9.4 Excercises for Chapters 8 and 9

1. (RL 10.5) Line emission is emitted from an optically thin, thermal source. Assuming that the only broadening mechanisms are Doppler and natural broadening, show that the observed half-width of the line is independent of the temperature T for $T \ll T_c$ and increases as the square root of T for $T \gg T_c$, where T_c is some critical temperature.

   For the Lyα line of hydrogen estimate T_c in terms of fundamental constants, and give its numerical value.

2. Consider the molecule HD. Its partial abundance in interstellar clouds with respect to H_2 is 10^{-5}. The energies E_J and statistical weights of its rotational levels are given by

$$E_J = B(J(J+1)), \tag{9.30}$$
$$g_J = 2J+1 \tag{9.31}$$

   where J is the rotational quantum number, $J = 0, 1, 2, \ldots$. The following values are given for the rotation constant B, the Einstein A coefficent A (J=1–0), and the collisional deexcitation rate coefficent q (J=1–0),

$$B = 44.665\,\mathrm{cm}^{-1} \tag{9.32}$$
$$A = 5.12 \times 10^{-8}\,\mathrm{s}^{-1} \tag{9.33}$$
$$q = 1.5 \times 10^{-11}\,\mathrm{cm}^3\,\mathrm{s}^{-1}. \tag{9.34}$$

   In spectroscopic literature, energy units of cm^{-1} (wavenumbers = waves per centimeter) are commonly used. To go to Joule, convert from wavenumber to wavelength, from wavelength to frequency, and from frequency to energy ($h\nu$). In the cm^{-1} units, the Boltzman constant has the value $k = 0.695\ \mathrm{cm}^{-1}\ \mathrm{K}^{-1}$.

   a. Calculate the energies of the J=1 and J=2 levels. Convert these energies to K. What are the frequencies and wavelengths of the 1–0 and 2–1 lines? Can these lines be observed from the ground?

   b. What is the critical density of the 1–0 transition?

   c. Calculate the relative level population n_1/n_0 for a molecular cloud with H_2 density $n = 5 \times 10^4\ \mathrm{cm}^{-3}$ and kinetic temperature $T_{\mathrm{kin}} =$

80 K. Use the 2-level approximation and assume that any line emission is fully optically thin.

d. Calculate the emission coefficient j for the 1–0 line under these conditions, integrated over the line profile. What is the emergent intensity for a cloud with line-of-sight dimension 0.05 pc?