

# Appendix F

## Solution: Line Processes

- Regions in front of the star absorb stellar continuum radiation at blueshifted wavelengths; regions to the side of the star emit line photons around the rest wavelength; redshifted line emission is blocked by the star. See figure.
- We need to find the location where the line-of-sight velocity  $v_{\parallel} = v(r) \cos \theta = \text{constant}$ . For the three cases this becomes, respectively,  $x = \text{constant}$ ;  $x \propto r$ ; and a combination for both, depending whether  $v_{\parallel} = v_{\text{max}}$  or  $v_{\parallel} < v_{\text{max}}$ . See figure.
- Simply fill in the equation to get  $\dot{M} = 2.3 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ .
- Fill in the equation to get  $\Gamma = 0.45$
- Setting  $\Gamma \leq 1$  gives  $M \geq 27 M_{\odot}$ .
- The dominant source of electrons in neutral-hydrogen gas are *metals*. Therefore, if the metals are less abundant, the electron fraction will be

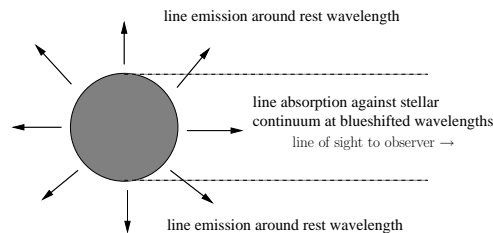


Figure F.1: Location of origin of various parts of the P Cygni profile.

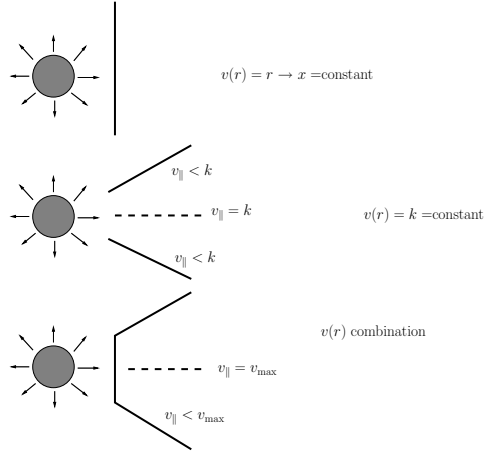


Figure F.2: Locations of constant velocity.

lower. As a result the scattering cross section *per unit mass* will be smaller, and the Eddington luminosity higher.

d.

$$g_{\text{eff}} = \frac{GM}{R^2} \left(1 - \frac{L}{4\pi GcM}\right) \quad (\text{F.1})$$

$$\rightarrow \frac{1}{2}v_{\text{esc}}^2 = \frac{GM}{R} \left(1 - \frac{L}{4\pi GcM}\right) \quad (\text{F.2})$$

$$\rightarrow v_{\text{esc}} = \sqrt{\frac{M}{R} 2G(1 - \Gamma)} \quad (\text{F.3})$$

h.  $v_{\text{esc}} = 929, 913, < 0 \text{ km s}^{-1}$  for, respectively,  $\zeta$  Pup,  $\tau$  Sco, and a red supergiant. The  $< 0$  value indicates that the Eddington limit is already violated for this latter object! In all three cases the wind velocity exceeds the escape velocity of the stars.

i. Respectively, 5332, 2243, and  $6.9 \times 10^6$  s.

j. We have  $L/c \geq \dot{M}v_{\infty}$ , and find  $\dot{M} \leq L/cv_{\infty}$ .

k.

$$\tau_l = \int_r^{\infty} \kappa_l \rho dr \quad (\text{F.4})$$

$$= \kappa_l \rho(r) \Delta r \quad (\text{F.5})$$

$$= \kappa_l \rho(r) \frac{v_{\text{th}}}{dv/dr}. \quad (\text{F.6})$$

l. Using  $\tau_i = \kappa_i \rho \Delta r$  and the definition of  $N_{\text{eff}}$ , we get

$$g_l = \sum_i \frac{\Delta \nu_i F_{\nu_i} \kappa_i}{c \tau_i} \quad (\text{F.7})$$

$$= \sum_i \frac{4\pi r^2 F_{\nu_i} \nu_i}{L} \frac{L}{4\pi r^2 \nu_i} \frac{\Delta \nu_i \kappa_i}{c \tau_i} \quad (\text{F.8})$$

$$= \frac{LN_{\text{eff}} \Delta \nu_i \kappa_i}{4\pi c r^2 \nu_i \tau_i} \quad (\text{F.9})$$

$$= \frac{LN_{\text{eff}} \Delta v \kappa_i}{4\pi c^2 r^2 \tau_i} \quad (\text{F.10})$$

$$= \frac{LN_{\text{eff}}}{4\pi c^2 r^2 \rho} \frac{\Delta v}{\Delta r}. \quad (\text{F.11})$$

m. Using the equations for  $\dot{M}$  and  $v \frac{dv}{dr}$ , and substituting the expression derived under (l) for  $g_l$ , some rearranging yields a new expression for  $\dot{M} \propto L$ .

n.

$$\int_{v_0}^{v_\infty} \frac{\epsilon}{1-\epsilon} v dv = \int_{R_\star}^{\infty} \frac{GM(1-\Gamma)}{r^2} dr \quad (\text{F.12})$$

$$\Leftrightarrow \frac{\epsilon}{1-\epsilon} \frac{1}{2} (v_\infty^2 - v_0^2) = GM(1-\Gamma) \frac{-1}{R_\star} \quad (\text{F.13})$$

$$\Leftrightarrow v_\infty^2 = \frac{1-\epsilon}{\epsilon} \frac{-2GM(1-\Gamma)}{R_\star} + v_0^2 \quad (\text{F.14})$$

$$\approx \frac{1-\epsilon}{\epsilon} \frac{-2GM(1-\Gamma)}{R_\star} \quad (\text{F.15})$$

$$\Leftrightarrow v_\infty = \sqrt{\frac{1-\epsilon}{\epsilon}} v_{\text{esc}}. \quad (\text{F.16})$$