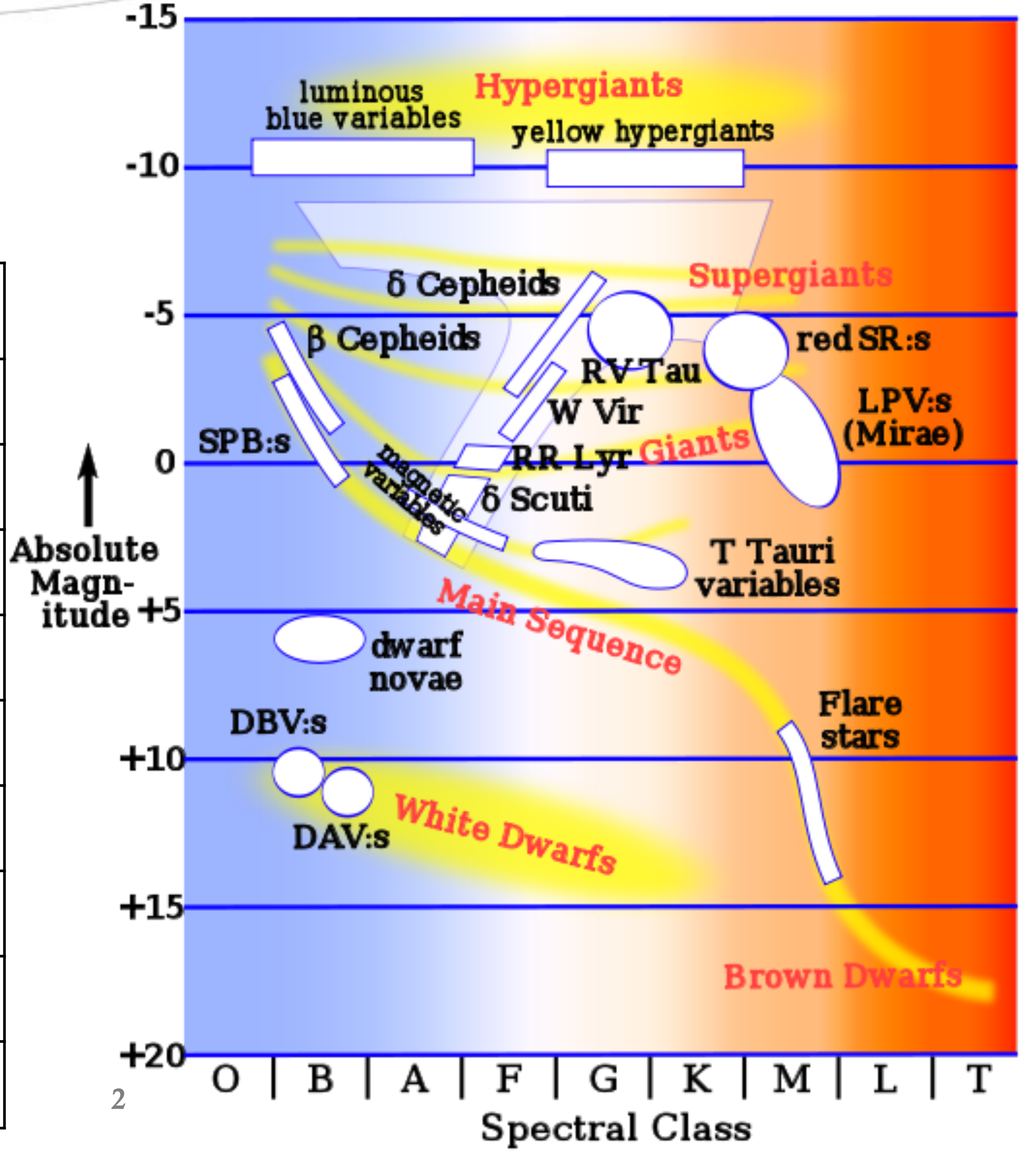
Lecture 7.1: Pulsating Stars

Literature: KWW chapter 41

a) Classes of pulsating stars

Many stars
Intrinsically variable
Subset: regular pulsation

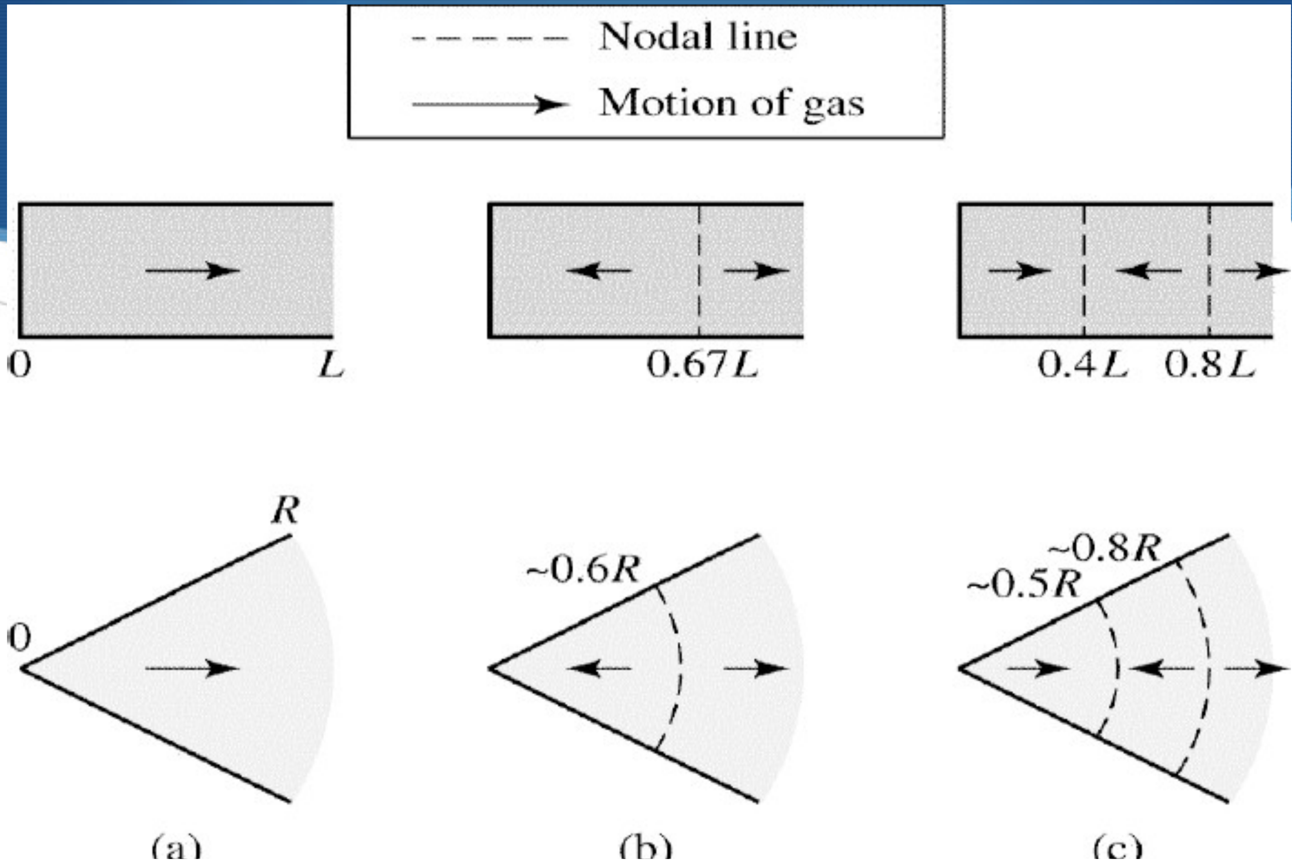
Type	Period (d)
RR Lyrae	0.3-0.9
Cepheids	1-50
W Virginis	2-50
RV Tauri	60-200
[?] Cep	0.2
[?] Scuti	0.2
LPV	100-700
Semi-regulars	100-200
Mira's	150-700



Pulsating stars lie in well-defined regions of the HRD

- Instability strip in HR diagram: narrow range in Teff spanning a wide range of luminosities
- Whenever stars pass through this instability strip in their evolution, they become pulsational unstable
- Stellar pulsations are due to sound waves resonating in their interiors
- Stellar pulsations depend on stellar structure

b) Period-mean density relation



Radial pulsations: standing waves; fundamental & overtones; nodes

Pulsation $\boxed{?}$ density perturbation with wavelength equal to stellar diameter $\boxed{?}$ regular pulsation $\boxed{?}$ standing wave

First order: period $\Pi \approx \frac{2R}{\bar{\upsilon}_s}$ with $\bar{\upsilon}_s$ mean sound speed $\upsilon_s = \sqrt{\Gamma_1 \frac{P}{\rho}}$

Assume: star oscillates around hydrostatic equilibrium

$\boxed{?}$ use virial theorem (Lecture 2 slides 7 &10) to estimate $\bar{\upsilon}_s$:

$$-E_g = 2E_i$$

$$q\frac{GM^2}{R} = 3\int \frac{P}{\rho} dm = 3\int \frac{\upsilon_s^2}{\Gamma_1} dm \equiv 3\overline{\left(\frac{\upsilon_s^2}{\Gamma_1}\right)} M \approx 3\frac{\overline{\upsilon_s^2}}{\Gamma_1} M$$

$$\Pi \approx \frac{2R}{\bar{\upsilon}_s} = 2\left(\frac{3}{q\Gamma_1}\right)^{1/2}\left(\frac{R^3}{GM}\right)^{1/2} = 2\left(\frac{3}{q\Gamma_1}\right)^{1/2}\frac{3^{1/2}}{(4\pi G\bar{\rho})^{1/2}} \propto \frac{1}{\sqrt{G\bar{\rho}}}$$

period-mean density relation $\Pi\sqrt{\frac{\bar{\rho}}{\bar{\rho}_O}} = Q \approx 0.04 \text{ day}$ (Eddington)

We will come back to oscillation frequencies when discussing asteroseismology (lecture 7.2)

c) Physics of stellar pulsations: k-mechanism

Eddington's valve

Pulsations are oscillations around an equilibrium position

1) Consider a layer that loses temporarily hydrostatic support and collapses
2) The layer will compress and heat up
3) The opacity will increase blocking radiation from below
4) Temperature and pressure below it will build up
5) Eventually, the layer will be pushed out again, expand, cool, and become more transparent
6) Radiation from below can escape
7) Cycle starts over

Steam engine: Radiation = steam; layer =piston; opacity=valve

Key is that opacity increases with compression

Kramer opacity: Normally, opacity decreases with compression as temperature increase wins over density increase $\kappa \propto \frac{\rho}{T^{3.5}}$

Partial ionization zones: Upon compression, internal energy is stored into increased ionization and density increase leads to increased opacity, Upon expansion, stored internal (ionization) energy is released and decreased density leads to decreased opacity

Three partial ionization zones:
1) HI, HeI: 10,000–15,000K
2) HeII: 40,000K
3) Iron: 200,000K

Approximate opacity as: $\kappa = \kappa_0 \rho^n T^{-s} \quad n, s > 0$ (Kramers: n=1, s=3.5)

Driving a pulsation needs heat exchange: non-adiabatic process
Use adiabatic relations to get approximate results

Write: $\dfrac{dT}{T} = (\Gamma_3 - 1)\dfrac{d\rho}{\rho} \Rightarrow T \propto \rho^{\Gamma_3 - 1} \Rightarrow \kappa \approx \rho^{1-3.5(\Gamma_3-1)}$

If $1 - 3.5(\Gamma_3 - 1) > 0$ $\boxed{?}$ increases during compression and decreases during expansion

$\boxed{?}$ pulsation can be driven when $\Gamma_3 < 1.28$

<u>Implications</u>:

Fully ionized monatomic ideal gas has $\Gamma_3 = 5/3$: no pulsation driven by $\boxed{?}$-mechanism

In ionization zone Γ_3 decreases to ~1.1 (Lecture 3-2, slide 38)

$\boxed{?}$ $\boxed{?}$-mechanism can operate: during compression energy is use

to increase degree of ionization, T rises only little

d) "Real estate" of pulsation

H-He ionization zone: thickness ~ 10^4K
He^+ zone: thickness ~2×10^4K: largest heat capacity

Stable pulsation possible when H, He, He^+ zones lie at such a depth that

Envelope not so massive that pulsation is damped

Zone has sufficient heat capacity

These conditions are fulfilled in classical instability strip

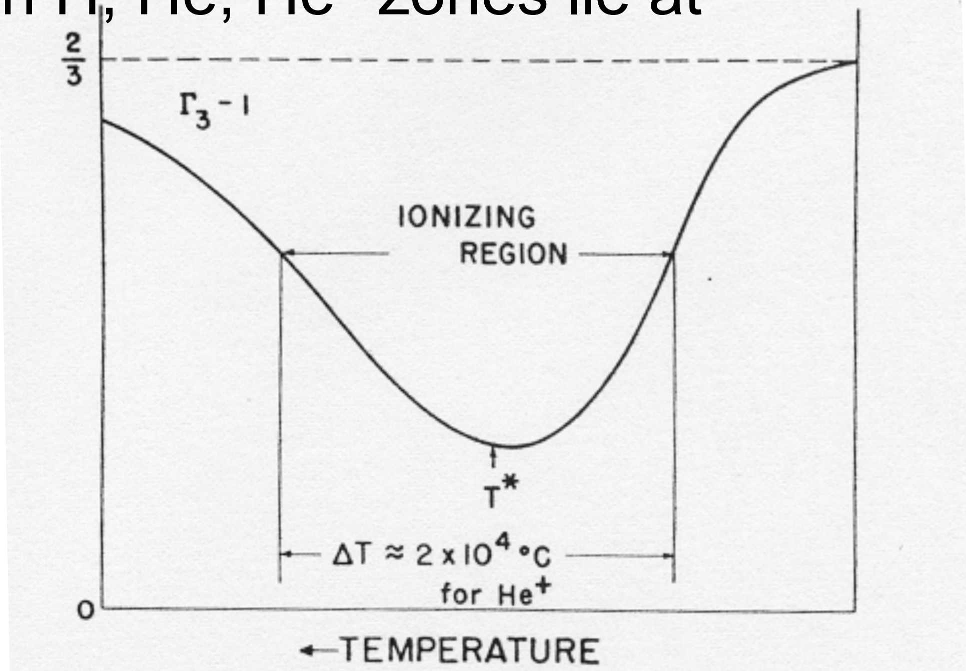


Fig. 27.4 $\Gamma_3 - 1$ vs. temperature (schematic) in the region of He^+ ionization in the equilibrium model of a stellar envelope.

Computed properties of Cepheids agree well (Iben 1991 ApJS, 76)

1) Partial ionization zones are prone to pulsational instability
2) Real estate agents mantra: Location, location, location
    1) PIZ at the surface, heat readily escapes
    2) PIZ deeply inside, overlaying layers dampen pulsations
3) Location of PIZ: bottom of zone, radiative energy dammed upon compression. Top of zone: energy can be radiated in a single period. Blue edge given by (red edge, convection cuts in): $\frac{\langle C_V T_{piz} \rangle \Delta m_{piz}}{\Pi L} \approx 1, L \propto \Delta \mathrm{m}_{piz} / \Pi$

$$\text{hydrostatic equilibrium: } P \approx GM \Delta \mathrm{m}_{piz} / R^4$$

$$\text{Envelope with Kramers opacity: } P \propto (M/L)^{1/2}$$

(lecture 5-1, slide 7)

$$\text{Thus, } \Delta \mathrm{m}_{piz} \propto R^4 / (ML)^{1/2} \text{ and } L \propto R^{5/3}$$

$$L \propto R^2 T_{eff}^4 \text{ yields an instability strip with: } L \propto T_{eff}^{-20}$$

Work done during one cycle

Cepheids, RR Lyrae

- ~60% energy He^+ zone
- ~40% energy: H-He zone

Large dissipation below

- Small dissipation above

Phase lag:

- Effect of inertia of upper layers
- 5-10% of L temporarily stored in outer layers and released later

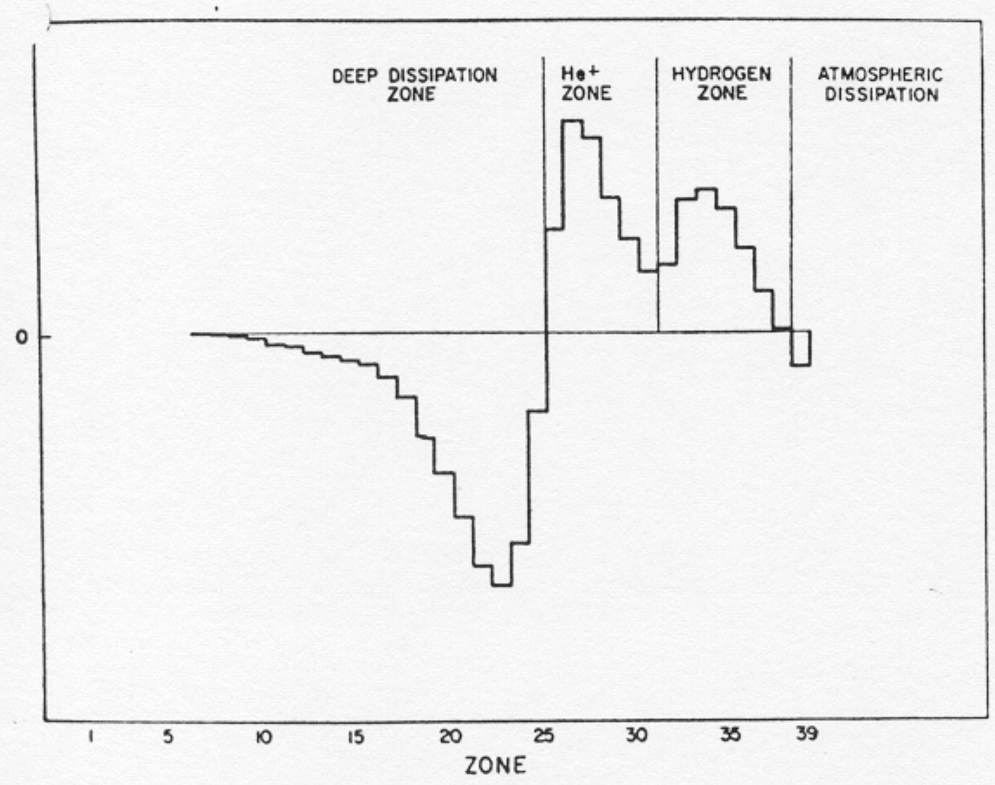


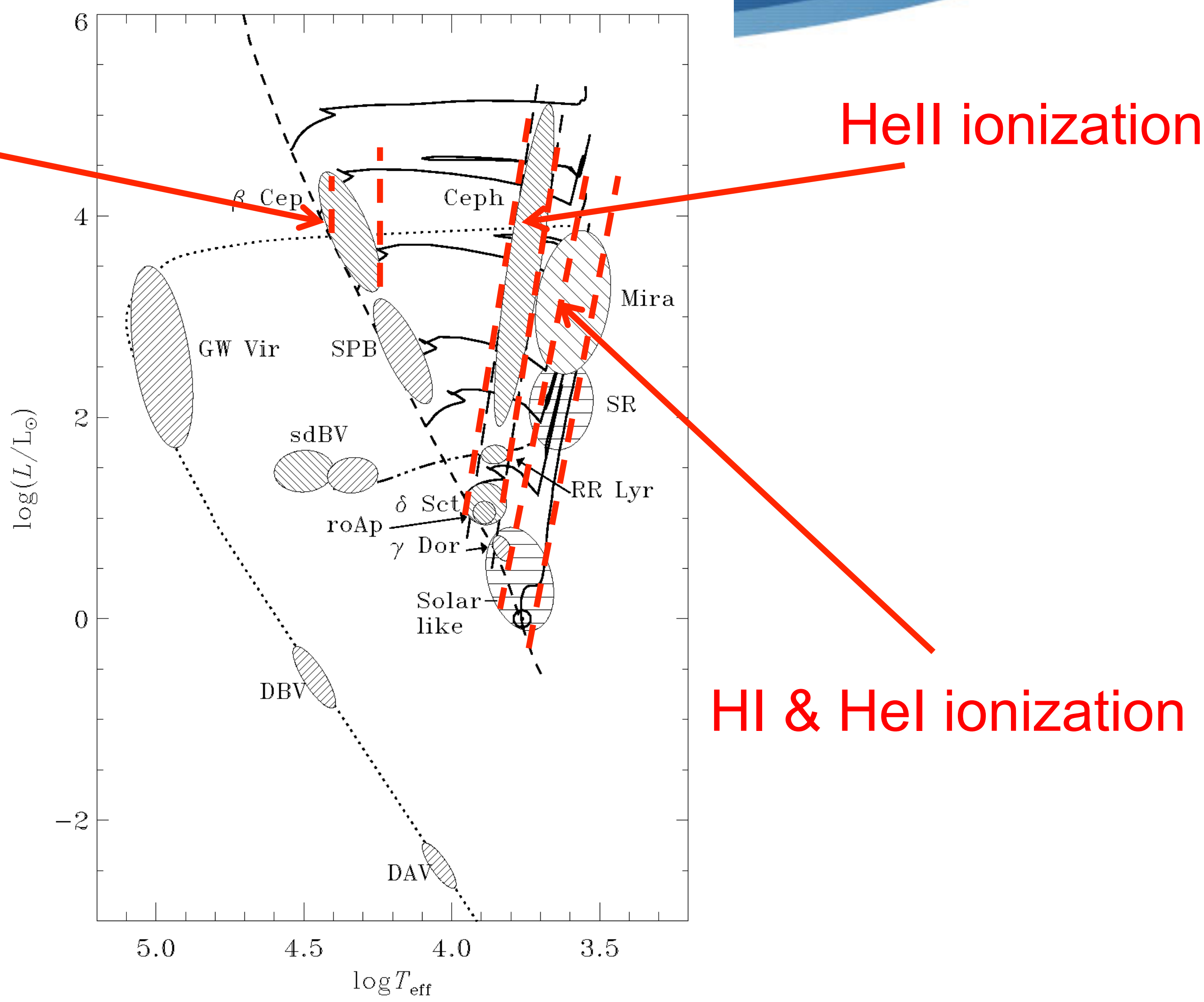
Fig. 27.15 The work W_i (on an arbitrary scale) done by the i^{th} mass shell on its surroundings around a complete period (see (27.194)) versus i for one of Christy's [Ch66a] unstable RR Lyrae models at limiting amplitude in the fundamental mode. (Parameters for this model are given in the text.) Positive values of W_i connote "driving" regions, negative values "damping" regions.

Pulsations & Partial Ionization Zones

iron ionization

Red edge due to location of partial ionization zone

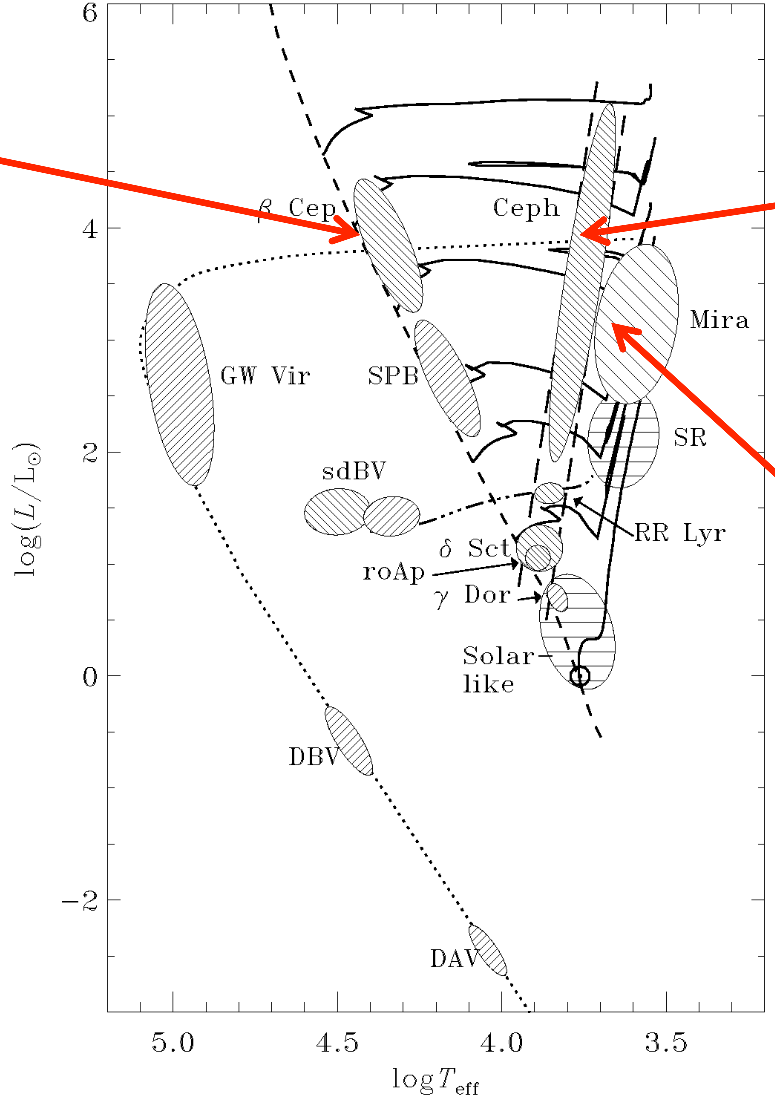
Blue edge due to convection reaching this zone



Pulsations & Partial Ionization Zones

iron ionization

HeII ionization



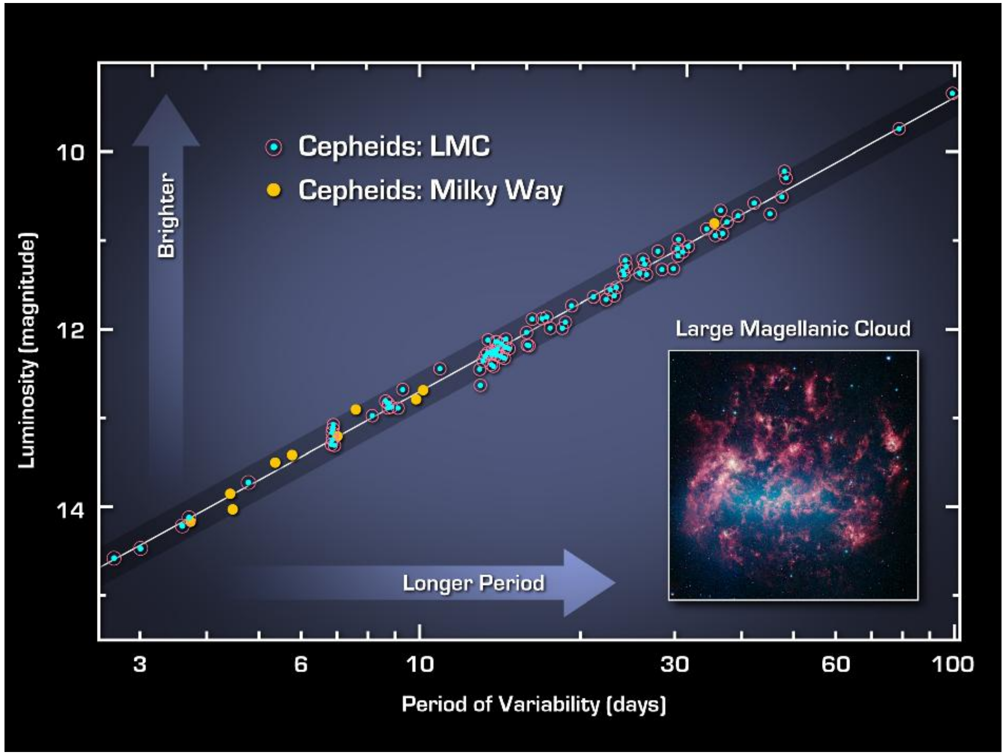
HI & HeI ionization

e) Application

Period-luminosity relation

Leavitt discovered P-L relation by study of Cepheid variables

In Magellanic Clouds



$\Pi \propto 1/\sqrt{\rho}$

$L \propto R^{5/3}$ (slide 6) &

$L \propto M^{11/2}/R^{1/2}$ (Radiative equilibrium & Kramers opacity)

$\Pi \propto L^{2/3}$ observed: $\Pi \propto L^{0.9}$