

0.4 Units and Physical Constants

Table 1: Units

Angstrom	1 Å = 10^{-10} m
Wavenumber	1 cm^{-1} = c Hz
Charge	1 C = 2.997925×10^9 (e.s.u.)
Electric dipole	1 C m = 2.997925×10^{11} cm (e.s.u.)
Dipole moment	1 Debye = 10^{-18} cm (e.s.u.) = 3.33564×10^{-30} C m
Magnetic field	1 T = 10^4 gauss
Energy	1 J = 10^7 erg
Energy	1 eV = $1.60217733(49) \times 10^{-19}$ J = 11604.45(10) K
Flux	1 Jy = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1} = 10^{-26} W m^{-2} Hz^{-1}
Astronomical unit	1 AU = $1.4959787066 \times 10^{11}$ m
Parsec	1 pc = 3.0856776×10^{16} m
Force	1 N = 10^5 dyn
Pressure	1 Pa = 10 dyn cm^{-2}

0.4.1 SI and Gaussian cgs units

In the astronomical literature two unit systems are often used: SI and Gaussian cgs (plus ‘natural’ units such as parsec, solar mass, etc). Rybicki & Lightman use cgs. For most quantities this means that the units are expressed using centimeters, grams, and seconds rather than the meters, kilograms, and seconds of SI. However, for the Maxwell equations the choice of units is important.

In SI the *magnetic permeability* is defined, $\mu_0 = 4\pi 10^{-7}$ H m^{-1} . Because in all systems $\epsilon_0\mu_0 = 1/c^2$, in SI the *dielectrical constant* becomes $\epsilon_0 = 10^7/(4\pi c^2)$ F m^{-1} with $c = 3.0 \times 10^8$ m s^{-1} . In SI units the Maxwell equations take the form

$$\begin{aligned} \vec{\nabla} \cdot \vec{D} &= \rho & \vec{\nabla} \times \vec{E} &= \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 & \vec{\nabla} \times \vec{H} &= \vec{j} + \frac{\partial \vec{D}}{\partial t} \end{aligned} \quad (1)$$

These equations are supplemented by the definition of the Lorentz force

Table 2: Physical Constants

Speed of light	$c = 2.99792458 \times 10^8 \text{ m s}^{-1}$
Elementary charge	$e = 1.60217733(49) \times 10^{-19} \text{ C}$ $e = 4.8032068(15) \times 10^{-10} \text{ esu}$
Electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Proton mass	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Dielectric constant	$\epsilon_0 = 8.85419 \times 10^{-12} \text{ F m}^{-1}$
Gravitational constant	$G = 6.67259(85) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Planck constant	$h = 6.6260755(40) \times 10^{-34} \text{ J s}$ $= 6.6260755(40) \times 10^{-27} \text{ erg s}$
Boltzmann constant	$k = 1.380658(12) \times 10^{-23} \text{ J K}^{-1}$ $= 1.380658(12) \times 10^{-16} \text{ erg K}^{-1}$
Stefan-Boltzmann	$\sigma = 5.67051(19) \times 10^{-5} \text{ erg cm}^{-2} \text{ K}^{-4} \text{ s}^{-1}$ $= 5.67051(19) \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Solar Luminosity	$L_\odot = 3.845(8) \times 10^{26} \text{ W}$ $= 3.845(8) \times 10^{33} \text{ erg s}^{-1}$
Solar Radius	$R_\odot = 6.995 \times 10^8 \text{ m}$
Solar Mass	$M_\odot = 1.9891 \times 10^{30} \text{ kg}$

and the definitions $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ and $\vec{H} = \vec{B}/\mu_0 - \vec{M}$. In vacuum, $\vec{D} = \epsilon_0 \vec{E}$ and $\vec{H} = \vec{B}/\mu_0$.

The cgs system comes in two flavors, e.s.u. and e.m.u. The first is defined as $\epsilon_0 = 1/4\pi$ and hence $|mu_0 = 4\pi/c^2$ with $c = 3.0 \times 10^{10} \text{ cm s}^{-1}$. The second is defined as $\mu_0 = 4\pi$ and hence $\epsilon_0 = 1/(4\pi c^2)$ with $c = 3 \times 10^{10} \text{ cm s}^{-1}$. The Gaussian cgs system is a combination, and uses e.s.u. for electrical quantities and e.m.u. for magnetic quantities. In Gaussian units the Maxwell equations take the form

$$\begin{aligned} \vec{\nabla} \cdot \vec{D} &= 4\pi\rho & \vec{\nabla} \times \vec{E} &= \frac{-1}{c} \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 & \vec{\nabla} \times \vec{H} &= \frac{4\pi}{2} \vec{J} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t} \end{aligned} \quad (2)$$

In vacuum, $\epsilon_0 = \mu_0 = 1$ in these units, and $\vec{D} = \vec{E}$ and $\vec{H} = \vec{B}$.

Conversion from SI to Gaussian units can be done by the following replacements

Replace	by	Replace	by
ϵ_0	$1/4\pi$	μ_0	$4\pi/c^2$
\vec{D}	$\vec{D}/4\pi$	\vec{H}	$c\vec{H}/4\pi$
χ_e	$4\pi\chi_e$	χ_m	$4\pi\chi_m$
\vec{B}	\vec{B}/c	V_m	$cV_m/4\pi$
Φ	Φ/c	\vec{m}	$c\vec{m}$
\vec{M}	$c\vec{M}$	P (pole)	cP
\vec{I}	$4\pi\vec{I}/c$		

Source: Duffin, *Electricity and Magnetism* (London: MacGraw-Hill).