Discussion section 4: molecular gas

1 Interstellar dust attenuation

Suppose that the dust grains in the ISM have a radius $r = 0.1\mu$m and a uniform number density of 2×10^{-6}m^{-3}. Assume an extinction efficiency $Q_{\rm ext} = 0.5$ at wavelength λ_0. Determine the extinction (in magnitudes) at wavelength λ_0 for a star at a distance of 1 kpc.

2 Heating of interstellar dust

Suppose that an interstellar dust grain is heated entirely through absorption of UV photons. Assume that the UV flux is 10^{10} photons/m^2/s/nm, the bandwidth is 100nm, and the mean photon energy is 9eV. If the grain cools by radiating like a blackbody with an efficiency of 0.1%, find the grain temperature $T_{\rm grain}$.

3 Linear molecule

The following expression is apropriate for a linear molecule: $A_{ul} = 1.165 \times 10^{-11}\ \mu_0^2\nu^3(J+1)/(2J+3)$, where ν is in GHz, μ_0 is in Debyes and J is the lower level in the transition from $J+1 \rightarrow J$. Use this to estimate the Einstein A coefficient for a system with a dipole moment of 0.1 Debye for a transition from the $J = 1$ level to the $J = 0$ level at 115.271 GHz.

4 Critical density

To determine whether a given level is populated, one frequently makes use of the concept of the 'critical density', $n^\star$: $A_{ul} = n^\star\langle\sigma v\rangle$. Here u is the quantum number of the upper level, and l is that for the lower level. If we take $\langle\sigma v\rangle$ to the $10^{-10}\,{\rm cm}^{-3}{\rm s}^{-1}$ from the following A_{ul} coefficients

CS $A_{10} = 1.8 \times 10^{-6}\,{\rm s}^{-1}$
CS $A_{21} = 2.2 \times 10^{-5}\,{\rm s}^{-1}$
C0 $A_{10} = 7.4 \times 10^{-8}\,{\rm s}^{-1}$,

determine the critical density. Discuss the results and the implications!

5 CO line intensity

- For the CO $J = 1-0$ transition, derive $I_{10} = 4.35 \times 10^{-24} N_1\,{\rm erg\,cm^{-2}s^{-1}sr^{-1}}$ from $I_{ul} = N_u A_{ul} h\nu_{ul}/4\pi$ (use the table).
- The brightness temperature, T_B, of a body which emits light with intensity $I(\nu)$ at frequency ν is defined as $I(\nu) = B(\nu, T_B)$. Derive the relation between the brightness temperature and the observed integrated intensity of a line in the Rayleigh limit.
- Rewrite $I_{ul} = N_u A_{ul} h\nu_{ul}/4\pi$ in terms of the brightness temperature.
- Using the expression for the partition function of a linear molecule

$$N_1 = g_1(\frac{kT_{ex}}{hcB})N_{\rm CO}e^{-E_1/kT_{ex}}\ , \quad (1)$$

derive the relation between the observed brightness temperature of the CO $J = 1-0$ transition and the total column density of CO.

Table 2.4 *Characteristics of molecular cooling lines*

Species	Transition	ν_{ul}(GHz)	E_u(K)	A_{ul} (s^{-1})	n_{cr} (cm^{-3})
CO	1–0	115.3	5.5	7.2×10^{-8}	1.1×10^{3}
	2–1	230.8	16.6	6.9×10^{-7}	6.7×10^{3}
	3–2	346.0	33.2	2.5×10^{-6}	2.1×10^{4}
	4–3	461.5	55.4	6.1×10^{-6}	4.4×10^{4}
	5–4	576.9	83.0	1.2×10^{-5}	7.8×10^{4}
	6–5	691.2	116.3	2.1×10^{-5}	1.3×10^{5}
	7–6	806.5	155.0	3.4×10^{-5}	2.0×10^{5}
CS	1–0	49.0	2.4	1.8×10^{-6}	4.6×10^{4}
	2–1	98.0	7.1	1.7×10^{-5}	3.0×10^{5}
	3–2	147.0	14.0	6.6×10^{-5}	1.3×10^{6}
	5–4	244.9	35.0	3.1×10^{-4}	8.8×10^{6}
	7–6	342.9	66.0	1.0×10^{-3}	2.8×10^{7}
	10–9	489.8	129.0	2.6×10^{-3}	1.2×10^{8}
HCO^+	1–0	89.2	4.3	3.0×10^{-5}	1.7×10^{5}
	3–2	267.6	26.0	1.0×10^{-3}	4.2×10^{6}
	4–3	356.7	43.0	2.5×10^{-3}	9.7×10^{6}
HCN	1–0	88.6	4.3	2.4×10^{-5}	2.6×10^{6}
	3–2	265.9	26.0	8.4×10^{-4}	7.8×10^{7}
	4–3	354.5	43.0	2.1×10^{-3}	1.5×10^{8}
H_2CO	2_{12}–1_{11}	140.8	6.8	5.4×10^{-5}	1.1×10^{6}
	3_{13}–2_{12}	211.2	17	2.3×10^{-4}	5.6×10^{6}
	4_{14}–3_{13}	281.5	30	6.0×10^{-4}	9.7×10^{6}
	5_{15}–4_{14}	351.8	47	1.2×10^{-3}	2.6×10^{7}
NH_3	(1,1) inversion	23.7	1.1	1.7×10^{-7}	1.8×10^{3}
	(2,2) inversion	23.7	42	2.3×10^{-7}	2.1×10^{3}
H_2	2–0	1.06×10^{4} [a]	510	2.9×10^{-11}	10
	3–1	1.76×10^{4} [b]	1015	4.8×10^{-10}	300

[a] $\lambda = 28.2\,\mu m$.
[b] $\lambda = 17.0\,\mu m$.

Figure 1: Table 2.4 from Tielens.