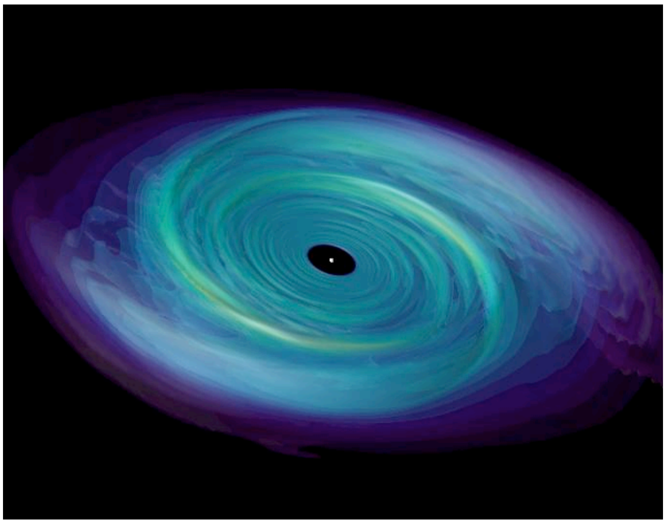
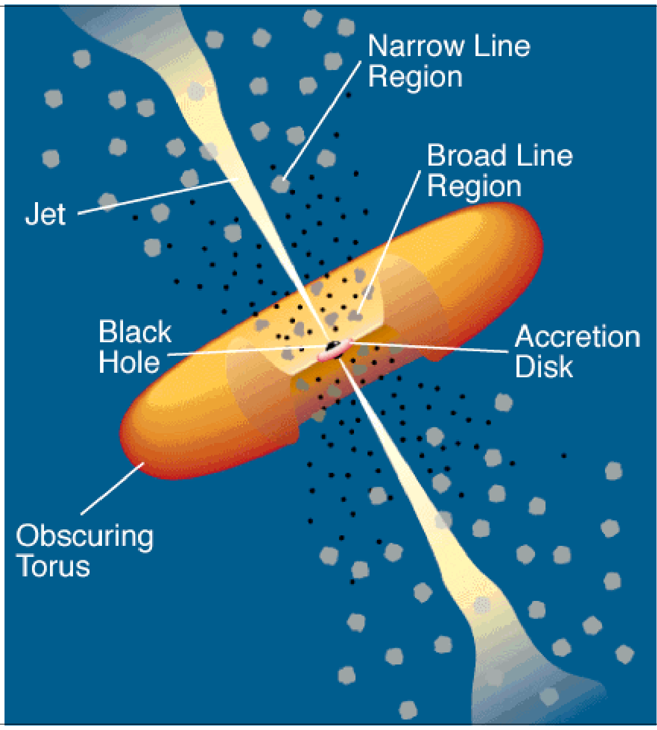
Accretion disks





Quantitative overview

- Gas orbits in nearly circular fashion
- Each gas element has a small inward motion due to viscous torques,
- resulting in an outward transportation of angular momentum
- Viscous stress generate viscous heat
- This heat is (partly) radiated away from both faces of the disk
- Potentially very efficient radiative process
  - Much more efficient that spherical accretion
- Major problem: proper description of viscosity

Accretion disk models

Quantitative calculation requires solution of the equations for viscous differentially rotating relativistic hydrodynamic flow around a black hole

This is a large area of research, with applications to AGN and to stellar mass compact objects (e.g., X-ray binaries)

Two main classes

Thin disk model
- thin, optically thick disk
- high efficiency: energy released by viscous stress immediately radiated
- quasi black-body spectrum $(T_{\rm eff} \propto (\dot{M}/M)^{1/4} \sim 10^5 - 10^7 \text{ K})$

Advection-dominated accretion flow (ADAF)
super-Eddington accretion rate
large optical depth traps radiation, carries the radiation inwards/advects it into the black hole
low radiation efficiency

Here:
- Basic principles
- Derivation of results for thin disks
- Some qualitative results for ADAFs

Definitions

Very obvious: $r, \Delta r, h, v_\phi, r_I, \dot{M}$

Plus:

f_ϕ: viscous stress
force per unit area
The stress exerted in the ϕ direction by fluid element at r on the neighbouring elements at *r+dr*

$\dot{j}+$: inward rate of angular momentum transport across radius r

$\dot{j}-$:rate at which angular momentum is consumed by the BH at the inner radius r_I

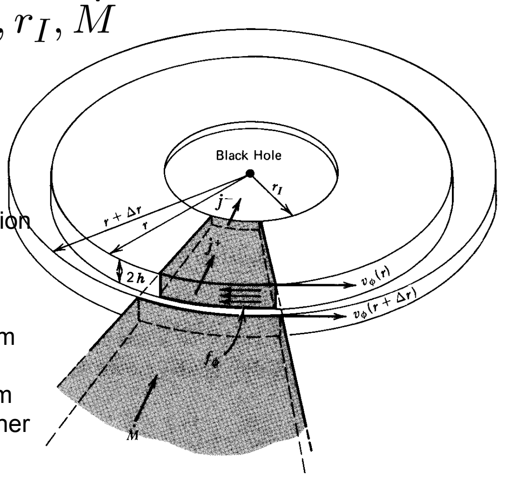


Figure 14.3 Slice of a thin, Keplerian accretion disk around a central black hole.

Quantitative description

- Assume a constant mass deposit rate $\dot{M}$
- Consider nearly-circular motion around mass M

$$v_r, v_z \ll v_\phi \Rightarrow v_{\text{orb}} \sim v_\phi$$

- In Keplerian motion, each element possesses specific angular momentum:

$$\ell = r v_\phi = r^2 \Omega = \sqrt{GMr}$$

- Near the BH, the angular momentum is much less than at the edge of the disk r_d
- In steady state, the loss of angular moment is:

$$\dot{J} = \dot{M}\ell(r_d) = \dot{M}\sqrt{GMr_d}$$

- Define
  - 2h: thickness of disk
  - Σ : surface density
- Then:

$$\Sigma = \int_{-h}^{h} \rho dz \sim 2h\rho(z=0)$$

- we will assume that disk is thin: h(r) << r
- later we'll see that this requires the disk to be cool: $kT << GMm_p/r$

Derivation of viscous stress (ST app. H)

Like pressure, viscous stresses in incompressible fluid are described by a tensor:

$$\mathbf{T} = t_{ij} = \eta \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}(\nabla \cdot \vec{v})\delta_{ij}\right]$$

which gives the rate at which momentum in *i*th direction is carried in direction *j* by viscosity. With η the coefficient for dynamic viscosity [g cm^{-1} s^{-1}]

The equations of motion for a viscous fluid:

$$\rho\frac{d\vec{v}}{dt} = -\nabla P + \nabla \cdot T$$

For a Keplerian disk v_r, $v_z \ll v_\phi$.

$$v_\phi = r\Omega = \left(\frac{GM}{r}\right)^{1/2}$$

In cylindrical coordinates, the stress tensor has only one non-zero component

$$t_{r\phi} = \eta\left(\frac{1}{r}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right)$$

Combined:

$$t_{r\phi} = -\frac{3}{2}\eta\Omega = -\frac{3}{2}\eta\left(\frac{GM}{r^3}\right)^{1/2} \qquad (ST14.5.11)$$

In words: the viscous force in the ϕ direction caused by rubbing of adjacent fluid elements generate a torque that carries the angular momentum outwards

The viscous stress force f_ϕ is related to the stress tensor according

.

$$f_\phi = -t_{r\phi}$$

Next:

- Conservations of mass
- Conservation of angular momentum
  - disk thickness
  - Vertical structure
- Energy conservation
- Vertical momentum conservation
- Viscosity law
- Temperature profile
- Spectrum

Conservation of mass

The continuity equation/equation for mass conservations is

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Sigma v_r) = 0 \qquad (1)$$

(Assuming no sources or sinks of matter in the disk)

Take $\partial\Sigma/\partial t = 0$ in continuity, then

$$\frac{\partial}{\partial r}(r\Sigma v_r) = 0 \quad \Rightarrow \quad \dot{M} = -2\pi r\Sigma v_r = constant$$

with $\dot{M}$ the constant mass accretion rate (otherwise matter would pile up at certain locations in the disk)

Angular momentum conservation

- Inward rate of of angular momentum transport at radius r:
  $$\dot{J}^{+} = \dot{M}(GMr)^{1/2}$$
- At the inner edge r_I of the disk:
  $$\dot{J}^{-} = \dot{M}(GMr_I)^{1/2}$$
- Angular momentum conservation:
  Net rate of change of angular momentum with r
  = torque exerted by the viscous stress:

torque = (force along $\mathbf{e}_\phi$ / area) x area x (r) = $\dot{J}^{+} - \dot{J}^{-}$

$$(f_\phi)(2\pi r 2h)(r) = \dot{M}\left[(GMr)^{1/2} - (GMr_I)^{1/2}\right] \qquad (ST14.5.14)$$

Note that the required stress f_ϕ is uniquely determined by M and $\dot{M}$

Energy conservation

- Using H.6 from ST, we see that the entropy (heat) that is generated by viscosity at a rate:
  $$\dot{Q} = -f_\phi t_{\phi r}/\eta$$
  With f_ϕ: viscous stress and $t_{r\phi}$ stress tensor and η coefficient of dynamic viscosity
- Using ST 14.5.11, 14.5.14 and 14.5.15:
  $$2h\dot{Q} = \frac{3\dot{M}}{4\pi r^2}\frac{GM}{r}\left[1 - \left(\frac{r_I}{r}\right)^{1/2}\right]$$
- Hence the flux from top or bottom as a function of r
  $$F(r) = \frac{1}{2} \times 2h\dot{Q} = \frac{3\dot{M}}{8\pi r^2}\frac{GM}{r}\left[1 - \left(\frac{r_I}{r}\right)^{1/2}\right]$$
- The total luminosity is:
  $$L = \int_{r_I}^{\infty} 2F \times 2\pi r dr = \frac{GM\dot{M}}{2r_I}$$
  Independent of viscosity.

Release of gravitational potential energy $\Rightarrow$
- Part to kinetic energy
- Part to outer regions (angular momentum transport)
- Half of the gravitational potential energy goes into heat

-Order of magnitude: take $r_{\rm m} = 6 r_{\rm g}$ for $a = 0$ black hole: $\epsilon = 0.083$

Vertical momentum conservation / disk thickness

- No net motion of the gas in the z direction.
- The pressure gradient is due to the z-component of the gravity field of the BH:

$$\frac{\partial P}{\partial z} = -\rho \frac{GM}{r^2}\frac{z}{r} \qquad (z << r)$$

- Simplify: $\Delta P \approx P, \qquad \Delta z \approx h$

$$h \approx \left(\frac{P}{\rho}\right)^{1/2} \left(\frac{r^3}{GM}\right)^{1/2} \approx \frac{c_s}{\Omega}$$

- For an ideal gas:

$$\frac{h^2}{r^2} = \frac{c_s^2}{v_{\rm orb}^2} = \frac{kT}{\mu}\frac{r}{GM} = \frac{kT}{\mu c^2}\frac{r}{r_{\rm g}}$$

with μ is mean mass per particle, and $r_{\rm g} = GM/c^2$
- Disk is thin as long as $kT \ll \mu c^2$ This is often valid since

$$kT \sim \mu c^2 \Rightarrow T \sim 10^{13}\ \rm K$$

Vertical structure (K. 7.3.4.2)

Equation for vertical hydrostatic equilibrium, take again:

$$\frac{\partial P}{\partial z} = \rho g_z = -\frac{GM\rho z}{r^3}$$

where

$$P = P_{\text{gas}} + P_{\text{rad}} = \frac{\rho kT}{\mu} + \frac{1}{3}aT^4 = \frac{\rho kT}{\mu} + \frac{\kappa\rho}{c}F_{\text{rad}}$$

for ideal gas. Here F_{rad} is radiative flux and κ is Rosseland mean opacity per unit mass:

$$\frac{1}{\kappa} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}\,d\nu$$

Gas pressure only

Assume T independent of height. Then

$$\frac{kT}{\mu}\frac{\partial\rho}{\partial z} = -\frac{GM\rho z}{r^3}$$

so that

$$\rho(z) = \rho(0)\exp\left[-\frac{GM\mu}{2r^3kT}z^2\right]$$

which means a Gaussian fall-off in density with

$$h = \left(\frac{2k_BTr^3}{GM\mu}\right)^{1/2} \sim \frac{c_s}{\Omega}$$

Radiation pressure only

Importance of radiation pressure increases inwards.
Assume $P = P_{\text{rad}}$ and $\kappa = \kappa_T$

$$h = \frac{3r_{\text{I}}}{4}\frac{L}{L_E}\left[1 - \left(\frac{r_{\text{I}}}{r}\right)^{1/2}\right]$$

Note that the radiation pressure puffs up the disk

The viscosity law

There is no satisfactory physical model for disk viscosity

Many possible/proposed mechanisms

- Turbulence (Shakura & Sunyaev 1973 AA 24 337 - SS73)
- Magnetic winds (Krolik 7.2.2)
- Turbulent electromagnetic stresses (Krolik 7.2.3)
- Photon viscosity (Loeb & Laor 1992 ApJ 384 115)
- Self-gravitating in z direction (Paczynski 1978 Acta Astron 28 241)

Here: take that in turbulent motion the coefficient of dynamic viscosity is given by (SS73)

$$\eta \approx \rho v_{turb} l_{turb}$$

where

v_{turb} is the velocity of the turbulent cells relative to the mean gas motion

l_{turb} is the size of the largest turbulent cells

Shocks will dissipate turbulent kinetic energy into heat whenever the motion is supersonic, hence: $v_{turb} < c_s$

cell sizes needs to be < h

- Using ST 14.5.10, ST 14.5.11 and ST 14.5.20:

$$f_\phi = -t_{\phi r} < (\rho c_s h)\Omega \approx \rho c_s^2 \approx P$$

- Parameterized:

$$f_\phi = \alpha P, \qquad \text{with } \alpha < 1$$

- Models using this parameterization are called α disks

Models with this *ad hoc* choice for α work remarkably well, and are called α-disks (Shakura & Sunyaev 1973 AA 24 337)

Eventually all of the matter in the ring [at radius r] ends up at the origin and all of the angular momentum is carried to infinite radius by none of the mass (Pringle 1981 ARAA 19, 137)

Temperature profile

Assume heat Q is released as thermal radiation from top and bottom of disk. Then:

$$\sigma T^4 = F(r) = \frac{3\dot{M}}{8\pi r^2}\frac{GM}{r}\left[1 - \left(\frac{r_I}{r}\right)^{1/2}\right]$$

For $r \gg r_I$

$$T \approx T_I\left(\frac{r}{r_I}\right)^{-3/4} \qquad (*)$$

With $T_I^4 = 3GM\dot{M}/8\pi r_I^3\sigma$, or

$$T_{\rm m} = 2\times10^5\left(\frac{M}{10^8 M_\odot}\right)^{1/4}\left(\frac{\dot{M}}{M_\odot {\rm yr}^{-1}}\right)^{1/4}\left(\frac{r_I}{10^{14}\,{\rm cm}}\right)^{-3/4} K$$

Comments
- T(r) independent of viscosity
- T(r) reaches maximum at inner radius r_m
- Typical radiation: UV photons

Spectrum

Planck spectrum at each T: $B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$

Integrated spectrum $I_\nu = 2\pi \int_{r_{\rm m}}^{r_{\rm out}} B_\nu(T[r]) r dr$

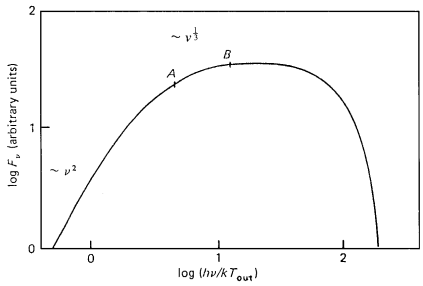


Fig. 5.3. The geometrically thin, optically thick, accretion disk spectrum as a function of frequency. The units are arbitrary. The approximate forms discussed in the text are indicated. Reproduced from FKR92.

Low ν spectrum is universal, high-ν cutoff depends on r_m

For $\nu \ll kT(r_{\rm out})/h \Rightarrow B_\nu \propto \nu^2 \Rightarrow I_\nu \propto \nu^2$ (Rayleigh-Jeans limit)

For $kT(r_{\rm out}) \ll h\nu \ll kT(r_{\rm m})$ spectrum dominated by $r \gg r_m$. Use eq.~(*, page 17), and substitute $y = h\nu/kT(r)$. Then $I_\nu \propto \nu^{1/3}$

For $\nu \gg kT(r_{\rm m})/h \Rightarrow B_\nu \propto \nu^3 \exp - h\nu/kT \Rightarrow I_\nu$ falls off exponentially (Wien limit)

Comparison with observations

This model for AGN accretion disk provides big blue bump in UV

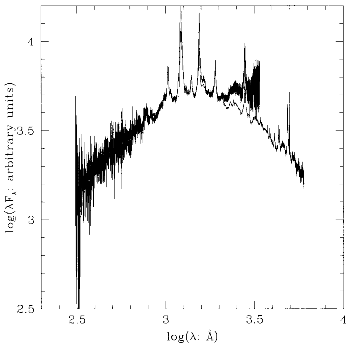


Fig. 7.10 The Francis et al. (1991) and Zheng et al. (1997) composite quasar spectral shapes, shifted so as to coincide in the vicinity of Lyα. In terms of $dF/d\ln\lambda$, there appears to be a broad, flat maximum in the vicinity of $\epsilon \simeq 10$ eV. The sharp peaks are emission lines (the subject of Chap. 10).

Thick disks

- At small radii electron scattering becomes dominant source of opacity -> disk geometrically thick in inner regions when $L \sim L_E$
- This thick disk provides funnel for collimation of jets
- Inner part of torus is pinched off into cusp, between marginally bound and marginally stable orbits
- Matter flowing in through cusp emits little radiation
- Accretion efficiency is low

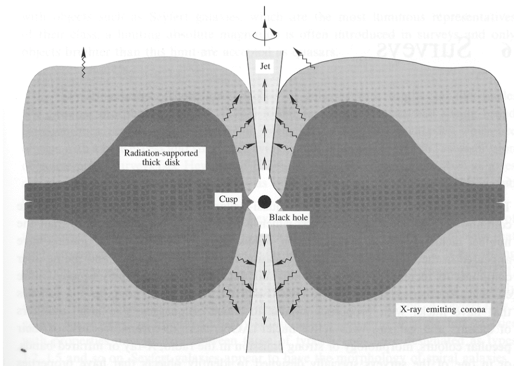


Fig. 5.4. Schematic diagram of a radiation-supported thick accretion disk. After Wiita (1991).

Advection dominated accretion disks

- If accretion rate low, then density low, so cooling rate low, and much of heat can be carried along rather than radiated ⇒ radiative efficiency of disk drops
- If heat retained ⇒ ions hot (virial temperature) ⇒ disk no longer thin but ∼ spherical
- Electrons radiate more easily: lower T
- If accretion rate high: photon diffusion takes long, and radiation trapped. Quasispherical when h ∼ r

See Narayan R., Mahadevan R., Quataert E., 1998, **astro-ph/980314**

<u>Steady axisymmetric accretion flow (p. 3+4 of Narayan et al 1998):</u>

Solve conservation of mass, angular momentum, and radial momentum:

- Energy advected $Q^{\mathrm{adv}} = fQ$

Q the rate of viscous heating

Self-similar solutions for f ∼ 1 (all energy generated advected) and $\alpha^2 \ll 1$:

$$\frac{v}{v_{\mathrm{ff}}} \simeq -\left(\frac{\gamma-1}{\gamma-5/9}\right)\alpha \qquad v_{\mathrm{ff}} \equiv \sqrt{\frac{GM}{r}}$$

$$\frac{\Omega}{\Omega_{\mathrm{K}}} \simeq \sqrt{\frac{2(5/3-\gamma)}{3(\gamma-5/9)}} \qquad \frac{c_s}{v_{\mathrm{ff}}} \simeq \sqrt{\frac{2(\gamma-1)}{3(\gamma-5/9)}}$$

With the Keplerian angular rotation: $\Omega_K = (GM/r)^{3/2}$

ADAF features

- Rapid infall: two-temperature ADAFs typically $\alpha \sim 0.2$ - $0.3 \Rightarrow v_r \geq 0.1\ v_{ff}$
- Sub-Keplerian rotation: $\gamma \rightarrow 5/3 \Rightarrow \ \Omega \rightarrow 0$

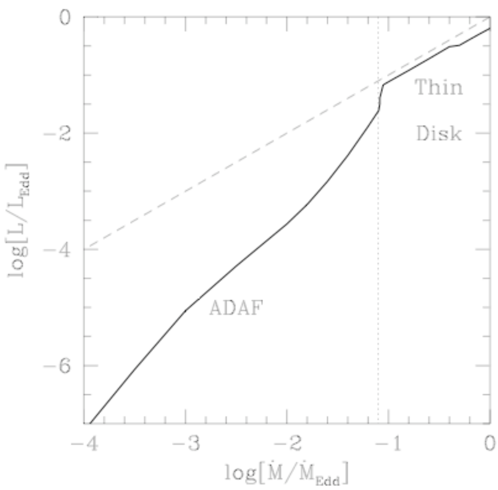
(As derived before: $\frac{h^2}{r^2} = \frac{c_s^2}{v_{\rm orb}^2} = \frac{kT}{\mu}\frac{r}{GM} = \frac{kT}{\mu c^2}\frac{r}{r_g}$)

-Quasi-spherical geometry: viscous energy is stored as internal energy: kT $\sim$ μ c^2 $\Rightarrow$ h $\sim$ r

-As a result the temperature is high, close the virial tempeture

-Disks ``puff up'': $H \sim c_s/\Omega_K \sim v_{ff} \sim R$

Combined model: inner ADAF, and outer thin accretion disk



Narayan p. 17

Thin disk:

$L \sim 0.1\dot{M}c^2$

ADAF:

$L \sim \dot{M}^2$

FIGURE 7. The bolometric luminosity vs. mass accretion rate according to the model developed by Esin et al. (1997). The vertical dotted line corresponds to $\dot{m}_{\rm crit}$ (for $\alpha = 0.3$). Above this $\dot{m}$, the accretion is via a thin disk and $L \propto \dot{M}$. Below $\dot{m}_{\rm crit}$, the accretion is via an ADAF at small radii and a thin disk at large radii (cf. §3.3). Here $L \propto \dot{M}^2$ because much of the viscously generated energy is advected into the black hole. The dashed line corresponds to $L = 0.1\dot{M}c^2$.

`Dead quasar problem' (Fabian & Canizares 1988 Nature 333 829)

Note

1. Drastic decrease in Quasar density since $z \sim 2$.
2.1 Local AGNs are underluminous for their inferred blackhole masses
2.2 BH mass in average galaxy is 10^7 M_0, but average galaxy not active

Maybe accretion switching from high-luminosity standard thin disk to low-luminosity ADAF due to decrease in fuel? Most of the energy is then advected Instead of radiated (Fabian and Rees, 1995 mnras 277, L55)

Assignment

- Read Fabian and Canizares (1988)
- On what physical basis did they estimate the upper limits to the masses of a central black holes (Table 1).
- How are the black hole masses for quasars estimated?
- Comparing the two mass estimates, what is the problem?
- What solutions are discussed for this problem?
- Read Fabian and Rees (1992)
- What solution do they propose here?
- How is this related to powerful radio sources normally being located in early type galaxies with a hot halo of gas?

Literature

General

- Khembavi & Narlikar, § 5.5
- Krolik, § 7.1, 7.2, 7.3
- Shapiro S.A., Teukolsky S.L § 14.5

More details:

- Shakura N.I., Sunyaev R.A., 1973 AA 24 337, Black holes in binary systems. Observational appearance
- Narayan R., Mahadevan R., Quataert E., 1998, astro-ph/9803141, Advection-Dominated Accretion around Black Holes