Polarization nulling interferometry for exoplanet detection

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Abstract: We introduce a new concept of nulling interferometer without any achromatic device, using polarization properties of light. This type of interferometer should enable a high rejection ratio in a theoretically unlimited spectral band. We analyze several consequences of the proposed design, notably, the possibility of fast internal modulation.

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References and links

1. Introduction
The first exoplanet has been discovered in 1995 by Mayor and Queloz [1]. Since that moment, more than one hundred and fifty planets have been detected within ten years. All of them were found by indirect methods [2, 3], which means that only some effects that the planet has on its star have been detected.

Direct detection of an Earth-like exoplanets is not an easy task. Indeed, if our solar system was seen from a distance of 10 pc, the angular separation between Earth and Sun would be equal to 0.5 μrad and the brightness contrast between the star and the planet would be 10^6 at 10 μm and significantly larger (10^{10}) in the visible.

Nulling interferometry [4] seems a quite promising technique up to now. It consists in observing a star-planet system with an array of telescopes, and then combining the light from
these telescopes in such a way that, simultaneously, destructive interference occurs for the star light and (partially) constructive interference for the planet light. The ratio between the intensities corresponding to constructive and destructive interference is called the rejection ratio. To be able to detect a planet, this ratio should be of the order of at least $10^6$, with the extra requirement that it should be achieved in a wide spectral band (6–18 $\mu$m or even wider [5]). Indeed, this wide band is required to obtain spectral information from the planet and to optimally exploit the photon flux from the planet.

To reach this high rejection ratio in a wide spectral band, most current nulling interferometers use a (achromatic) phase shifter. In this paper, we present a totally different approach that makes use of the polarization properties of light, leading to a new way to achieve nulling interferometry. Note that a similar analogy has been proposed in visible coronography [6].

In Section 2, we derive the generalized condition to have on-axis destructive interference for an $N$-telescope array, including the polarization effects. We apply this condition to a two- and a three-telescope configurations. In Section 3, we apply this concept in a wide spectral band and we propose a design of a new type of nulling interferometer. In Section 4, we look at the interference patterns that can be obtained with the proposed design. In Section 5, we establish a criterion to define an acceptable width of the spectral band. In Section 6, we look at the sensitivity of the proposed design with respect to some imperfections and misalignments. Our conclusions are then summarized in Section 7.

2. Generalized nulling condition

In this section, we derive the general condition to have on-axis destructive interference for an array of $N$ telescopes and we look at some implications of this condition, in the case of a two- and a three-beam nulling interferometer.

Let us consider an array of $N$ telescopes and let us assume that we can apply independent phases and amplitudes $\phi_j$ and $A_j$ to each beam before recombination. To cancel the light from the star, we need on-axis destructive interference. We can show [7] that the condition to have such a destructive interference (nulling condition) is given by

$$\sum_{j=1}^{N} A_j \exp(i\phi_j) = 0.$$  \hspace{1cm} (1)

We can divide both members of this equation by the factor $A_1 \exp(i\phi_1)$, in such a way that the amplitudes $A_j$ and the phases $\phi_j$ are defined relatively to the amplitude and phase of the first beam. Note that we assumed here that the relative amplitudes $A_j$ are not wavelength-dependent but we did not make any assumption about the absolute spectra of the beams. Note also that these relative amplitudes $A_j$ and phases $\phi_j$ could be wavelength-dependent [7]. A more general condition can be derived assuming independent states of polarization for each beam. Using Jones formalism [8] to describe polarization, the generalized condition is given by

$$\sum_{j=1}^{N} \vec{A}_j \exp(i\phi_j) = \sum_{j=1}^{N} \left( \begin{array}{c} A_{x,j} \\ A_{y,j} \end{array} \right) \exp(i\phi_j) = 0,$$  \hspace{1cm} (2)

where $A_{x,j}$ and $A_{y,j}$ are complex numbers. This generalized condition has very important consequences since it can lead to a new type of nulling interferometers, as discussed below.

2.1. Example 1: Two-beam nulling interferometer

In the case of a two-beam nulling interferometer, the generalized nulling condition in Eq. (2) simply amounts to

$$\vec{A}_1 \exp(i\phi_1) = -\vec{A}_2 \exp(i\phi_2).$$  \hspace{1cm} (3)
In most current nulling interferometers, this condition is satisfied by applying a \( \pi \)-phase shift between the two beams (\( \phi_2 = \phi_1 + \pi \)). The condition in Eq. (3) could also be fulfilled without any phase shift but considering a polarization rotation of \( \pi \) (\( \vec{A}_1 = -\vec{A}_2 \)). This is a fundamentally different approach, as it will appear more clearly in the following example.

2.2. Example 2: Three-beam nulling interferometer

In this case, we have the following nulling condition.

\[
\vec{A}_1 \exp(i\phi_1) + \vec{A}_2 \exp(i\phi_2) + \vec{A}_3 \exp(i\phi_3) = 0. \tag{4}
\]

If we assume that all the beams have the same phase, we have

\[
\vec{A}_1 + \vec{A}_2 + \vec{A}_3 = 0. \tag{5}
\]

This condition can be fulfilled by rotating the polarization of the beams. For example, if we impose a horizontal linear state of polarization on the first beam, we could satisfy the condition in Eq. (5) using

\[
\vec{A}_1 = A_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

\[
\vec{A}_2 = A_0 \begin{pmatrix} \cos \left( \frac{2\pi}{3} \right) & \sin \left( \frac{2\pi}{3} \right) \\ -\sin \left( \frac{2\pi}{3} \right) & \cos \left( \frac{2\pi}{3} \right) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A_0 \begin{pmatrix} \cos \left( \frac{2\pi}{3} \right) \\ -\sin \left( \frac{2\pi}{3} \right) \end{pmatrix},
\]

\[
\vec{A}_3 = A_0 \begin{pmatrix} \cos \left( \frac{4\pi}{3} \right) & \sin \left( \frac{4\pi}{3} \right) \\ -\sin \left( \frac{4\pi}{3} \right) & \cos \left( \frac{4\pi}{3} \right) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A_0 \begin{pmatrix} \cos \left( \frac{4\pi}{3} \right) \\ -\sin \left( \frac{4\pi}{3} \right) \end{pmatrix}. \tag{6c}
\]

This shows that we can satisfy the nulling condition without any phase shifter by only rotating the polarization and consequently cancel the light coming from an on-axis star.

If a planet is orbiting around that star, then the planetary light coming from the different telescopes will have different optical path lengths. For that reason, it is interesting to look at the detected intensity as a function of the optical path differences between the three beams. Let us first consider the monochromatic case. The detected amplitude as a function of the optical path differences is given, within a phase factor, by

\[
\vec{A}_{\text{tot}} = \vec{A}_1 + \vec{A}_2 \exp(i2\pi\text{OPD}_{21}/\lambda) + \vec{A}_3 \exp(i2\pi\text{OPD}_{31}/\lambda). \tag{7}
\]

where \( \text{OPD}_{21} \) and \( \text{OPD}_{31} \) are respectively the optical path differences between beams 2 and 1 and between beams 3 and 1. The detected intensity is then given by the square modulus of the amplitude in Eq. (7).

In the case of the example of Eqs. (6), we find the detected intensity depicted in Fig. 1. The rejection ratio, defined as the ratio between the maximal and minimal intensities of the interference pattern, is theoretically infinite.

It is usually thought that beams with different coherent states of polarization cannot interfere with a high contrast. Our example shows that we can make three differently-polarized coherent beams interfere with a theoretically perfect contrast. This is also true for \( N \) beams provided that \( N > 2 \). The second consequence is that, since the intensity depends on the optical path differences, it should be possible to have constructive interference for the light coming from the planet. The important fact is that the destructive interference takes place at the zero-OPD position. In that case, there is no wavelength-dependent phase difference between the beams.
OPD21 (in $\mu$m)

OPD31 (in $\mu$m)

Fig. 1. Normalized detected intensity (simulation) as a function of the optical path differences (OPD) between the three beams.

3. Applications in wide-band nulling interferometry

In this section, we look at the generalized condition in a wide spectral band in order to design a new type of nulling interferometer.

To reach the amplitudes in Eqs. (6) for every wavelength in the spectral band, we would need achromatic polarization rotators. This is possible by combining different waveplates to create achromatic half-wave plates [9] or by using zero-order gratings [10]. But we choose a different approach, as explained below.

Suppose a system of $N$ beams, initially horizontally linearly polarized (x direction). Each polarization is then changed using a simple waveplate whose principal axis makes an angle $\alpha$ with the horizontal (see Fig. 2). If $T_r$ and $T_\alpha$ are the complex transmission coefficients of the waveplate in its principal directions ($T_r = |T_r|$ and $T_\alpha = |T_\alpha| \exp(i\phi_{o-e})$, where $\phi_{o-e}$ is the phase difference between the ordinary and extraordinary axes), the Jones matrix of that waveplate at an angle $\alpha = 0$ is given by

$$M_w(0) = \begin{pmatrix} T_r & 0 \\ 0 & T_\alpha \end{pmatrix}. \quad (8)$$

For an orientation of the waveplate $\alpha$, the Jones matrix would then be

$$M_w(\alpha) = R(-\alpha)M_w(0)R(\alpha) = \begin{pmatrix} T_r \cos^2 \alpha + T_\alpha \sin^2 \alpha & \frac{1}{2} \sin 2\alpha (T_r - T_\alpha) \\ \frac{1}{2} \sin 2\alpha (T_r - T_\alpha) & T_r \sin^2 \alpha + T_\alpha \cos^2 \alpha \end{pmatrix}, \quad (9)$$

where $R(\alpha)$ is the rotation matrix at an angle $\alpha$. The polarization state after the waveplate is then given by

$$\vec{A} = A \begin{pmatrix} T_r \cos^2 \alpha + T_\alpha \sin^2 \alpha & \frac{1}{2} \sin 2\alpha (T_r - T_\alpha) \\ \frac{1}{2} \sin 2\alpha (T_r - T_\alpha) & T_r \sin^2 \alpha + T_\alpha \cos^2 \alpha \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A \begin{pmatrix} T_r \cos^2 \alpha + T_\alpha \sin^2 \alpha \\ \frac{1}{2} \sin 2\alpha (T_r - T_\alpha) \end{pmatrix}. \quad (10)$$

Since we want on-axis destructive interference without any phase difference between the beams and assuming that all the waveplates are exactly the same but with different orientations,
Fig. 2. The polarization of each beam (initially linear in the x direction (horizontal)) is changed after a waveplate whose principal axis makes an angle $\alpha$ with the horizontal.

we must satisfy

$$\sum_{j=1}^{N} A_j \left( T_r \cos^2 \alpha_j + T_\alpha \sin^2 \alpha_j \right) = \left( T_r \sum_{j=1}^{N} A_j \cos^2 \alpha_j + T_\alpha \sum_{j=1}^{N} A_j \sin^2 \alpha_j \right) = 0$$  \hspace{1cm} (11)

Using a simple waveplate in a wide spectral band, $T_r$ and $T_\alpha$ will be wavelength-dependent in such a way that the first component of the vector in Eq. (11) cannot be equal to zero for every wavelength. The second component, on the other hand, can be canceled achromatically by a good choice of the amplitudes $A_j$ and angles $\alpha_j$. If we add, for each beam, a perfect vertical linear polarizer after the wave plate, the amplitude of the $j^{th}$ beam is then given by

$$\vec{A}_j = \left( \begin{array}{c} 0 \\ \frac{1}{2} A_j (T_r - T_\alpha) \sin 2\alpha_j \end{array} \right),$$ \hspace{1cm} (12)

and the nulling condition simply amounts to

$$\sum_{j=1}^{N} A_j \sin 2\alpha_j = 0.$$ \hspace{1cm} (13)

This condition is wavelength-independent. Therefore the null is achromatic if we assume identical waveplates.

In this proposed type of nulling interferometers, each beam encounters a horizontal linear polarizer, a waveplate and a vertical linear polarizer (see Fig. 3). Thus, it should be possible to reach a high rejection ratio in a wide spectral band with simple commercially available components. For example, in the case of a two-beam nulling interferometer and choosing $A_1 = A_2$, the condition to have an achromatic null is that $\alpha_2 = \pi - \alpha_1$ or $\alpha_2 = \pi / 2 + \alpha_1$. In order to maximize the transmission of the interferometer, we can choose $\alpha_1 = \pi / 4$ and $\alpha_2 = 3\pi / 4$. This can also be easily applied to a three-beam nulling interferometer by choosing for example $A_1 = A_2 = A_3$, $\alpha_1 = \pi / 4$, $\alpha_2 = 7\pi / 12$ and $\alpha_3 = 11\pi / 12$.

Note that we would have obtained similar results if the beams were initially vertically linearly polarized. We then could use a polarizing beam splitter instead of the first linear polarizer and apply the same principle to both outputs of the beam splitter in order to use the whole incoming intensity.
4. Transmission and modulation

In this section, we look at several direct consequences of the proposed design in the case of a three-beam nulling interferometer.

4.1. Transmission map

Let us consider $N$ coplanar telescopes looking in the same direction $z$ (see Fig. 4). The position of the $j^{th}$ telescope is given in polar coordinates by $(L_j, \delta_j)$. For a point source located at an angular separation from the optical axis $\theta$ and at an azimuth angle $\varphi$, the detected complex amplitude $\vec{f}_\varphi(\theta)$ is given by

$$\vec{f}_\varphi(\theta) = \sum_{j=1}^{N} \vec{A}_j \exp(ikL_j \cos(\delta_j - \varphi))$$

$$= \sum_{j=1}^{N} \left( \begin{array}{cc} A_j \exp\left(\frac{1}{2}iL_j(T_r - T_\alpha)\sin2\alpha_j\right) \end{array} \right) \exp(ikL_j \cos(\delta_j - \varphi)).$$

Note that this expression is not general in the sense that the star which we point at lies on the $z$-axis. If this was not the case, there would be additional delays that are not taken into account here. Note also that this reasoning is valid for a space mission where longitudinal dispersion can be neglected. For any ground-based interferometer, this effect would decrease the performances of the interferometer. In principle, the function $\vec{f}_\varphi(\theta)$ should also include the polarization and birefringent properties of the individual telescopes. We have not included this effect here because we suppose that the telescopes behave identically in this respect and that, on recombination, these effects are canceled.

If we define the transmission map $T_\varphi(\theta)$ as the normalized detected intensity, we have

$$T_\varphi(\theta) = \frac{||\vec{f}_\varphi(\theta)||^2}{\max(||\vec{f}_\varphi(\theta)||^2)}.$$  

(15)

4.2. $\theta$-dependence of the transmission map

A star is not a point source but has some non-negligible finite size. For example, the angular diameter of our sun, seen from a distance of 10 pc, is of the order of 5 nrad. To detect an

Fig. 3. Design of a new type of nulling interferometer, each beam encounters a horizontal linear polarizer, a waveplate and a vertical linear polarizer.
exoplanet, we need not only a high rejection ratio for \( \theta = 0 \) but also for angular separations \( \theta \) of a few nrad. The flatter the transmission map around \( \theta = 0 \), the easier it will be to reach this “extended” rejection ratio. That is why a transmission map proportional to \( \theta^4 \) or, even better, to \( \theta^6 \) is preferred.

We can show [7] that, in order to have a \( \theta^4 \)-transmission map, we must satisfy, in addition to the nulling condition in Eq. (13),

\[
\sum_{j=1}^{N} A_j \sin 2 \alpha_j L_j \cos (\delta_j - \varphi) = 0.
\] (16)

Since this condition should be fulfilled for all angles \( \varphi \), Eq. (16) can be split into two different conditions

\[
\sum_{j=1}^{N} A_j \sin 2 \alpha_j L_j \cos \delta_j = 0,
\] (17a)

\[
\sum_{j=1}^{N} A_j \sin 2 \alpha_j L_j \sin \delta_j = 0.
\] (17b)

These conditions are different from the \( \theta^4 \)-conditions for other types of nulling interferometers [7]. However, we can show that, in the case of a three-telescope configuration, the only possible configuration to fulfill these conditions is a linear configuration, as it is the case for other nulling interferometers. For exoplanet detection, a linear configuration is less interesting because it only gives information in one direction. A solution to this lack of information would be to rotate the whole array of telescopes but this would give rise to slow modulation, as explained in the next section. We can then conclude, that no interesting three-telescope configuration can fulfill the \( \theta^4 \)-conditions.

4.3. Modulation

Another difficulty could prevent us from directly detecting an Earthlike exoplanet: the possible emission from exo-zodiacal dust near the orbital plane of the planet, as in our own solar system. We, a priori, do not know anything about the exo-zodiacal cloud, but we can assume that it is centro-symmetric. Because of this central symmetry, this problem could be handled by using

![Diagram](https://via.placeholder.com/150)

Fig. 4. Array of telescopes (dots) situated in the plane \( z = 0 \) and looking in the \( z \) direction. The angles \( \theta \) and \( \varphi \) define the direction of the incoming light. The position of the \( j \)th telescope is given in polar coordinates by \( (L_j, \delta_j) \).
modulation techniques. A possible solution is to use external modulation, which consists in rotating the whole telescope array around its center, but this gives rise to very slow modulation and it will considerably decrease the number of targets that we can observe during a space mission. A more convenient solution is internal modulation. With this technique, we do not change the positions of the telescopes. Via optical means, we create different transmission maps that we combine in order to create modulation maps.

By changing the angle $\alpha_j$, we can change the “weight” of the amplitude $A_j$. Thus, we can then change the ratio between the amplitudes of the different beams by simply rotating the waveplates, provided that the nulling condition in Eq. (13) is satisfied. This has two consequences: the first one is that, with this type of nulling interferometer, we do not need any extra amplitude-matching device, as it is the case in most of current nulling interferometers. The amplitude-matching is inherent to the design and is simply produced by a rotation of the waveplate; the second and much more important consequence is that, since we can change the ratio between the amplitudes of the beams, we can have a continuous set of transmission maps, which could be used for fast modulation.

Fig. 5 shows an example of a set of six transmission maps in the three-telescope case that have been obtained by only rotating the waveplates. In these transmission maps, the maximal intensity has been normalized to a value given by $\left(\sum_{j=1}^{N} |A_j/A_1 \sin2\alpha_j|\right)^2$. Note that this is just an example, out of a continuous range of transmission maps. Nevertheless, it can be shown that any of these transmission maps can be represented by a linear combination of three others. Therefore, three different transmission maps are sufficient to get the whole continuous set (for example, Fig. 5(a), 5(c) and 5(e) or 5(b), 5(d) and 5(f)). We also have to think, in more details, about a modulation strategy between these transmission maps.

5. Spectral response

In some applications, besides the detection of an Earthlike exoplanet, spectral information of the light coming from the planet is needed in order to study its atmosphere. In this case, a wide spectral band is required.

In the proposed design, if we make the assumption of perfect polarizers and exactly identical waveplates, there is absolutely nothing in the nulling condition in Eq. (13) that limits the spectral band, so a high rejection ratio in an infinitely-wide spectral band is not unthinkable. However, in practice, this is not true since polarizers and waveplates are not perfect and are spectrally limited. Furthermore, as we will see in this section, the response of the interferometer is not the same for all wavelengths, that is, the detected intensity is wavelength-dependent.

If we assume identical waveplates for each beam, the detected intensity will be proportional to

$$I \propto |T_r - T_\alpha|^2,$$

independently of the optical path length differences between the beams. The intensity for the constructive interference is then also proportional to Eq. (18), which, in the case of a perfect wave plate is proportional to

$$I \propto |1 - \exp(i\Delta\phi)|^2 = 4 \sin^2 \frac{\Delta\phi}{2},$$

where $\Delta\phi$ is the phase difference between the two states of polarization induced by the waveplate. If we furthermore consider conventional waveplates (as opposed to achromatic waveplates), we have

$$\Delta\phi = \frac{2\pi}{\lambda} (n_e(\lambda) - n_o(\lambda)) d = \frac{2\pi}{\lambda} B(\lambda),$$

where $n_e(\lambda)$ and $n_o(\lambda)$ are the extraordinary and ordinary indices of refraction, respectively, $d$ is the thickness of the waveplate, and $B(\lambda)$ is the birefringence of the waveplate.
Fig. 5. Simulated three-telescope transmission maps corresponding to different waveplate orientations. All these maps have been calculated with the following parameters: $A_1 = A_2 = A_3$, $L_1 = L_2 = L_3 = 25m$ and $\delta_1 = 0$, $\delta_2 = 2\pi/3$, $\delta_3 = 4\pi/3$, and for a spectral band going from 500 to 650nm. (a) $2\alpha_1 = 0, 2\alpha_2 = 2\pi/3, 2\alpha_3 = 4\pi/3$, (b) $2\alpha_1 = \pi/6, 2\alpha_2 = \pi/6 + 2\pi/3, 2\alpha_3 = \pi/6 + 4\pi/3$, (c) $2\alpha_1 = 2\pi/6, 2\alpha_2 = 2\pi/6 + 2\pi/3, 2\alpha_3 = 2\pi/6 + 4\pi/3$, (d) $2\alpha_1 = 3\pi/6, 2\alpha_2 = 3\pi/6 + 2\pi/3, 2\alpha_3 = 3\pi/6 + 4\pi/3$, (e) $2\alpha_1 = 4\pi/6, 2\alpha_2 = 4\pi/6 + 2\pi/3, 2\alpha_3 = 4\pi/6 + 4\pi/3$, (f) $2\alpha_1 = 5\pi/6, 2\alpha_2 = 5\pi/6 + 2\pi/3, 2\alpha_3 = 5\pi/6 + 4\pi/3$.
where $\lambda$ is the wavelength, $n_{e}(\lambda)$ and $n_{o}(\lambda)$ are the extraordinary and ordinary refractive indices, $d$ is the thickness and $B(\lambda)$ is the birefringence of the waveplate.

The intensity is then maximum for $\Delta \phi = (2n + 1) \pi$ (half-wave plate) and equal to zero for $\Delta \phi = 2n\pi$, where $n$ is an integer. This shows that some wavelengths will be well transmitted, while others will not be transmitted at all.

The criterion that we chose to define the acceptable spectral band is then that all the wavelengths should be transmitted with at least half the maximal intensity, which leads to the following condition

$$
(4n + 1) \frac{\pi}{2} \leq \Delta \phi = \frac{2\pi}{\lambda} B(\lambda) \leq (4n + 3) \frac{\pi}{2},
$$

(21)

We assume that the birefringence is constant in the spectral band. This is not a very realistic assumption but we can show, in the example of quartz, that it does not drastically affect the criterion. In Fig. 6, we compare the spectral response of the interferometer in the visible domain in the case of quartz waveplates and in the case of constant-birefringence approximation. We can see that the approximation is not very good but in both cases, the acceptable spectral band is of the same order of magnitude. Furthermore, the birefringence is chosen in such a way that the waveplate is a half-wave plate for the wavelength $\lambda_{0}$. We then have

$$
B = (2n + 1) \frac{\lambda_{0}}{2}.
$$

(22)

The minimal and maximal wavelengths in the acceptable spectral band are then given by

$$
\lambda_{\text{min}} = \frac{4n + 2}{4n + 3} \lambda_{0} \quad \text{and} \quad \lambda_{\text{max}} = \frac{4n + 2}{4n + 1} \lambda_{0}.
$$

(23)

We can characterize the bandwidth by defining

$$
M = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} = \frac{4n + 3}{4n + 1}.
$$

(24)

We can see that the bandwidth is maximum if we use zero-order waveplates ($n = 0$), which in this case gives $M = 3$. For example, in the infrared region, this technique would allow us to work from 6 to 18 $\mu m$. The spectral band will then probably be limited by the polarizers. Obviously, the acceptable spectral band can be wider if we use achromatic waveplates (see Fig. 6).

Note that the presence of polarizers does not necessarily imply losses if we use achromatic waveplates and a polarizing beam splitter to separate the two orthogonal states of polarization. In this way, both outputs of the beam splitter can be used in an apart set-up.

6. Sensitivity to imperfections and misalignments

In this section, we will see how sensitive the proposed set-up is with respect to misalignments and imperfections and how these defects affect the rejection ratio.

Let us first derive a general expression for the on-axis detected intensity. We assume that each polarizer could have its own imperfection, that is its own extinction ratio, $\varepsilon_{k,j}, k = 1$ or 2 ($\varepsilon_{k,j}$ represents the imperfection of the first or second polarizer for the $j$th beam) and that each waveplate could be different for each beam ($T_{r,j}$ and $T_{a,j}$). The amplitude of the $j$th beam is given by

$$
\tilde{A}_{j} = A_{j} \left( \begin{array}{c} \varepsilon_{2,j} \\ 0 \\ 1 \end{array} \right) \left( \begin{array}{ccc} T_{r,j} \cos^{2} \alpha_{j} + T_{a,j} \sin^{2} \alpha_{j} & \frac{1}{2} \sin 2 \alpha_{j} (T_{r,j} - T_{a,j}) \\ \frac{1}{2} \sin 2 \alpha_{j} (T_{r,j} - T_{a,j}) & T_{r,j} \sin^{2} \alpha_{j} + T_{a,j} \cos^{2} \alpha_{j} \end{array} \right) \left( \begin{array}{c} 1 \\ \varepsilon_{1,j} \end{array} \right).
$$

(25)
After some calculations, we find that the on-axis detected intensity is given by

\[ I_{\lambda} = \left| \sum_{j=1}^{N} A_j e_{2,j} [Tr, j \cos^2 \alpha_j + T_{\alpha,j} \sin^2 \alpha_j + \frac{1}{2} e_{1,j} \sin 2\alpha_j (Tr, j - T_{\alpha,j})] \right|^2 + \left| \sum_{j=1}^{N} A_j \left[ \frac{1}{2} \sin 2\alpha_j (Tr, j - T_{\alpha,j}) + e_{1,j} (Tr, j \sin^2 \alpha_j + T_{\alpha,j} \cos^2 \alpha_j) \right] \right|^2. \]  

(26)

Hereafter, we will analyze each defect separately. For all the simulations, we used the following parameters for the perfect case

- \( A_1 = A_2 = A_3 = 1 \),
- \( 2\alpha_1 = \frac{7\pi}{6}, 2\alpha_2 = \frac{11\pi}{6}, 2\alpha_3 = \frac{\pi}{2} \),
- \( e_{1,1} = e_{1,2} = e_{1,3} = 0 \),
- \( e_{2,1} = e_{2,2} = e_{2,3} = 0 \),
- \( Tr, 1 = T_{r,2} = T_{r,3} = Tr = 1 \),
- \( T_{\alpha,1} = T_{\alpha,2} = T_{\alpha,3} = T_{\alpha} = \exp(i5\pi\lambda_0/\lambda) \),
- \( \lambda_{\min} = 500\text{nm}, \lambda_{\max} = 650\text{nm}, \lambda_0 = 562\text{nm} \).

6.1. Amplitude and phase mismatchings

Let us first consider an amplitude mismatching \( A_j = A_{j,0} + \delta A_j \). We can show that the on-axis detected intensity is then given by

\[ I_{\lambda} = |\delta A_j \sin 2\theta_j|\left(\frac{Tr - T_{\alpha}}{4}\right)^2 \approx |\delta A_j|^2. \]  

(28)

The rejection ratio obtained with an amplitude mismatching for the first beam is depicted in Fig. 7. The amplitude mismatching should be of the order of \( 10^{-3} \) or lower in order to have a rejection ratio of \( 10^6 \).
If we consider now a phase mismatching $A_l = A_{l,0} \exp(i\delta\phi_l)$, we will find the same expression for the on-axis intensity,

$$I_\lambda = |\delta\phi_l \sin 2\theta_l|^2 \left( T_r - T_\alpha \right)^2 \approx |\delta\phi_l|^2.$$  \hspace{1cm} (29)

We can then draw the same conclusion for the rejection ratio (not depicted here since it is identical to the rejection ratio obtained with an amplitude mismatching). The phase mismatching should be lower than $10^{-3}$, which is easily obtained in the infrared.

![Fig. 7. Rejection ratio as a function of an amplitude-mismatching $\delta A_1$.](image)

6.2. Imperfections of the polarizers

Imperfections of the polarizers are modeled by setting $\varepsilon_{1,j}$ and $\varepsilon_{2,j}$ different from zero. These numbers can be complex, giving rise to a certain ellipticity in the polarization state.

We chose all the $\varepsilon_{1,j}$ and $\varepsilon_{2,j}$ in such a way that their moduli are a random number between 0 and $10^{-3}$ and the phases are also randomly chosen. We then look at the average rejection ratio that can be obtained. The results are plotted in Fig. 8.

The rejection ratio is in average slightly higher than $10^6$. The “amplitude imperfections” of the polarizers should then also be of the order of $10^{-3}$, which is not easy to satisfy in a wide spectral band. Note that this requirement is less stringent if we use achromatic waveplates instead of conventional waveplates. Indeed, if we consider identical quasi-achromatic waveplates ($T_{\alpha,1} = T_{\alpha,2} = T_{\alpha,3} = T_{\alpha} = \exp(i(\pi + \delta))$, where $\delta << \pi$) and identical imperfect polarizers ($\varepsilon_{1,1} = \varepsilon_{1,2} = \varepsilon_{1,3} = \varepsilon_{2,1} = \varepsilon_{2,2} = \varepsilon_{2,3} = \varepsilon << 1$), we can show that the on-axis intensity is then proportional to

$$I \propto (\varepsilon \delta)^2,$$  \hspace{1cm} (30)

which shows that imperfections of the polarizers can be compensated by very achromatic waveplates and inversely, chromatic waveplates can be used combined with very good polarizers.

6.3. Rotation of the waveplates

We consider a small additional angle in the rotation of the $l^{th}$ waveplate, we then have $\alpha_l = \alpha_{l,0} + \delta \alpha_l$. After calculations, we find

$$I_\lambda = |T_r - T_\alpha|^2 |A_l (\delta \alpha_l \cos 2\alpha_l + \delta \alpha_l^2 \sin 2\alpha_l)|^2.$$  \hspace{1cm} (31)
Fig. 8. Rejection ratio with randomly-chosen $\varepsilon_{1,j}$ and $\varepsilon_{2,j}$.

This shows that the rejection ratio will be much less sensitive to waveplate rotations if $\cos 2\alpha_j = 0$, as shown in Fig. 9(a). The required accuracy in waveplate rotation should not be the limiting factor in an actual set-up.

Fig. 9. (a) Rejection ratio when one of the waveplates is rotated with respect to its normal position for $2\alpha_1 = 7\pi/6$ (solid line) and $2\alpha_3 = \pi/2$ (dash-dot line) and (b) rejection ratio with a differential birefringence $\delta B$.

6.4. Differential birefringence

Suppose that the birefringence of the $l^{th}$ waveplate is slightly different from the birefringence of the other waveplates, $B_l = B_0 + \delta B$. The detected intensity is then given by

$$I_\lambda = \left| A_l \sin 2\alpha_l \frac{\pi \delta B}{\lambda} \right|^2. \quad (32)$$

As shown in Fig. 9(b), we should have $\delta B/\lambda \leq 5 \times 10^{-4}$ in order to have a rejection ratio higher than $10^6$, which should be easier to reach in the infrared than in the visible region of the spectrum.
7. Conclusions

We have derived an expression for the generalized $N$-beam nulling condition, which includes amplitude, phase and polarization. We have applied this condition to the cases of a two- and a three-beam nulling interferometer. We have shown that interferometry with beams with different coherent states of polarization is possible with a theoretically perfect contrast.

We have seen that we can theoretically reach an infinite rejection ratio in the monochromatic case without any phase shifter, using only polarization rotation. In a wide spectral band, we can still have a high rejection ratio without any phase shifter, but we need an extra device, which could be an achromatic polarization rotator. The approach we show involves only commercial elements: linear polarizers and waveplates. We thus have introduced a totally new type of nulling interferometers, which should allow a high rejection ratio, without any achromatic device. We have derived an expression for the nulling condition of the newly-designed nulling interferometer, which should be fulfilled only by rotating waveplates.

We have looked at the $\theta$-dependence of the transmission map and have seen that, as in regular nulling interferometers, the only three-telescope configuration to have a $\theta^4$-transmission map is the linear configuration, which is not very interesting for exoplanet detection. As conclusion, the proposed design does not need any amplitude-matching device and is also very suitable for fast internal modulation. Indeed, it allows to obtain a continuous set of transmission maps, by only rotating the waveplates. But a modulation strategy still has to be investigated.

Another interesting aspect of these calculations is that there is no theoretical limit for the width of the spectral band for which we reach a deep null. However, the transmission is not equal for all wavelengths. We have established a criterion to define the acceptable spectral band in such a way that the transmission over the whole spectral band is higher than half the maximal transmission. We have seen that this criterion could lead to a relatively wide band in the infrared. The ultimate limit of the spectral band will probably be set by the transmission of the linear polarizers and the waveplates.

In order to complete the analysis, we also have looked at the sensitivity of the proposed design to some imperfections and misalignments. We have seen that the factor limiting the rejection ratio will probably be the imperfections of the polarizers. Indeed, rotations can be reached very accurately. Phase mismatching and differential birefringence can be a problem in the visible region but are much less important in the infrared. In conclusion, using the approach presented in this paper, we have shown that it should be possible to reach a high rejection ratio without any achromatic device, thus opening a new promising way to detect Earthlike exoplanets.

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