 Å and redward of 9500 Å using the Pickles medium-resolution library. Ages are indicated next to the spectra (in Gyr).

2) Integrated Stellar Population Analyses

How would we expect the colors or mass-to-light ratios to change with time for a simple stellar population?

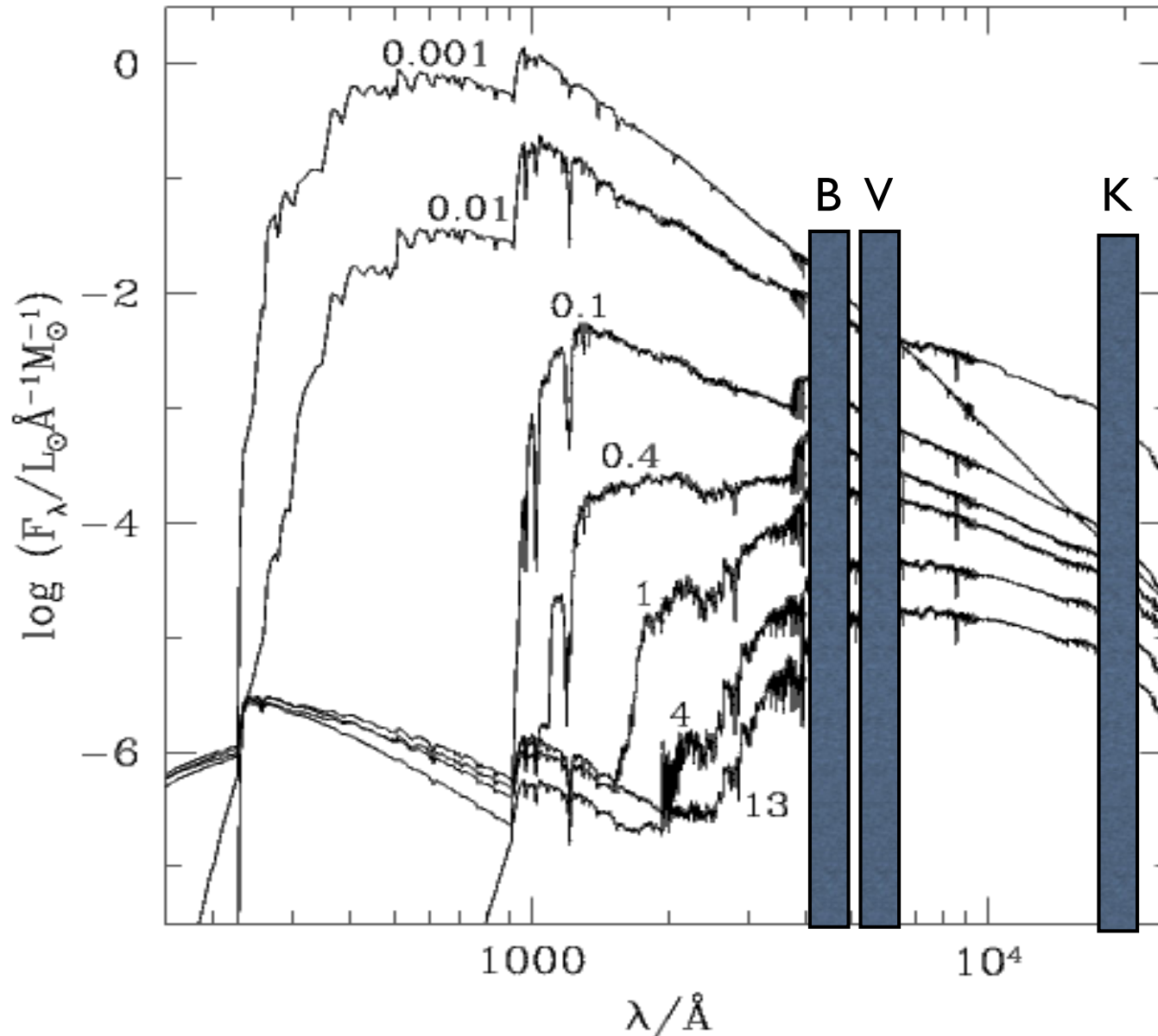
Here are the result for a few different metallicities:

- 0.2 Z_{solar}
- 1 Z_{solar}
- 2.5 Z_{solar}



2) Integrated Stellar Population Analyses

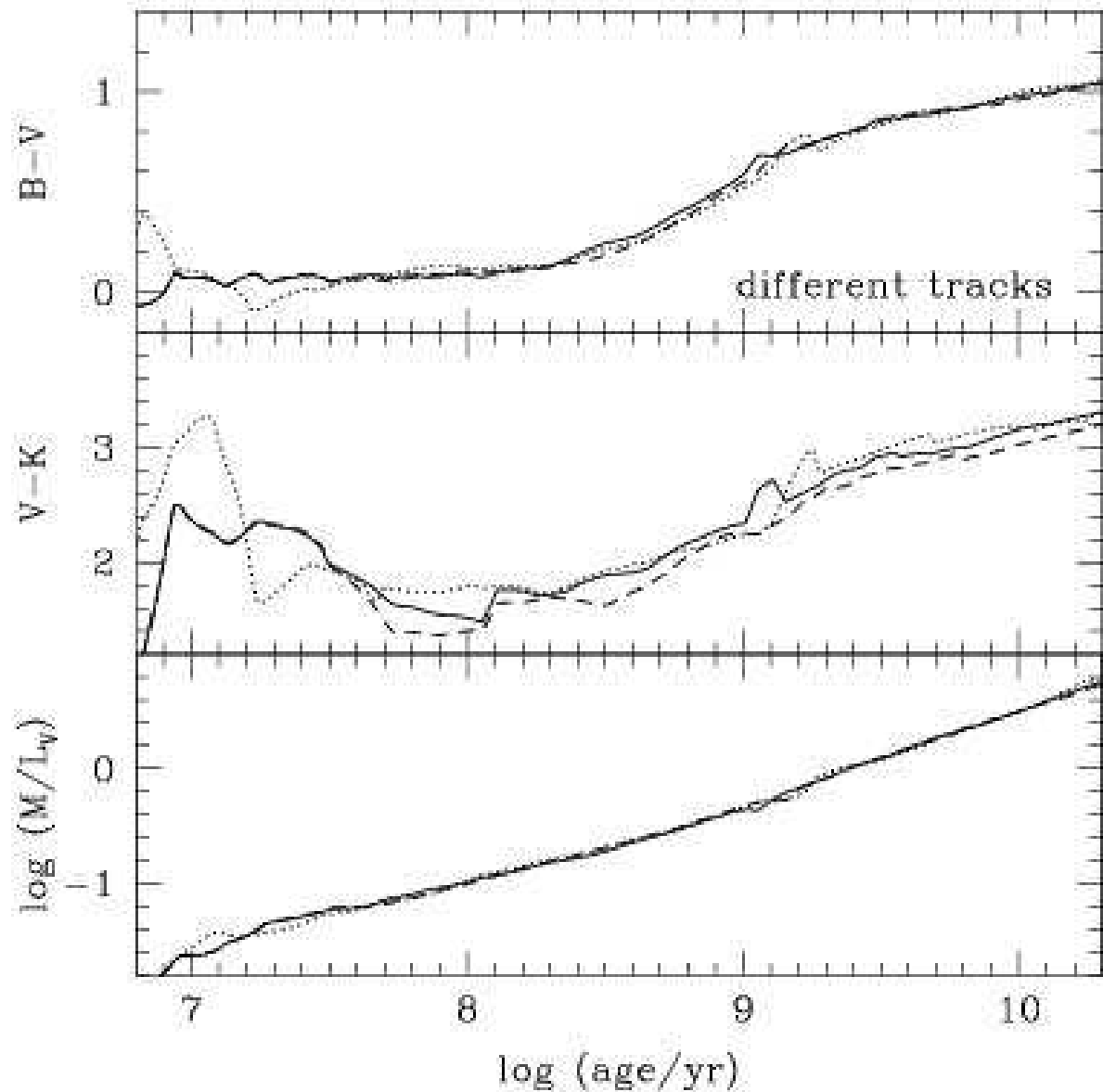
BACKGROUND INFORMATION (for previous slide): At which wavelengths do the B, V, and K bands refer to?



2) Integrated Stellar Population Analyses

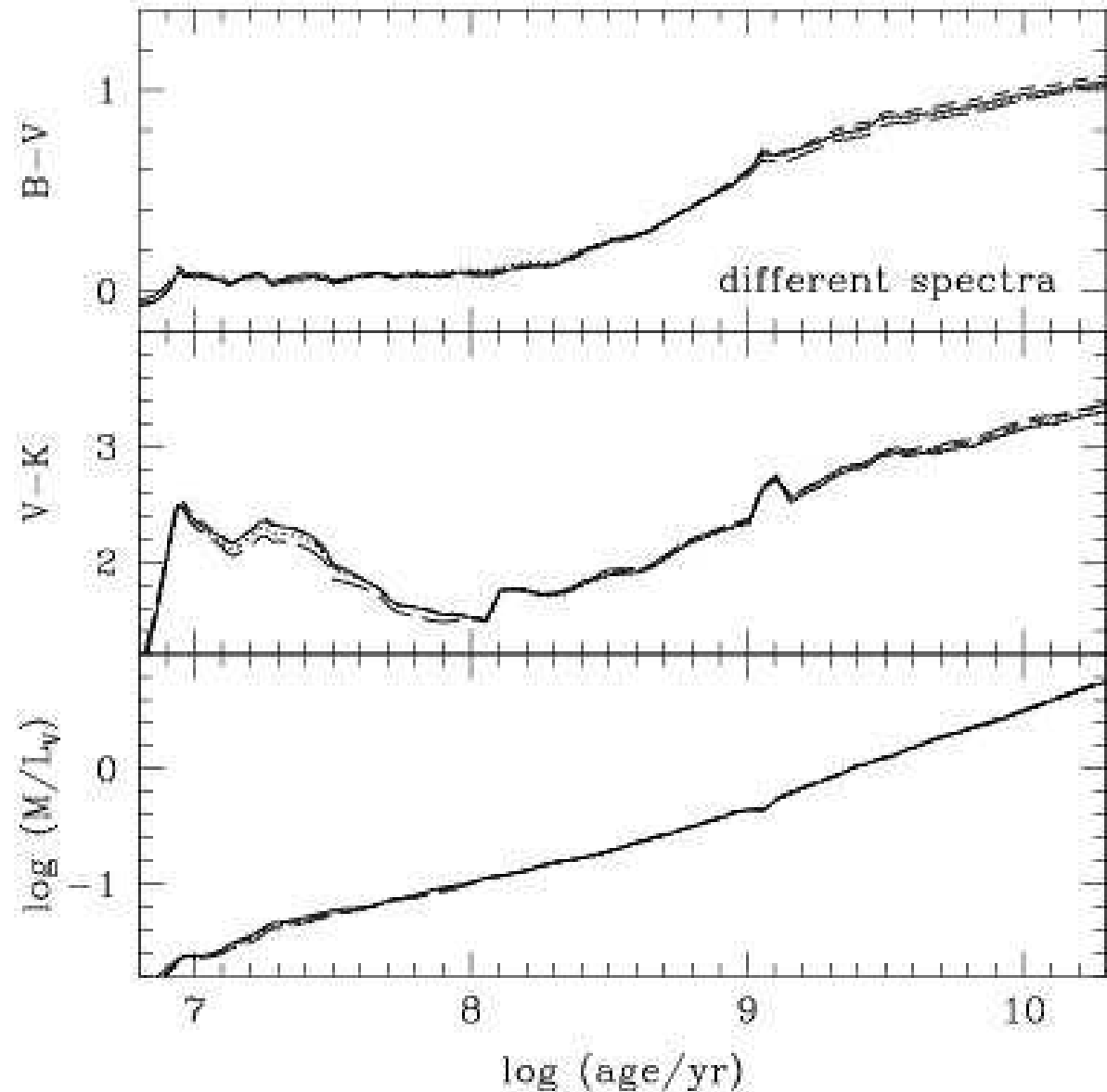
How do these results depend on the stellar tracks calculated for a simple stellar population?

Recall that the stellar tracks tell us how the luminosity, temperature, etc., of a star changes with time



2) Integrated Stellar Population Analyses

How do these results depend on the theoretical or empirical spectra one puts together with the stellar tracks?

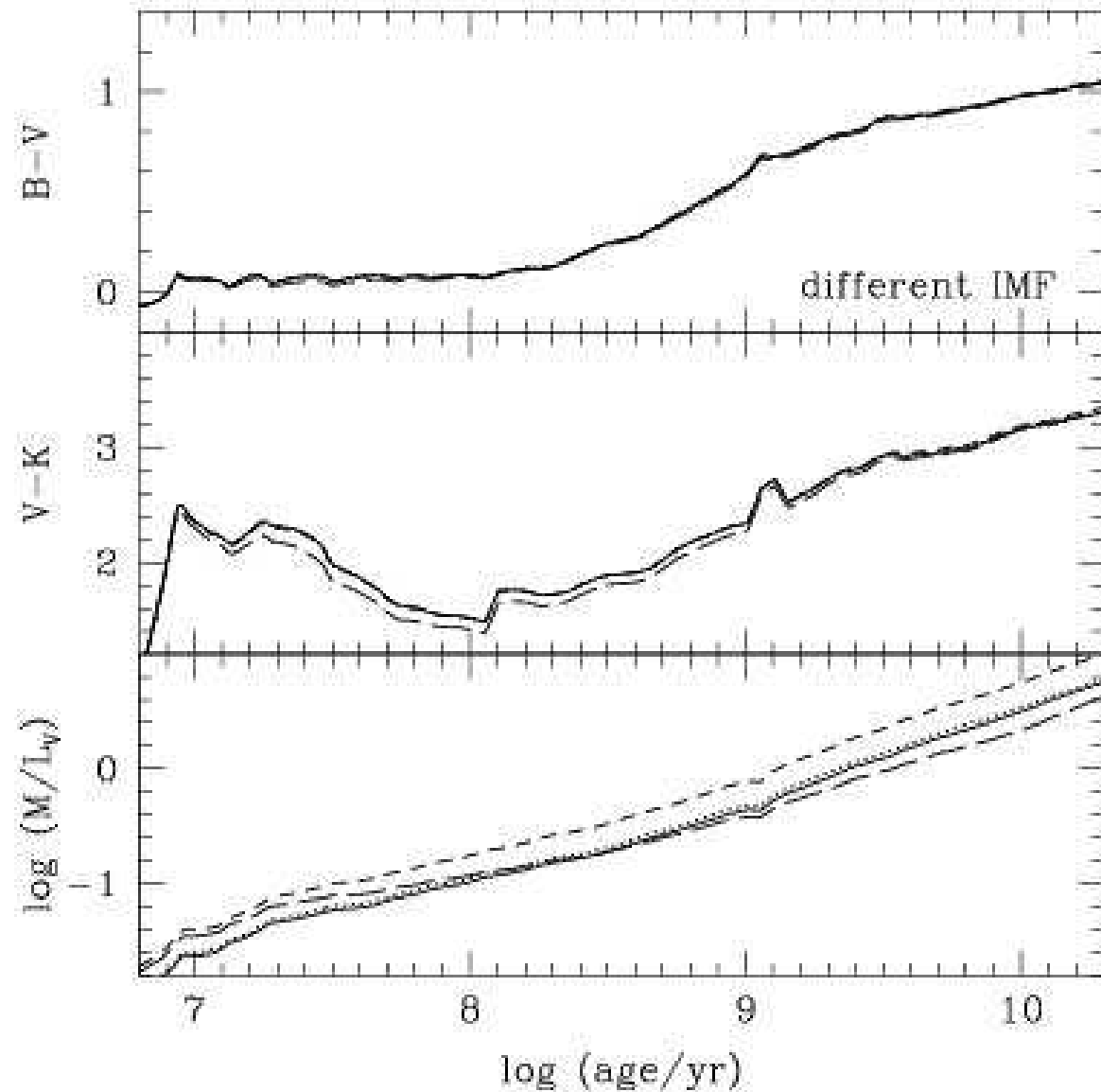


2) Integrated Stellar Population Analyses

Results shown for
different IMFs

Salpeter,
Kroupa,
Scalo,
Chabrier

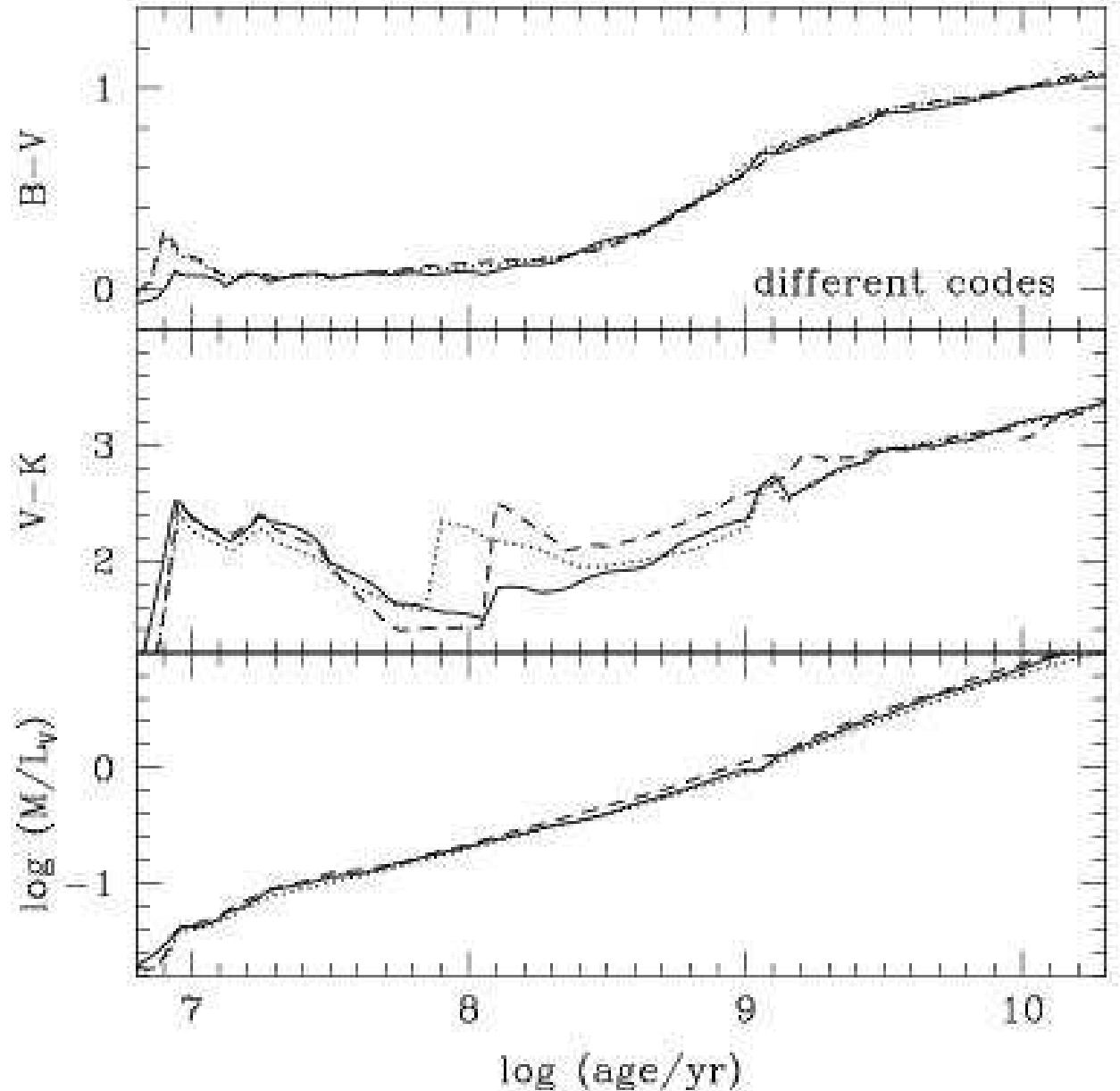
The lower three
IMFs all have fewer
low mass stars than
Salpeter



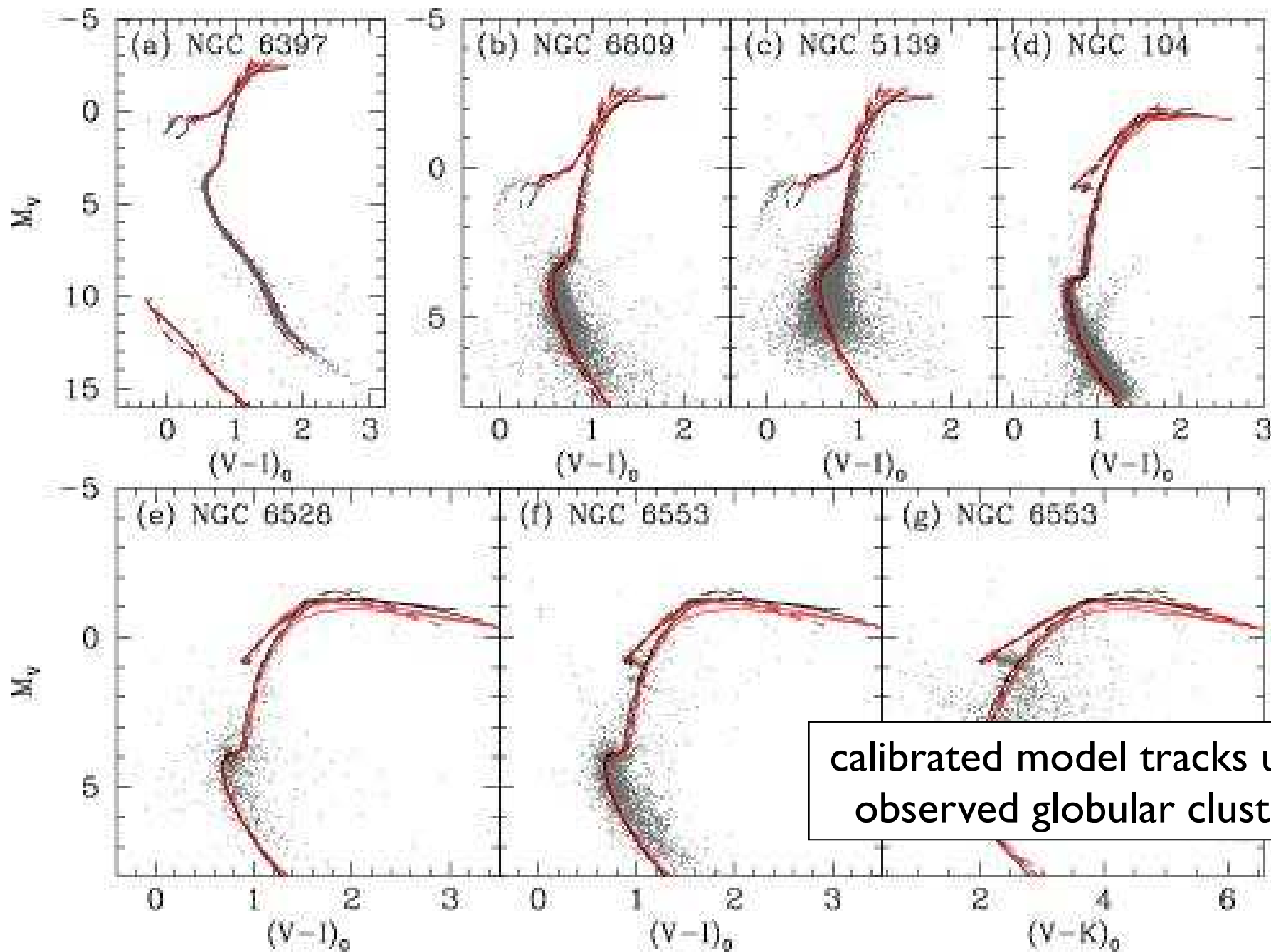
2) Integrated Stellar Population Analyses

How do the results depend on the code which puts together the results?

Differences generally occur depending on how one treats the asymptotic giant branch stars.



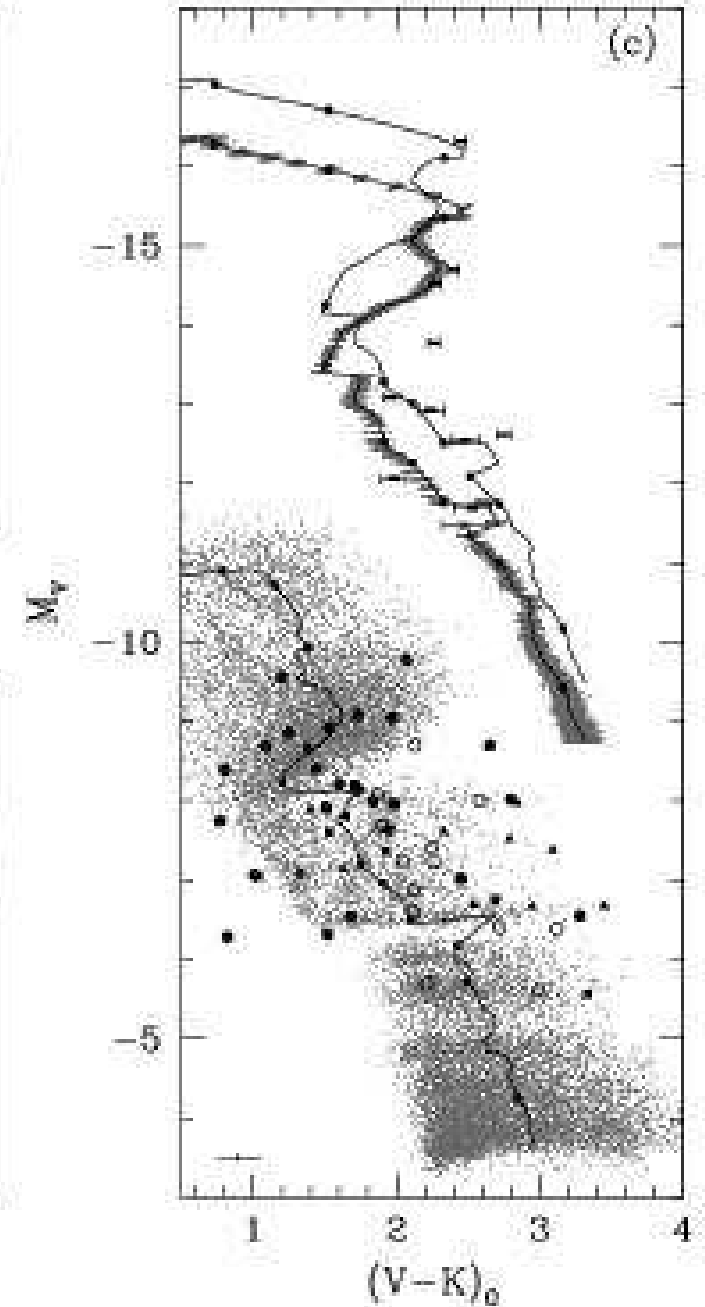
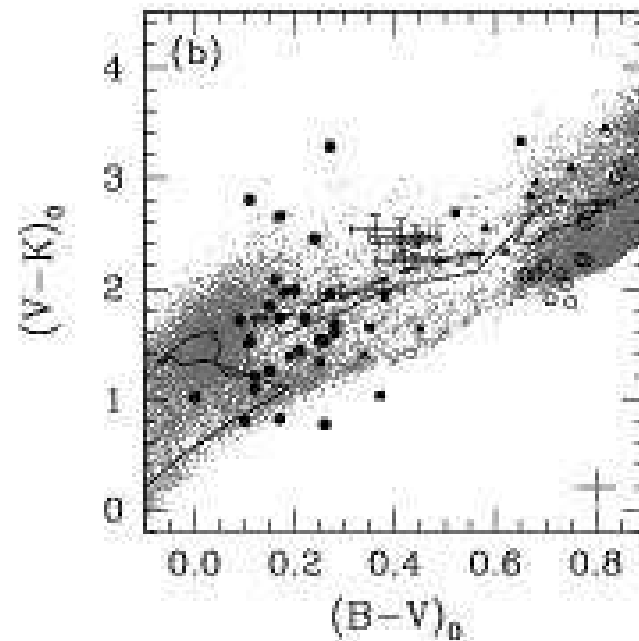
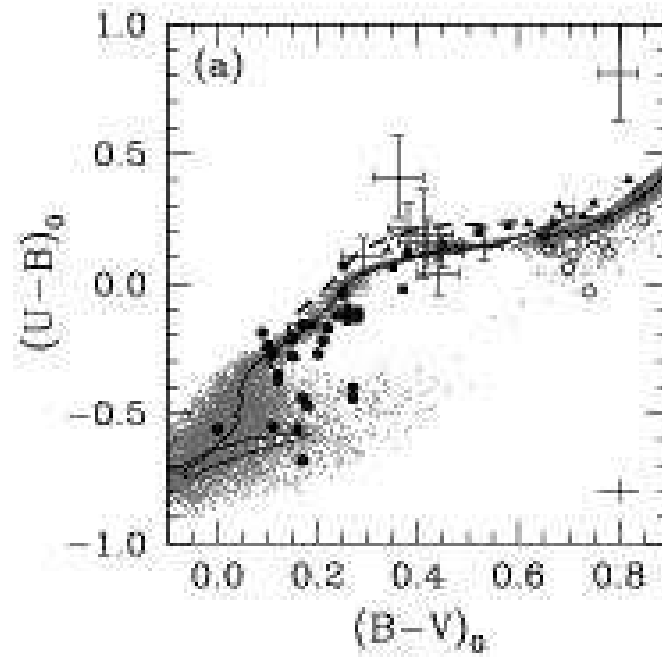
2) Integrated Stellar Population Analyses



calibrated model tracks using
observed globular clusters

2) Integrated Stellar Population Analyses

calibrate using
colors from
observed star
clusters



Age-Metallicity Degeneracy

A wide variety of different ages and metallicities for a stellar population produce approximately the same integrated spectrum:

It can therefore be quite challenging to determine both the age and metallicity of a galaxy uniquely.

There are subtle differences between these spectra.

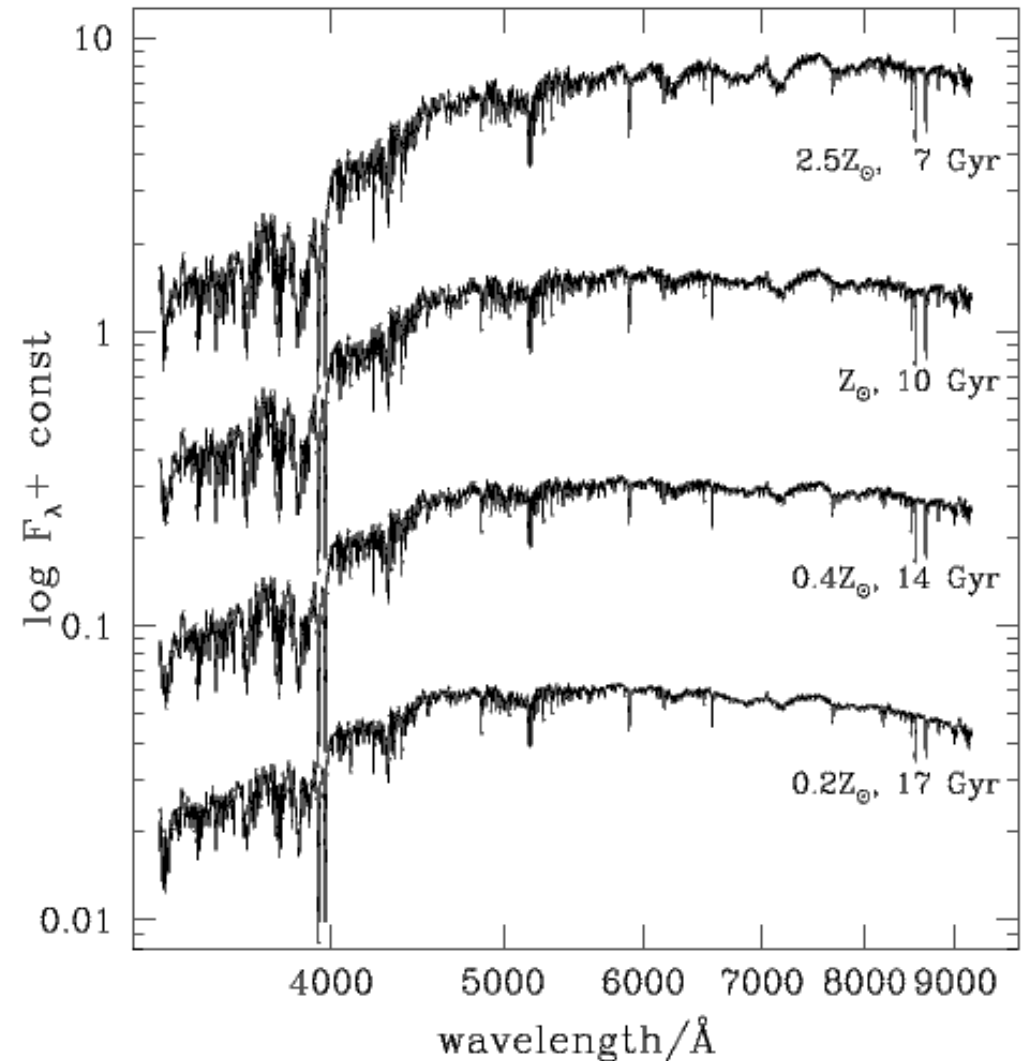


Figure 10. Spectra of the standard SSP model of Section 3 at different ages for different metallicities, as indicated. The prominent metallic features show a clear strengthening from the most metal-poor to the most metal-rich models, even though the shape of the spectral continuum is roughly similar in all models.

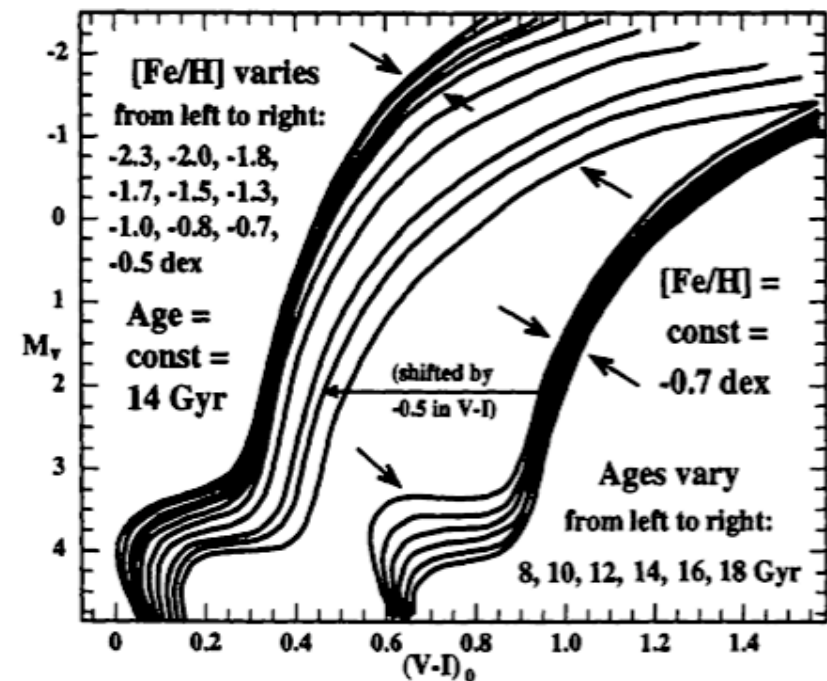
The Age-Metallicity Degeneracy

THE DREADED DEGENERACY

AGE -- Increasing age reddens the population by adding more luminosity to the RGB, removing hot stars from the MS.

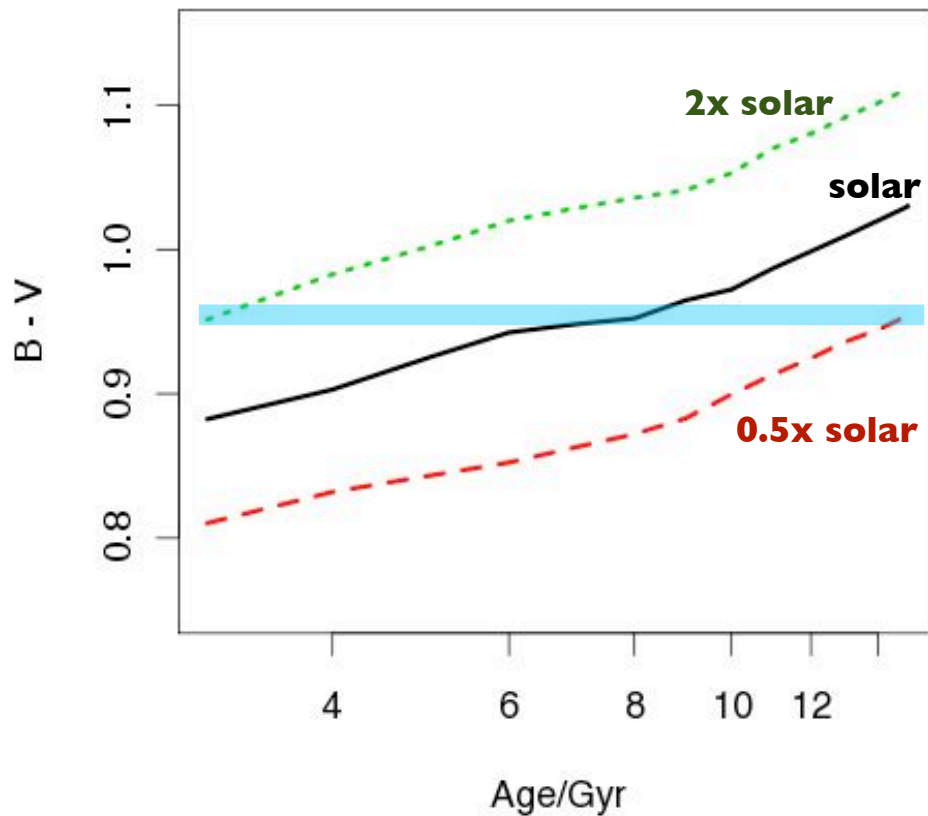
METALLICITY -- Increasing Metallicity reddens the population by changing the high-temperature opacities.

(Metallicity also reddens the population through increased line blanketing in cool phases.)



Credit: Russell Smith

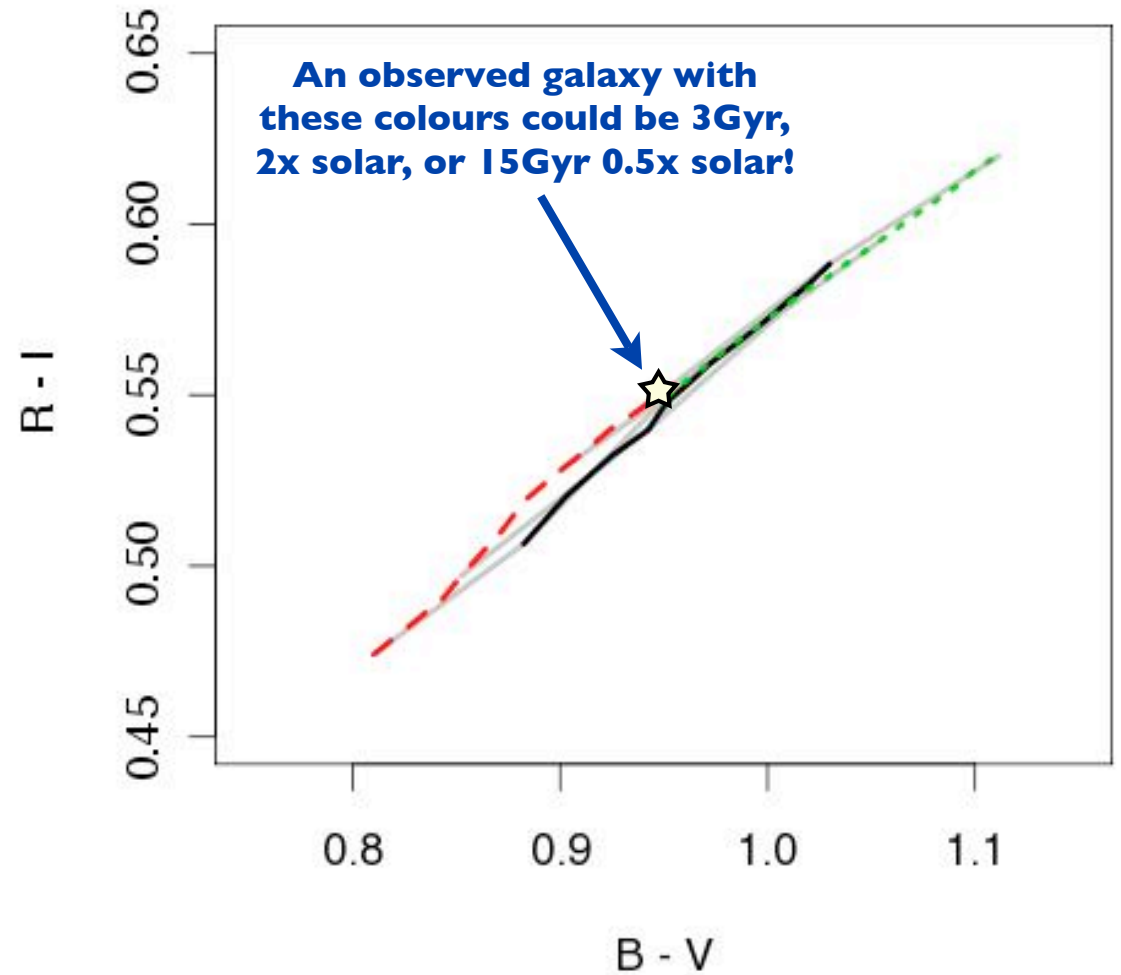
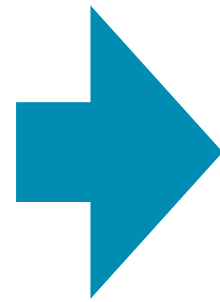
Age-Metallicity Degeneracy



Because both age and metallicity cause the population to redden, a single colour is not enough to disentangle the parameters.

Credit: Russell Smith

Age-Metallicity Degeneracy



A pair of (optical) colours is no help either!

Credit: Russell Smith

Disentangling age and metallicity

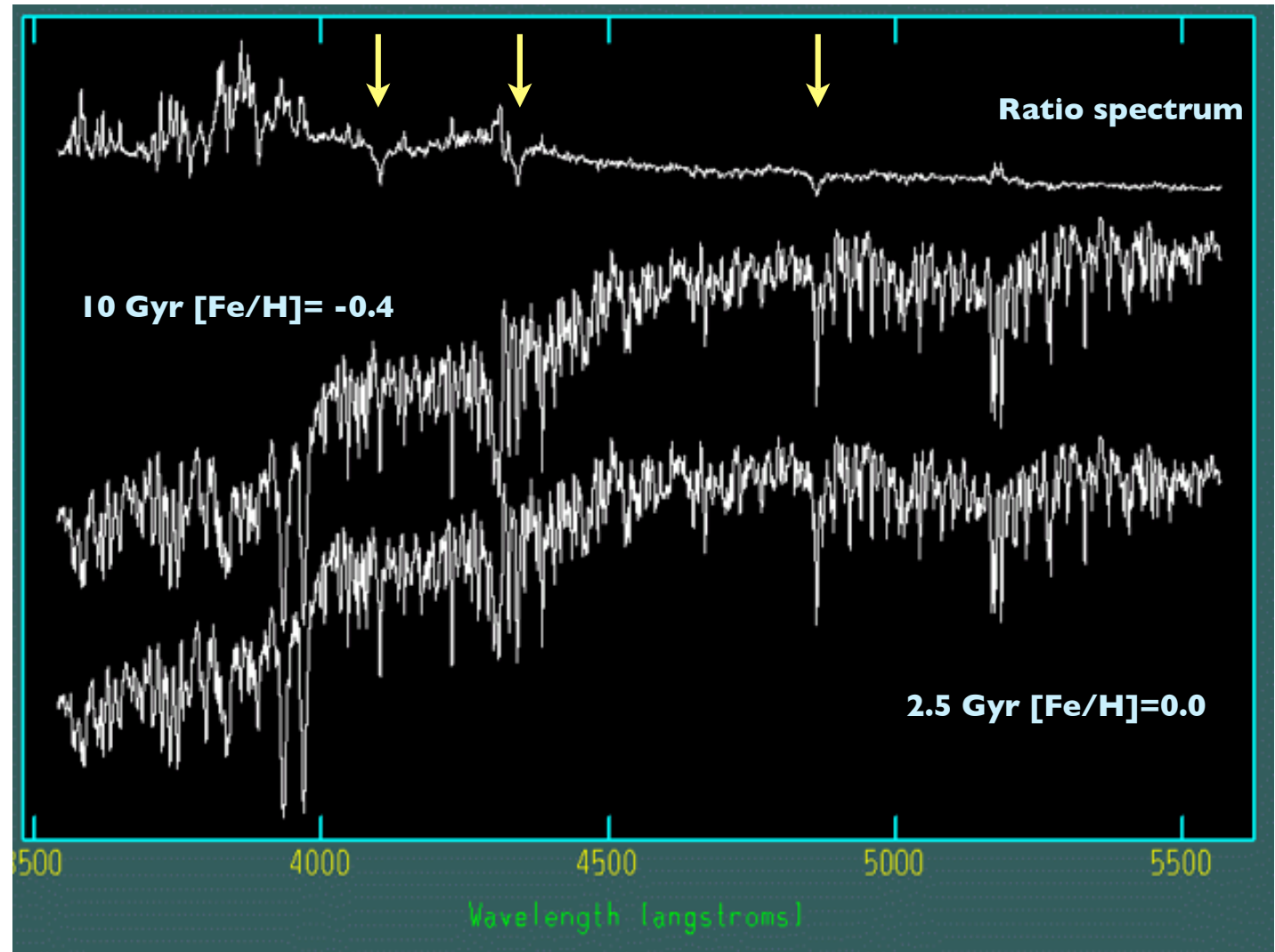
BREAKING THE DEGENERACY WITH SPECTRA

Two spectra with age and metallicity chosen to produce same broad-band colours.

Similar spectra, but differences in detail at the Balmer lines. Also differences at $\lambda < 4000 \text{ \AA}$.

We can exploit this localized spectral information to beat the age-metallicity degeneracy.

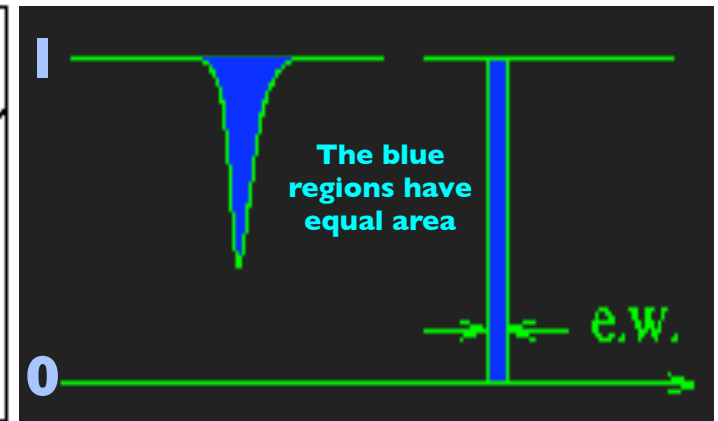
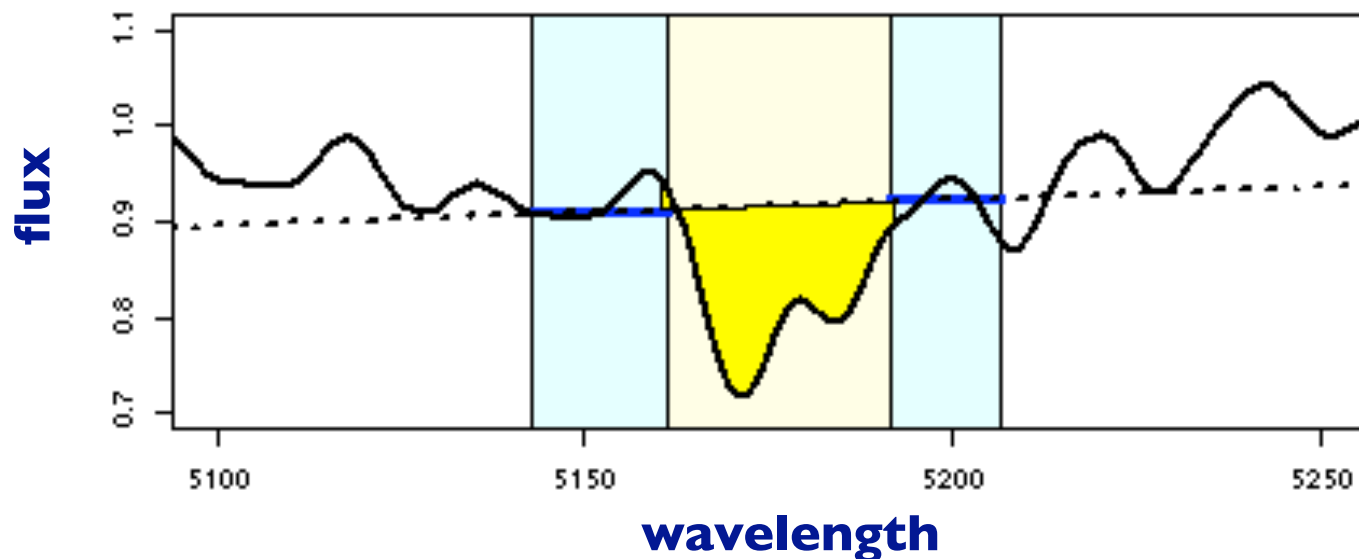
But how?



Vazdekis et al. (2007) models from MILES library

Credit: Russell Smith

Line indices (e.g. “Lick” indices)



EW is the width of a square, completely-absorbed segment of the spectrum with same integrated absorption as the line itself.

We have seen that the degeneracy-breaking power of spectra is localised to particular features.

So define “indices” which isolate these features and so carry most of the information in the spectra.

Cannot see “true” continuum. Use neighbouring region to define “pseudo-continua”.

Express absorbed flux as an equivalent width.

Credit: Russell Smith

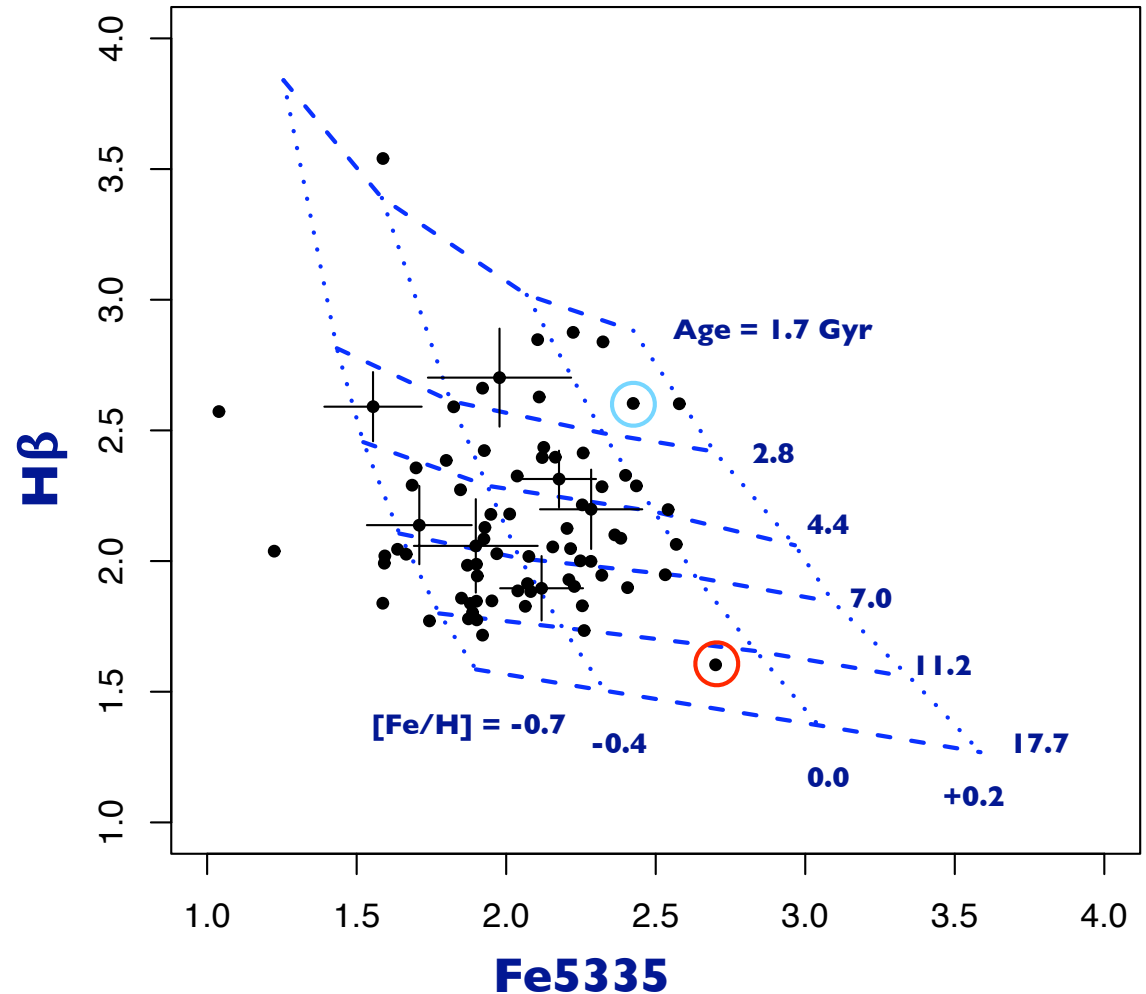
Predicting Indices

Either: Sum library spectra along isochrone and measure indices on the synthetic spectra (e.g. Vazdekis, Coelho, Percival).

Or: Measure indices on the library stars and compute luminosity-weighted average index along the isochrone (e.g. Worthey, Schiavon)

Result: Balmer-vs-metallic grids widely separate the constant-age and constant-metallicity tracks. So we can “read off” the results for an observed galaxy.

Many pairs of indices could be chosen: do they all give the same results for a given galaxy?



Grids: Schiavon et al. (2007).
Data for Coma cluster dEs from RJS et al. (2008)

Credit: Russell Smith

How does the luminosity of galaxies evolve with time?

After red giant stars start to form, the luminosity of galaxies is dominated by red giant stars.

$$L_{\text{galaxy}} \propto N_{\text{RGB}}$$

where N_{RGB} is the number of red giant stars.

How many stars are on the red giant branch?

During the typical lifetime of a red giant star T_{RGB} , calculate the number of stars that will turn off the main sequence and become red giant stars.

The typical lifetime of a main sequence star scales with mass as follows:

$$t_{\text{MS}} / (10 \text{ Gyr}) = (M/M_{\odot})^{-2.5}$$

so M_{turnoff} scales as $t_{\text{MS}}^{-0.4}$

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How much does the turn-off mass change during some time $dt = T_{\text{RGB}}$?

$$dM_{\text{turnoff}} \propto -0.4 t^{-1.4} T_{\text{RGB}} \propto t^{-1.4} T_{\text{RGB}}$$

How many stars are in this mass interval M_{turnoff} to $M_{\text{turnoff}} + dM_{\text{turnoff}}$?

Let's assume an IMF of the form $dN/dM = cM^{-(1+x)}$ where $x = 1.35$:
(salpeter)

$$dN = dM_{\text{turnoff}} (cM^{-(1+x)}) \propto t^{-1.4} T_{\text{RGB}} M^{-(1+x)} \propto (M^{-2.5})^{-1.4} T_{\text{RGB}} M^{-(1+x)}$$
$$dN \propto M^{2.5-x} \propto (t^{-0.4})^{2.5-x} \propto t^{-1+0.4x}$$

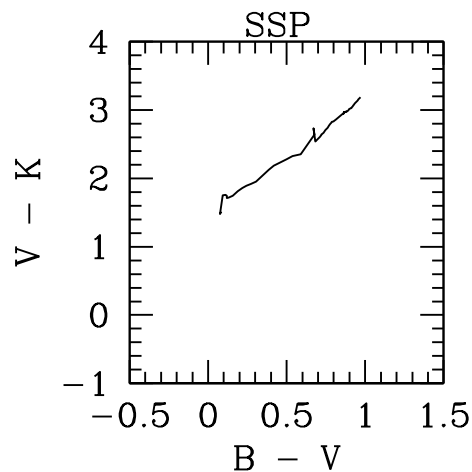
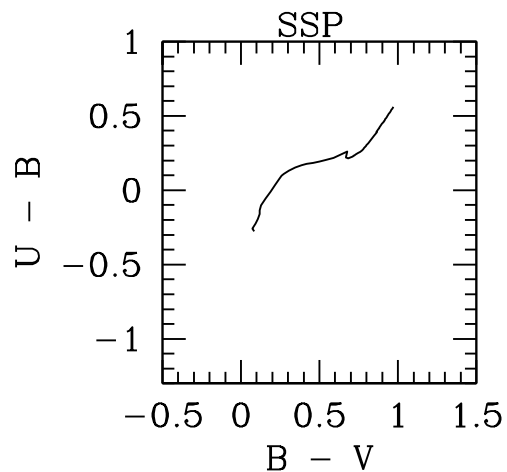
Therefore the luminosity of galaxy scales as follows:

$$L \propto dN \propto t^{-1+0.4x} \propto t^{-0.46}$$

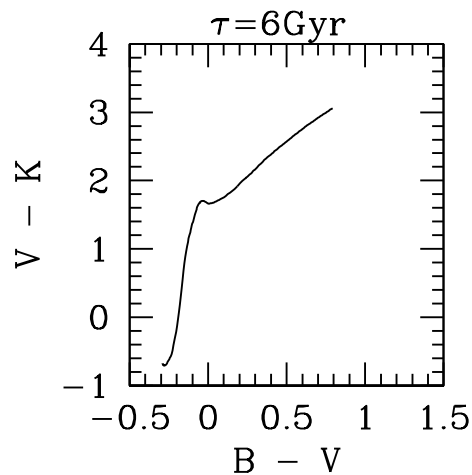
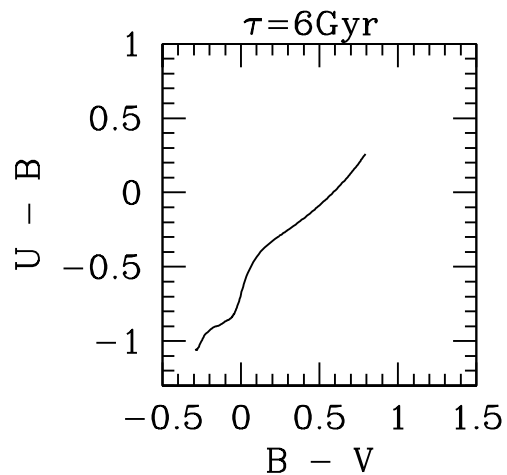
So, the luminosity is a power-law function of time and depends on the slope of the IMF near the main-sequence turnoff mass

Correlation between color and M/L Ratio

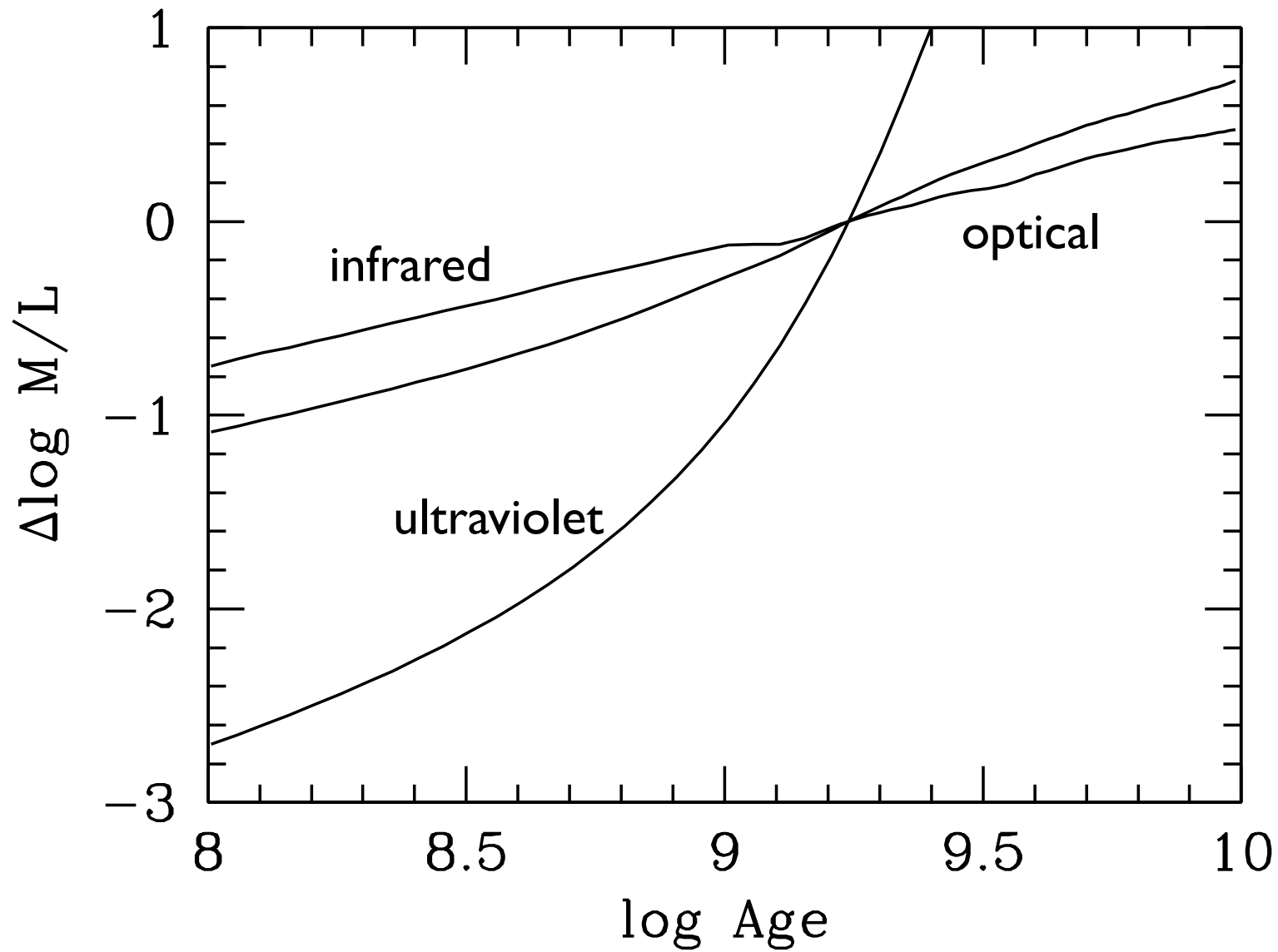
Since the color of galaxies evolve with time, the evolution of the M/L ratio must depend on the band in which one views a galaxy.



Note: SSP = all stars formed at the same point in time



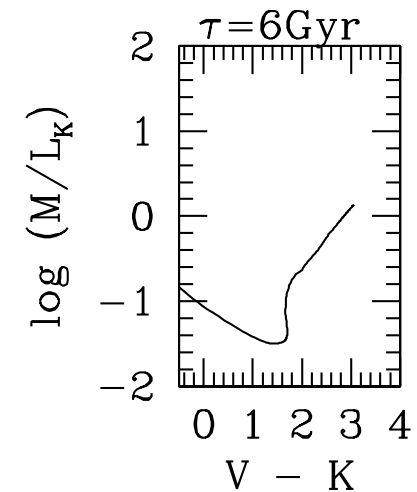
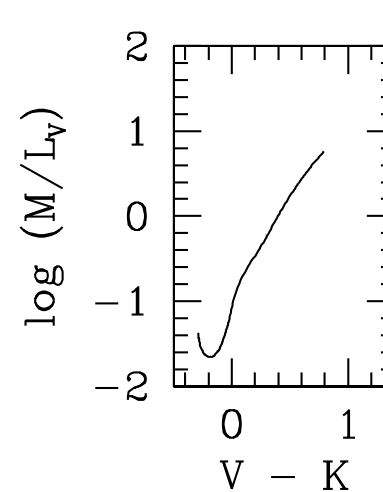
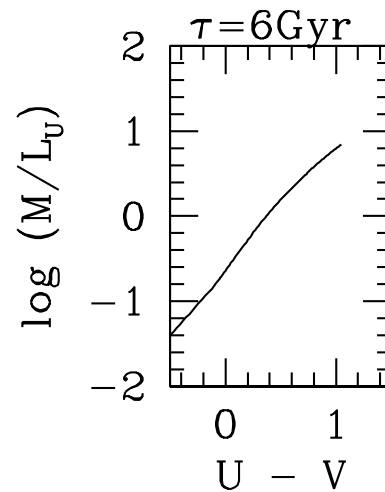
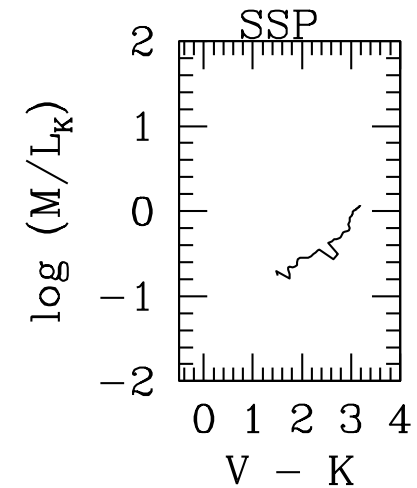
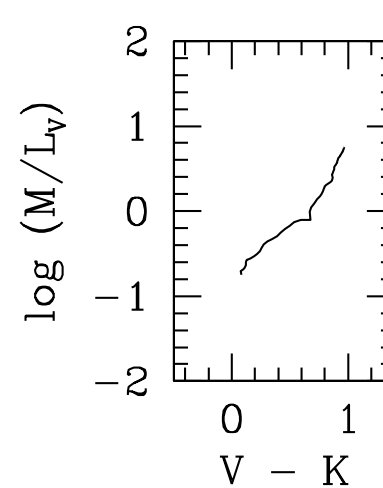
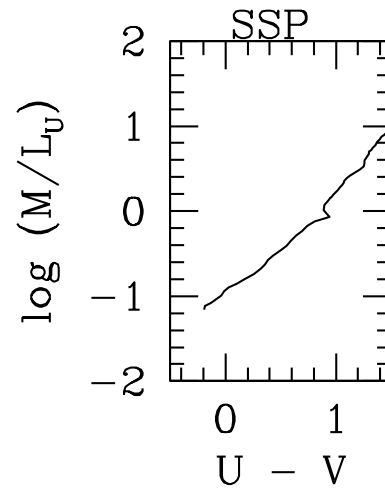
Evolution of the M/L with time...



Correlation between color and M/L Ratio

Mass-to-light ratio of galaxies is well correlated with the color.

Notice this is also true for different star formation histories.



There is an important result, since it will allow us to calculate the total mass in stars for both distant and nearby galaxies.

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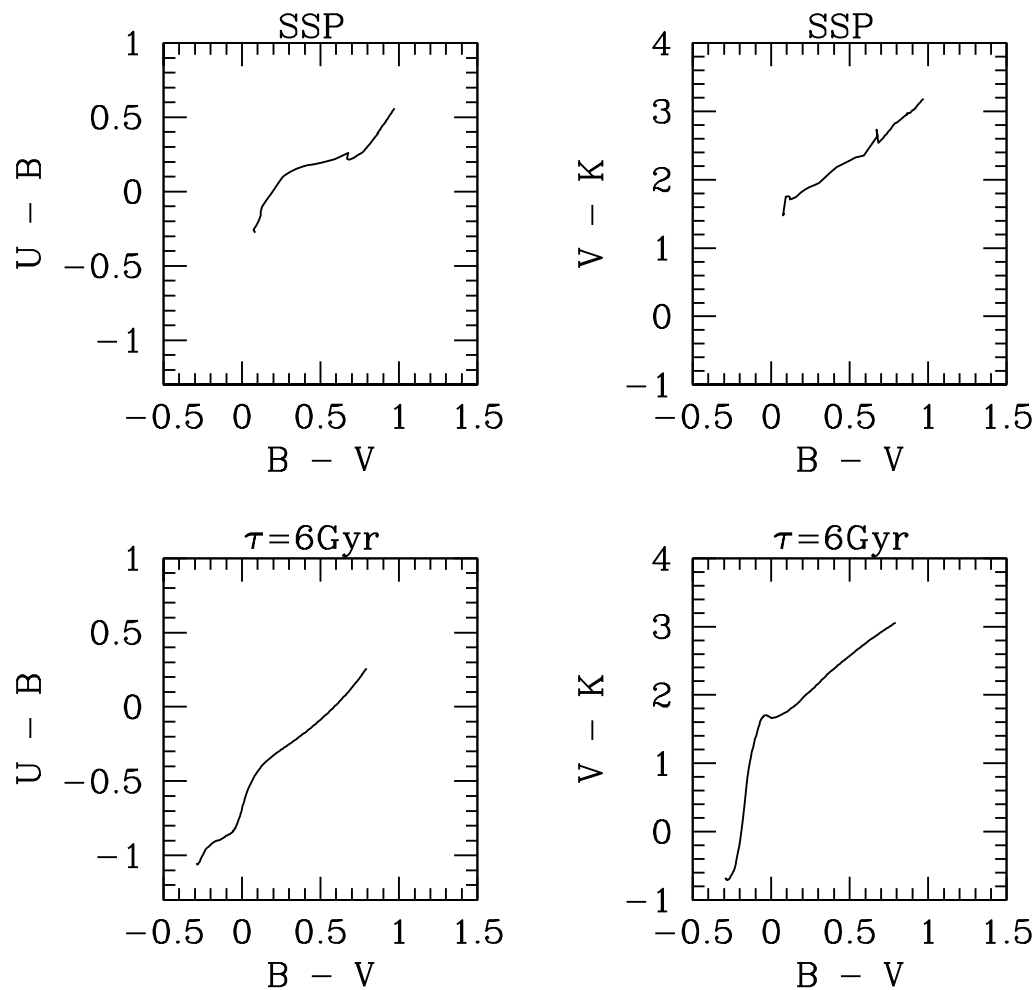
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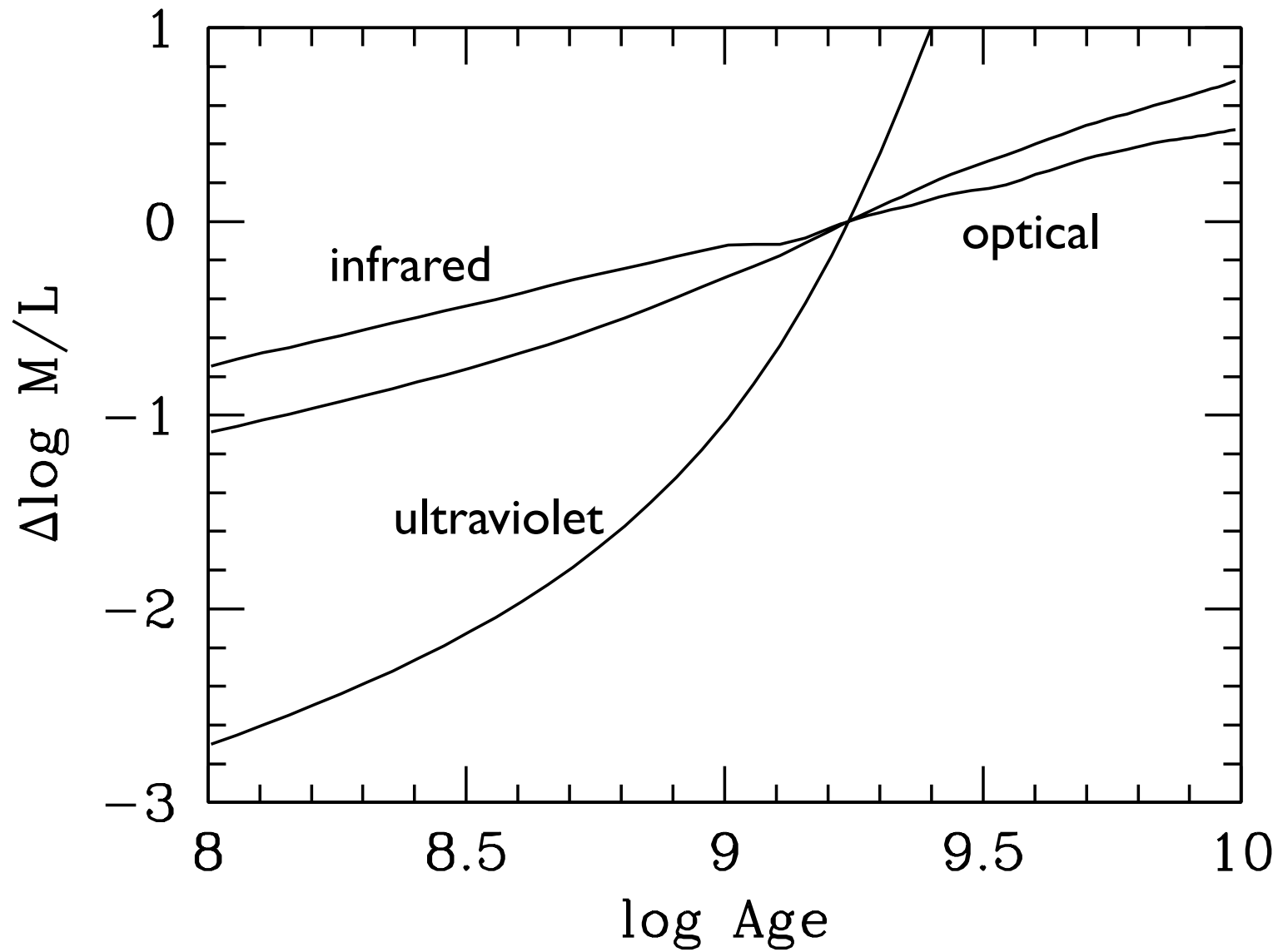
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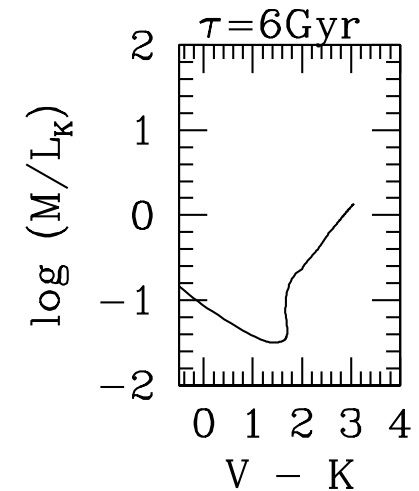
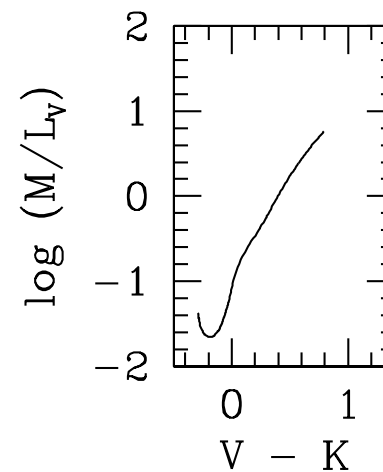
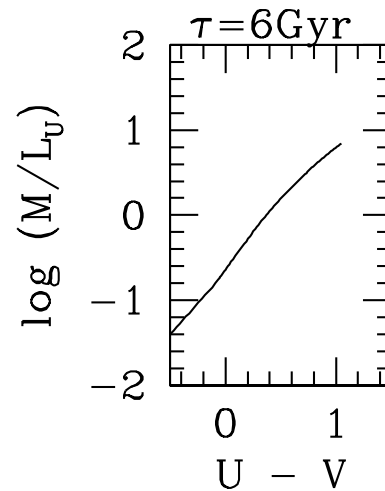
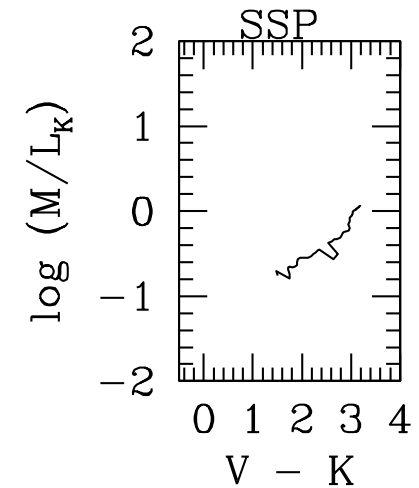
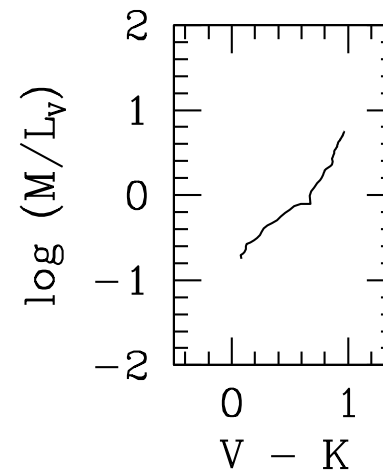
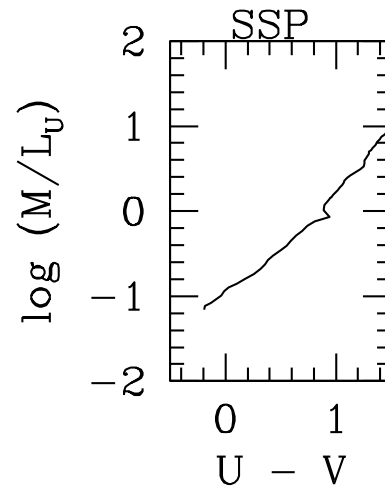
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Stellar Population Codes

Prospector: Stellar Population Inference from Spectra and Photometry

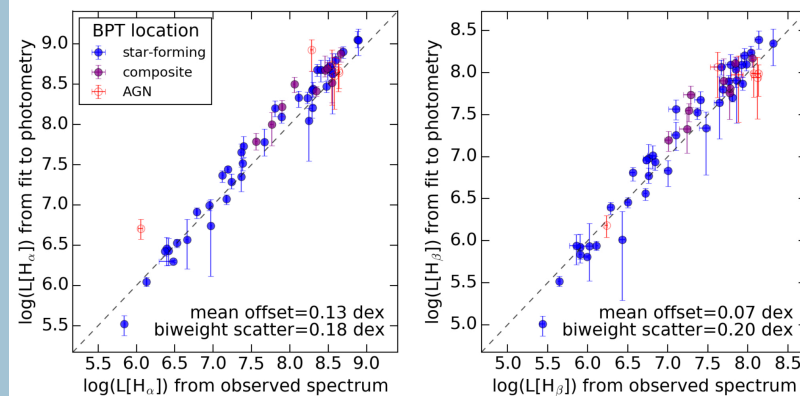
Benjamin Johnson (benjamin.johnson@cfa.harvard.edu)
with Charlie Conroy, Nell Byler, and Joel Leja

<http://git.com>
Documentation

(Other commonly used codes: BAGPIPES, FAST, CIGALE, LePhare, MAGPHYS, Beagle)

Description: Prospector is a package for inference of stellar population parameters given observed spectra and/or photometry. Prospector uses Monte Carlo sampling of the posterior probability distribution of the parameters, combined for both spectroscopic and photometric data. Stellar population models are generated on-the-fly at each iteration of the MCMC sampling. This allows for higher dimensionality and more flexibility than grid based approaches. Prospector is pure Python, open source, flexible, and easily extensible.

Predicting Balmer lines from broadband SED fitting:



After fitting non-parametric star formation histories to broadband UV through FIR photometry for nearby, nearly normal galaxies from [2], we can predict the observed H α luminosity (including dust effects) and equivalent width with 0.2 dex scatter [8]. The Balmer decrement is also well recovered.

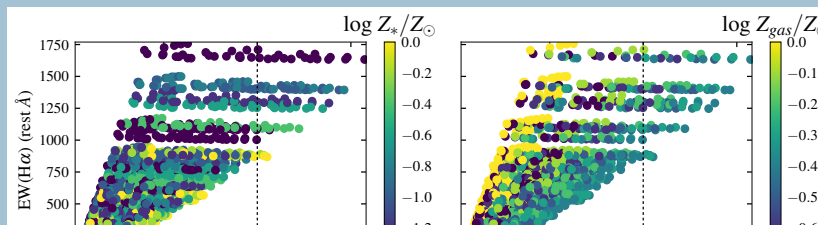
Stellar Population Models:

The stellar population models are currently provided by the `python-FSPS` bindings to the `FSPS v3.0` code [5]. Nearly all of the features available in FSPS are available in python-FSPS including:

- dust attenuation and emission (including R_V and f_{bump} parameters)
- nebular emission (from Cloudy via [3])
- AGN emission
- various parameterized SFHs
- SSP specific parameters (e.g. AGBs)

Any of the ~60 parameters controlling the stellar population can be specified or fit for

Nebular Emission in Broadband Photometry: Nebular continuum and line emission are from Cloudy models with self consistent ionizing populations (via [3]). Nebular emission is included in and can substantially affect broadband photometry. It is important for photometric SED fitting to be able to reproduce extreme equivalent widths – $\text{EW}(\text{OIII} + \text{H}\beta) > 1000\text{\AA}$ – observed at high redshift (e.g. [1, 7]).



H α and OIII+H β equivalent widths for a grid of Padova07 /Geneva + Cloudy models. In these models it is only possible to generate extreme equivalent widths with young rising (delay- τ)

Stellar Population Codes

- redshift and resolution
- noise properties
- non-parametric SFHs

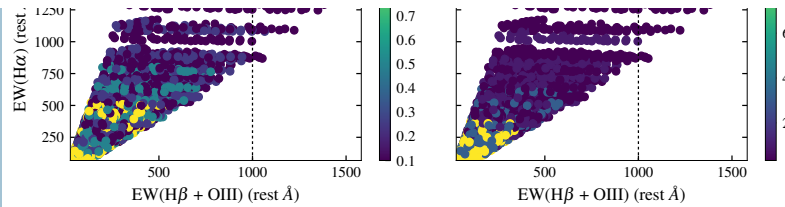
Other stellar population models can be included through the modular structure !

Additional Features:

- *Joint Spectra and Photometry fitting:* Simultaneous self-consistent fitting of combined spectral and photometric data.
- *Noise modeling.* Non-trivial noise modeling, including correlated additive or multiplicative (calibration) noise in the spectral and/or photometric data, or in the models.
- *Flexible parameters.* Parameters can be transformed to new parameters, tied to fitted parameters, and user defined priors can be placed (e.g. from photo-z PDFs)
- *Isochrones.* FSPS v3.0 includes the PARSEC, Padova2007, Geneva, BaSTI, and new MIST isochrones at very low metallicity. α -enhanced models coming.
- *Parallelization.* Can be parallelized with MPI across many cores and nodes.
- *Sampling.* Optimization, in addition to the ensemble MCMC sampling provided by emcee [6] and nested sampling from nestle [9]

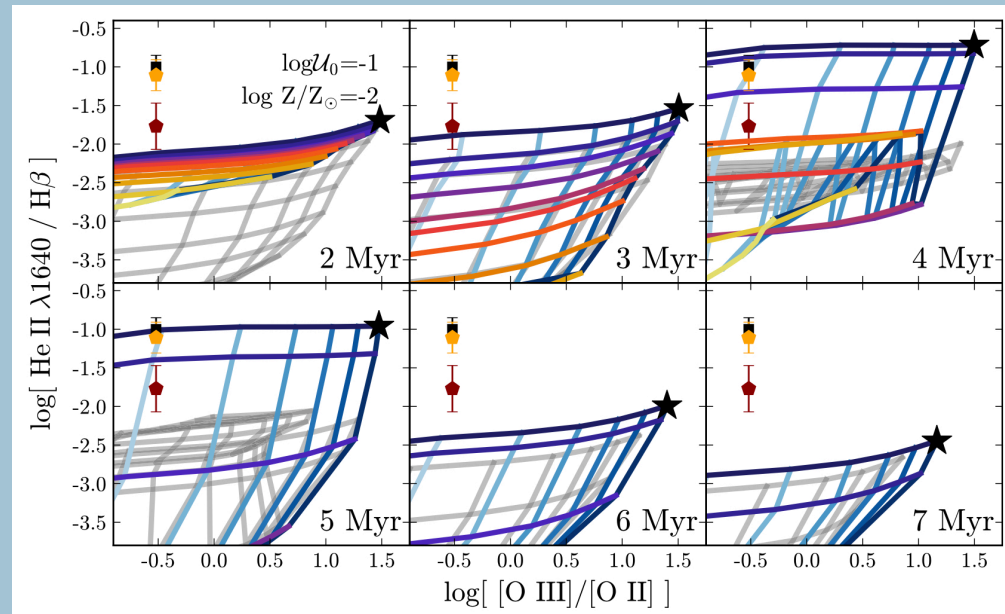
References

- | | |
|--|---|
| [1] Brammer et al. 2012, ApJ 758 | [6] Foreman-Mackey et al. 2012 |
| [2] Brown et al. 2014 ApJS 212, 18 | [7] Labbe, I. 2013 ApJL 777 |
| [3] Byler, N. et al. 2012 ApJ 754, 98 | [8] Leja, J. et al. 2017 ApJ 837 |
| [4] Choi, Conroy, & Byler 2017 ApJ | [9] github.com/kbarbary/nestle |
| [5] Conroy, Gunn & White 2009 ApJ 699, 486 | [10] Steidel et al. 2016 ApJ 826 |



explained by α -enhanced abundance patterns (e.g. [10]), which are a subject of active development in the stellar models, as are expanded grids of gas-phase abundance patterns.

MIST Isochrones with Rotation: The MIST isochrones include rotation, which results in longer lived and hotter stars. Compared to the Padova2007/Geneva isochrones this leads to substantially increased HeII ionizing photons [4] and HeII 1640 emission at selected ages and low Z [3], though still less than binary models. Balmer EWs are also increased, and inferred SFRs are affected.



From [3]: HeII emission vs O32 is shown for single age populations with MIST isochrones for a grid of ionization parameter and metallicity. Metallicity increases from purple to orange and ionization from light blue to dark blue. The Padova07+Geneva models are shown in gray. The pentagons are the $z=2.4$ composite from [10] with stellar emission removed.

NEW TOPIC:

**Lessons about Galaxy
Formation from Large
Samples in Nearby Universe**

What can we learn about
the structure, formation
and evolution of galaxies by
putting together a large
survey of galaxies in the
nearby universe?

Let us consider spectra, colors, luminosities for 100,000s of galaxies in the nearby universe such as we have from the Sloan Digital Sky Survey.

What general conclusions can we draw about galaxies from these observations?

THE SDSS

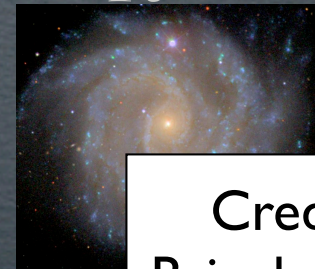
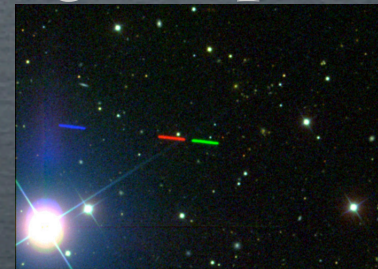
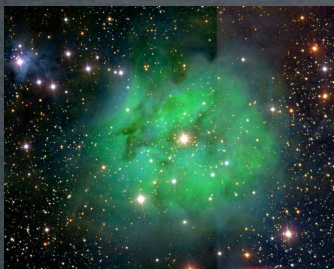


The most ambitious survey of the sky ever undertaken.

Imaging survey of 8600 square degrees.

Redshifts of more than 1,000,000 galaxies & QSOs.

Robotic 2.5m telescope - imaging & Spectroscopy



Credit:
Brinchmann

THE SDSS DR7 - (AUTUMN 2008)

Legacy

- 5 band imaging over 8423 deg² down to $r \sim 21.5$
- ~230 million objects
- Spectra covering 3800Å-9200Å with $R \sim 2000$.
- 928,567 galaxy spectra, 109,862 QSO spectra with $z < 2.3$ and 8,802 high- z QSOs
- Median seeing 1.54"

<http://www.sdss.org/dr7>

SEGUE

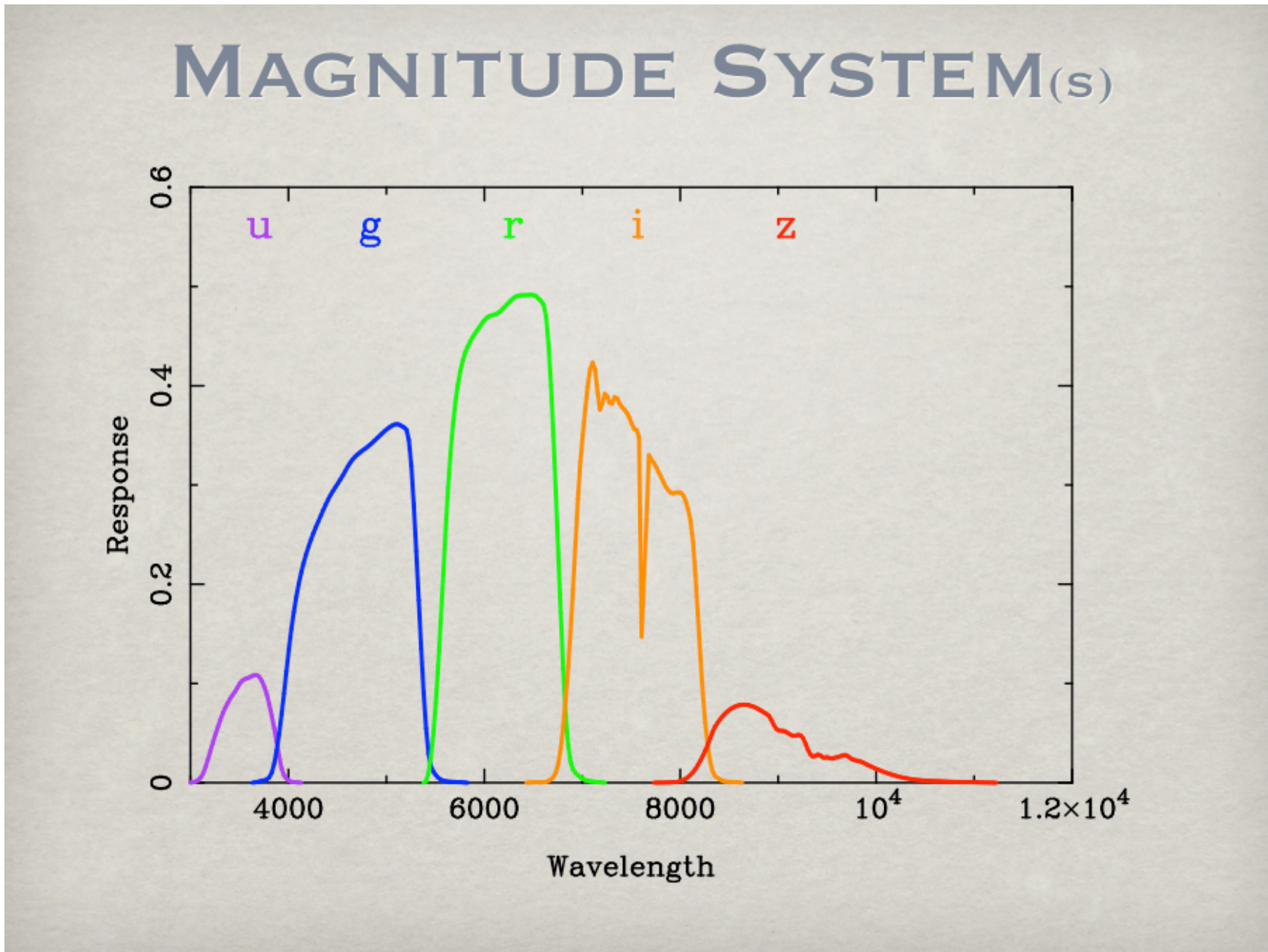
- 5 band imaging over 3240 deg²
- ~127 million objects.
- 229,466 spectra of stars of type K and earlier, and 7,922 M stars and later.
- $\log g$, T_{eff} , $[\text{Fe}/\text{H}]$ and

And: SuperGalaxy Survey (SGS) 31 and other spectroscopic surveys in Galactic latitude run

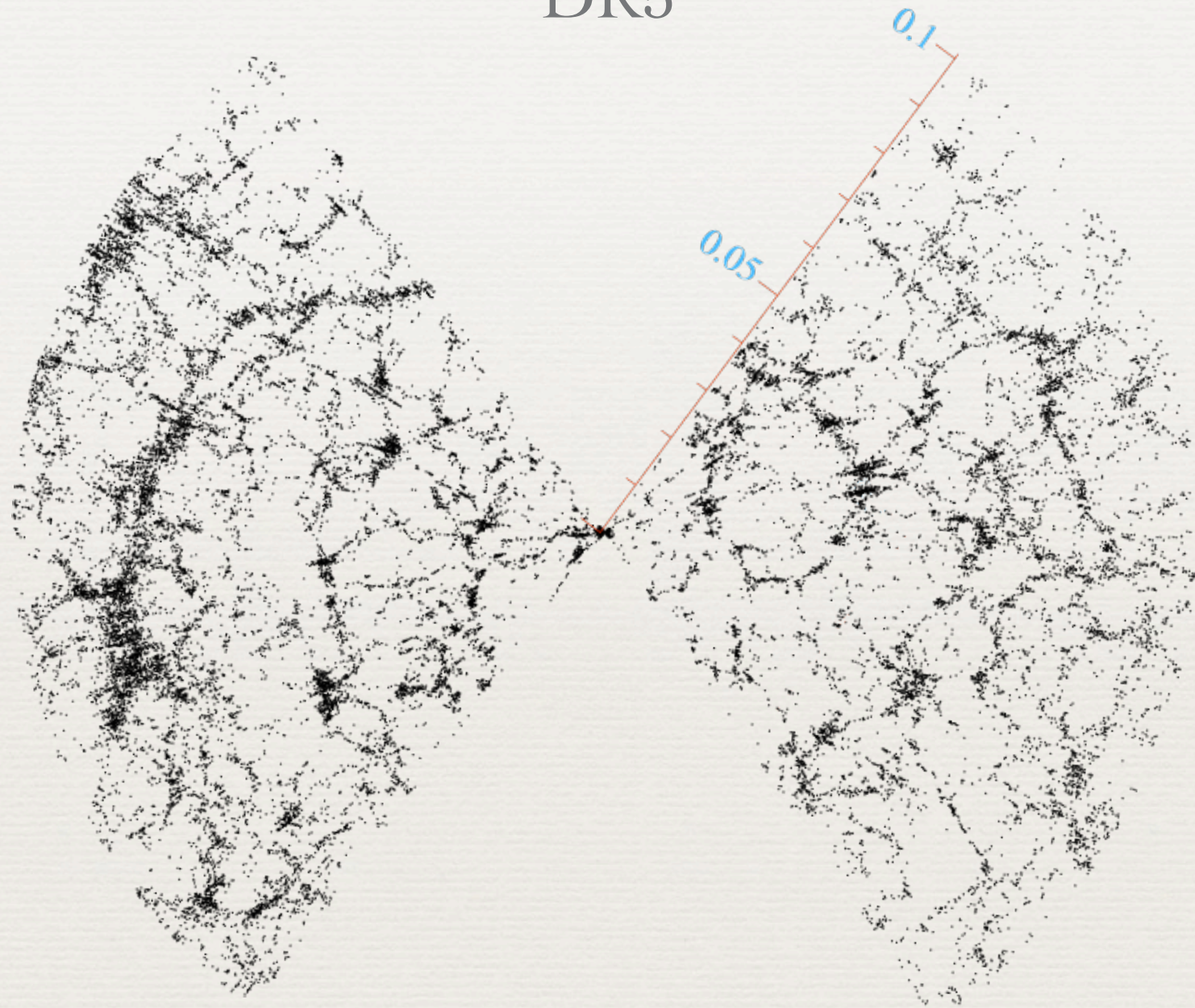
Sloan Extension for Galaxy Understanding and Exploration

Credit:
Brinchmann

The Sloan Digital Sky Survey acquired images of the sky with the following five filters:

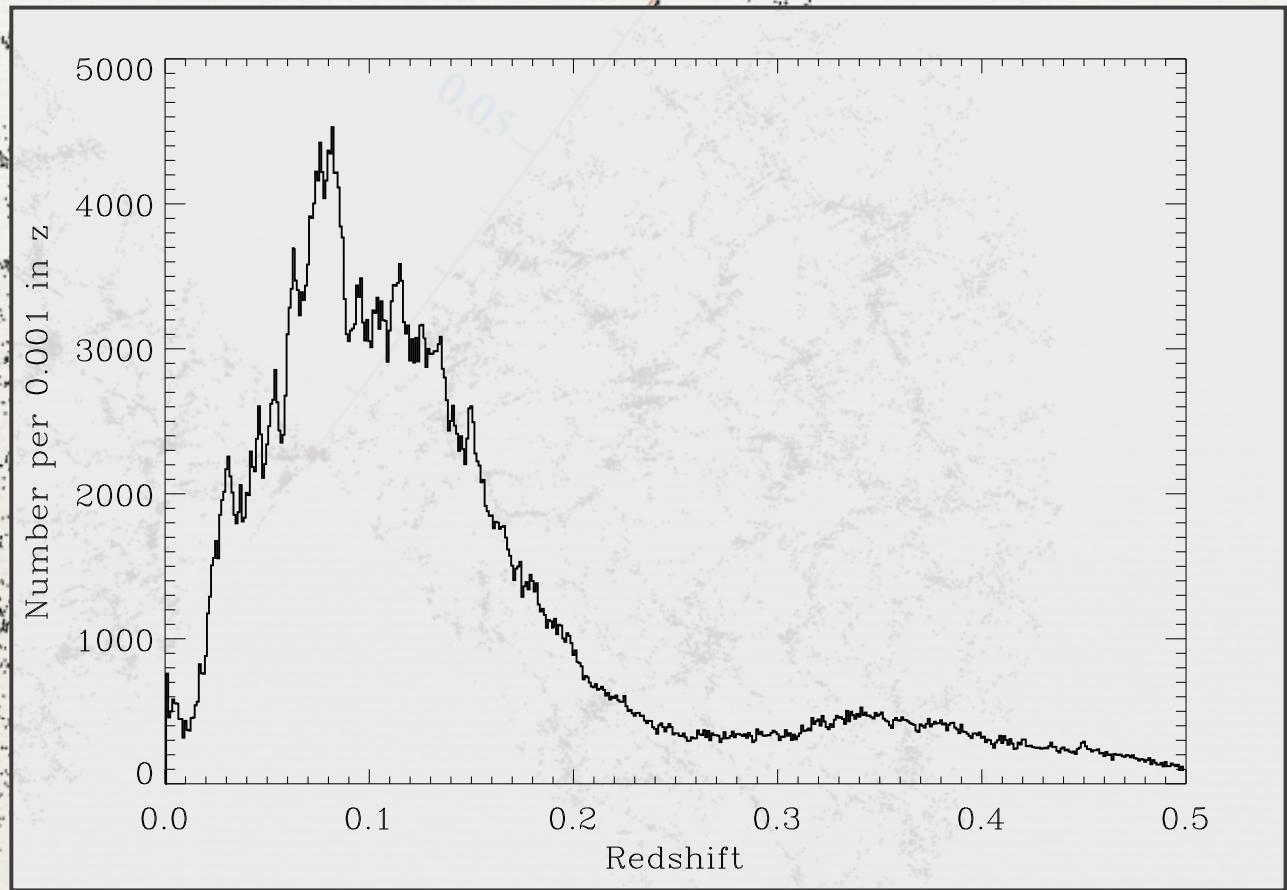
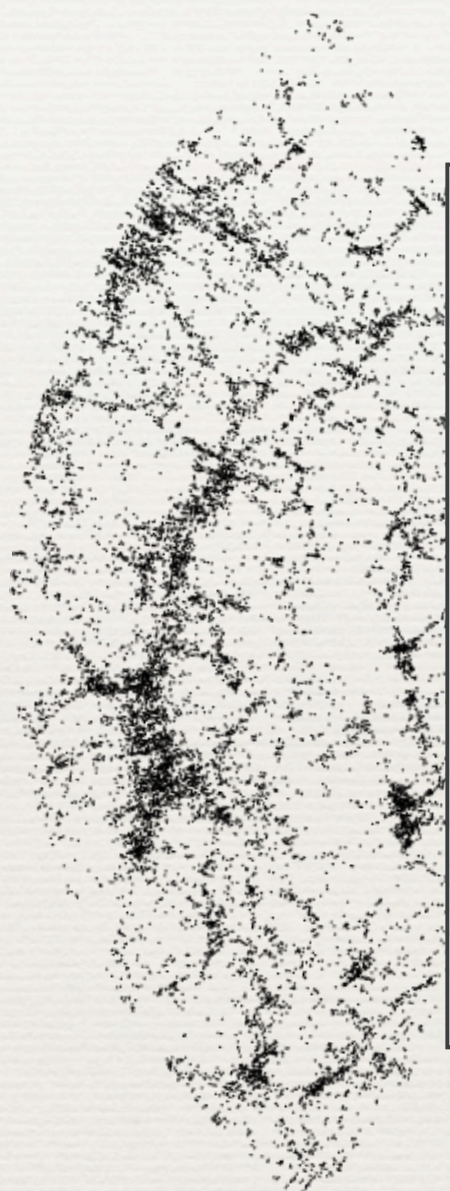


DR5



Credit:
Brinchmann

DR5



Credit:
Brinchmann

DID THE SDSS FIND ANYTHING NEW?

The low- z universe has been well studied up through the years - was there anything new to be found?

Tully-Fisher
Faber-Jackson
Luminosity functions
 Mg_2 - σ
Colour-Magnitude
etc.

Yes!

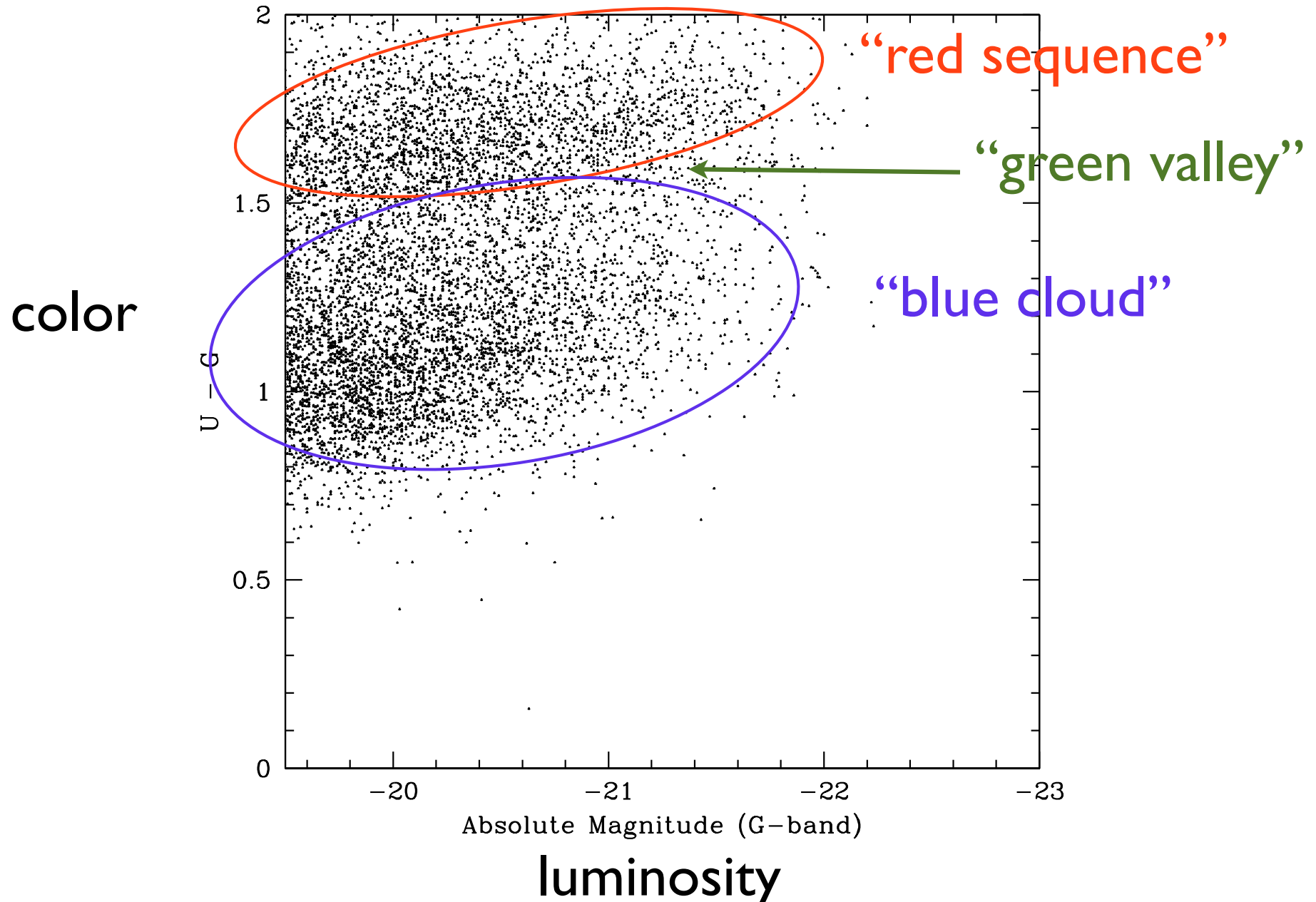
Main progress areas:

- ✓ Well-understood selection function allows the construction of **distribution functions**.
- ✓ The vastness of the sample provides large samples of **extreme objects**.
- ✓ Well-known trends can be studied with very **high precision**.

Credit:
Brinchmann

What general conclusions can we draw about galaxies from these large samples?

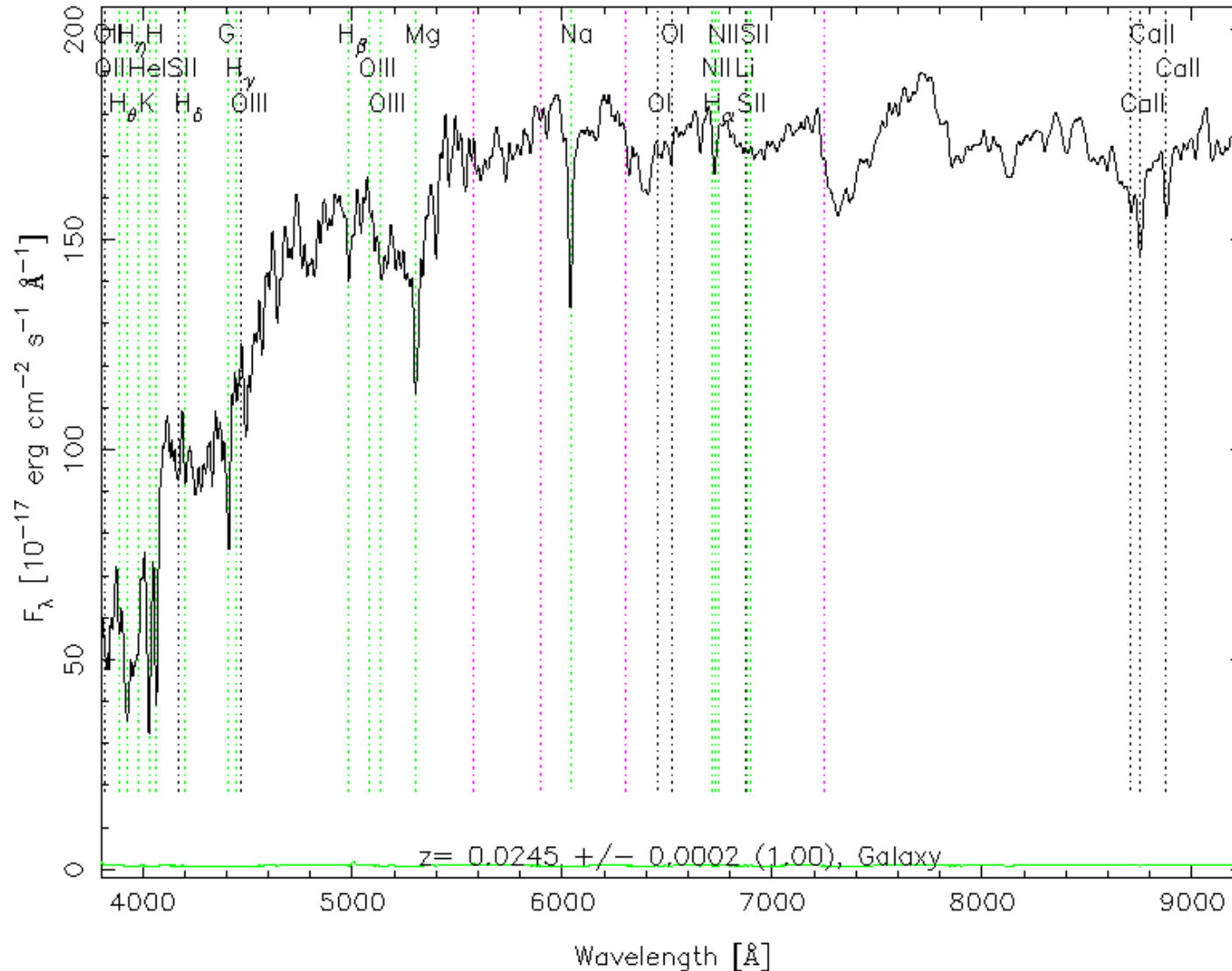
How are galaxies distributed in color and luminosity?



Clear ***bimodality*** to the distribution!

What is the typical spectrum of a galaxy on the red sequence?

RA=194.35151, DEC=27.49782, MJD=53823, Plate=2240, Fiber=157

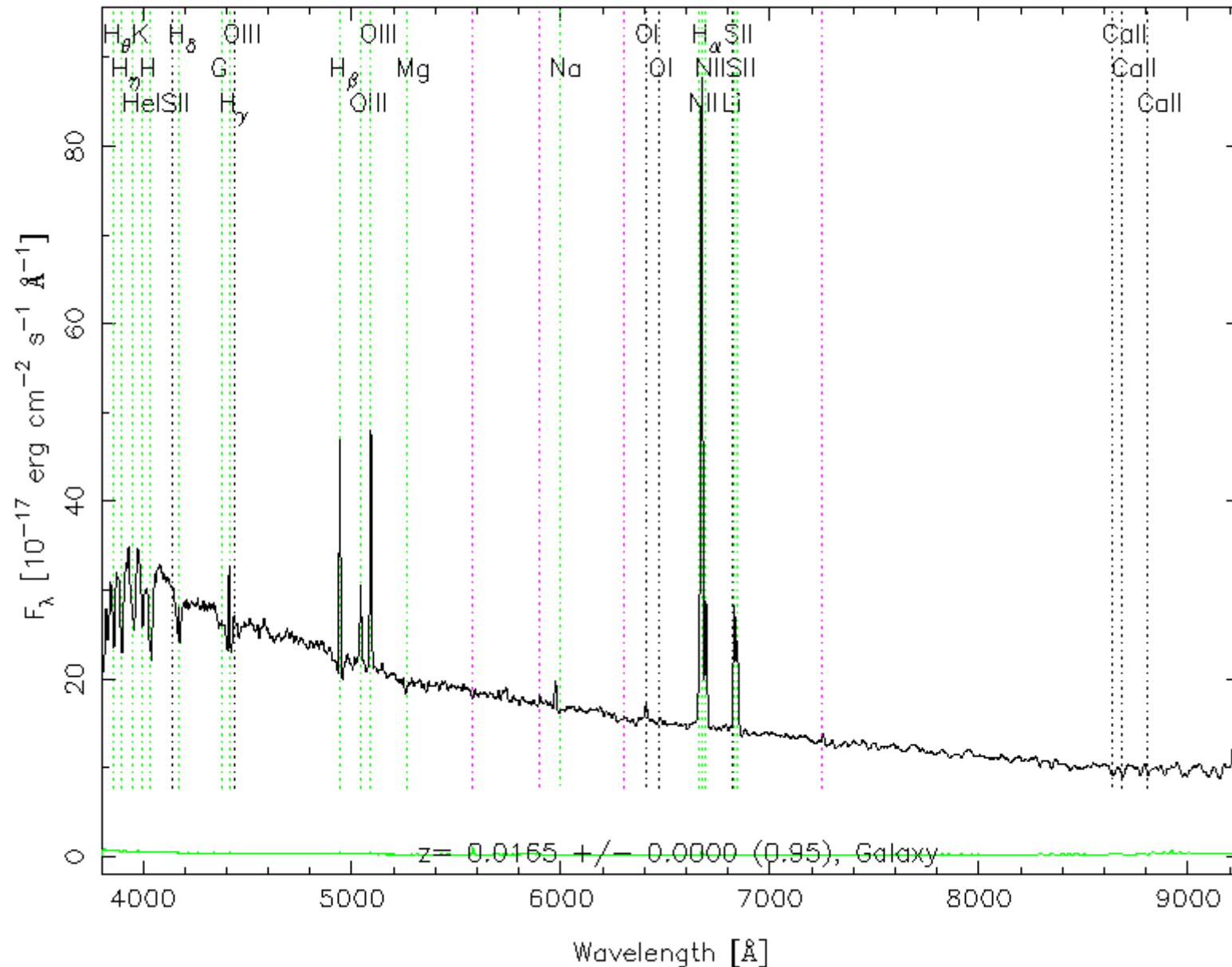


Large 4000
Angstrom break

Magnesium
absorption lines

Clearly a very
old galaxy!

What is the typical spectrum of a galaxy on the blue cloud?



Strong Balmer emission lines (H_α , H_β , H_γ) come from hot ionized bubbles around O and B stars.

Earlier today, we saw the types of spectra we would predict for galaxies if all the stars in a galaxy formed at some time in the past

If star formation stops in a galaxy, it will have a strong 4000 Angstrom break after ~1-2 billion years

Clear similarity between the predicted spectra and those observed

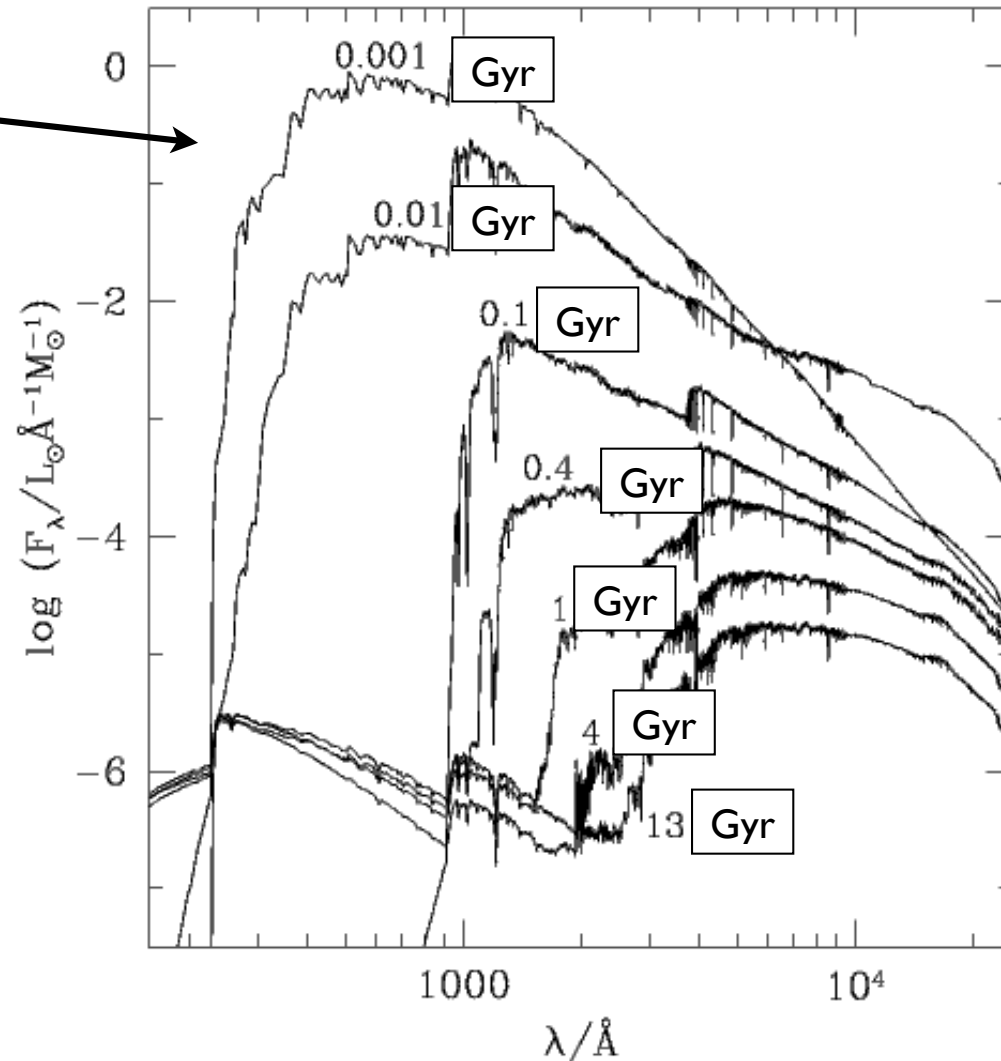


Figure 9. Spectral evolution of the standard SSP model of Section 3 for the solar metallicity. The STELIB/BaSeL 3.1 spectra have been extended blueward of 3200 \AA and redward of 9500 \AA using the Pickles medium-resolution library. Ages are indicated next to the spectra (in Gyr).

What is the distinction between galaxies in the red sequence and the blue cloud?

It would appear to be whether the galaxies are still actively undergoing star formation or not.

But this isn't the only factor, as both dust and metallicity can make normal star-forming galaxies appear as if they are on the red sequence

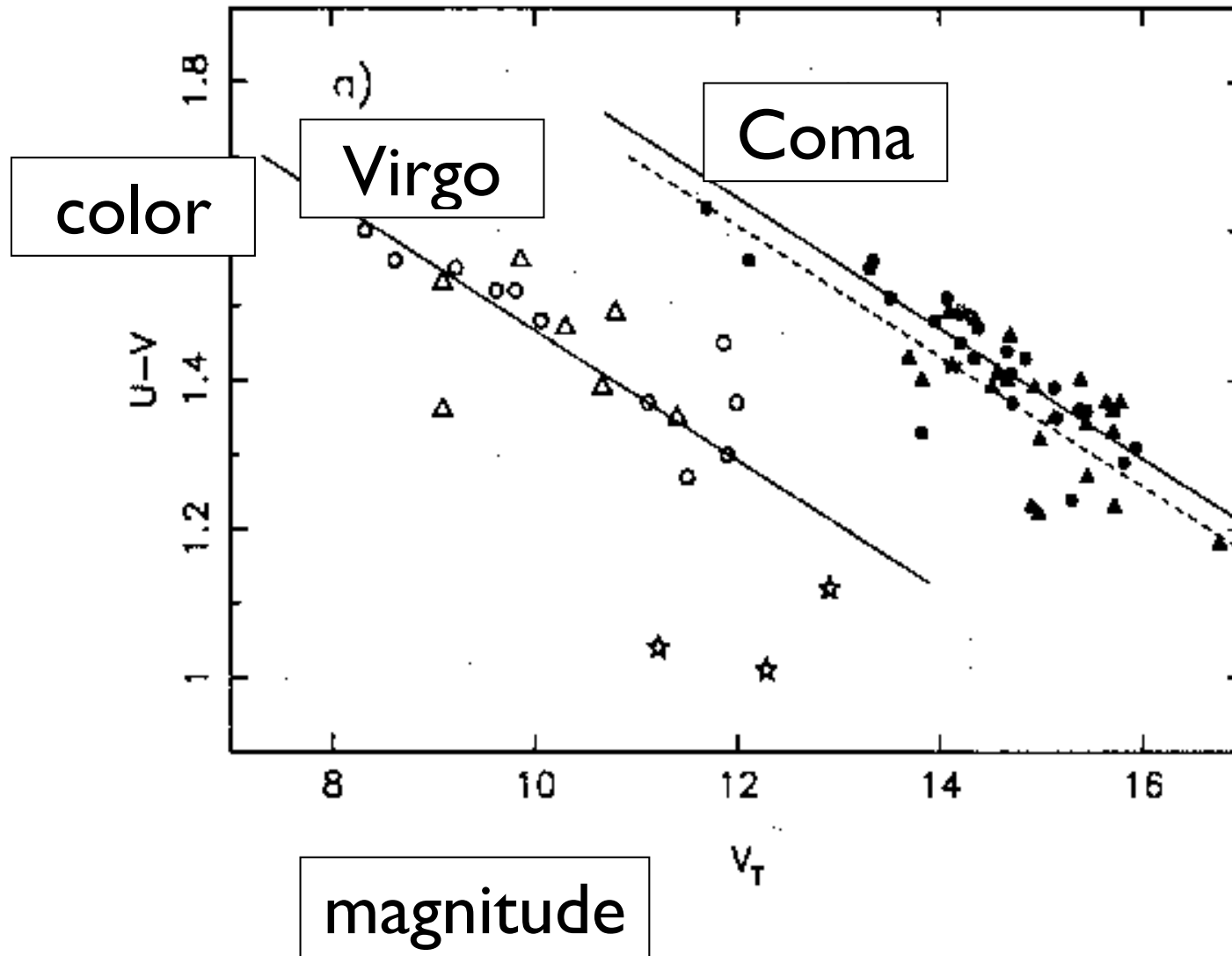
Can we conclude anything about “red sequence” galaxies based on their color?

Nominally, stars in the galaxy would appear to be 10 billion years old.

But there are other things which can make galaxies redder, e.g., dust, metals...

Fortunately, we can circumvent such degeneracies (age-dust-metal) by measuring the age of the stellar populations in galaxies from their absorption line features in a way that is relatively insensitive to the dust content

The tightness of the red sequence gives us an independent constraint on the age of galaxies on this sequence.



The tightness of the color-luminosity relation for the red sequence is particularly obvious in the galaxy clusters

If stars in galaxies on the red sequence formed relatively recently, one might not expect the sequence to be so tight.

Measured scatter in the U-V color of the red sequence is 0.05 mag. After removing the scatter due to noise in the measurements themselves, the intrinsic scatter is 0.04 mag.

What range in the ages of the stellar populations would produce this sort of scatter?

Earlier this lecture, we found the following formula is approximately true:

$$\text{color} = a \log_{10}(\text{time}) + b$$

For the U-V color, $a \sim 0.65$, so that a 0.04 scatter in U-V color translates into a 0.06 dex scatter in age (i.e., $\sim 15\%$)

This means galaxies on the red sequence have a spread of $\sim 15\%$ in their ages

The only easy way is to easily accommodate such a small spread in ages is to suppose “red sequence” galaxies are very old

Systematic Analysis of Galaxy Spectra from the Sloan Digital Sky Survey

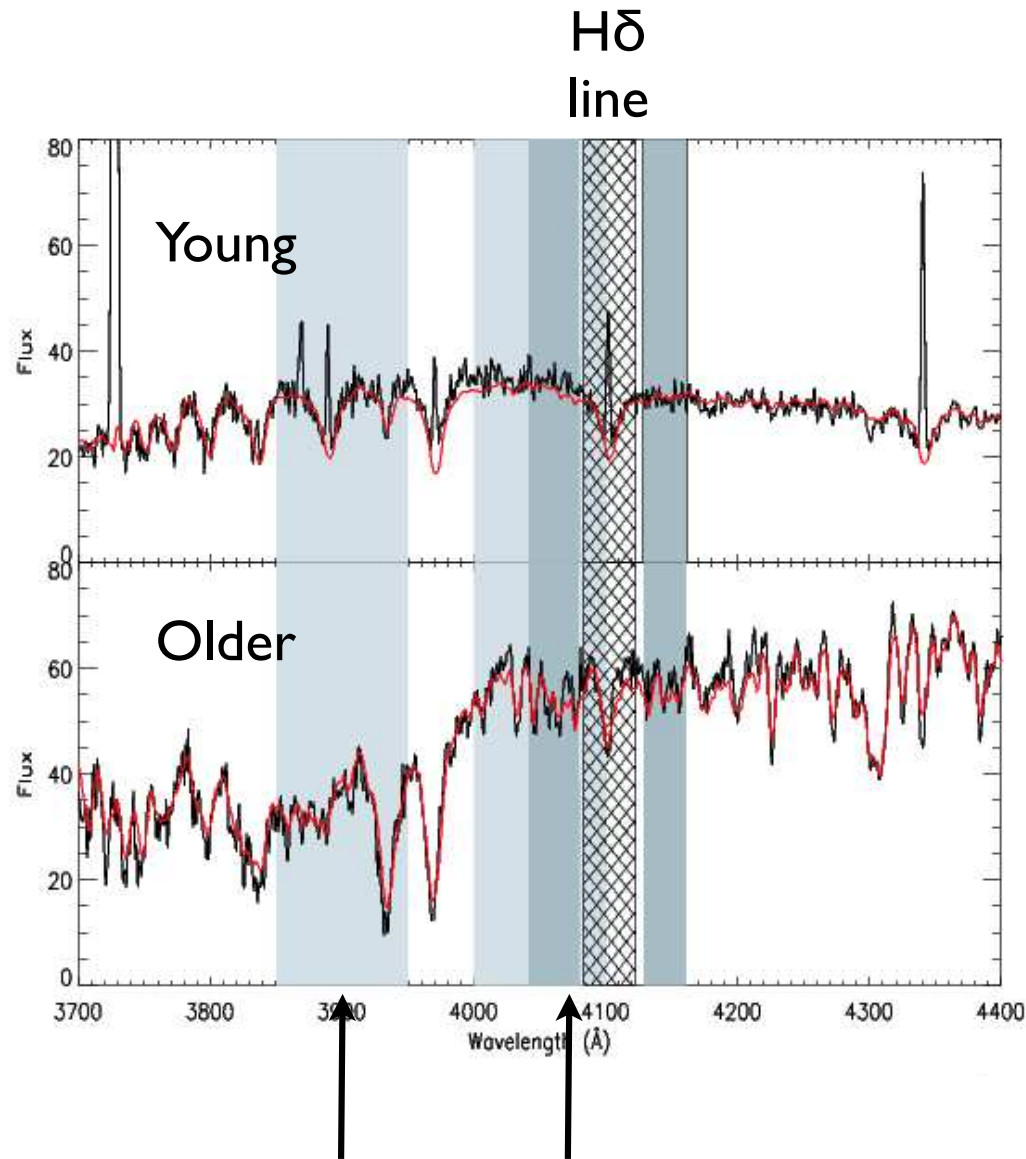
One can study the properties of galaxies much more robustly making use of the spectra and looking at specific features in the spectra.

One very thorough analysis of the spectra of $>100,000$ galaxies from the Sloan Digital Sky Survey was conducted by Kauffmann, Brinchmann, Charlot, Heckman, etc.

Most of the analysis is given in Kauffmann+2003a,b,c; Kauffmann+2004; Brinchmann+2004; Tremonti+2004

Two features they made extensive use of were the $H\delta$ line and the magnitude of the 4000 Angstrom break $D_n(4000)$

$H\delta$ line can show up in emission or in absorption



Almost no
4000
Angstrom
break

Measurable
4000
Angstrom
break

$D_n(4000)$ break measured by
comparing these two spectral regions

How do these features vary with the age of the stellar population in a galaxy?

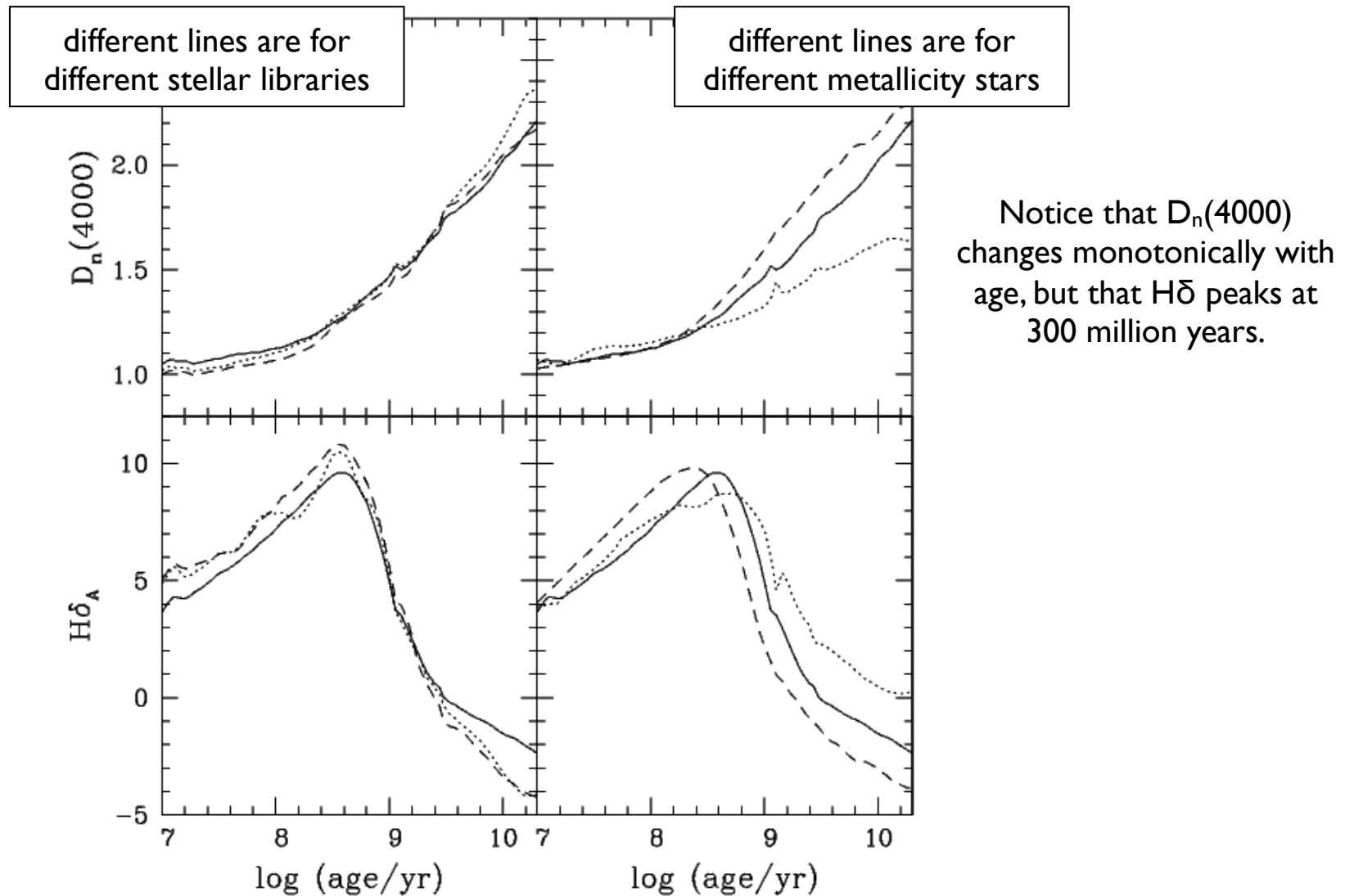
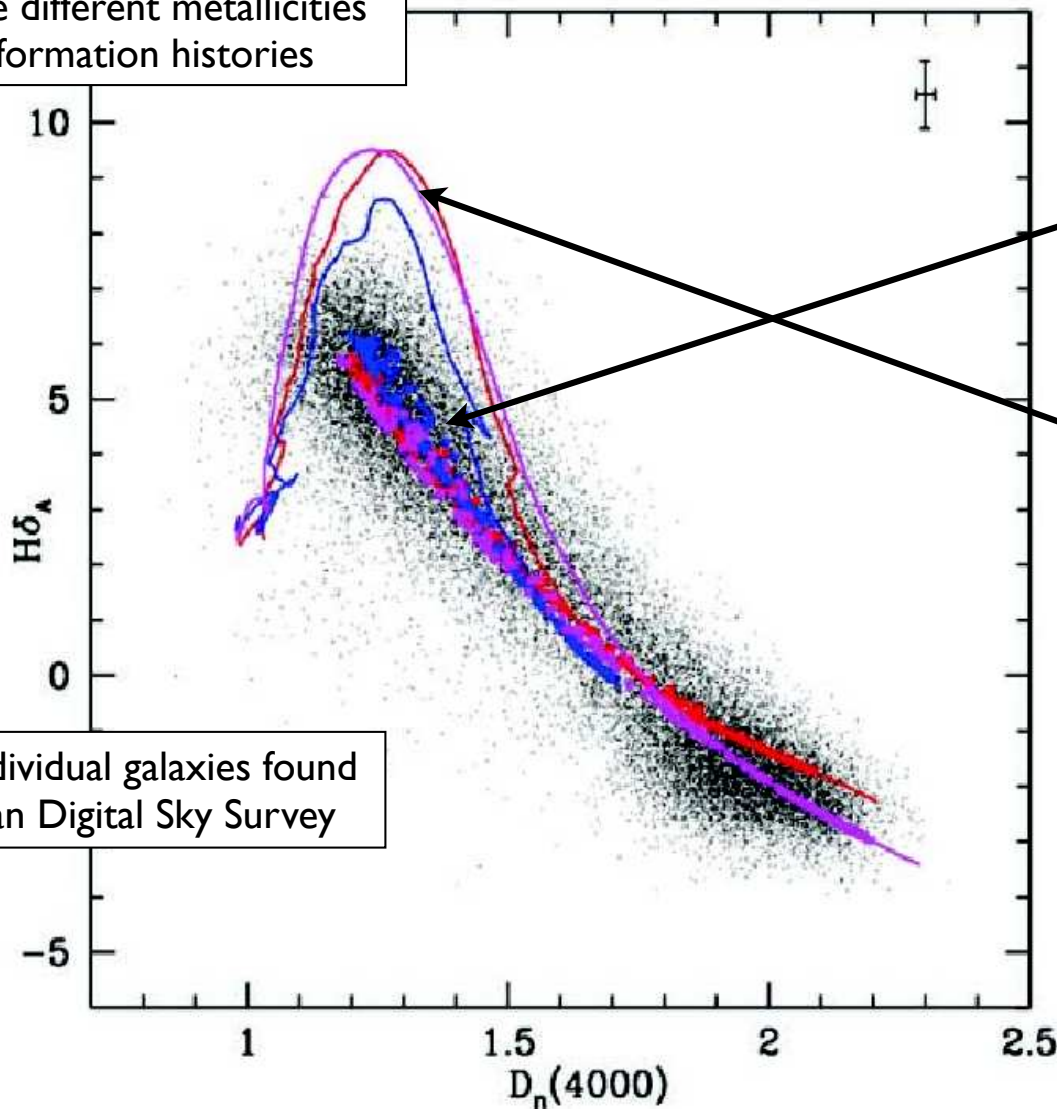


Figure 2. Left: the evolution of $D_n(4000)$ and $H\delta_A$ following an instantaneous, solar-metallicity burst of star formation. Solid lines show results from BC2003+STELIB, the dotted line shows results if the Pickles (1998) library is used, and the dashed line is for the Jacoby et al. (1984) library. Right: the evolution of $D_n(4000)$ and $H\delta_A$ for bursts of different metallicity. The solid line is a solar metallicity model, the dotted line is a 20 per cent solar model and the dashed line as a 2.5 solar model.

What values for $H\delta$ and $D_n(4000)$ do we measure for galaxies in SDSS?

lines show the expectations assuming galaxies have different metallicities and star formation histories



For simple star formation histories (where the rate of star formation diminishes with time), there is good agreement with the observations.

However, if one invokes super bursts of star formation on top of a continuous star forming models, this disagrees quite significantly with the observations.

dots are individual galaxies found in the Sloan Digital Sky Survey

Figure 3. $H\delta_A$ is plotted as a function of $D_n(4000)$ for 20 per cent solar, solar and 2.5 times solar metallicity bursts (blue, red and magenta lines), and for 20 per cent solar, solar and 2.5 solar continuous star formation histories (blue, red and magenta symbols). A subset of the SDSS data points with small errors are plotted as black dots. The typical error bar on the observed indices is shown in the top right-hand corner of the plot.

One can also derive the dust extinction for galaxies based on the colors -- since the age and metallicity can be constrained from $D_n(4000)$ and $H\delta$

Here is the derived dust extinction A_z for a large sample of galaxies:

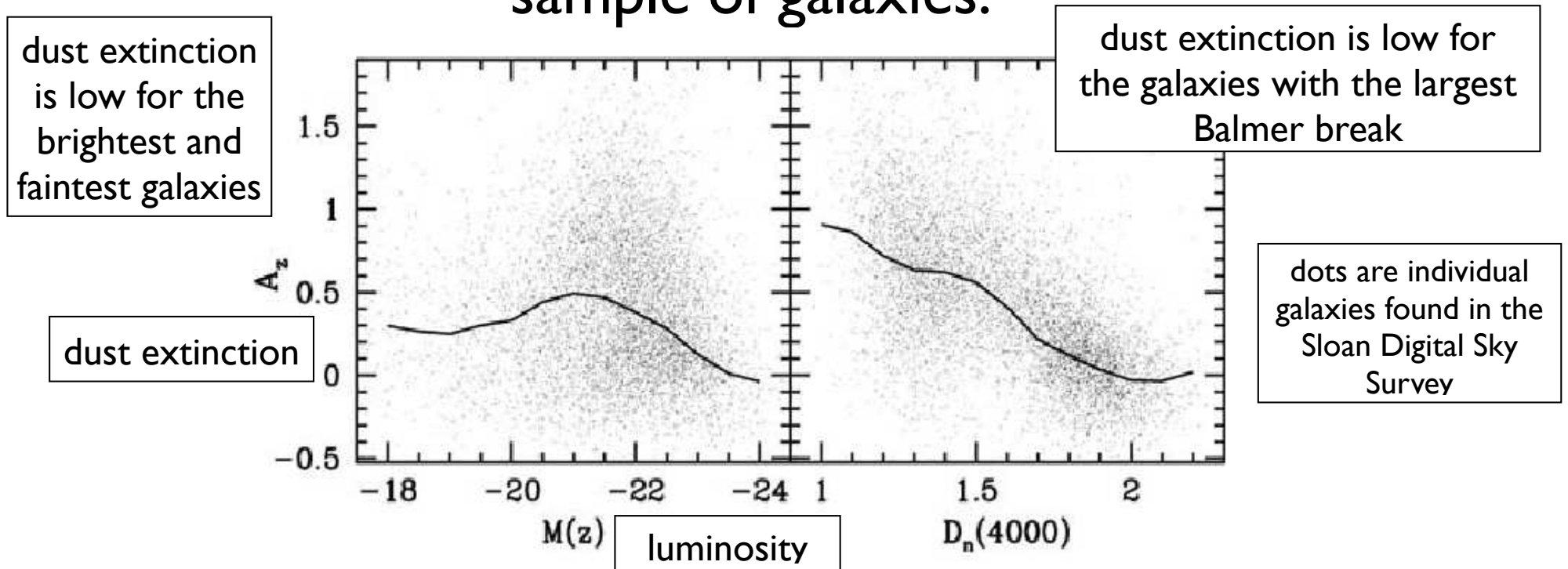
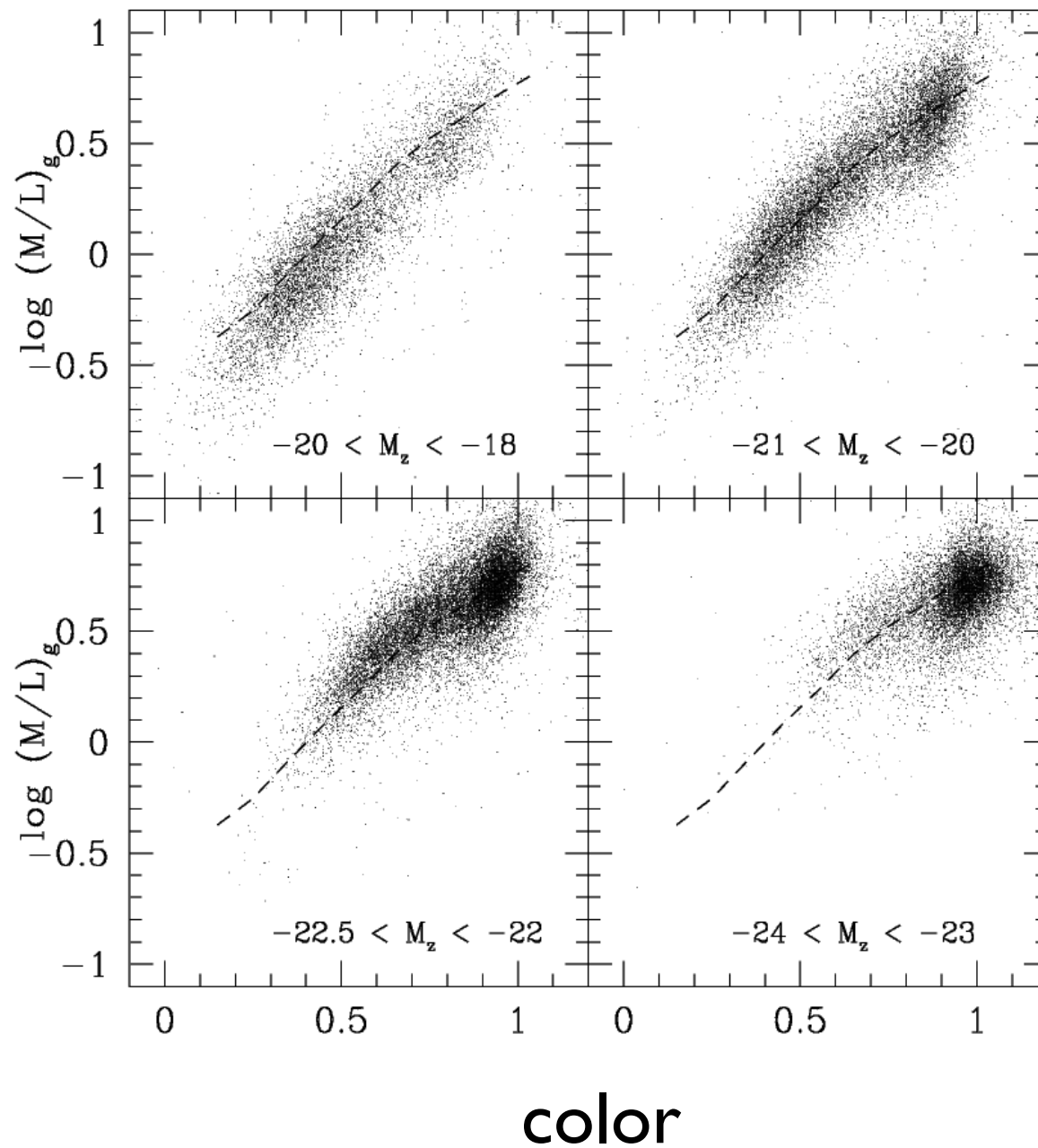


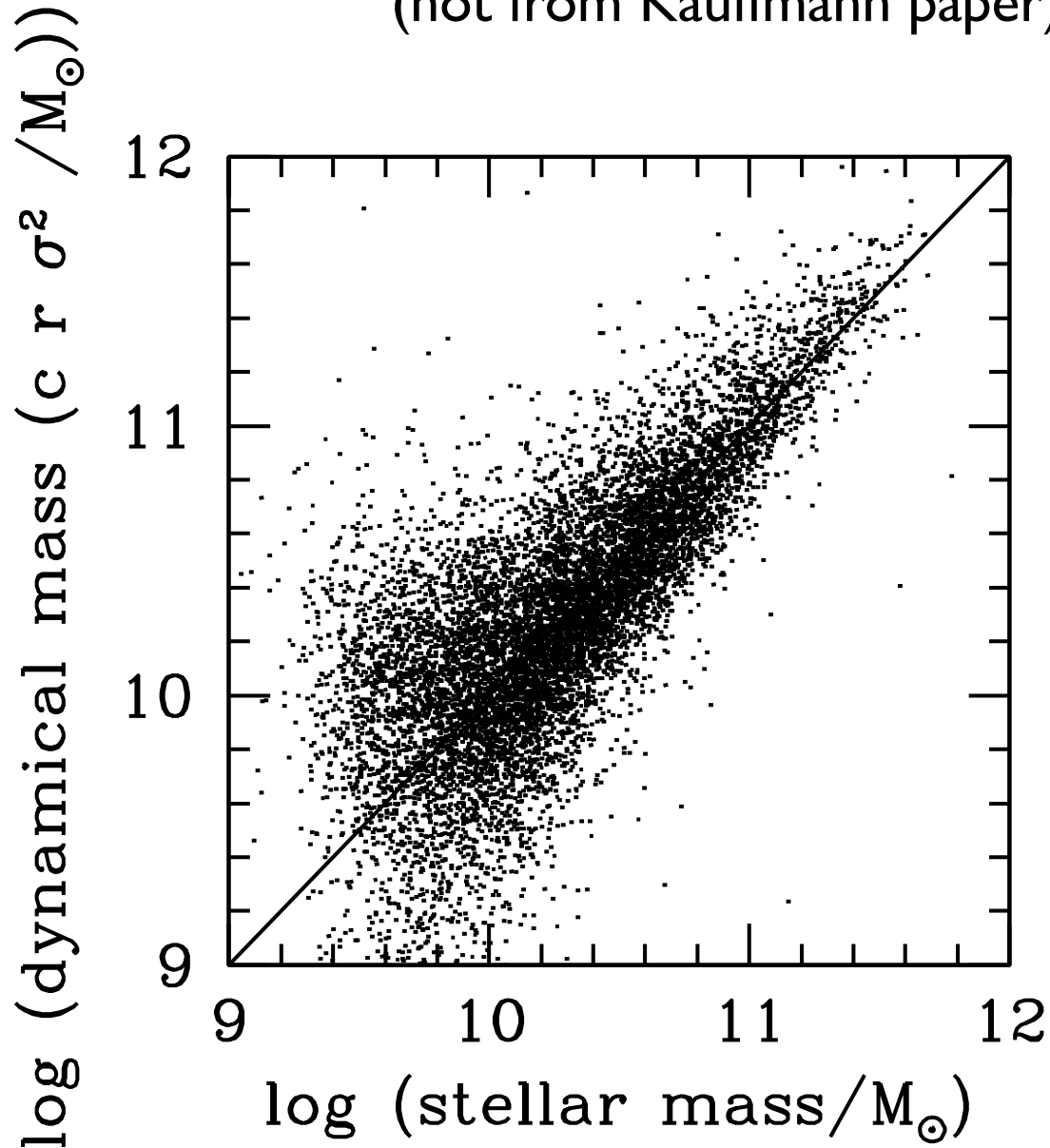
Figure 12. Left: A_z is plotted as a function of z -band absolute magnitude for a random subsample of galaxies. The solid line shows the running median of the distribution for the full sample. Right: A_z is plotted as a function of $D_n(4000)$.

Good correlation between the mass-to-light ratio of the stars to the observed color of galaxies:



Dynamical masses of galaxies found to correlate well with the stellar masses derived based on the colors:

(not from Kauffmann paper)



$$\text{Stellar Mass} = (M/L)_z L_z$$

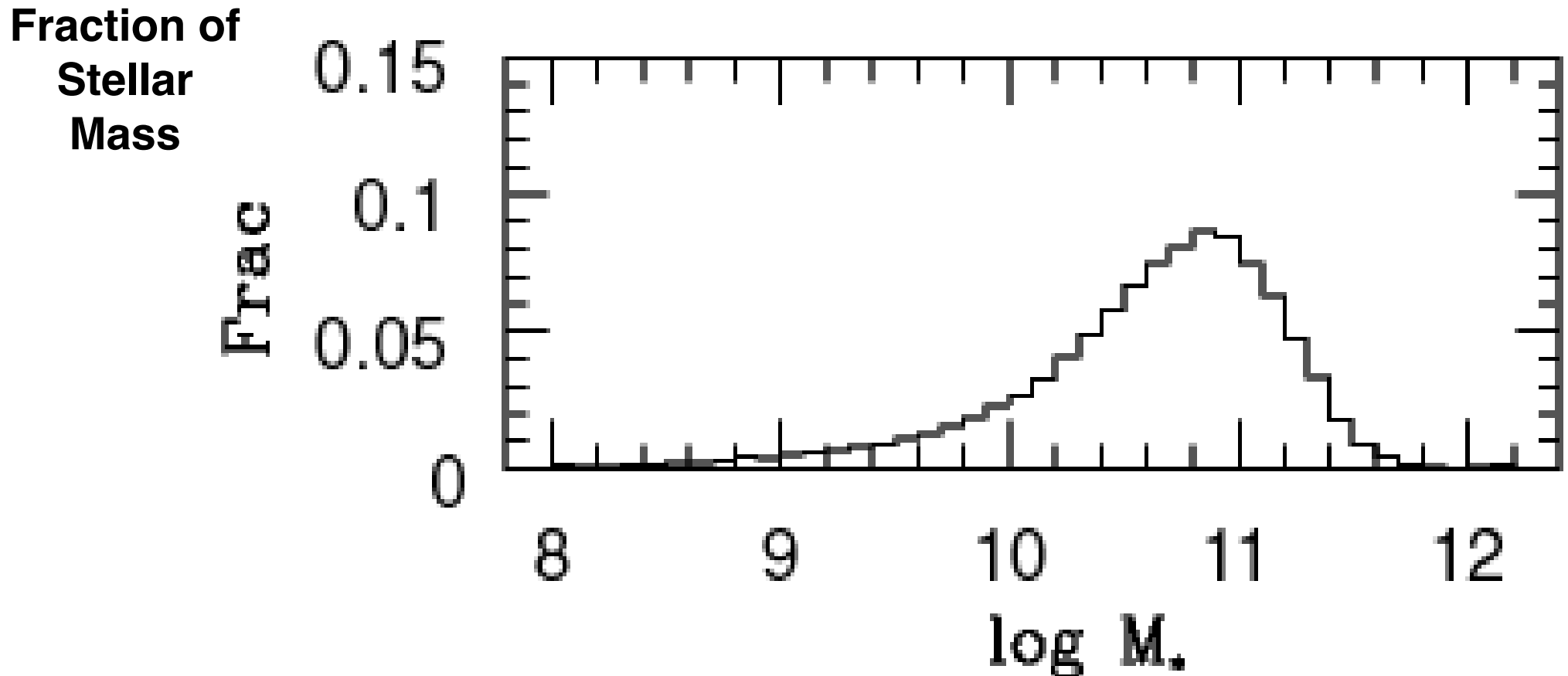
from the photometry

from the colors

from the luminosity

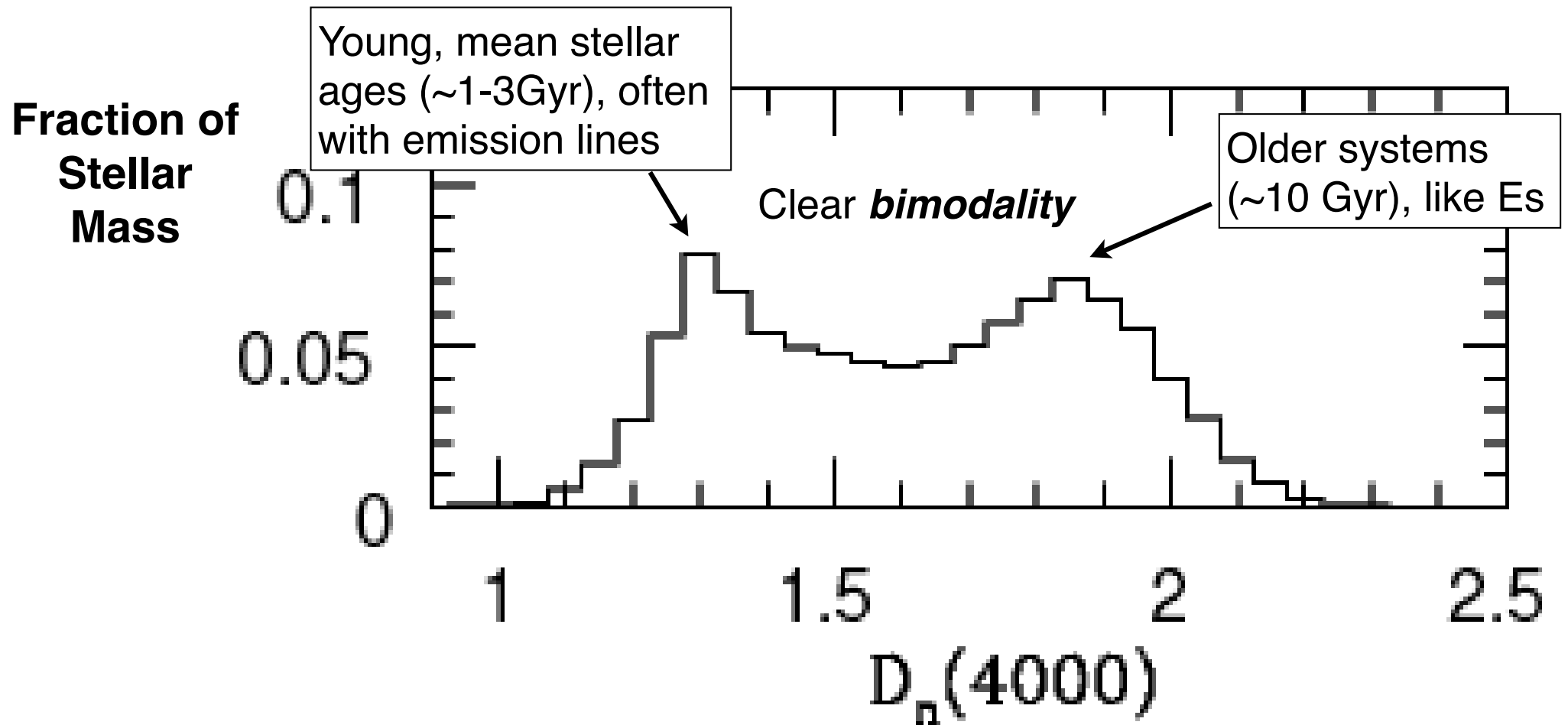
Where is the most of mass in stars in the nearby universe?

What fraction of the stellar mass in the universe exist in objects with specific properties?

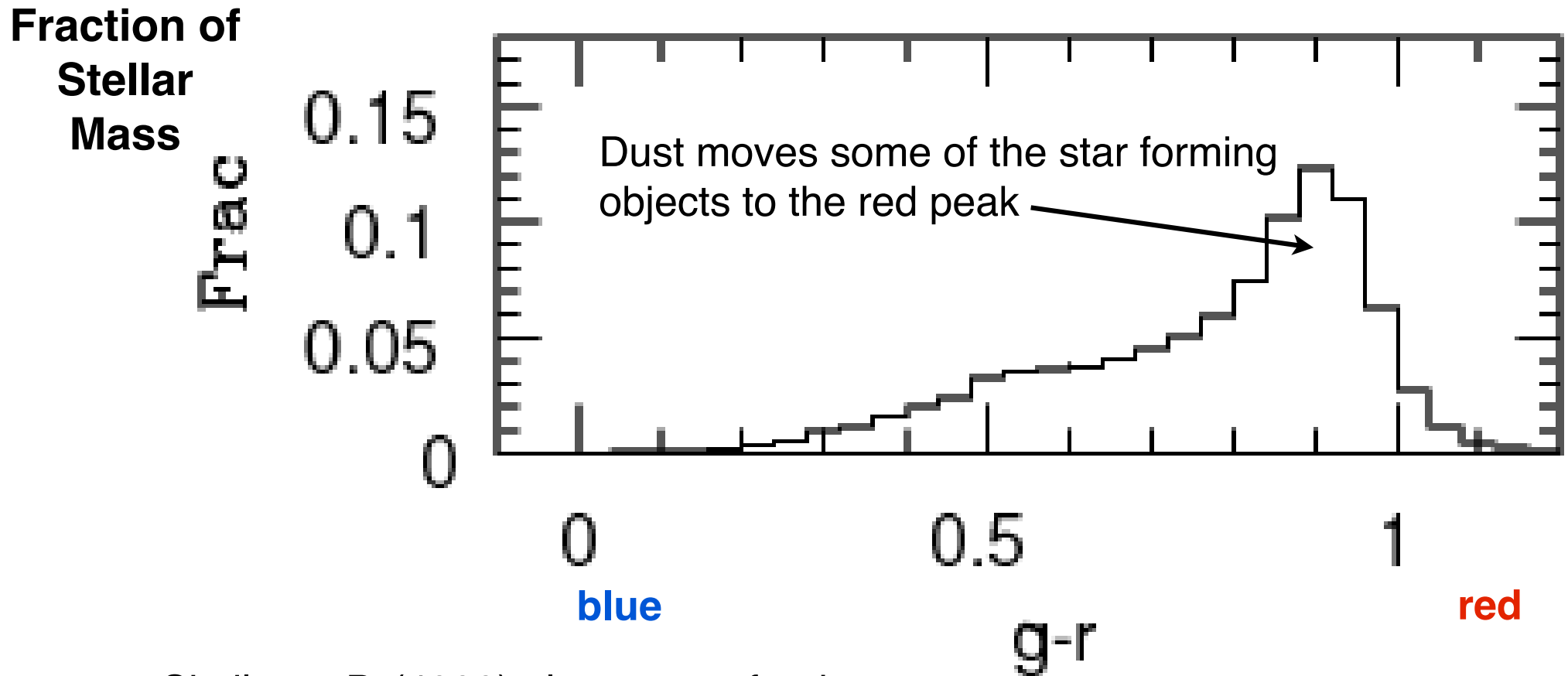


The stellar mass budget of the Universe dominated by galaxies within a factor 10 of the mass of Milky Way.

What fraction of the stellar mass in the universe exist in objects with specific properties?

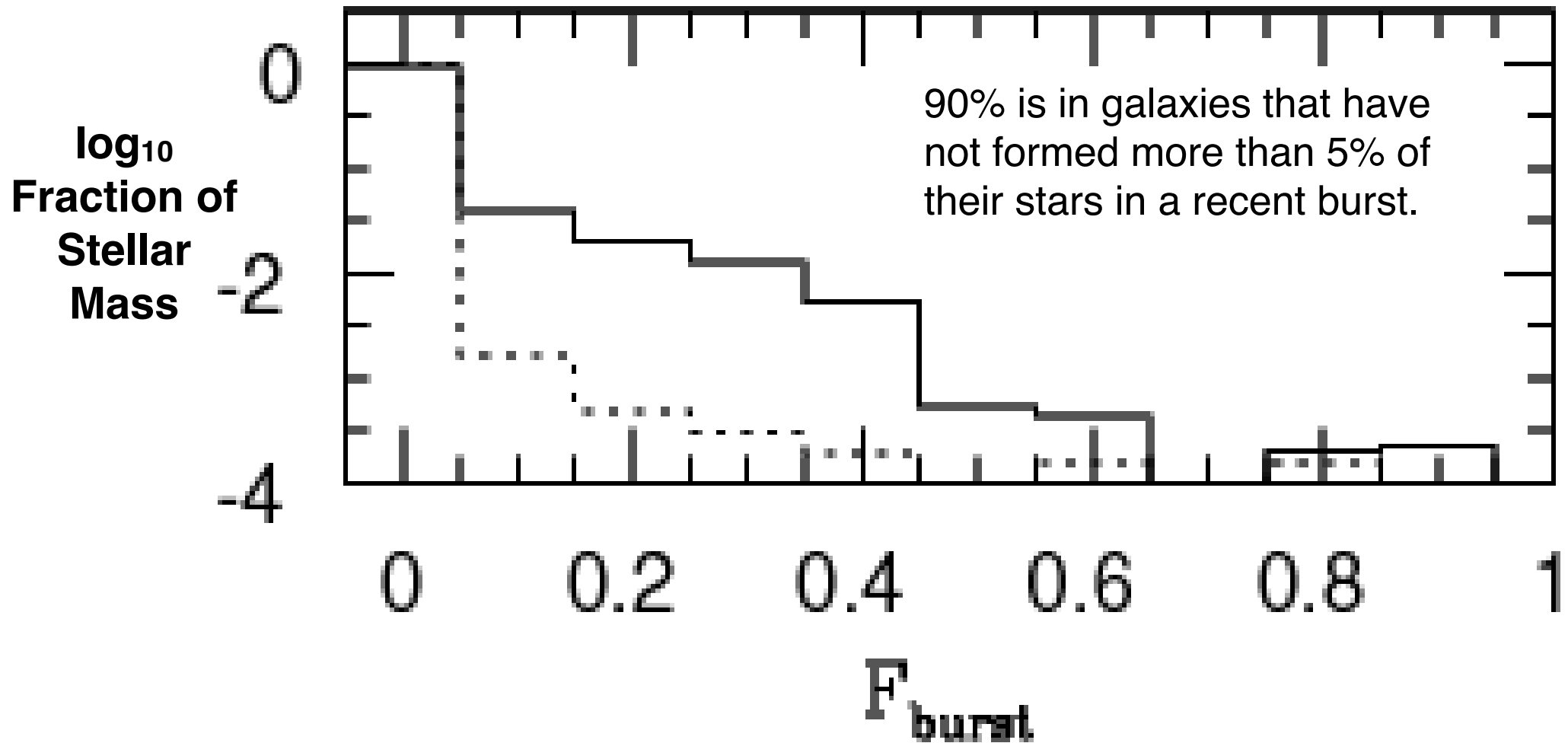


What fraction of the stellar mass in the universe exist in objects with specific properties?



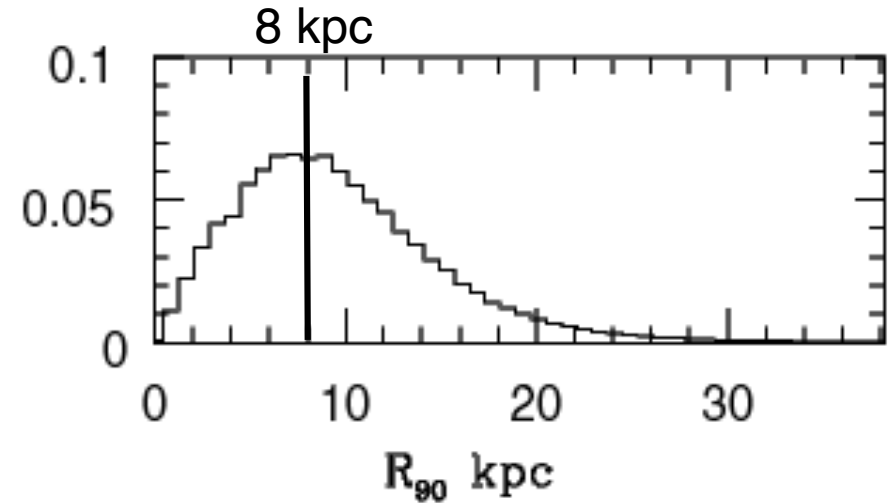
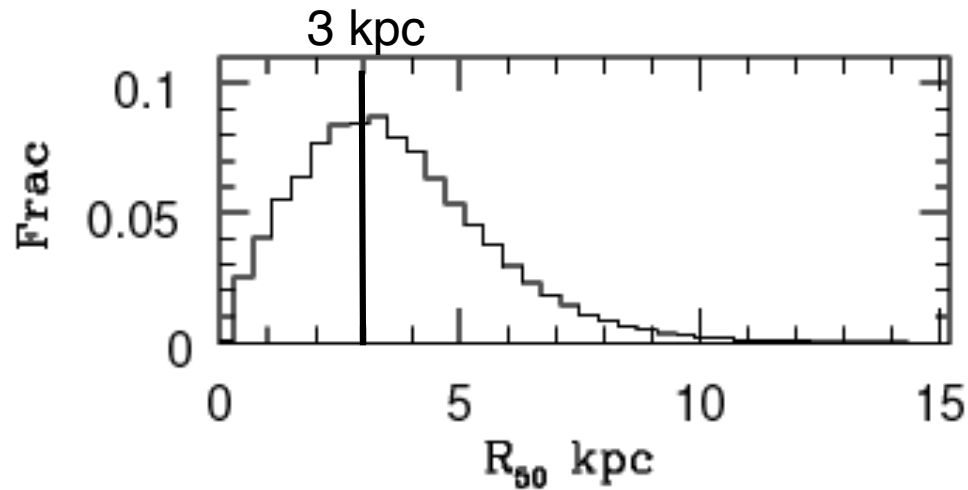
Similar to $D_n(4000)$ plot, except for the impact of dust

What fraction of the stellar mass in the universe exist in objects with specific properties?



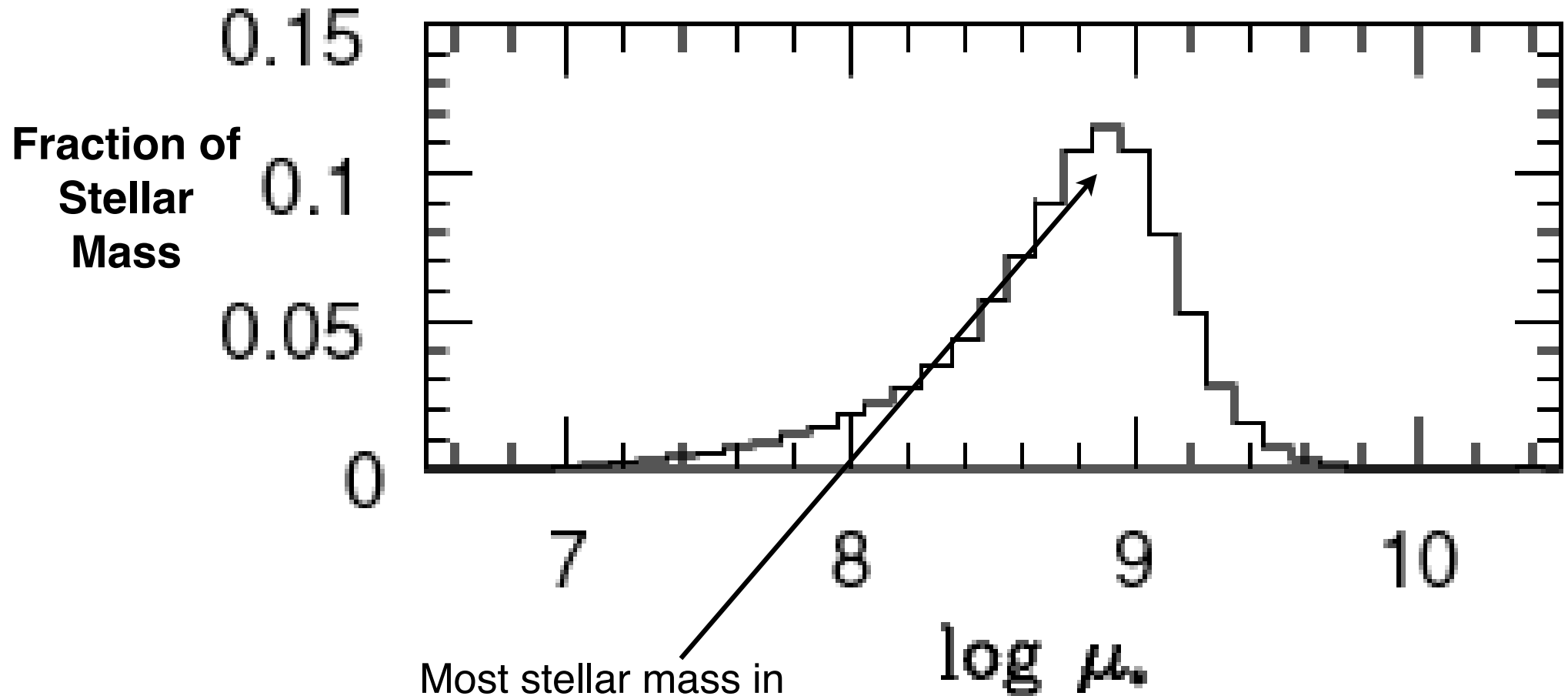
Fraction of stellar mass formed in starbursts during last 2Gyr

What fraction of the stellar mass in the universe exist in objects with specific properties?



More than 90% of the stellar mass in the universe is within galaxies that have sizes with a factor of 3 of these values

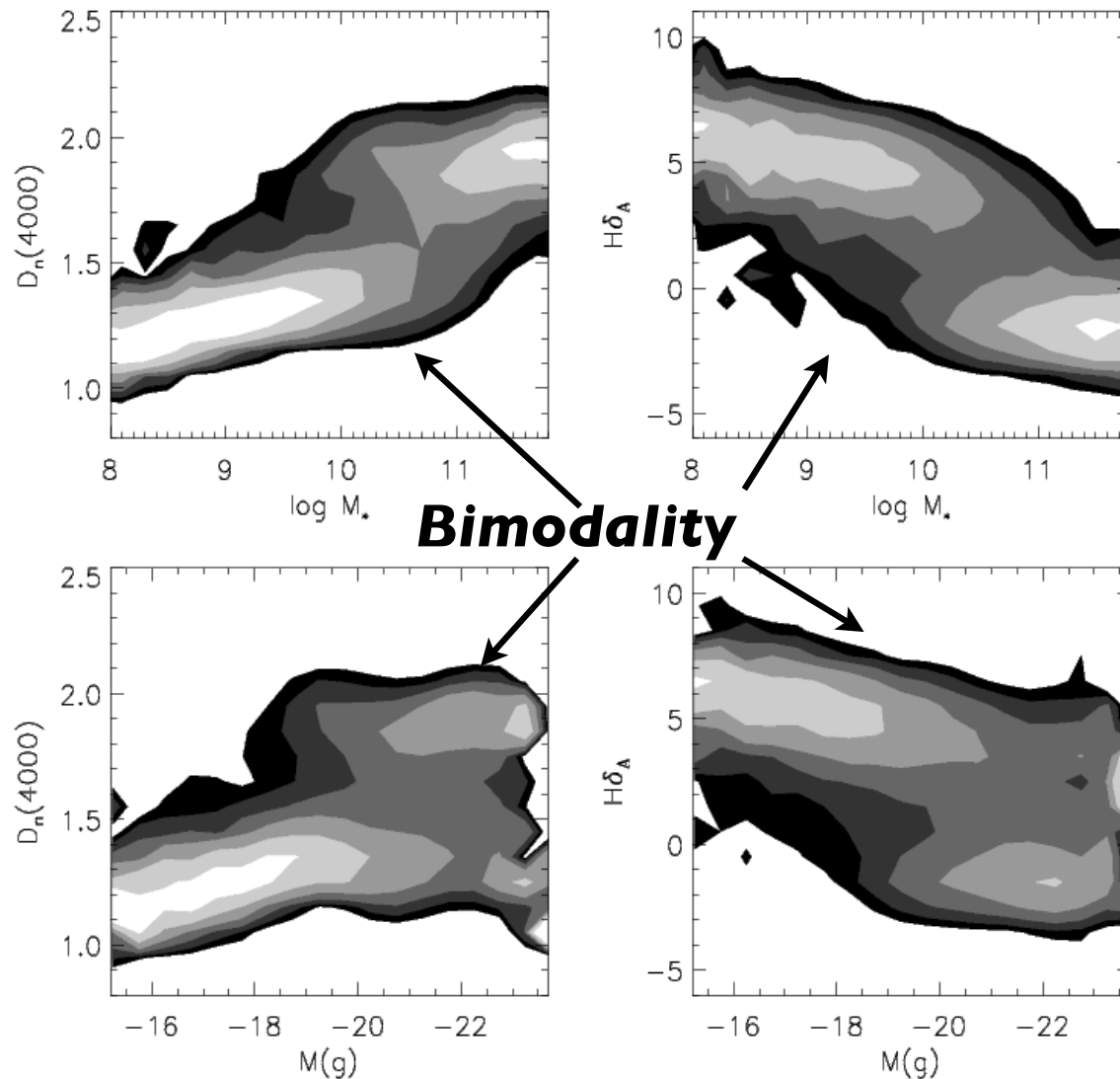
What fraction of the stellar mass in the universe exist in objects with specific properties?



Most stellar mass in Universe has surface stellar mass density within a factor two of this

How do the properties of galaxies depend on their stellar mass?

How do the spectral properties of galaxies, i.e., $D_n(4000)$ and $H\delta$, depend on their mass?



what is striking is a **bimodality** in the distribution

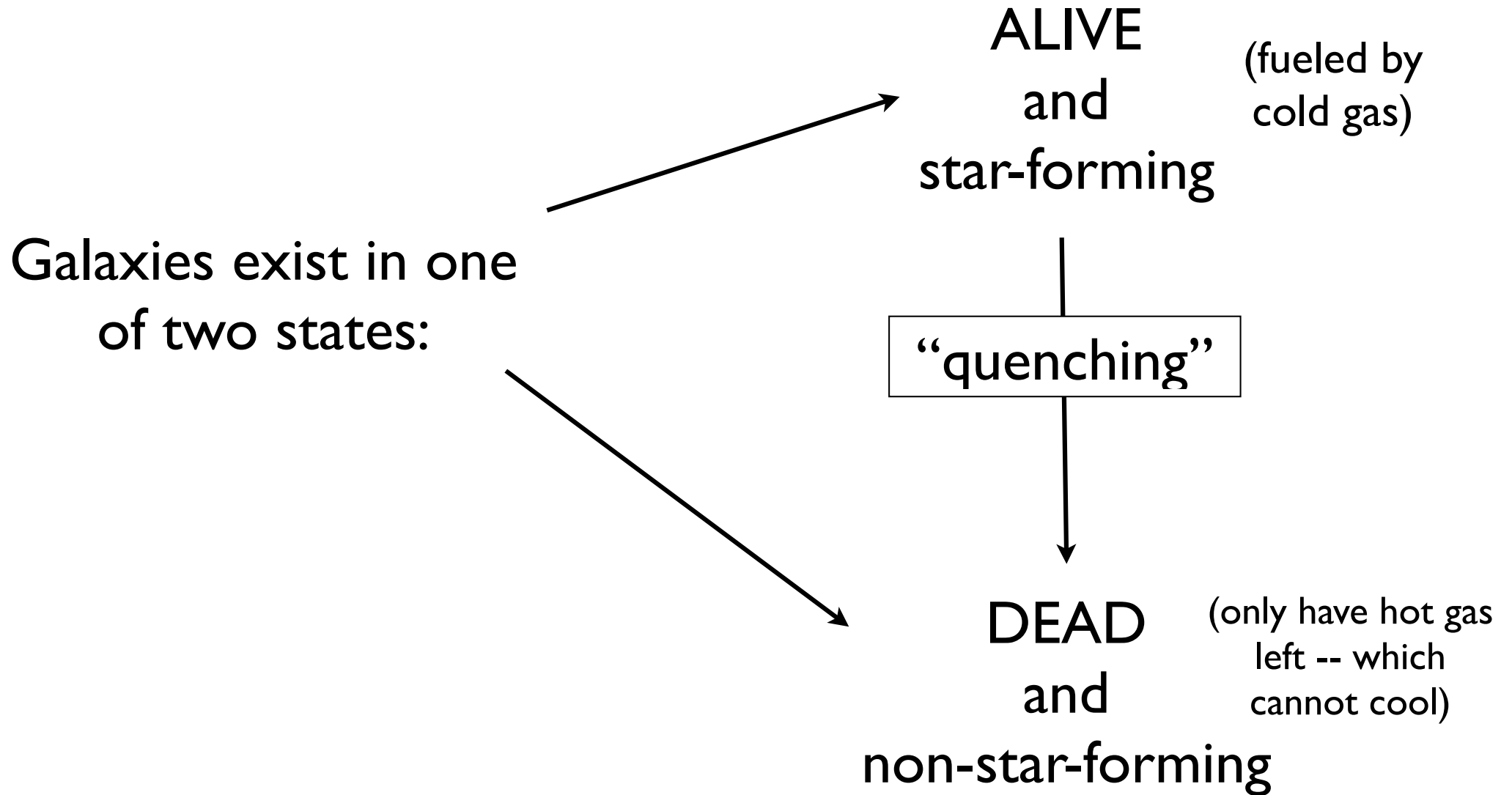
it occurs around a solar mass of $3 \times 10^{10} M_{\text{solar}}$

galaxies that are less massive than $3 \times 10^{10} M_{\text{solar}}$ show low $D_n(4000)$

galaxies that are more massive than $3 \times 10^{10} M_{\text{solar}}$ have high $D_n(4000)$

Figure 1. Conditional density distributions showing trends in the stellar age indicators $D_n(4000)$ and $H\delta_A$ as functions of the logarithm of stellar mass and of g -band absolute magnitude. Galaxies have been weighted by $1/V_{\text{max}}$ and the bivariate distribution function has been normalized to a fixed number of galaxies in each bin of $\log M_*$ or $M(g)$. Here and in all subsequent contour plots, each contour represents a factor of 2 change in density.

Why is there a bimodality?



This can also be seen in the following figure showing distribution of galaxies vs. $D_n(4000)$

Fraction

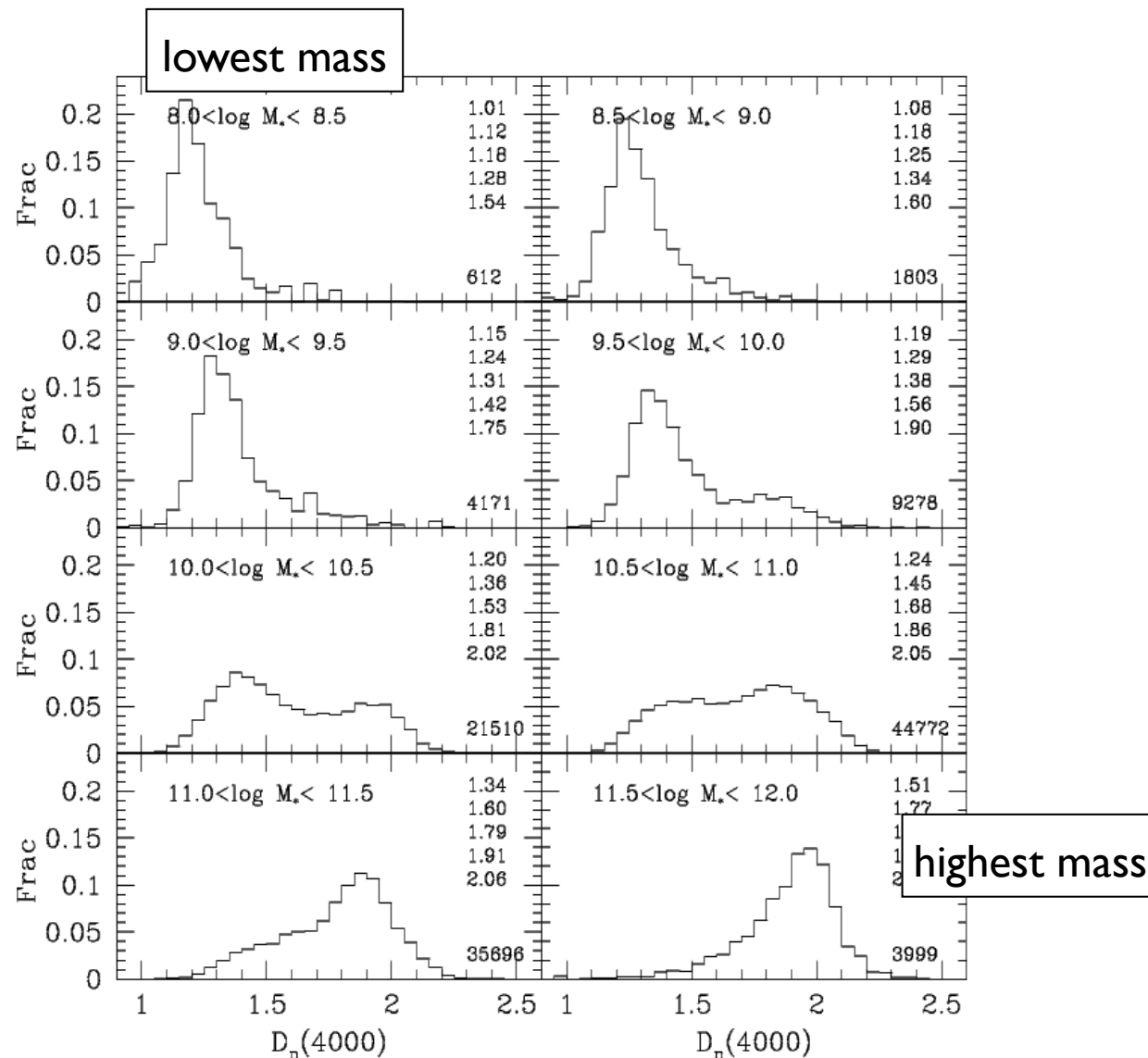
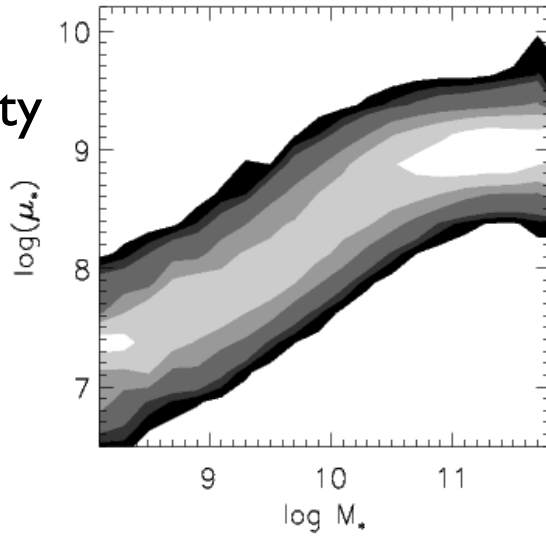


Figure 2. Histograms showing the fraction of galaxies as a function of $D_n(4000)$ in eight different ranges of stellar mass. The numbers in the upper right-hand corner of each panel list, from top to bottom, the fifth, 25th, 50th, 75th and 95th percentiles of the distribution. The number in the lower right-hand corner is the number of galaxies contributing to the histogram.

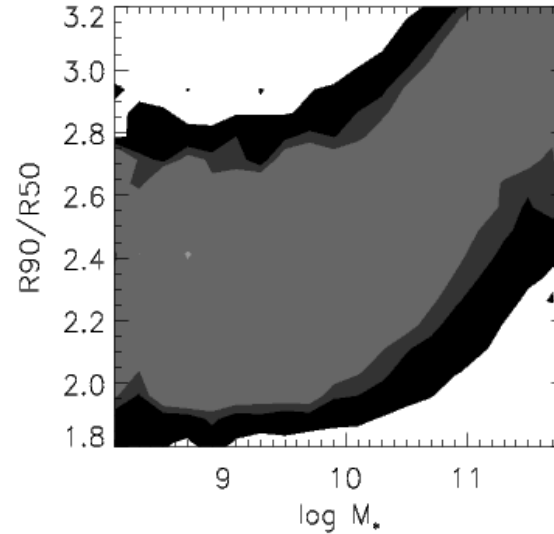
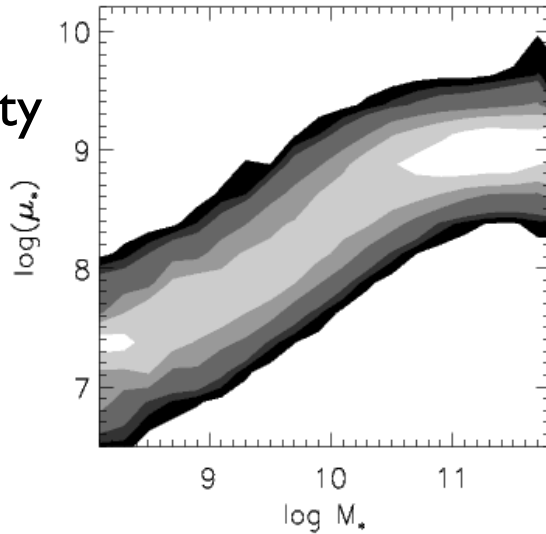
Other structural properties of galaxies also depend on their mass

μ_* = surface density of stars = M_* / radius^2



Other structural properties of galaxies also depend on their mass

μ_* = surface density of stars = M_* / radius^2



“concentration of light”

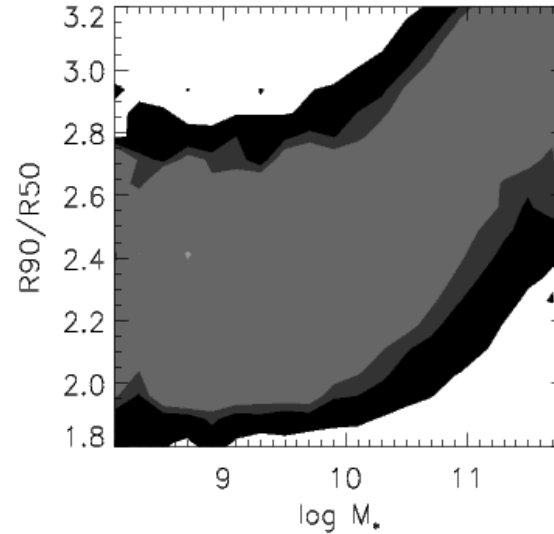
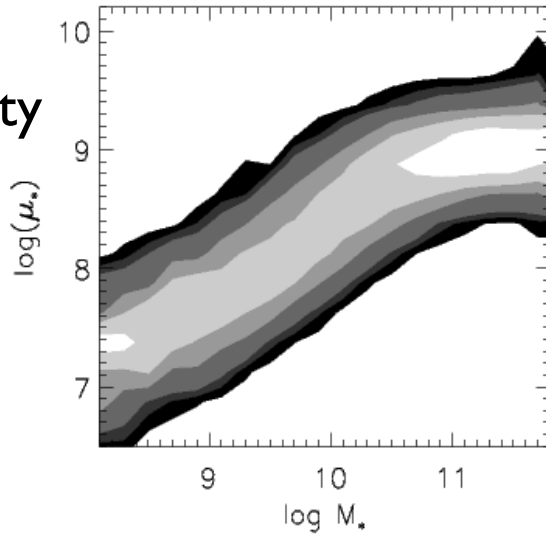
R_{90} / R_{50} = radius containing 90% of light / radius containing 50% of light

related to the Sersic index of galaxies

low mass galaxies have exponential disks while high mass galaxies have $r^{1/4}$ profiles

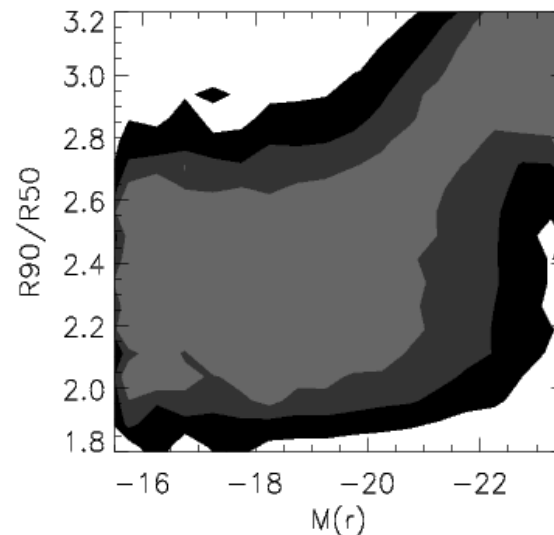
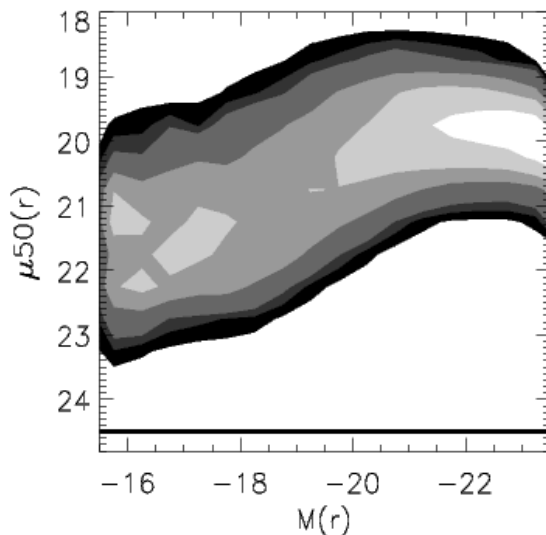
Other structural properties of galaxies also depend on their mass

μ_* = surface density of stars = M_* / radius^2



“concentration of light”

R_{90} / R_{50} = radius containing 90% of light / radius containing 50% of light



related to the Sersic index of galaxies

low mass galaxies have exponential disks while high mass galaxies have $r^{1/4}$ profiles

Figure 8. Conditional density distributions showing trends in the structural parameters μ_* , $\mu_{1/2}$ and $C = R_{90}/R_{50}$ as a function the logarithm of stellar mass and as a function of r -band absolute magnitude. Galaxies have been weighted by $1/V_{\text{max}}$ and the bivariate distribution function has been normalized to a fixed number of galaxies in each bin of $\log M_*$ and of r -band absolute magnitude. The line in the bottom left-hand panel indicates the surface brightness completeness limit of the SDSS survey.

There is a good connection between the Spectral Properties of Galaxies ($D_n(4000)$ and $H\delta$) and Structural Properties (μ^* and R_{90}/R_{50})

$D_n(4000)$ correlates well with surface density of stars and also with the concentration R_{90}/R_{50} .

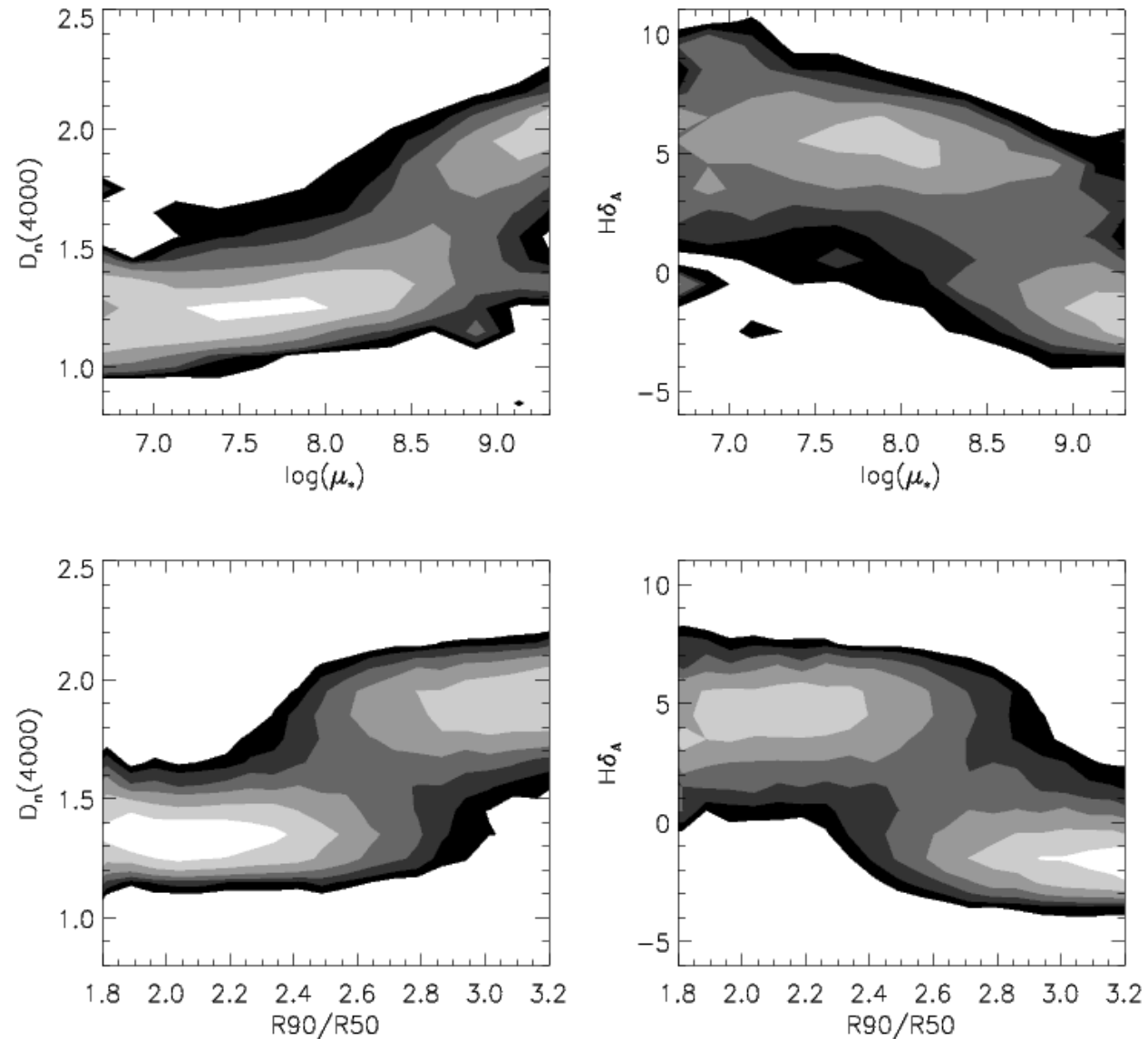


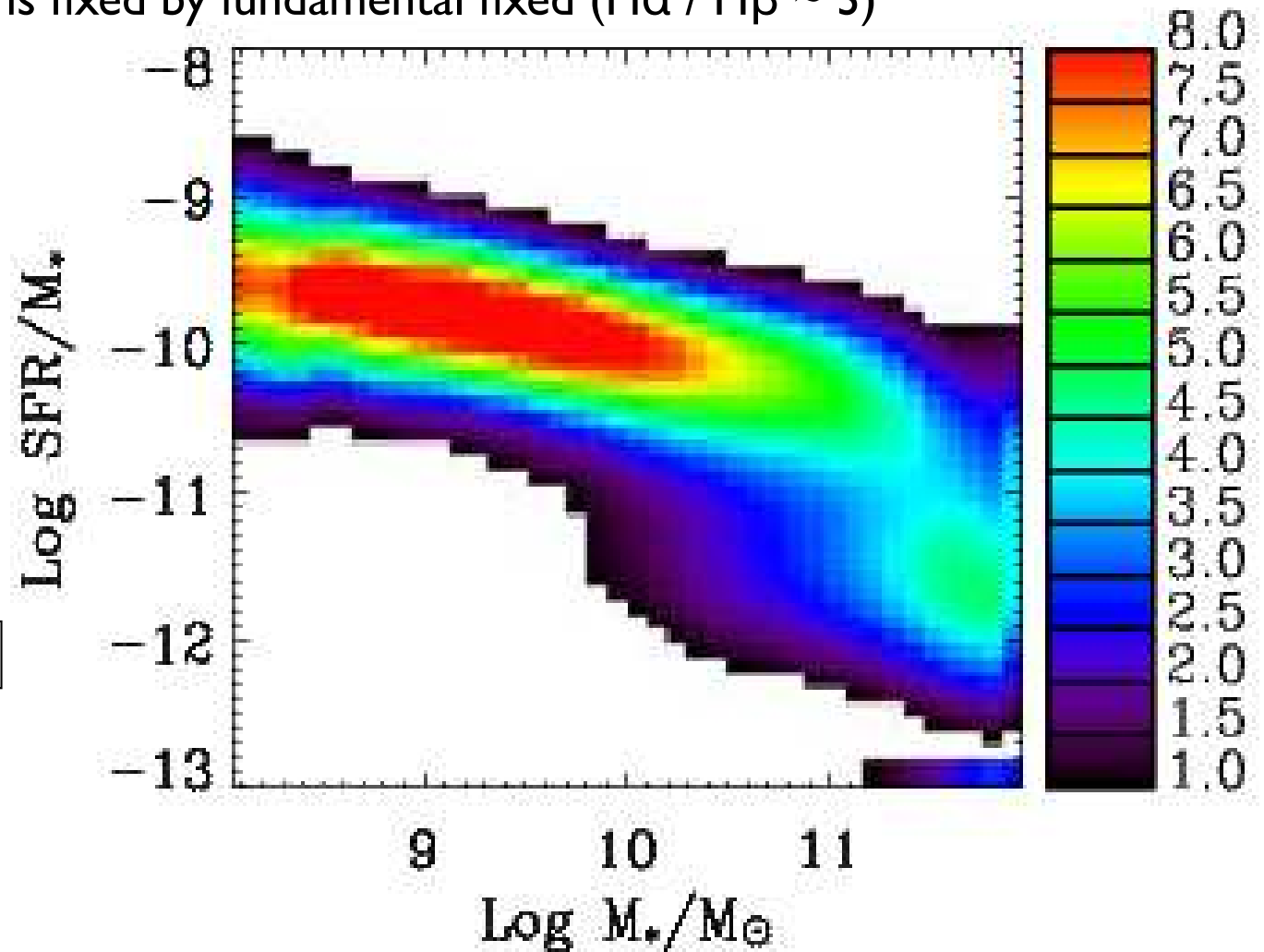
Figure 12. Conditional density distributions showing trends in the stellar age indicators $D_n(4000)$ and $H\delta_A$ as functions of the logarithm of the surface mass density μ_* and of the concentration index C .

Can compute the star formation rate for galaxies in the nearby universe from the strength of H α , H β lines

and correcting for extinction realizing that intrinsic ratio of H α , H β is fixed by fundamental fixed (H α / H β \sim 3)

Specific Star Formation
Rate
= Star Formation Rate /
M* (stellar mass)

Brinchmann et al. 2004



Galaxies with lower stellar mass tend to be star-forming, whereas galaxies with higher stellar masses appear to have stopped forming stars.

Very massive galaxies formed most of their stars long ago in the past!